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Naturalness implies intra-generational degeneracy for decoupled squarks and sleptons

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The supersymmetry (SUSY) flavor, *CP*, gravitino and proton-decay problems are all solved to varying degrees by a decoupling solution wherein first/second generation matter scalars would exist in the multi-TeV regime. Recent models of natural SUSY presumably allow for a coexistence of naturalness with the decoupling solution. We show that if sfermions are heavier than ∼10 TeV, then a small first/second generation contribution to electroweak fine-tuning requires a rather high degree of intra-generational degeneracy of either 1. (separately) squarks and sleptons, 2. (separately) left- and right-type sfermions, 3. members of $SU(5)$ multiplets, or 4. all members of a single generation, as in $SO(10)$. These (partial) degeneracy patterns required by naturalness hint at the necessity of an organizing principle and highlight the limitations of models such as the phenomenological minimal supersymmetric standard model in the case of decoupled first/second generation scalars.

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I. INTRODUCTION

Weak scale supersymmetry provides a solution to the notorious gauge hierarchy problem by ensuring the cancellation of quadratic divergences endemic to scalar fields which are otherwise unprotected by a symmetry [1]. While realistic and natural SUSY models of particle physics can be constructed in accordance with all experimental constraints, especially those arising from recent LHC searches, they are subject to a host of open questions [2]. Included amongst these are

(i) the SUSY flavor problem [3], wherein unfettered flavor-mixing soft terms lead to e.g. large $K-\bar{K}$ mass difference and anomalous contributions to flavor-changing decays such as $b\to s\gamma$ and $\mu\to e\gamma$,
(ii) the SUSY *CP* problem [3], in which unfettered *CP* violating phases lead to large contributions to electron and various atomic electric dipole moments,
(iii) the SUSY gravitino problem [4], wherein thermally produced gravitinos in the early Universe may decay after BBN, thus destroying the successful prediction of light element abundances created in the early universe, and
(iv) the SUSY proton decay problem [5], wherein even in R-parity conserving grand unified theories (GUT), the proton is expected to decay earlier than recent bounds from experimental searches.

While there exist particular solutions to each of these problems (e.g. degeneracy [6] or alignment [7] for the flavor problem, small phases for the *CP* problem, low T_R for the gravitino problem [8], and cancellations for proton decay [9]), there is one solution which potentially tames all four: decoupling of squarks and sleptons [10–12].[1] For the decoupling solution, squark and slepton masses $\gtrsim$ a few TeV are sufficient for the SUSY *CP* problem while $m_{3/2}\gtrsim 5$ TeV allows for gravitino decay before the onset of BBN. For the supersymmetry (SUSY) flavor problem, first/second generation scalars ought to have mass $\gtrsim$5–100 TeV depending on which process is examined, how large of flavor-violating soft terms are allowed and the possible GUT relations amongst GUT scale soft terms [13]. For proton decay, again multi-TeV matter scalars seem sufficient to suppress decay rates depending on other GUT scale parameters [14,15].

Naively, the decoupling solution seems in conflict with notions of SUSY naturalness, wherein sparticles are expected at or around the weak scale [6] typified by the recently discovered Higgs mass $m_h=125.5\pm0.5$ GeV [16,17]. To move beyond this, we require the necessary (although not sufficient) condition for naturalness, quantified by the measure of electroweak fine-tuning (EWFT) which requires that there be *no large cancellations within the weak scale contributions to* m_Z *or to* m_h [14,18–21].

Recall that minimization of the one-loop effective potential $V_{\rm tree}+\Delta V$ leads to the well-known relation

[1]In the case of the gravitino problem, we tacitly assume gravity-mediation of SUSY breaking, wherein the scalar mass parameters as well as the gravitino mass $m_{3/2}$ arise from a common source of SUSY breaking in a hidden sector. In this case, the scalar mass parameters all have magnitudes comparable to $\sim m_{3/2}$.

