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Electroweak baryogenesis in the MSSM with vectorlike superfields

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Introducing heavy particles with strong couplings to the Higgs field can strengthen electroweak phase transition through the entropy release mechanism from both bosons and fermions. We analyze the possibility of electroweak baryogenesis in the minimal supersymmetric Standard Model (MSSM) with new vectorlike superfields. The new vectorlike particles belong to the representation $5+\bar{5}+10+\overline{10}$ of $SU(5)$. By analyzing in detail the effective potential at finite temperature, we show that a strongly first-order electroweak phase transition in this model is ruled out by a combination of the 125 GeV Higgs requirement, the bound for exotic quarks, the gluon fusion Higgs production rate, and the Higgs diphoton decay rate, as well as the electroweak precision measurement.

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I. INTRODUCTION

The origin of the matter-antimatter asymmetry of our Universe remains unclear. The three Sakharov conditions [1] can be fulfilled in high-scale mechanisms such as leptogenesis [2,3] and Grand Unified Theory (GUT) baryogenesis [4–7] but are difficult to test by electroweak (EW) scale experiments. By contrast, electroweak baryogenesis (EWBG) [8], relying on weak-scale physics, provides an alternative solution which requires a strongly first-order phase transition (SFOPT) [9]. Unfortunately, the EW phase transition (EWPT) is too weak in the Standard Model (SM) with large a Higgs mass [10,11], and the CP violation is too small [12].

Extensions of the SM with new EW-scale physics can lead to a SFOPT, in all of which new particles beyond the SM are needed. On the other hand, the ATLAS and CMS collaborations at the CERN Large Hadron Collider (LHC) reported observation of a SM-like Higgs boson with a mass of 125–126 GeV [13,14]. If we require the EWBG mechanism to account for the matter-antimatter asymmetry, the new fields introduced for a SFOPT can induce significant corrections to the SM-like Higgs mass as well as production and decay rates, which will be strongly constrained. For example, in the minimal supersymmetric Standard Model (MSSM), the light stop scenario [15,16] has been severely constrained [17].

Based on what physics is responsible for generating the barrier between the symmetric and broken phases, there are three EWPT model classes in general [18]. In this paper, we focus on the thermally driven case. In addition to the effect induced by terms cubic in ϕ in the bosonic high-temperature expansion, the phase transition can be strengthened by introducing heavy particles with strong couplings to the Higgs fields, such as the SM extension with TeV Higgsinos, winos, and binos [19,20]. That is, after the electroweak symmetry breaking (EWSB), the new particles get Yukawa masses and become heavier, they approximately decouple from the thermal plasma and transfer their entropy into the thermal bath. In this paper we consider a different model, namely in addition to the MSSM, adding several vectorlike (VL) superfields. This kind of model [21,22] has been extensively studied and found interesting, for it can relax the naturalness problem raised by the Higgs mass, be consistent with gauge-coupling unification and precision EW measurements, and have a rich phenomenology. So it is interesting to explore its possibility to realize the SFOPT in detail.

The added exotic particles belong to the representation $5+\bar{5}+10+\overline{10}$ of $SU(5)$, which consists of four new quarks, two new charged leptons, two left-handed neutrinos, and the corresponding sparticles with total degree of freedom 120. The model is the MSSM with two new supersymmetric generations, while VL mass terms are introduced between the two to escape the experimental fourth-generation search bound. In search for a SFOPT, we analyze in detail the zero-temperature potential, the one-loop zero-temperature potential, and the finite-temperature potential. To search for a viable parameter region, we also impose all conventional constraints: the SM-like Higgs mass is about 125 GeV, no new light quarks of a few hundred GeV exist [23], the gluon fusion Higgs production rate and the Higgs diphoton decay rate are not significantly changed [13,14], and the Peskin-Takeuchi parameters T and S [24] are small.

We find generally that a SFOPT combining with a 125 GeV Higgs requirement will lead to a exotic fermion/scalar that is too light. In order to make them heavy enough to escape the direct search bound, the VL

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masses should be about 500 GeV, but the VL Yukawa are also pushed to large values near the perturbativity bound. We find an almost supersymmetric VL sector with large $\tan\beta$ and no scalar mixing as our best solution, which can satisfy the 125 GeV Higgs requirement without changing the Higgs gluon fusion rate or the Higgs diphoton decay rate. However, it is still in tension with the direct light new particle search and eventually ruled out by contributing a very large Peskin-Takeuchi T parameter. So in all, the possibility of EWBG induced by supersymmetric VL generations in our setup is fairly ruled out.

The outline of the rest of the paper is as follows: We will define the model precisely in Sec. II. In Secs. III and IV, we investigate the zero-temperature potential (as well as the Higgs mass) and the finite-temperature potential separately. Section V contains our final results and discussions of various constraints. A brief summary is given in the last section.

II. THE MSSM WITH VECTORLIKE SUPERFIELDS

As mentioned above, new particles beyond the MSSM are represented by two new generations $5+\bar{5}+10+\overline{10}$ of $SU(5)$. Here we do not take the singlet right-hand neutrino into account, so there will be no Yukawa couplings of the VL neutrinos, and the neutrinos do not contribute to EWSB. Moreover, the model almost preserves gauge-coupling unification [25], so it is also UV motivated.

The corresponding quantum numbers of VL superfields under $\mathrm{SU}(3)_c\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ are given as

$$\begin{aligned}&Q\left(3,2,\frac{1}{3}\right),\quad U\left(3,1,\frac{4}{3}\right),\quad D\left(3,1,-\frac{2}{3}\right),\quad L(1,2,-1),\quad E(1,1,-2),\\&\bar{Q}\left(\bar{3},2,-\frac{1}{3}\right),\quad \bar{U}\left(\bar{3},1,-\frac{4}{3}\right),\quad \bar{D}\left(\bar{3},1,\frac{2}{3}\right),\quad \bar{L}(1,2,1),\quad \bar{E}(1,1,2).\end{aligned}\tag{1}$$

And the superpotential is

$$\begin{aligned}W={}&W_{\mathrm{MSSM}}+M_Q\bar{Q}Q+M_U\bar{U}U+M_D\bar{D}D+M_L\bar{L}L+M_E\bar{E}E\\&+k_1H_uQ\bar{U}+k_2H_u\bar{Q}D+k_3H_u\bar{L}E-k_1'H_d\bar{Q}U-k_2'H_dQ\bar{D}-k_3'H_dL\bar{E}.\end{aligned}\tag{2}$$

Note that in general, there is mixing between the new vectorlike superfields and the MSSM superfields. The related Yukawa couplings with the first-/second-family MSSM fields are strongly constrained by the EW phenomenology such as flavor-changing neutral current [26], which needs to be less than 10^{-3}. The constraint on the couplings with the third-family MSSM fields is relatively loose and can be of order 0.1. We ignore the effect of these terms just in the EWPT calculation for simplicity.

