Generating nonperturbative physics from perturbation theory

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In a large variety of quantum mechanical systems, we show that the full nonperturbative expression for energy eigenvalues, containing all orders of perturbative, nonperturbative, and quasi-zero-mode terms, may be generated directly from the perturbative expansion about the perturbative vacuum, combined with a single global boundary condition. This provides a dramatic realization of the principle of "resurgence," that the fluctuations about different semiclassical saddle points are related to one another in a precise quantitative manner. The analysis of quantum mechanics also generalizes to certain calculable regimes of quantum field theory.

It is well known that perturbation theory is generically divergent and that this fact leads to a deep relationship between perturbation theory and nonperturbative physics [1–3]. Here we report a new and deeper level of correspondence. We show that perturbation theory contains, in an efficiently coded form, all information about all orders of nonperturbative physics. The decoding of this information (explained below) requires only that perturbation theory be combined with a global boundary condition that specifies how one degenerate vacuum connects to another. Our result applies to a wide class of potentials with degenerate vacua, including the double-well (DW) potential and the periodic sine-Gordon (SG) potential. These potentials are paradigms for the physics of tunneling, level splitting, and band splitting, instantons, and multiinstantons and so have broad applications in many branches of physics.

Our result extends the previous state of the art, due to Zinn-Justin and Jentschura (ZJJ) [4], who showed that all orders of the perturbative and nonperturbative expansion can be generated using a (conjectured) exact quantization condition, together with two functions, $B(E, g)$ and $A(E, g)$, of the energy E and perturbation parameter g. In ZJJ's approach, $B(E, g)$ is equivalent to the perturbative expansion, and $A(E, g)$ is a nonperturbative single-instanton function including fluctuations. We prove our result by showing that $A(E, g)$ can be deduced immediately from $B(E, g)$ and moreover that ZJJ's quantization condition is the natural global boundary condition in a uniform Wentzel-Kramers-Brillouin (WKB) approach.

For both the DW and SG systems, at leading instanton order, the perturbative energy levels are split, but this is just the tip of the iceberg: there is also an infinite ladder of higher-order multi-instanton effects. "Resurgence" is a formalism that unifies perturbative and nonperturbative analysis, in such a way that the terms in this ladder are intricately related to one another and can all be expressed in a compact coded form [3,5,6]. Here we demonstrate explicitly how resurgence appears in these paradigmatic quantum problems. We show that the entire (nonperturbative) ladder

can be generated from perturbation theory about the perturbative vacuum.

Although our result is derived in quantum mechanics, it has implications for asymptotically free quantum field theories [5–7], such as two-dimensional nonlinear sigma models and four-dimensional Yang–Mills. A striking property of these theories is adiabatic continuity upon compactification (with suitable boundary conditions or deformation) [8]. This permits a continuous connection between the incalculable strong coupling regime and the weak coupling calculable regime, without a phase transitions or rapid crossover, where the results of our current work also apply. Such connections between strong and weak coupling have broad physical applications. Resurgent expansions have also been explored recently for string theory and matrix models [9–12].

For potentials with degenerate harmonic minima, perturbation theory about any minimum is not Borel summable [13,14], and this leads to ambiguous nonperturbative imaginary terms, first seen at the two-instanton level. These terms are canceled by corresponding ambiguous imaginary terms in the instanton/anti-instanton sector [15,16]. This cancellation, which we refer to as the Bogomolny–Zinn-Justin mechanism, persists to all nonperturbative orders, producing a real and unambiguous result. For example, the fluctuation about the single instanton is itself a divergent non-Borel-summable series, generating an ambiguous imaginary term, which is canceled by a term in the three-instanton sector. This infinite ladder of inter-related perturbative and nonperturbative terms is written as a "resurgent trans-series" expansion of the Nth energy eigenvalue [3]

$$E^{(N)}(g) = \sum_{\pm} \sum_{k=0}^{\infty} \sum_{l=1}^{k-1} \sum_{p=0}^{\infty} c_{k,l,p}^{\pm} \frac{e^{-k\frac{S}{g}}}{g^{k(N+\frac{1}{2})}} \left(\ln \left[\mp \frac{2}{g} \right] \right)^l g^p. \quad (1)$$