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$$\frac{M_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2, \quad (1)$$

where Σ_u^u and Σ_d^d are radiative corrections that arise from the derivatives of ΔV evaluated at the potential minimum. Noting that all entries in Eq. (1) are defined at the weak scale, the *electroweak fine-tuning measure*

$$\Delta_{\rm EW} \equiv \max_i |C_i|/(m_Z^2/2) \quad (2)$$

may be constructed, where $C_{H_d} = m_{H_d}^2/(\tan^2\beta - 1)$, $C_{H_u} = -m_{H_u}^2 \tan^2\beta/(\tan^2\beta - 1)$ and $C_\mu = -\mu^2$. Also, $C_{\Sigma_u^u(k)} = -\Sigma_u^u(k)\tan^2\beta/(\tan^2\beta - 1)$ and $C_{\Sigma_d^d(k)} = \Sigma_d^d(k)/(\tan^2\beta - 1)$, where k labels the various loop contributions included in Eq. (1). Expressions for the Σ_u^u and Σ_d^d are given in the Appendix of the second paper of Ref. [19]. The contributions from $\Sigma_u^u(k)$ are almost always much more important than the $\Sigma_d^d(k)$ since the $\Sigma_d^d(k)$ are suppressed by the factor $1/\tan^2\beta$. Typically, the dominant radiative corrections to Eq. (1) come from the top-squark contributions $\Sigma_u^u(\tilde{t}_{1,2})$. By adopting a large value of the weak scale trilinear soft term A_t, each of $\Sigma_u^u(\tilde{t}_1)$ and $\Sigma_u^u(\tilde{t}_2)$ can be minimized while lifting up m_h into the 125 GeV regime [18].

For first/second generation sfermions, neglecting the small Yukawa couplings, we find the contributions

$$\Sigma_{u,d}^{u,d}(\tilde{f}_{L,R}) = \mp \frac{c_{\rm col}}{16\pi^2} F(m_{\tilde{f}_{L,R}}^2)(-4g_Z^2(T_3 - Q_{em}x_W)), \quad (3)$$

where T_3 is the weak isospin, Q_{em} is the electric charge assignment (taking care to flip the sign of Q_{em} for R-sfermions), $c_{\rm col} = 1(3)$ for color singlet (triplet) states, $x_W \equiv \sin^2\theta_W$, and where

$$F(m^2) = m^2\left(\log\frac{m^2}{Q^2} - 1\right). \quad (4)$$

We adopt an optimized scale choice $Q^2 = m_{\rm SUSY}^2 \equiv m_{\tilde{t}_1} m_{\tilde{t}_2}$.[2] The explicit first generation squark contributions to Σ_u^u (neglecting the tiny Yukawa couplings) are given by

$$\begin{aligned}
\Sigma_u^u(\tilde{u}_L) &= \frac{3}{16\pi^2} F(m_{\tilde{u}_L}^2)\left(-4g_Z^2\left(\frac{1}{2} - \frac{2}{3}x_W\right)\right)\\
\Sigma_u^u(\tilde{u}_R) &= \frac{3}{16\pi^2} F(m_{\tilde{u}_R}^2)\left(-4g_Z^2\left(\frac{2}{3}x_W\right)\right)\\
\Sigma_u^u(\tilde{d}_L) &= \frac{3}{16\pi^2} F(m_{\tilde{d}_L}^2)\left(-4g_Z^2\left(-\frac{1}{2} + \frac{1}{3}x_W\right)\right)\\
\Sigma_u^u(\tilde{d}_R) &= \frac{3}{16\pi^2} F(m_{\tilde{d}_R}^2)\left(-4g_Z^2\left(-\frac{1}{3}x_W\right)\right).
\end{aligned} \quad (5)$$

These contributions, arising from electroweak D-term contributions to masses, are frequently neglected since the various contributions cancel amongst themselves in the limit of mass degeneracy due to the fact that weak isospins and electric charges (or weak hypercharges) sum to zero in each generation. However, if squark and slepton masses are in the multi-TeV regime but are *nondegenerate* within each generation, then the contributions may be large and noncanceling. In this case, they may render a theory which is otherwise considered to be natural, in fact, unnatural.