By assuming universality of the mass-squared terms and the alignment of the B terms, the soft mass terms and the trilinear soft terms of all the VL scalar partners are given by

$$\begin{aligned}-\mathcal{L}_{\mathrm{soft}}={}&m_Q^2|\tilde{Q}|^2+m_{\bar{Q}}^2|\tilde{\bar{Q}}|^2+m_U^2|\tilde{U}|^2+m_{\bar{U}}^2|\tilde{\bar{U}}|^2+m_D^2|\tilde{D}|^2+m_{\bar{D}}^2|\tilde{\bar{D}}|^2\\&+m_L^2|\tilde{L}|^2+m_{\bar{L}}^2|\tilde{\bar{L}}|^2+m_E^2|\tilde{E}|^2+m_{\bar{E}}^2|\tilde{\bar{E}}|^2+(B_QM_Q\tilde{Q}\,\tilde{\bar{Q}}+B_UM_U\tilde{U}\,\tilde{\bar{U}}\\&+B_DM_D\tilde{D}\,\tilde{\bar{D}}+B_LM_L\tilde{L}\,\tilde{\bar{L}}+B_EM_E\tilde{E}\,\tilde{\bar{E}}+A_{k_t}k_1H_u\tilde{Q}\,\tilde{\bar{U}}+A_{k_b}k_2H_u\tilde{\bar{Q}}\,\tilde{D}\\&-A_{k_t'}k_1'H_d\tilde{\bar{Q}}\,\tilde{U}-A_{k_b'}k_2'H_d\tilde{Q}\,\tilde{\bar{D}}-A_{k_\tau'}k_3'H_d\tilde{L}\,\tilde{\bar{E}}+\mathrm{c.c.}).\end{aligned}\tag{3}$$

From Eqs. (2) and (3), the new charged fermions' field-dependent mass matrices are

$$\mathcal{M}_U(\phi)=\begin{pmatrix}M_Q & k_1\frac{\phi_u}{\sqrt{2}}\\ k_1'\frac{\phi_d}{\sqrt{2}} & M_U\end{pmatrix},\qquad \mathcal{M}_D(\phi)=\begin{pmatrix}M_Q & k_2'\frac{\phi_d}{\sqrt{2}}\\ k_2\frac{\phi_u}{\sqrt{2}} & M_D\end{pmatrix},\qquad \mathcal{M}_E(\phi)=\begin{pmatrix}M_L & k_3'\frac{\phi_d}{\sqrt{2}}\\ k_3\frac{\phi_u}{\sqrt{2}} & M_E\end{pmatrix}.\tag{4}$$

We have defined[1] $\langle\phi_d\rangle=v_d=c_\beta v$ and $\langle\phi_u\rangle=v_u=s_\beta v$, and $v\simeq 246$ GeV. The corresponding field-dependent sfermion squared-mass matrix, for the new up-type squark for instance, is

[1]In this paper, we use s_β, c_β for $\sin\beta$, $\cos\beta$.

$$\mathcal{M}^2_{\tilde{U}} = \begin{pmatrix} m^2_{\tilde{t}_{L'}} & m^2_{X_{t'}} & B_Q M_Q & M_Q^* k_1 \frac{\phi_u}{\sqrt{2}} + M_U k_1' \frac{\phi_d}{\sqrt{2}} \\ m^2_{X_{t'}} & m^2_{\tilde{t}_{R'}} & M_U k_1 \frac{\phi_u}{\sqrt{2}} + M_Q^* k_1' \frac{\phi_d}{\sqrt{2}} & B_U M_U \\ B_Q M_Q & M_U k_1 \frac{\phi_u}{\sqrt{2}} + M_Q^* k_1' \frac{\phi_d}{\sqrt{2}} & m^2_{\tilde{t}_{L'}} & m^2_{X_{t'}} \\ M_Q^* k_1 \frac{\phi_u}{\sqrt{2}} + M_U k_1' \frac{\phi_d}{\sqrt{2}} & B_U M_U & m^2_{X_{t'}} & m^2_{\tilde{t}_{R'}} \end{pmatrix}, \quad (5)$$

in which the basis is $(\bar{Q}^*, U, Q, \bar{U}^*)$, and we have defined

$$\begin{aligned} m^2_{\tilde{t}_{L'}}(\phi) &= M_Q^2 + m_{\bar{Q}}^2 + \frac{1}{2}k_1'^2\phi_d^2 + D^2_{\tilde{t}_{L'}}(\phi), \\ m^2_{\tilde{t}_{R'}}(\phi) &= M_U^2 + m_U^2 + \frac{1}{2}k_1'^2\phi_d^2 + D^2_{\tilde{t}_{R'}}(\phi), \\ m^2_{\tilde{t}_{L''}}(\phi) &= M_Q^2 + m_Q^2 + \frac{1}{2}k_1^2\phi_u^2 + D^2_{\tilde{t}_{L''}}(\phi), \\ m^2_{\tilde{t}_{R''}}(\phi) &= M_U^2 + m_{\bar{U}}^2 + \frac{1}{2}k_1^2\phi_u^2 + D^2_{\tilde{t}_{R''}}(\phi), \\ m^2_{X_{t'}}(\phi) &= k_1'\left(A_{k_{t'}}\frac{\phi_d}{\sqrt{2}} - \mu\frac{\phi_u}{\sqrt{2}}\right), \\ m^2_{X_{t''}}(\phi) &= k_1\left(A_{k_t}\frac{\phi_u}{\sqrt{2}} - \mu\frac{\phi_d}{\sqrt{2}}\right), \\ D^2_{\tilde{t}_{L'}}(\phi) &= -D^2_{\tilde{t}_{L''}}(\phi) = -\left(\frac{g^2}{8} - \frac{g'^2}{12}\right)(\phi_d^2 - \phi_u^2), \\ D^2_{\tilde{t}_{R'}}(\phi) &= -D^2_{\tilde{t}_{R''}}(\phi) = -\frac{g'^2}{6}(\phi_d^2 - \phi_u^2). \end{aligned} \quad (6)$$

The squared-mass matrices for the down-type squark and charged slepton are similar. After diagonalization, we get two new Dirac up-type quarks $t'_{1,2}$, two new Dirac down-type quarks $b'_{1,2}$, two new Dirac charged leptons $\tau'_{1,2}$, and two new left-handed neutrinos $\nu'_{1,2}$, as well as their superpartners $\tilde{t}'_{1,2,3,4}$, $\tilde{b}'_{1,2,3,4}$, $\tilde{\tau}'_{1,2,3,4}$, and $\tilde{\nu}'_{1,2}$.[2]

In the following calculation, we neglect all the D terms and B terms in the mass matrices.[3] For simplicity, we further assume at low scale [namely, without renormalization group equation (RGE) running]

$$\begin{aligned} m_Q^2 &= m_{\bar{Q}}^2 = m_U^2 = m_{\bar{U}}^2 = m_D^2 = m_{\bar{D}}^2 \\ &= m_L^2 = m_{\bar{L}}^2 = m_E^2 = m_{\bar{E}}^2 = m^2, \\ M_Q &= M_U = M_D = M_L = M_E = M_V, \\ A_{k_{t,b,\tau}} &= A_{k_{t',b',\tau'}} = A \end{aligned} \quad (7)$$