In physical terms, the trans-series is a sum over all instanton contributions, perturbative fluctuations about each instanton sector, and log terms coming from quasizero modes, starting

at the two-instanton level. The $\pm$ is correlated with the nature of the quasizero mode integration, yielding $+/-$ for repulsive/attractive interactions.

We adopt the normalization conventions of ZJJ [4], writing potentials for the DW and SG models as $V(x) = x^2(1 - \sqrt{g}x)^2/2$, and $V(x) = \sin^2(2\sqrt{g}x)/(8g)$. Degenerate vacua are separated by a distance $\sim 1/\sqrt{g}$, with a barrier height $\sim 1/g$. When $g = 0$, we have isolated harmonic oscillator wells, so for nonzero g, it is natural to use a uniform WKB approach [17], writing the wave function in terms of a harmonic oscillator wave function (the parabolic cylinder function [18]): $\Psi(x) = D_\nu(u(x)/\sqrt{g})/\sqrt{u'(x)}$. When $g = 0$, the ansatz parameter ν reduces to the integer harmonic oscillator level number N. For example,

$$E(\nu, g) = \sum_{n=0}^{\infty} E_n(\nu)g^n, \qquad (2)$$

where the nth perturbative coefficient $E_n(\nu)$ is a polynomial of degree $(n + 1)$ in the parameter ν. When ν is an integer, N, this is precisely the standard Rayleigh–Schrödinger perturbative expansion about the Nth harmonic level. This expansion is divergent and not Borel summable [13,14] and therefore incomplete. This is not surprising since perturbation theory does not specify a boundary condition to relate one well to another. For the DW, this requires writing the upper (lower) level as an odd (even) function about the barrier midpoint, while for SG,

we impose a Bloch condition, which can also be specified at a barrier midpoint [18]. The inherent ambiguity in performing a Borel sum of the perturbative expansion means that we must analytically continue $g \to g \pm i\epsilon$, which in turn implies that the semiclassical limit requires the asymptotic behavior of the parabolic cylinder functions slightly off the real axis. This entails a balance between two different exponential terms [19], leading to the boundary conditions (for DW, the $\epsilon = \pm 1$ refers to the upper/lower level, and for SG, θ is the Bloch angle):

$$\text{DW}: \frac{1}{\Gamma(-\nu)}\left(\frac{e^{\pm i\pi}2}{g}\right)^{-\nu} = -\epsilon H_0(\nu, g)\frac{e^{-S/g}}{\sqrt{\pi g}} \qquad (3)$$

$$\text{SG}: \frac{1}{\Gamma(-\nu)}\left(\frac{2}{g^2}\right)^{-\nu} = -\cos\theta H_0(\nu, g)\frac{e^{-S/g}}{\sqrt{\pi g}}$$
$$\mp i\frac{\pi}{2\Gamma(1+\nu)}\left(\frac{e^{\pm i\pi}2}{g^2}\right)^{+\nu}\left[H_0(\nu, g)\frac{e^{-S/g}}{\sqrt{\pi g}}\right]^2, \quad (4)$$

where $S = \frac{1}{6}$ for the DW and $S = \frac{1}{2}$ for SG. Consider the DW result (3). The rhs contains the single-instanton factor $\xi \equiv e^{-S/g}/\sqrt{\pi g}$, multiplied by a fluctuation factor $H_0(\nu, g)$. When $g = 0$, the rhs vanishes, forcing $\nu = N$. For nonzero g, expanding the Gamma function shows that ν is only exponentially close to N:

$$\nu_{\text{DW}} = N + \frac{(\frac{2}{g})^N}{N!}H_0(N, g)\xi - \frac{(\frac{2}{g})^{2N}}{(N!)^2}\left[H_0\frac{\partial H_0}{\partial N} + \left(\ln\left(\frac{e^{\pm i\pi}2}{g}\right) - \psi(N + 1)\right)H_0^2\right]\xi^2 + O(\xi^3) \qquad (5)$$

$$\nu_{\text{SG}} = N - \cos\theta\frac{(\frac{2}{g})^N}{N!}H_0(N, g)\xi - \frac{(\frac{2}{g})^{2N}}{(N!)^2}\left\{\left[H_0\frac{\partial H_0}{\partial N} + \left(\ln\left(\frac{2}{g}\right) - \psi(N + 1)\right)H_0^2\right]\cos 2\theta \pm i\pi H_0^2\right\}\xi^2 + O(\xi^3). \qquad (6)$$

Notice the appearance of the $\ln g$ factors at the two-instanton level: $O(\xi^2)$. Substituting the expansion (5,6) into the energy (2) generates the full trans-series (1).

The conditions (3,4) are identical to the conjectured exact quantization conditions of ZJJ, but with a different interpretation. (To convert to the notation of ZJJ [4], write $B \equiv \nu + \frac{1}{2}$, and $e^{-A/2} \equiv \sqrt{\pi g}\xi H_0$.) ZJJ view $\nu = \nu(E, g)$ as a function of E, rather than E as a function of ν, as in Eq. (2); they also express $H_0 = H_0(E, g)$ as a function of energy. The ZJJ strategy is to separately compute the perturbative function $B(E, g)$ and the nonperturbative function $A(E, g)$, and then insert them into the exact quantization condition (3,4), obtaining an implicit transcendental expression for E as a function of g, for which the small g expansion yields the trans-series (1). An advantage of our uniform WKB approach is that it reveals a simple relation between the functions B and A.

To illustrate this, we first note that the perturbative expansion $E = E(\nu, g)$ in Eq. (2) agrees precisely, using the identification $B \equiv \nu + \frac{1}{2}$, with the inversion of the ZJJ expressions for $B = B(E, g)$ in Ref. [4]:

$$E_{\text{DW}}(B, g) = B - g\left(3B^2 + \frac{1}{4}\right) - g^2\left(17B^3 + \frac{19}{4}B\right)$$
$$- g^3\left(\frac{375}{2}B^4 + \frac{459}{4}B^2 + \frac{131}{32}\right) - \ldots \qquad (7)$$

$$E_{\text{SG}}(B, g) = B - g\left(B^2 + \frac{1}{4}\right) - g^2\left(B^3 + \frac{3B}{4}\right)$$
$$- g^3\left(\frac{5B^4}{2} + \frac{17B^2}{4} + \frac{9}{32}\right) - \ldots \qquad (8)$$

Next, we use these expressions for $E(B, g)$ to write ZJJ's nonperturbative function $A(E, g)$ [4] as $A(B, g)$:

$$A_{\text{DW}}(B,g) = \frac{1}{3g} + g\left(17B^2 + \frac{19}{12}\right) + g^2\left(125B^3 + \frac{153B}{4}\right)$$
$$+ g^3\left(\frac{17815}{12}B^4 + \frac{23405}{24}B^2 + \frac{22709}{576}\right) + \dots \quad (9)$$

$$A_{\text{SG}}(B,g) = \frac{1}{g} + g\left(3B^2 + \frac{3}{4}\right) + g^2\left(5B^3 + \frac{17B}{4}\right)$$
$$+ g^3\left(\frac{55B^4}{4} + \frac{205B^2}{8} + \frac{135}{64}\right) + \dots \quad (10)$$