The first generation slepton contributions to Σ_u^u are given by

$$\begin{aligned}
\Sigma_u^u(\tilde{e}_L) &= \frac{1}{16\pi^2} F(m_{\tilde{e}_L}^2)\left(-4g_Z^2\left(-\frac{1}{2} + x_W\right)\right)\\
\Sigma_u^u(\tilde{e}_R) &= \frac{1}{16\pi^2} F(m_{\tilde{e}_R}^2)(-4g_Z^2(-x_W))\\
\Sigma_u^u(\tilde{\nu}_L) &= \frac{1}{16\pi^2} F(m_{\tilde{\nu}_{eL}}^2)\left(-4g_Z^2\left(\frac{1}{2}\right)\right);
\end{aligned} \quad (6)$$

these may also be large for large $m_{\tilde{\ell}}^2$, although again they cancel amongst themselves in the limit of slepton mass degeneracy.

Our goal in this article is to examine the case where the scalar masses are large, as suggested by the decoupling solution, but where the masses are not necessarily degenerate. In models such as radiatively driven natural SUSY [19], where $m_{H_u}^2$, μ^2 and $\Sigma_u^u(\tilde{t}_{1,2})$ are all ~100–200 GeV, then for nondegenerate first generation squarks and sleptons, the $\Sigma_u^u(\tilde{q}_i)$ and $\Sigma_u^u(\tilde{\ell}_i)$ may be the dominant radiative corrections. And if they are sufficiently large, then large cancellations will be needed amongst independent contributions to yield a value of m_Z of just ~91.2 GeV: i.e. the model will become highly electroweak fine-tuned. Alternatively, requiring electroweak naturalness (low $\Delta_{\rm EW} \lesssim 30$) will require a rather high degree of intragenerational degeneracy amongst decoupled matter scalars.

II. RESULTS

To a very good approximation, the masses of first and second generation sfermions (whose Yukawa couplings can be neglected) are given by

$$m_{\tilde{f}_i}^2 = m_{F_i}^2 + m_{f_i}^2 + M_Z^2\cos 2\beta(T_3 - Q_{em}\sin^2\theta_W) \simeq m_{F_i}^2, \quad (7)$$

where $m_{F_i}^2$ is the corresponding weak scale soft-SUSY breaking parameter, and the sign of Q_{em} is flipped for R-sfermions, as described just below Eq. (3). The latter approximate equality holds in the limit of large soft masses (decoupling), where D-term contributions are negligible.

In the limit of negligible hypercharge D terms and $m_{f_i}^2$, the elements of each squark and slepton doublet are essentially mass degenerate; in this case, the weak isospin contributions to Eq. (3) cancel out, and one is only left with the possibility of noncanceling terms which are

[2]The optimized scale choice is chosen to minimize the log contributions to $\Sigma_u^u(\tilde{t}_{1,2})$ which occur to all orders in perturbation theory.

proportional to electric charge. The summed charge contributions (multiplied by $c_{\rm col}$) of each multiplet are then $Q(Q_1) = +1$, $Q(U_1) = -2$, $Q(D_1) = +1$, $Q(L_1) = -1$ and $Q(E_1) = +1$. To achieve further cancellation, one may then cancel the $Q(U_1)$ against any two of $Q(Q_1)$, $Q(D_1)$ and $Q(E_1)$. The remaining term may cancel against $Q(L_1)$. Thus, the possible cancellations break down into four possibilities:

(1) separate squark and slepton degeneracy: $m_{U_1} = m_{Q_1} = m_{D_1}$ and $m_{L_1} = m_{E_1}$,
(2) separate right- and left- degeneracy: $m_{U_1} = m_{D_1} = m_{E_1}$ and $m_{L_1} = m_{Q_1}$,
(3) $SU(5)$ degeneracy: $m_{U_1} = m_{Q_1} = m_{E_1} \equiv m_{10_1}$ and $m_{L_1} = m_{D_1} \equiv m_{5_1}$ and
(4) $SO(10)$ degeneracy: $m_{U_1} = m_{Q_1} = m_{E_1} = m_{L_1} = m_{E_1} \equiv m_{16_1}$.