[2]Strictly speaking, (s)neutrinos do not need diagonalization.
[3]At phase transition, the D terms are comparable with the top-squark thermal mass in Eq. (45), which we are not ignoring, but here we have a more important contribution from VL Yukawa couplings anyway.

and define the VL scalar squared-mass average and the mass mixing parameter as

$$\begin{aligned} M_S^2 &= M_V^2 + m^2, \\ X_1 &= A - \mu\cot\beta, \\ X_2 &= A - \mu\tan\beta. \end{aligned} \quad (8)$$

We choose $\tan\beta = 10$ as our benchmark. Note that the Yukawa $k_{1,2,3}$ are always combined with ϕ_u and the Yukawa $k'_{1,2,3}$ with ϕ_d; the latter is always suppressed by $\tan\beta$. We actually set $k'_{1,2,3}$ to zero (see the discussion of the gluon fusion and Higgs diphoton deacy), and then ϕ_d decouples. Arising from the first mass matrix in Eq. (4), the field-dependent squared-mass eigenvalues of $t'_{1,2}$ can be simplified as

$$m^2_{t'_{1,2}}(\phi_u,\phi_d) = M_V^2 + \frac{1}{4}k_1^2\phi_u^2 \mp \frac{1}{4}\sqrt{k_1^4\phi_u^4 + 8M_V^2k_1^2\phi_u^2}, \quad (9)$$

and the four field-dependent squared-mass eigenvalues, arising from Eqs. (5) and (6), are

$$m^2_{\tilde{t}'_1}(\phi_u,\phi_d) = M_S^2 + \frac{1}{4}k_1^2\phi_u^2 - \frac{1}{2\sqrt{2}}k_1\phi_u X_1 - \frac{1}{2}\sqrt{\left(\frac{1}{2}k_1^2\phi_u^2 - \frac{1}{\sqrt{2}}k_1\phi_u X_1\right)^2 + 2M_V^2k_1^2\phi_u^2}, \quad (10)$$

$$m^2_{\tilde{t}'_2}(\phi_u,\phi_d) = M_S^2 + \frac{1}{4}k_1^2\phi_u^2 + \frac{1}{2\sqrt{2}}k_1\phi_u X_1 - \frac{1}{2}\sqrt{\left(\frac{1}{2}k_1^2\phi_u^2 + \frac{1}{\sqrt{2}}k_1\phi_u X_1\right)^2 + 2M_V^2k_1^2\phi_u^2}, \quad (11)$$

$$m^2_{\tilde{t}'_3}(\phi_u,\phi_d) = M_S^2 + \frac{1}{4}k_1^2\phi_u^2 - \frac{1}{2\sqrt{2}}k_1\phi_u X_1 + \frac{1}{2}\sqrt{\left(\frac{1}{2}k_1^2\phi_u^2 - \frac{1}{\sqrt{2}}k_1\phi_u X_1\right)^2 + 2M_V^2k_1^2\phi_u^2}, \quad (12)$$

$$m^2_{\tilde{t}'_4}(\phi_u,\phi_d) = M_S^2 + \frac{1}{4}k_1^2\phi_u^2 + \frac{1}{2\sqrt{2}}k_1\phi_u X_1 + \frac{1}{2}\sqrt{\left(\frac{1}{2}k_1^2\phi_u^2 + \frac{1}{\sqrt{2}}k_1\phi_u X_1\right)^2 + 2M_V^2 k_1^2\phi_u^2}. \quad (13)$$

For field-dependent masses of new down-type quarks, new charged leptons, and their superpartners, one just needs to substitute $k_1 \to k_2$, k_3.

At the end of this section, we give the direct search limits on new particles. As mentioned before, the exotic heavy fermions can decay into SM particles when kinematically allowed through the mixing Yukawa couplings [21,22]. Direct searches set limits on the exotic fermions in such decay modes. Limits on sparticles depend on the mixing angles of the mass eigenstates and the mass splittings between them and the lightest neutralino. The strongest current limits on the extra quarks, leptons, and their scalar particles are given as [23]

$$m_{t'} > 685 \text{ GeV}, \qquad m_{\tilde{t}'} > 95.7 \text{ GeV}, \quad (14)$$

$$m_{b'} > 685 \text{ GeV}, \qquad m_{\tilde{b}'} > 89 \text{ GeV}, \quad (15)$$

$$m_{\tau'} > 100.8 \text{ GeV}, \qquad m_{\tilde{\tau}'} > 81.9 \text{ GeV}, \quad (16)$$

$$m_{\nu'} > 39.5 \text{ GeV}, \qquad m_{\tilde{\nu}'} > 94 \text{ GeV}. \quad (17)$$

However, when considering various combinations of decay modes of new fermions without being limited to a special mass constraint for scalars, the above bounds are relaxed. We will see later that the masses of charged exotic fermions are important to an acceptable SFOPT, so here in our work we consider some optimistic mass limits for new charged fermions. Namely, we consider $m_{t'} > 415$ GeV for t' [27], which is achieved by scanning the exotic decay branching ratio triangle, and $m_{b'} > 360$ GeV [21] for b', and $m_{\tau'} > 63.5$ GeV [28] for τ'. The mass limits for other new particles still take the values shown above.

III. ZERO-TEMPERATURE POTENTIAL AND HIGGS MASS

In this model, the zero-temperature effective potential at one-loop level is given by

$$V(\phi_u,\phi_d,T=0) = V_0(\phi_u,\phi) + V_1(\phi_u,\phi_d), \quad (18)$$

in which V_0 is the tree-level potential, and V_1 is the zero-temperature renormalized one-loop potential.

A. Tree-level potential

The zero-temperature tree-level potential here in our model is the same as in the MSSM, which is given as

$$V_{\rm MSSM} = \frac{1}{2}m_{11}^2\phi_d^2 + \frac{1}{2}m_{22}^2\phi_u^2 - m_{12}^2\phi_d\phi_u + \frac{1}{4}\lambda_1\phi_d^4 + \frac{1}{4}\lambda_2\phi_u^4 + \frac{1}{2}\lambda_3\phi_d^2\phi_u^2, \quad (19)$$

in which

$$m_{11}^2 = m_{H_d}^2 + \mu^2, \quad (20)$$

$$m_{22}^2 = m_{H_u}^2 + \mu^2, \quad (21)$$

$$m_{12}^2 = b\mu, \quad (22)$$

$$\lambda_1 = \lambda_2 = -\lambda_3 = \frac{1}{8}(g^2 + g'^2). \quad (23)$$

B. The renormalization-group improved Higgs potential and the SM-like Higgs mass

The third-generation MSSM particles and the new VL particles will induce significant corrections to the Higgs potential. Here we are interested in the complete one-loop improved Higgs potential, because it determines the SM-like Higgs mass. We follow Ref. [29] to write it as

$$V_{\rm MSSM} = \frac{1}{2}(m_{11}^2 + \Delta m_{11}^2)\phi_d^2 + \frac{1}{2}(m_{22}^2 + \Delta m_{22}^2)\phi_u^2 - (m_{12}^2 + \Delta m_{12}^2)\phi_d\phi_u + \frac{1}{4}(\lambda_1 + \Delta\lambda_1)\phi_d^4 + \frac{1}{4}(\lambda_2 + \Delta\lambda_2)\phi_u^4 + \frac{1}{2}(\lambda_3 + \Delta\lambda_3)\phi_d^2\phi_u^2 + \Delta\lambda_6\phi_d^3\phi_u + \Delta\lambda_7\phi_d\phi_u^3, \quad (24)$$

where $\Delta\lambda_6\phi_d^3\phi_u$ and $\Delta\lambda_7\phi_d\phi_u^3$ are the one-loop-potential induced terms which do not exist in the tree-level potential. The expressions for the corrections are listed in the Appendix.