Inspection of Eqs. (7,8,9,10) reveals the simple relations:

$$\frac{\partial E_{\text{DW}}}{\partial B} = -6Bg - 3g^2\frac{\partial A_{\text{DW}}}{\partial g} \quad (11)$$

$$\frac{\partial E_{\text{SG}}}{\partial B} = -2Bg - g^2\frac{\partial A_{\text{SG}}}{\partial g}. \quad (12)$$

Similar relations arise by inverting and reexpanding the expressions for $B(E,g)$ and $A(E,g)$ in Ref. [4] for the Fokker–Planck and $O(d)$ anharmonic oscillator potentials. We can write Eq. (11,12) as the general expression,

$$\frac{\partial E}{\partial B} = -\frac{g}{2S}\left(2B + g\frac{\partial A}{\partial g}\right), \quad (13)$$

where S is the numerical coefficient of the instanton action in $\xi \equiv e^{-S/g}/\sqrt{\pi g}$. Equation (13) is our main result. This relation (13) has a dramatic computational consequence: we can deduce $A = A(B,g)$ immediately from the perturbative expansion (2) for $E(B,g)$; thus, the instanton computation for $A(B,g)$ is actually unnecessary, as it is already encoded in the perturbative expression for $E(B,g)$. Furthermore, the overall normalization factor S is encoded

in the leading nonalternating large-order growth $n!/(2S)^n$ of the ground state perturbative expansion or is easily computed from the single instanton integral. This proves our claim that the entire trans-series can be generated from perturbation theory, together with the global boundary condition. The relation (13) is not at all obvious in the ZJJ expressions where B and A are written as functions of E because it requires an inversion and reexpansion. For the DW, a related form of Eq. (11) was noted in Ref. [20], but its physical meaning and consequences were not pursued.

We now explain the resurgent origin, and implications, of this general relation (13). The key step is to note that the trans-series (1) arises from combining the formal perturbative expansion (2) with the global condition that determines ν to be exponentially close to its perturbative value N, as, e.g., in Eqs. (5,6). Therefore, we expand

$$E_N(g) = E(N,g) + (\delta\nu)\left[\frac{\partial E}{\partial\nu}\right]_N + \frac{(\delta\nu)^2}{2}\left[\frac{\partial^2 E}{\partial\nu^2}\right]_N + \dots \quad (14)$$

Each correction term involves two factors. One is a perturbative factor of derivatives with respect to ν of the perturbative expression $E(\nu,g)$, evaluated at $\nu = N$. The other is a nonperturbative factor of powers of the nonperturbative shift $\delta\nu$ of ν from its integer value N, as in Eqs. (5,6). But $\delta\nu$ is expressed entirely in terms of ξ and H_0, and because of the general relation (13), each of these is easily deduced from the perturbative expansion. From Eq. (13) and the identification $e^{-A/2} \equiv \sqrt{\pi g}\xi H_0$, the instanton fluctuation factor is derived from $E(B,g)$ as

$$H_0 = \exp\left[S\int_0^g \frac{dg}{g^2}\left(\frac{\partial E}{\partial B} - 1 + \frac{Bg}{S}\right)\right]. \quad (15)$$

For example,

$$H_0^{\text{DW}}(N,g) = 1 - \frac{(102N^2 + 102N + 35)}{12}g + \frac{(10404N^4 + 2808N^3 - 9456N^2 - 11868N - 3779)}{288}g^2 + \dots \quad (16)$$

$$H_0^{\text{SG}}(N,g) = 1 - \frac{3(2N^2 + 2N + 1)}{4}g + \frac{(36N^4 - 8N^3 - 48N^2 - 92N - 35)}{32}g^2 + \dots \quad (17)$$

As an immediate application, consider the first order in the instanton expansion, which gives the level/band splitting. For the Nth level, we have

$$\Delta E_N(g) \sim 2\frac{\left(\frac{2}{g}\right)^N}{N!}\frac{e^{-S/g}}{\sqrt{\pi g}}F_0(N,g). \quad (18)$$