We assume that the gaugino masses are small enough that splittings caused by the renormalization of the mass parameters between the GUT scale and the SUSY scale are negligible so that these relations may equally be taken to be valid at the GUT scale. Any major deviation from the first three of these patterns (which implies a deviation to the fourth $SO(10)$ pattern) can lead to unnaturalness in models with decoupled scalars. In models such as the phenomenological minimal supersymmetric standard model, or pMSSM, where $m_{U_1}, m_{Q_1}, m_{E_1}, m_{L_1}$ and m_{E_1} are all taken as independent, a decoupling solution to the SUSY flavor, CP, gravitino and proton-decay problems would likely be unnatural.

In this connection, it is worth mentioning that D-term contributions associated with a reduction of rank when a GUT group is spontaneously broken to the SM gauge symmetry can lead to intra-generational splittings [22]. Assuming that weak hypercharge D terms are negligible, the splitting of the MSSM sfermions can be parametrized in terms of the $vevs$ of the D terms associated with $U(1)_X$ and $U(1)_S$ (in the notation of the last paper of Ref. [22]). The $SU(5)$ splitting pattern 3. is automatically realized for arbitrary values of D_X and D_S, while patterns 1. and 2. do not appear to emerge from the GUT framework.

To illustrate the growth of $\Delta_{\rm EW}$ for *ad hoc* sfermion masses, in Fig. 1 we plot the green curve as the summed contribution to $\Delta_{\rm EW}$ from first generation matter scalars by taking all soft masses $m_{F_i} = 20$ TeV except m_{U_1}, which varies from 5–30 TeV. The summed $\Sigma_u^u(\tilde{f}_1)$ contributions to $\Delta_{\rm EW}$ for $m_{U_1} = 5$ TeV begin at ~250 and slowly decrease with increasing m_{U_1}. The summed contributions reach zero at $m_{U_1} = 20$ TeV, where complete cancellation amongst the various squark/slepton contributions to $\Delta_{\rm EW}$ is achieved. A nominal value of low EWFT adopted in Ref. [19] is 30: higher values of $\Delta_{\rm EW}$ require worse than $\Delta_{\rm EW}^{-1} = 3\%$ EWFT. We see from the plot that for $\Delta_{\rm EW} < 30$, $m_{U_1} \sim 19$–21 TeV; i.e. a rather high degree of degeneracy of m_{U_1} in one of the above four patterns is required by naturalness.

In Fig. 1, we also plot the blue curve (with red dashes lying atop) as $\Delta_{\rm EW}$ for all scalar soft masses = 20 TeV

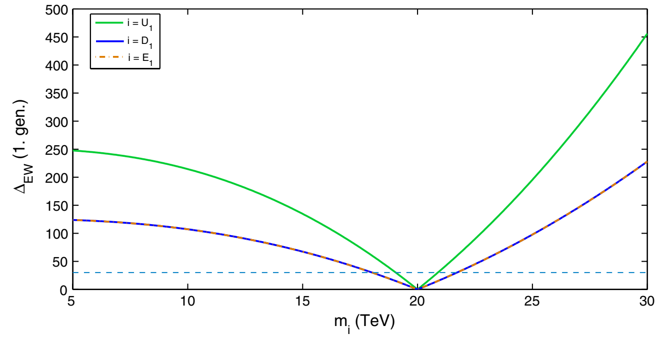

FIG. 1 (color online). Contribution to $\Delta_{\rm EW}$ from first generation squarks and sleptons where all scalar soft masses are set to 20 TeV except m_{U_1} (green), m_{D_1} (blue) or m_{E_1} (orange-dashed) with $m_{\rm SUSY} = 2.5$ TeV and $\tan\beta = 10$.

except now varying m_{D_1}. The contributions to $\Delta_{\rm EW}$ are much reduced due to the lower d-squark charge, but are still significant: in this case, $m_{D_1} \sim 18$–22 TeV is required for $\Delta_{\rm EW} < 30$. We also show the dashed red curve as the contribution to $\Delta_{\rm EW}$ from first generation scalars where we take soft masses = 20 TeV but now vary m_{E_1}. The curve lies exactly atop the varying m_{D_1} curve since the color factor of three in Eq. (5) exactly compensates for the increased electric charge by a factor of three in Eq. (6). Thus, for $m_{F_1} = 20$ TeV, $m_{E_1} \sim 18$–22 TeV is required to allow for electroweak naturalness. Requiring $\Delta_{\rm EW}$ as low as 10, as can occur in radiatively driven natural SUSY [19,21], requires even tighter degeneracy.