With the renormalization-group (RG) improved Higgs potential, the SM-like Higgs mass can be written as

$$m_{h_0}^2 = m_Z^2\cos^2 2\beta + 2\Delta\lambda_1 v^2\sin^4\beta + 2\Delta\lambda_2 v^2\cos^4\beta + 4\Delta\lambda_3 v^2\sin^2\beta\cos^2\beta + 8\Delta\lambda_6 v^2 \sin\beta\cos^3\beta + 8\Delta\lambda_7 v^2\sin^3\beta\cos\beta. \quad (25)$$

In order to get a simple analytical expression, we set the parameters as mentioned before and further set

$$k_1 = k_2 = k_3 = k. \quad (26)$$

Then the SM-like Higgs mass can be simplified as

$$m_{h_0}^2 = m_Z^2\cos^2 2\beta + \frac{3v^2}{4\pi^2}y_t^4\left[\ln\left(\frac{\tilde{m}_t}{m_t}\right) + \frac{X_t^2}{2\tilde{m}_t^2}\left(1-\frac{X_t^2}{12\tilde{m}_t^2}\right)\right] + \frac{7v^2}{8\pi^2}k^4 s_\beta^4\left[\ln\frac{M_S^2}{M_V^2} - \frac{1}{6}\left(5-\frac{M_V^2}{M_S^2}\right)\left(1-\frac{M_V^2}{M_S^2}\right) + \hat{X}_1^2\left(1-\frac{M_V^2}{3M_S^2}\right) - \frac{\hat{X}_1^4}{12}\right]. \tag{27}$$

We can see that new heavy particles give extra contributions, permitting a relatively lighter stop mass, which can loosen the tension of the naturalness problem.

C. Zero-temperature one-loop-level potential

In the above analysis, we actually run the RGE top down from the supersymmetry-breaking scale in order to fix the low-energy Higgs mass to be the observed value. However, as we go to higher scales where the EW phase transition takes place, the RGE running is backwards from that of the low-energy potential Eq. (24). We describe this process in the way of (zero-temperature) one-loop potential, which is equivalent to the RGE.[4] The zero-temperature one-loop potential is given by

$$V_1(\phi_u,\phi_d) = \frac{1}{64\pi^2}\sum_i n_i m_i^4(\phi_u,\phi_d)\left[\log\frac{m_i^2(\phi_u,\phi_d)}{Q^2} - c_i\right], \tag{28}$$

where $m_i(\phi_u,\phi_d)$ are the field-dependent masses and Q is the renormalization scale.[5] Here i stands for the particles which can contribute to the effective potential, n_i is the particle degree of freedom, and c_i's are constants which are $5/6$ for gauge bosons and $3/2$ for fermions and scalars. In our work, we include the large one-loop corrections induced by the top, stop, and all the vectorlike particles, as well as the EW gauge bosons. The corresponding degrees of freedom are $n_t = n_{t'_{1,2}} = n_{b'_{1,2}} = 3n_{\tau'_{1,2}} = -12$, $n_{\tilde{t}_{1,2}} = n_{\tilde{t}'_{1,2,3,4}} = n_{\tilde{b}'_{1,2,3,4}} = 3n_{\tilde{\tau}'_{1,2,3,4}} = 6$, $n_{W_L} = 2$, $n_{W_T} = 4$, $n_{Z_L} = 1$, $n_{Z_T} = 2$, where the subscripts L and T signify the longitudinal and transverse modes, respectively.

As stressed above, the one-loop potential should be renormalized in a way which preserves the low-energy Higgs VEV and the Higgs mass. In the one-loop-potential language it is easy to implement, namely by requiring

$$\left(\frac{\partial}{\partial\phi_u},\frac{\partial}{\partial\phi_d},\frac{\partial^2}{\partial\phi_u^2},\frac{\partial^2}{\partial\phi_d^2},\frac{\partial^2}{\partial\phi_u\partial\phi_d}\right) \times (V_1(\phi_u,\phi_d) + V_1^{\text{c.t.}}(\phi_u,\phi_d))|_{\phi_d=v_d,\phi_u=v_u} = 0. \tag{29}$$

Here we introduce the finite "counterterms" $V_1^{\text{c.t.}}$ to protect the one-loop potential from shifting the Higgs VEV and CP-even Higgs mass matrix. We have five equations, so we can determine up to five coefficients of the counterterm polynomial; here we choose them to be

$$V_1^{\text{c.t.}} = \frac{1}{2}\delta m_{11}^2\phi_d^2 + \frac{1}{2}\delta m_{22}^2\phi_u^2 + \frac{1}{4}\delta\lambda_1\phi_d^4 + \frac{1}{4}\delta\lambda_2\phi_u^4 + \frac{1}{2}\delta\lambda_3\phi_d^2\phi_u^2. \tag{30}$$

And the corresponding total zero-temperature one-loop potential is

$$V_1^{\text{re}}(\phi_u,\phi_d)_i = V_1(\phi_u,\phi_d)_i + V_1^{\text{c.t.}}(\phi_u,\phi_d)_i = \frac{n_i}{64\pi^2}\left[m_i^4(\phi_u,\phi_d)\log\frac{m_i^2(\phi_u,\phi_d)}{Q^2} + \alpha_i^u\phi_u^2 + \alpha_i^d\phi_d^2 + \beta_i^u\phi_u^4 + \beta_i^d\phi_d^4 + 2\beta_i^{ud}\phi_u^2\phi_d^2\right]. \tag{31}$$

The solution of Eq. (29) is unique, namely

$$\alpha_i^u = \left(-\frac{3}{2}\frac{\omega_i\omega_i^{u'}}{v_u} + \frac{1}{2}\omega_i^{u'2} + \frac{1}{2}\omega_i\omega_i^{u'}\right)\log\frac{\omega_i}{Q^2} - \frac{3}{4}\frac{\omega_i\omega_i^{u'}}{v_u} + \frac{3}{2}\omega_i^{u'2} + \frac{1}{2}\omega_i\omega_i^{u'} - \beta_i^{ud}v_d^2, \tag{32}$$

$$\alpha_i^d = \left(-\frac{3}{2}\frac{\omega_i\omega_i^{d'}}{v_d} + \frac{1}{2}\omega_i^{d'2} + \frac{1}{2}\omega_i\omega_i^{d'}\right)\log\frac{\omega_i}{Q^2} - \frac{3}{4}\frac{\omega_i\omega_i^{d'}}{v_d} + \frac{3}{2}\omega_i^{d'2} + \frac{1}{2}\omega_i\omega_i^{d'} - \beta_i^{ud}v_u^2, \tag{33}$$