The leading factor is the familiar textbook single-instanton result, while the nontrivial fluctuation factor, $F_0(N,g) = H_0(N,g)[\frac{\partial E}{\partial\nu}]_N$, can be deduced entirely from the perturbative expansion $E(\nu,B)$, using Eqs. (2,15):

$$F_0^{\text{DW}}(N,g) = 1 - \frac{(102N^2 + 174N + 71)}{12}g + O(g^2) \quad (19)$$

$$F_0^{\text{SG}}(N,g) = 1 - \frac{(6N^2 + 14N + 7)}{4}g + O(g^2). \quad (20)$$

This DW result agrees with available ($N = 0$) results from instanton calculus [21,22], which is obtained by evaluating the Feynman diagrams up to the two-loops order in the instanton background. Similarly, for SG, we agree with

results from the Mathieu equation for any N in Refs. [18,23]. It is striking that our formalism correctly produces the perturbative fluctuations around an instanton background *without* evaluating the Feynman diagrams in that background. This is the power of resurgence.

Now consider the two-instanton sector. From Eqs. (5,6), the first imaginary term [24] enters at $O(\xi^2)$ from $\delta\nu$:

$$\text{Im} \, E_N(g) \sim \frac{2^{2N}}{(N!)^2} \frac{e^{-2S/g}}{g^{2N+1}} H_0^2(N,g) \left[\frac{\partial E}{\partial \nu}\right]_N. \quad (21)$$

A dispersion relation for the energy gives the leading large-order growth of the perturbative coefficients, as a function of the level number N:

$$c_n^{(N)} \sim \frac{1}{\pi} \int_0^\infty \frac{dg}{g^{n+1}} \text{Im} \, E_N(g)$$

$$\sim -\frac{1}{\pi} \frac{2^{2N}}{(N!)^2} \frac{\Gamma(n+2N+1)}{(2S)^{n+2N+1}} \left[1 + O\left(\frac{2S}{n+2N}\right)\right]. \quad (22)$$
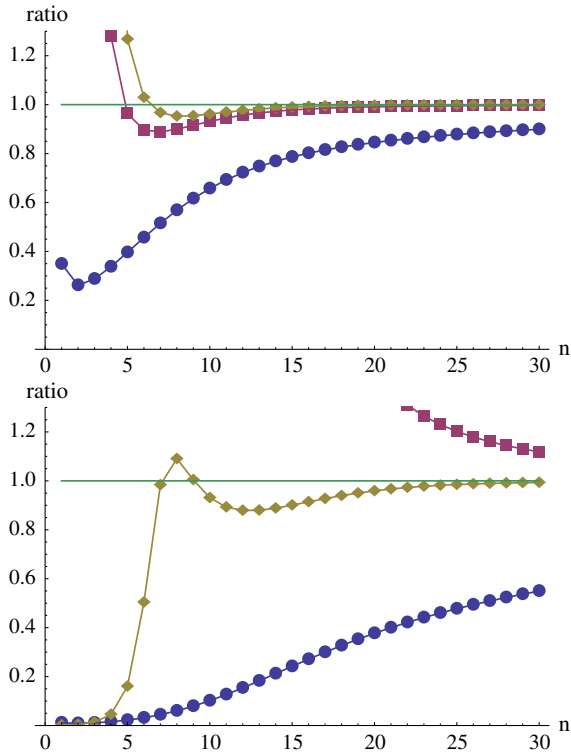

FIG. 1 (color online).   Plots of the ratio of the exact perturbative coefficients to the large-order growth (22), for the double-well potential, for the $N = 0$ ground state (upper plot) and $N = 1$ first excited state (lower plot). The solid (blue) circles, (red) squares, and (gold) diamonds denote the leading, subleading, and sub-subleading large n behavior, respectively, including increasing information from Eqs. (15,21,22) concerning the fluctuations about the instanton/anti-instanton sector.