Adopting a variant on the degenerate $SO(10)$ case with all sfermions but the $\tilde{u}_R$ squark having the same mass, in Fig. 2 we plot color-coded regions of first generation squark contributions to $\Delta_{\rm EW}$ in the m_{U_1} vs m_{F_1} plane, where m_{F_a} stands for the common sfermion mass other than m_{U_1}. The regions in between the lightest grey bands (which have $27 < \Delta_{\rm EW} < 37$) would mark the rough boundary of

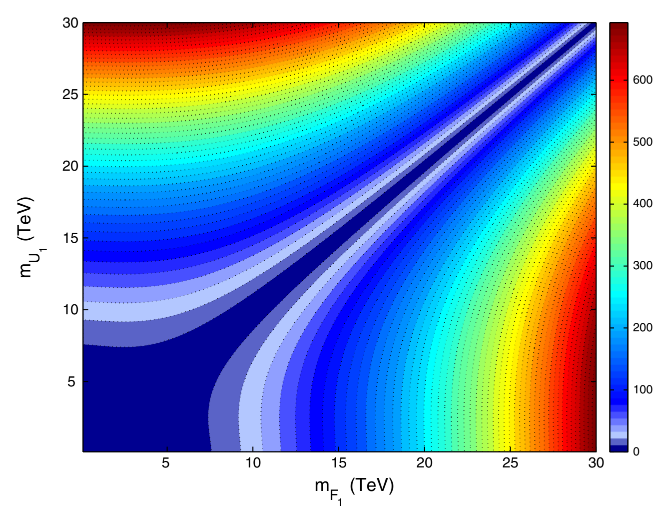

FIG. 2 (color online). Plot of contours of $\Delta_{\rm EW}(\tilde{f}_1)$ (summed over just first generation sfermions) in the m_{U_1} vs m_{F_1} plane with $m_{\rm SUSY} = 2.5$ TeV and $\tan\beta = 10$.

the natural region. From the plot, we see that if weak scale soft squark masses are below ∼10 TeV, then the $\Sigma_u^u(\tilde{f}_i)$ are all relatively small, and there is no naturalness constraint on nondegenerate sfermion masses. As one moves to much higher sfermion masses in the $\gtrsim$10–15 TeV regime, then the sfermion soft masses within each generation are required to be increasingly degenerate in order to allow for EW naturalness.

Similarly, we can show contributions to $\Delta_{\rm EW}$ from first generation sleptons in the m_{L_1} vs m_{E_1} mass plane. The various regions have qualitatively similar shapes (but different widths, reflecting the different coefficient $Q(L_1)$ that enters in the calculation) to Fig. 2 with the replacements $m_{U_1} \to m_{L_1}$: a high degree of left-slepton mass degeneracy with another multiplet is required by naturalness once slepton masses reach above about 10–15 TeV.

III. CONCLUSIONS

The SUSY flavor, CP, gravitino and proton-decay problems are all solved to varying degrees by a decoupling solution wherein first/second generation matter scalars would exist in the multi-TeV regime. In this case, where matter scalar masses exist beyond the ∼10–15 TeV level, intra-generation degeneracy following one of several patterns appears to be necessary for electroweak naturalness; i.e. $\Delta_{\rm EW} \lesssim$ 10–30. Such degeneracy is not necessarily expected in generic SUSY models such as the pMSSM unless there is a protective symmetry: for instance, $SU(5)$ or $SO(10)$ GUT symmetry provides the required degeneracy provided additional contributions (such as running gauge contributions) are not very large. Our results seem to hint at the existence of an additional organizing principle if a decoupling solution (with sfermions heavier than ∼10 TeV) to the SUSY flavor, CP, gravitino and proton-decay problems is invoked along with electroweak naturalness. This could well be a grand unification symmetry in accordance with recent calculations of flavor changing contributions to Δm_K where $SO(10)$ mass relations also contribute to suppress flavor violation [13].

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