$$\beta_i^u = \frac{1}{v_u^2}\left[\left(\frac{1}{4}\frac{\omega_i\omega_i^{u'}}{v_u} - \frac{1}{4}\omega_i^{u'2} - \frac{1}{4}\omega_i\omega_i^{u'}\right)\log\frac{\omega_i}{Q^2} + \frac{1}{8}\frac{\omega_i\omega_i^{u'}}{v_u} - \frac{3}{8}\omega_i^{u'2} - \frac{1}{8}\omega_i\omega_i^{u'}\right], \tag{34}$$

$$\beta_i^d = \frac{1}{v_d^2}\left[\left(\frac{1}{4}\frac{\omega_i\omega_i^{d'}}{v_d} - \frac{1}{4}\omega_i^{d'2} - \frac{1}{4}\omega_i\omega_i^{d'}\right)\log\frac{\omega_i}{Q^2} + \frac{1}{8}\frac{\omega_i\omega_i^{d'}}{v_d} - \frac{3}{8}\omega_i^{d'2} - \frac{1}{8}\omega_i\omega_i^{d'}\right], \tag{35}$$

[4]We choose to present the one-loop issue in this awkward way because this is the way we do the numerical work: the Coleman-Weinberg-form one-loop potential is always implemented by a built-in function in the code CosmoTransition [30], so the low-scale parameters consistent with Higgs mass and VEV need to be run down from the supersymmetry breaking scale at first.

[5]A variation of Q induces variation of the ϕ^2 and ϕ^4 terms; in Eqs. (29)–(31)), we see that the combination of them together with counterterms are determined by the renormalization condition, so the value of Q is immaterial.

$$\beta_i^{ud} = -\left(\frac{\omega_i^{u'}\omega_i^{d'} + \omega_i\omega_i^{ud'}}{4v_u v_d}\right)\log\frac{\omega_i}{Q^2} + \frac{3\omega_i^{u'}\omega_i^{d'} + \omega_i\omega_i^{ud'}}{2v_u v_d}, \tag{36}$$

where we define $\omega_i = m_i^2(v_u, v_d)$, $\omega_i^{u(d)'} = \frac{\partial m_i^2(\phi_u,\phi_d)}{\partial\phi_{u(d)}}|_{(v_u,v_d)}$, $\omega_i^{u(d)''} = \frac{\partial^2 m_i^2(\phi_u,\phi_d)}{\partial^2\phi_{u(d)}}|_{(v_u,v_d)}$, and $\omega_i^{ud''} = \frac{\partial^2 m_i^2(\phi_u,\phi_d)}{\partial\phi_u\partial\phi_d}|_{(v_u,v_d)}$. These are the generalization of expressions in Ref. [19] to the two-Higgs-doublet model.

IV. FINITE-TEMPERATURE POTENTIAL

The temperature-dependent potential at the one-loop level is given by

$$\Delta V(\phi_u,\phi_d,T) = \Delta V_1(\phi_u,\phi_d,T) + \Delta V_{\text{daisy}}(\phi_u,\phi_d,T), \tag{37}$$

where ΔV_1 is the finite-temperature one-loop potential [11], and ΔV_{daisy} is the finite-temperature effect coming from the resummation of the leading infrared-dominated higher-loop contributions [10]. The specific formulas are

$$\Delta V_1(\phi_u,\phi_d,T) = \frac{T^4}{2\pi^2}\left\{\sum_{i=\text{bosons}} n_i J_B\left[\frac{\bar{m}_i(\phi_u,\phi_d)}{T}\right] + \sum_{i=\text{fermions}} n_i J_F\left[\frac{m_i(\phi_u,\phi_d)}{T}\right]\right\}, \tag{38}$$

$$\Delta V_{\text{daisy}}(\phi_u,\phi_d,T) = -\frac{T}{12}\sum_{i=\text{bosons}} n_i[\bar{m}_i^3(\phi_u,\phi_d,T) - m_i^3(\phi_u,\phi_d)], \tag{39}$$

with definitions and high-temperature expansions

$$J_B\left[\frac{m}{T}\right] = \int_0^\infty dx\, x^2\log\left[1 - e^{-\sqrt{x^2+\frac{m^2}{T^2}}}\right] = -\frac{\pi^4}{45} + \frac{\pi^2}{12}\frac{m^2}{T^2} - \frac{\pi}{6}\left(\frac{m^2}{T^2}\right)^{\frac{3}{2}} + \mathcal{O}\left(\frac{m^4}{T^4}\right), \tag{40}$$

$$J_F\left[\frac{m}{T}\right] = \int_0^\infty dx\, x^2\log\left[1 + e^{-\sqrt{x^2+\frac{m^2}{T^2}}}\right] = \frac{7\pi^4}{360} - \frac{\pi^2}{24}\frac{m^2}{T^2} + \mathcal{O}\left(\frac{m^4}{T^4}\right), \tag{41}$$

in which the thermal mass $\bar{m}_i^2(\phi_u,\phi_d,T) = m_i^2(\phi_u,\phi_d) + \Pi_i(T)$, and $\Pi_i(T)$ is the leading T dependent self-energy. To leading order, only bosons receive thermal mass corrections. Only the longitudinal components of W and Z receive the daisy corrections.

The thermal masses of the MSSM particles are well known. For the EW gauge bosons, the field- and temperature-dependent masses are

$$m_W^2(\phi_u,\phi_d,T) = \frac{1}{2}g^2(\phi_d^2+\phi_u^2) + \Pi_{W^\pm}, \tag{42}$$

$$\mathcal{M}_{Z\gamma}^2(\phi_u,\phi_d,T) = \begin{pmatrix} \frac{1}{2}g^2(\phi_d^2+\phi_u^2)+\Pi_{W^3} & -\frac{1}{2}gg'(\phi_d^2+\phi_u^2) \\ -\frac{1}{2}gg'(\phi_d^2+\phi_u^2) & \frac{1}{2}g'^2(\phi_d^2+\phi_u^2)+\Pi_B \end{pmatrix} \tag{43}$$

in which the thermal masses $\Pi_{W^\pm} = \Pi_{W^3} = \frac{9}{2}g^2T^2$ and $\Pi_B = \frac{9}{2}g'^2T^2$ for the longitudinal modes, and for the transverse modes all the thermal masses are zero. The field and-temperature-dependent mass of the MSSM third-generation stops are given by

$$\mathcal{M}_{\tilde{t}}^2(\phi_u,\phi_d,T) = \begin{pmatrix} M_{\tilde{t}}^2 + \frac{1}{2}y_t^2\phi_u^2 + \Pi_{\tilde{t}_L} & \frac{1}{\sqrt{2}}y_t\phi_u X_1 \\ \frac{1}{\sqrt{2}}y_t\phi_u X_1 & M_{\tilde{t}}^2 + \frac{1}{2}y_t^2\phi_u^2 + \Pi_{\tilde{t}_R} \end{pmatrix}, \tag{44}$$

where

$$\Pi_{\tilde{t}_L} = \frac{2}{3}g_s^2T^2 + \frac{1}{72}g^2T^2 + \frac{3}{8}g'^2T^2 + \frac{1}{4}y_t^2T^2, \tag{45}$$

$$\Pi_{\tilde{t}_R} = \frac{2}{3}g_s^2T^2 + \frac{2}{9}g'^2T^2 + \frac{1}{2}y_t^2T^2. \tag{46}$$

All the thermal masses are derived from Ref. [31]. On the other hand, all the new VL particles' thermal masses are neglected in our work, for both simplicity and nonexistence in literature. If included, naively, it will further rise by an order of $g_s^2T^2$ or k^2T^2 contribution to the M_S^2 terms, which is probably large and makes a SFOPT even more difficult according to the following discussion.