Figure 1 shows excellent agreement with the numerically computed large-order growth, from the first 30 terms in the perturbative expansion, also including subleading corrections to Eq. (22) that come from the fluctuations about the instanton/anti-instanton sector, for the DW potential, using Eqs. (16,17,21,22). Analogous results apply to the SG potential.

This large-order growth is not directly evident from the perturbative expansion (2), as it involves the large-order growth of the polynomials $E_n(B)$, for different values of B. The result (22) requires the input of the general relation (13) and its immediate corollary (15). An interesting example is the Fokker–Planck potential [4], the supersymmetry (SUSY) quantum mechanical double-well potential, with ground state energy that is perturbatively zero. Here the global condition leads to $B \sim N + \delta B$, and SUSY is broken nonperturbatively at the two-instanton level: $E^{(0)} \sim \frac{1}{2\pi} e^{-\frac{1}{3g}}$. The ground state perturbation expansion is convergent (equal to 0), and correspondingly the two-instanton term has no imaginary part for $N = 0$. For all excited states $N \geq 1$, the perturbative expansion is not Borel summable, with associated imaginary parts in the two-instanton sector. All this information is encoded in the perturbative expansion (2), via relation (13).

A deeper manifestation of the relation (13) comes from consideration of the high orders of the perturbative fluctuations $F_0(N, g)$ about the single-instanton term in Eq. (18). Like the perturbation around the vacuum, these fluctuations are divergent and not Borel summable, producing ambiguous nonperturbative imaginary terms that must be cancelled by other terms in the trans-series. The leading large-order growth of $\frac{\partial E}{\partial \nu}$ can be deduced from Eq. (22):

$$\frac{\partial c_n^{(N)}}{\partial N} \sim -\frac{1}{\pi} \frac{2^{2N+1}}{(N!)^2} \frac{\Gamma(n+2N+1)}{(2S)^{n+2N+1}} \ln(n+2N+1). \quad (23)$$

Notice the $(n! \ln n)$ large-order growth, different from the conventional $n!$ behavior. The origin of this lies in the resurgent structure of the trans-series. These perturbative fluctuations about the single instanton produce an imaginary term $\sim e^{-2S/g}$, which, when combined with the single instanton factor ξ, become $O(e^{-3S/g})$, which must be cancelled by a term in the three-instanton sector, coming from instanton/instanton/anti-instanton quasizero modes terms, as encoded in the "resurgence triangle" [6,19]. Indeed, in the three-instanton sector of the trans-series (1), because of the $(\ln(-2/g))^2$ term, the trans-series has an imaginary part proportional to $e^{-3S/g} \ln g$, and this corresponds to this novel $(n! \ln n)$ large-order growth in the perturbative fluctuations around the single instanton. Again, this correspondence relies crucially on the general relation (13).

To conclude, we have shown that the remarkable results of Zinn-Justin and Jentschura [4] can be extended even further: the full nonperturbative trans-series expansion of

energy eigenvalues is encoded already in the perturbative expansion $E(\nu, g)$. Practically speaking, this eliminates the necessity of computing the complicated multi-instanton function $A(E, g)$ in the ZJJ approach, as it can be deduced immediately from Eq. (13). We have further shown that this relation is itself a statement of resurgence, required in order to produce the correct cancellations between the zero and two-instanton sectors, the one- and three-instanton sectors, and so on. Ultimately, our result is a manifestation of resurgent structure [6,9–11,25,26] in the quantum path integral: the fluctuations about different saddle points (vacuum and different multi-instanton sectors) are quantitatively related so tightly that the entire trans-series, to all nonperturbative orders, can be encoded in terms of just the perturbative saddle point.

By the idea of adiabatic continuity mentioned in the introduction, resurgent structure is also present in the weak coupling calculable regimes of quantum field theory. It is important to understand how this result can be generalized to strongly coupled quantum fields.

ACKNOWLEDGMENTS

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