We calculate the thermal functions $J_{B/F}$ in Eq. (40) numerically instead of using a high-temperature expansion, which is crucial for our purpose. The change in $J_{B/F}$ includes the information of continuous variation of entropy density induced by the new VL particles; see Fig. 2 in the next section.

V. RESULTS AND DISCUSSION

We use the public code CosmoTransition [30] for a numerical evaluation of the phase transition and perform several scans of the parameter space. As mentioned before, we choose $\tan\beta = 10$; we also choose a CP-odd Higgs mass $m_A = 2000$ GeV for a typical decoupling Higgs sector. For the MSSM top/stop sector, we want a small contribution to the SM-like Higgs mass (so that a large contribution from the VL sector and a large VL Yukawa coupling are possible), so we choose $M_{t_L} = 700$ GeV, $M_{t_R} = 500$ GeV, and $A_t = 500$ GeV for the soft breaking parameters and $\mu = 500$ GeV. However, our results are not sensitive to the values of the MSSM parameters, because with $X = 0$ we can (as we actually do) choose arbitrarily degenerated fermions and sfermions, $\frac{M_S^2}{M_V^2} \to 1$, to reduce their contribution to the Higgs mass through the factor

$$\log\left(\frac{M_S^2}{M_V^2}\right) - \frac{1}{6}\left(5 - \frac{M_V^2}{M_S^2}\right)\left(1 - \frac{M_V^2}{M_S^2}\right) \to \frac{1}{3}\left(\frac{M_S^2}{M_V^2} - 1\right). \tag{47}$$

As for the VL parameters, for simplicity in all our scans we set the parameters as mentioned in Eqs. (7), (8), and (26), and all the new down-type Yukawa couplings k' and the down-type mass mixing parameter X_2 are taken to be zero. In scan we have checked that the up-type mass mixing parameter X_1 prefers zero in order to have a larger phase-transition strength, so we also fix $X_1 = 0$, which also reduces other contributions to add to the factor in Eq. (47) to enable a large Yukawa.

A. SFOPT

In Fig. 1, we show two scans of phase-transition strength with the Yukawa coupling k and the VL mass M_V. We also show the constraint of Higgs mass $m_{h^0} \sim 124$–127 GeV and the lightest new fermion mass contours. In the left panel, we fix $M_S/M_V = 1.5$. We can see that for such a range of Higgs mass, the SFOPT can only be achieved for $k \simeq 1.6$ and VL mass $M_V \lesssim 100$ GeV. In the right panel, we fix $M_S/M_V = 1.1$, for which the combination of SFOPT with about 125 GeV Higgs can only be generated for $k \simeq 2.6$ and VL mass $M_V \lesssim 230$ GeV.

First, we can see, as far as the SFOPT is concerned, that the larger the VL mass M_V is, the larger the Yukawa coupling k needs to be. Because a Boltzmann suppression effect of a few hundred GeV M_V may decouple the new particle in the symmetric phase, significant entropy release effects for a SFOPT can only be guaranteed by a large Yuwaka mass and a large $m(\phi)/T$ shift.

Comparing this to the entropy release effect in Ref. [19], we can see that for a SFOPT our required degree of freedom is much larger.[6] This is quantitatively the most significant point of our analysis. To see clearly, with Eqs. (16)–(20) we can write the new fermion mass squares as $M_{f_{1,2}}^2 = M_V^2 + \frac{1}{4}k^2\phi_u^2 \mp \frac{1}{4}\sqrt{k^4\phi_u^4 + 8M_V^2k^2\phi_u^2}$, or equivalently

$$M_{f_{1,2}} = \sqrt{M_V^2 + \frac{1}{8}k^2\phi_u^2} \mp \frac{1}{2\sqrt{2}}k\phi_u. \tag{48}$$

The new sfermion mass has a similar behavior. We can understand it in the following interesting picture: After EWSB, the fermion masses jump from M_V to $\sqrt{M_V^2 + \frac{1}{4}k^2\phi_u^2}$, and on this basis become split. The mass-splitting terms $\frac{1}{2\sqrt{2}}k\phi_u$ make half of the VL fermions lighter than those in the symmetric phase, overcoming the common shift $M_V \to \sqrt{M_V^2 + \frac{1}{4}k^2\phi_u^2}$, while the other half are heavier. In Fig. 2, we show the fully calculated finite-temperature potential contribution J_B/F instead of only the high-temperature expansions. We can refer to the J_B, J_F curves to see the potential change.

A shift of $M_V/T \to \sqrt{M_V^2 + \frac{1}{4}k^2\phi_u^2}/T$ is exactly the entropy release effect in Ref. [19], with an effect of the representative point rise on the J_F curve, or the thermal potential rise. Here the further new splitting of $\frac{1}{2\sqrt{2}}k\phi_u$ for the heavy particle will raise the $m(\phi,T)/T$ more and release more entropy, while unfortunately, the $-\frac{1}{2\sqrt{2}}k\phi_u$ for the light particle will have an opposite effect. A little bit more quantitative analysis indicates that because the slope of the J_B/J_F curve is less at higher m/T (for example, 4) than at lower m/T (say, 1), the backward splitting $-\frac{1}{2\sqrt{2}}k\phi_u$ to lower masses always induces a larger thermal potential drop $\Delta J_B/\Delta J_F$ than the forward splitting $+\frac{1}{2\sqrt{2}}k\phi_u$, and the net effect is a drop, unable to trigger the SFOPT [20]. This opposite effect will significantly compensate the $M_V/T \to \sqrt{M_V^2 + \frac{1}{4}k^2\phi_u^2}/T$ effect, making the contribution to phase transition strength in our scenario much smaller than that with merely the same degree of freedom, the same soft mass and the same Yukawa, but without splitting. We will give a more general analysis in our next paper.

B. Higgs mass and light exotic particle constraints

Apparently, with SFOPT requirements, the first two scans always give too light a new fermion, so they are ruled out. As we have already discussed, the direction we can go is to increase M_V and k. In Ref. [21], an infrared quasi–fix point is pointed out, as $k \simeq 1.0$ and $h \simeq 1.2$. Here we ignore this bound, but the bottom line is the perturbativity bound $k \lesssim 4$. We choose to saturate the bound, and then we get an almost unbroken supersymmetric VL sector (Fig. 3).[7]

[6] We note a convention difference, and our $k = 4$ corresponds to $h = 2$ in Ref. [19].

[7] With our MSSM parameter choices, we get $\frac{M_S}{M_V} = 1.019$.

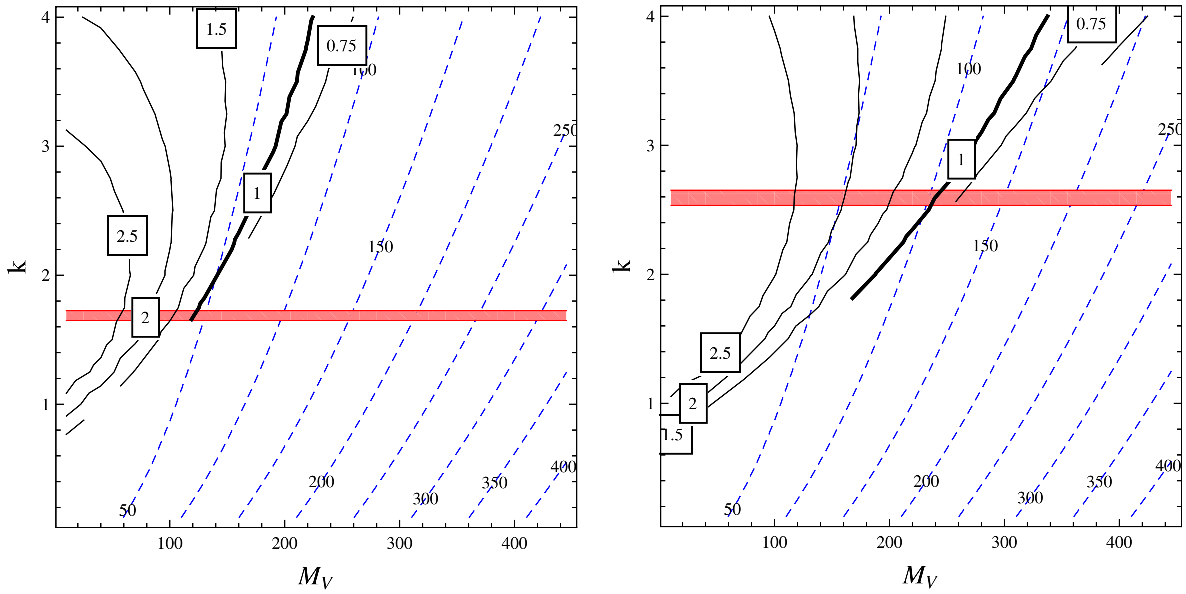

FIG. 1 (color online). EWBG, Higgs mass and the lightest new fermion mass contours in the MSSM extension with vectorlike superfields. Red/solid curves are the EWPT strength ϕ_c/T_c, and blue/dotted curves are the lightest new fermion mass. The pink/gray band is the SM-like Higgs mass region 124–127 GeV. In the left panel, we fix $M_S/M_V = 1.5$; in the right panel, we fix $M_S/M_V = 1.1$.

The best lightest fermion mass we get is about 240 GeV, which is still generally ruled out by heavy t' and b' quark searches, even by optimistic bounds, as mentioned in Sec. II. We will not discuss the possibility of aligned Yukawa matrix in generation basis, which makes the decay mode nonstandard. On the other hand, there is a possibility to relax the degeneracy between the quarks and the leptons, to make the quark sector M_V and M_S larger to accommodate heavier new quarks. However, at first, it is naively against our model assumption of $5 + \bar{5} + 10 + \overline{10}$ of $SU(5)$ GUT, which predicts $M_Q = M_U = M_E$ and $M_D = M_L$ at the GUT scale. Further, we numerically find that due to large zero-temperature corrections, for separate quark and lepton (or generally two-set) corrections, the potential usually does not even run away from symmetric phase even at zero temperature. So we will not go into detail on that possibility.

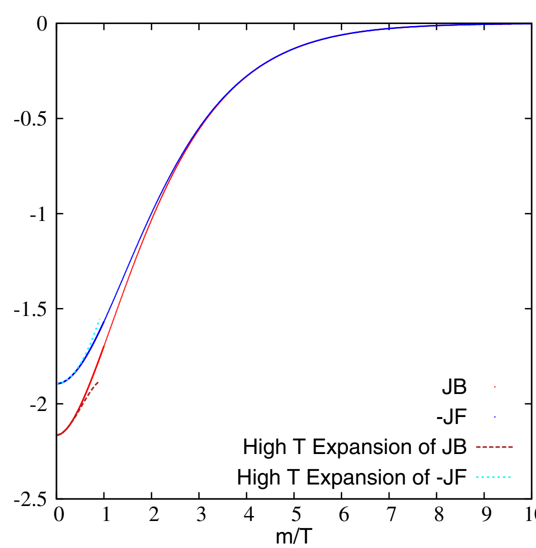

FIG. 2 (color online). The complete thermal one-loop potential contribution J_B (red/light gray curve) and $-J_F$ (blue/dark gray curve) as defined in Eq. (40) and the comparison with their high-temperature expansions (brown/long dashed and cyan/short dashed, respectively).

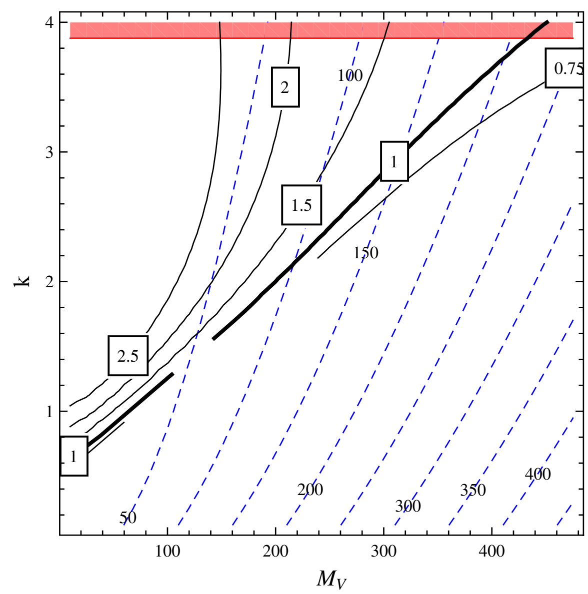

FIG. 3 (color online). Same as in Fig. 1, but for $M_S/M_V = 1.019$.

TABLE I. Input parameters which can realize both the SFOPT and 124–127 GeV Higgs, and the corresponding new particle masses.

M_V	M_S	k	$m_{f'_1}$	$m_{f'_2}$	$m_{\tilde{f}'_1}$	$m_{\tilde{f}'_2}$	$m_{\tilde{f}'_3}$	$m_{\tilde{f}'_4}$	ϕ_c/T_c	m_{h_0}
70	105	1.6	17	293	80	80	304	304	2.1	126.5
100	150	1.6	32	309	116	116	329	329	1.46	126.9
230	253	2.4	102	517	116	116	528	528	0.93	125.7
475	484	4.0	241	934	259	259	939	939	0.94	126.0

C. Gluon fusion and Higgs diphoton decay constraints

We use the low-energy theorem [32] for an estimation. The contributions to the loop amplitude are all proportional to $\frac{\partial}{\partial \ln v}$ det $\mathcal{M}$, where $\mathcal{M}$ is any of the mass matrices in Eqs. (4) or (5). As can be seen clearly in the fermion mass matrix, setting all k''s to be zero eventually makes all the determinants independent of the Higgs VEV. The two masses of Eq. (9) are actually from the matrix $\mathcal{M}\mathcal{M}^\dagger$ or $\mathcal{M}^\dagger\mathcal{M}$, and $\det(\mathcal{M}\mathcal{M}^\dagger) = m_{t'_1}^2 m_{t'_2}^2 = M_V^4$ is independent of ϕ_u, ϕ_d. With $X_1 = X_2 = 0$, the scalar sector has a similar behavior, but there is a residual contribution proportional to supersymmetry-breaking soft parameter $m^2 = M_S^2 - M_V^2$, namely det $\mathcal{M}^2_{\tilde{U},\tilde{D},\tilde{E}} = (M_S^4 + \frac{1}{2} m^2 k^2 \phi_u^2)^2$. Since we are interested in an almost supersymmetric VL sector and $m^2 \to 0$, the corrections to gluon fusion and Higgs diphoton decay amplitudes also vanish in this limit. So the gluon fusion Higgs production rate and Higgs diphoton decay rate are not affected. This discussion also justifies our parameter choices: $k' = 0$, $X_1 = 0$, and $X_2 = 0$.

D. Peskin-Takeuchi T and S parameters

We perform a numerical calculation. The fermionic contribution agrees with the formulas in Ref. [21]:

$$\Delta T = \frac{N_c}{480\pi s_W^2 M_W^2 M_V^2}\left(\frac{13}{4}(k^4 v_u^4 + k'^4 v_d^4) + \frac{1}{2}(k^3 k' v_u^3 v_d + k'^3 k v_d^3 v_u) + \frac{9}{2} k^2 k'^2 v_u^2 v_d^2\right), \tag{49}$$

$$\Delta S = \frac{N_c}{30\pi M_V^2}\left(2(k^2 v_u^2 + k'^2 v_d^2) + k k' v_u v_d \left(\frac{3}{2} + 10 Y_\phi\right)\right), \tag{50}$$

with $Y_\phi = -\frac{1}{3}$ for our model, while the scalar part gives nearly the same contribution for nearly unbroken supersymmetry. In particular, for the last point in Table I, we get $T \simeq 32.5$ and $S \simeq 0.2$, which apparently makes it excluded. Such a large-T parameter contribution occurs because it is proportional to k^4, and only suppressed by M_V^2. On the other hand, the form of the superpotential in Eq. (2) determines that the custodial symmetry is violated in the maximal way—namely, a light left-hand up-quark component always finds a heavy left-hand down-quark component.

VI. SUMMARY

We have discussed EWBG in the MSSM extension with vectorlike superfields belonging to the representation $5 + \bar{5} + 10 + \bar{10}$ of $SU(5)$ in detail. We find that the SFOPT has been ruled out by a combination of the 125 GeV Higgs requirement, the direct search for the exotic fermions, the gluon fusion rate, and the Higgs diphoton decay rate, as well as the EW precision measurement. However, the general contribution from a (nearly) supersymmetric sector to SFOPT with minimal effect to Higgs phenomenology is still interesting.

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ONE-LOOP CORRECTIONS TO THE QUARTIC COUPLING COEFFICIENTS

Under the parameter assumptions mentioned above, the one-loop corrections to quadratic and quartic coupling coefficients in the zero-temperature potential are given by

$$\Delta m_{11}^2 = \sum_i \frac{n_i}{64\pi^2} \alpha_i^d, \tag{A1}$$

$$\Delta m_{22}^2 = \sum_i \frac{n_i}{64\pi^2} \alpha_i^u, \tag{A2}$$

$$\Delta\lambda_1 = \sum_i \frac{n_i}{64\pi^2}\beta_i^d + \frac{1}{16\pi^2}\left\{-3y_t^4\frac{\mu^4}{12M_s^4} + \sum_i N_{C_i}\left[k_i'^4\left(\ln\frac{M_S^2}{M_V^2} - \frac{1}{6}\left(5-\frac{M_V^2}{M_S^2}\right)\left(1-\frac{M_V^2}{M_S^2}\right) + \hat{X}_1^2\left(1-\frac{M_V^2}{3M_S^2}\right) - \frac{\hat{X}_1^4}{12}\right)\right]\right\}, \quad (A3)$$

$$\Delta\lambda_2 = \sum_i \frac{n_i}{64\pi^2}\beta_i^u + \frac{1}{16\pi^2}\left\{3y_t^4\left[\ln\left(\frac{\tilde{m}_t^2}{m_t^2}\right) + \frac{A_t^2}{\tilde{m}_t^2}\left(1-\frac{A_t^2}{12\tilde{m}_t^2}\right) - \frac{\mu^4}{12\tilde{m}_t^4}\right] + \sum_i N_{C_i}\left[k_i^4\left(\ln\frac{M_S^2}{M_V^2} - \frac{1}{6}\left(5-\frac{M_V^2}{M_S^2}\right)\left(1-\frac{M_V^2}{M_S^2}\right) + \hat{X}_1^2\left(1-\frac{M_V^2}{3M_S^2}\right) - \frac{\hat{X}_1^4}{12}\right)\right]\right\}, \quad (A4)$$

$$\Delta\lambda_3 = \sum_i \frac{n_i}{64\pi^2}\beta_i^{ud} + \frac{1}{16\pi^2}\left\{3\left[\frac{y_t^4}{2}\frac{\mu^2}{\tilde{m}_t^2}\left(1-\frac{A_t^2}{2\tilde{m}_t^2}\right)\right] + \sum_i N_{C_i}\left[k_i^2k_i'^2\left(-\left(1-\frac{M_V^2}{M_S^2}\right)^2 - \frac{1}{3}(\hat{X}_1+\hat{X}_2)^2\right)\right]\right\}, \quad (A5)$$

$$\Delta\lambda_6 = \frac{1}{16\pi^2}\left\{3y_t^4\frac{\mu^3A_t}{12\tilde{m}_t^4} + \sum_i N_{C_i}\left[k_i'^3k_i\left(-\frac{2}{3}\left(2-\frac{M_V^2}{M_S^2}\right)\left(1-\frac{M_V^2}{M_S^2}\right) - \frac{1}{3}(2\hat{X}_2^2+\hat{X}_1\hat{X}_2)\right)\right]\right\}, \quad (A6)$$

$$\Delta\lambda_7 = \frac{1}{16\pi^2}\left\{3y_t^4\frac{\mu A_t}{\tilde{m}_t^2}\left(-\frac{1}{2}+\frac{A_t^2}{12\tilde{m}_t^2}\right) + \sum_i N_{C_i}\left[k_i^3k_i'\left(-\frac{2}{3}\left(2-\frac{M_V^2}{M_S^2}\right)\left(1-\frac{M_V^2}{M_S^2}\right) - \frac{1}{3}(2\hat{X}_1^2+\hat{X}_1\hat{X}_2)\right)\right]\right\}.146-+ \quad (A7)$$

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