Limits on Lorentz and *CPT* violation from double beta decay

Jorge S. Díaz

Physics Department, Indiana University, Bloomington, Indiana 47405, USA

Deviations from Lorentz and *CPT* invariance in the neutrino sector and their observable effects in double beta decay are studied. For two-neutrino double beta decay, a spectral distortion and its properties are characterized for different isotopes. Majorana couplings for Lorentz violation are studied and shown to trigger neutrinoless double beta decay even for negligible Majorana mass. Existing data are used to obtain first limits of 5×10^{-9} for 18 individual coefficients, and attainable sensitivities in current and future experiments are presented.

I. INTRODUCTION

The development of Fermi's theory of beta decay [1] was rapidly followed by important ideas involving weak interactions. In 1935, Goeppert-Mayer proposed the possibility that two neutrons in a nucleus could simultaneously decay into two protons, two electrons, and two antineutrinos [2], estimating that this rare decay would have a half-life greater than 10^{17} years. This two-neutrino double beta decay is a second-order weak process of the form $(A, Z) \rightarrow (A, Z + 2) + 2e^- + 2\bar{\nu}_e$ that has been observed in many nuclei [3,4]. Other authors also proposed the possibility of another mode of double beta decay, in which the neutrino would be absent in the final state. This neutrinoless double beta decay $(A, Z) \rightarrow (A, Z + 2) + 2e^-$ could be possible if neutrinos are their own antiparticles [5]. Today, there is a dedicated experimental program searching for neutrinoless double beta decay. A careful study of two-neutrino double beta decay is also performed by these experiments because it constitutes a background for the neutrinoless mode. The high precision of many experiments has motivated the formulation of different modes of double beta decay so that experiments can also look for new physics through unconventional decay modes [4].

In the present work, we propose to use experiments studying double beta decay as probes of Lorentz invariance. The spontaneous breakdown of this spacetime symmetry is an interesting feature that can be accommodated by many candidate theories of quantum gravity, such as string theory [6]. Even though direct studies of physics at quantum-gravity energies remain inaccessible for current experiments, we can use low-energy experiments as tools to explore suppressed signals of new physics at the Planck scale. The effects of violations of Lorentz symmetry in beta decay have been characterized in Ref. [7], which briefly describes the experimental signals of Lorentz violation in double-beta-decay experiments. In this work, we assume that Lorentz-violating effects only affect neutrinos. The corresponding effects arising from Lorentz violation in the gauge and Higgs sectors are active topics of research [8],

which shows the growing interest on testing fundamental symmetries using weak decays [9]. Couplings with weak gravitational fields are discussed in Ref. [10], and *CPT* violation without breaking Lorentz invariance is presented in Ref. [11].

The general framework that incorporates operators that break Lorentz invariance in the Standard Model (SM) is the SM Extension (SME) [12]. This effective field theory parametrizes generic deviations from Lorentz invariance in the form of coordinate-invariant terms contracting operators of conventional fields with controlling coefficients for Lorentz violation. This construction guarantees invariance of the theory under observer transformations, whereas particle Lorentz symmetry is broken. It should be noted that a subset of operators in the SME also break *CPT* symmetry [13]. The development of the SME has led to a worldwide experimental program searching for violations of Lorentz invariance, whose results are summarized in Ref. [14].

The study of the neutrino sector of the SME [15] has characterized the high sensitivity of neutrino-oscillation experiments. The development of methodologies to perform systematic searches for Lorentz violation in these experiments [16,17] has motivated several experimental searches using neutrinos and antineutrinos [18–23]. Additionally, the SME has been used to construct alternative models for neutrino oscillations that can accommodate the established data and also some of the anomalous results reported by different experiments [24]. Some of these models based on the SME offer elegant and interesting solutions to neutrino anomalies. More recently, the observation of very-high-energy neutrinos [25] has served to determine stringent constraints on *CPT*-even SME coefficients [26].

Even though the interferometric nature of neutrino oscillations makes them sensitive tools to search for new physics, the study of weak decays offers access to some operators that are unobservable using neutrino mixing. Operators of arbitrary dimension in the theory can be studied using the methods introduced in Ref. [27].

Nonetheless, of particular interest are those whose observable effects escape detection through sensitive measurements such as neutrino oscillations and time of flight. These so-called *countershaded* effects [7,28] arise due to oscillation-free operators of mass dimension three, whose effects are controlled by the four independent components of the coefficient denoted $(a_{of}^{(3)})^\alpha$ [27].

II. TWO-NEUTRINO DOUBLE BETA DECAY

The unconventional spinor solutions of the modified Dirac equation and the form of the neutrino phase space produce observable effects recently studied in the context of tritium decay, neutron decay, and double beta decay [7]. In this section, we describe a detailed presentation of the relevant experimental signature in the two-neutrino mode of double beta decay.

Denoting the 4-momentum of the two electrons and the two antineutrinos by $p_j^\alpha = (E_j, \boldsymbol{p}_j)$ and $q_j^\alpha = (\omega_j, \boldsymbol{q}_j)$, respectively ($j = 1, 2$), the relevant matrix element for the two-neutrino mode of double beta decay is given by

$$i\mathcal{M} = iG_F^2 V_{ud}^2 [\bar{u}(p_1)\gamma^\mu(1-\gamma_5)v(q_1)]$$
$$\times [\bar{u}(p_2)\gamma^\nu(1-\gamma_5)v(q_2)]J_{\mu\nu} - (p_1 \leftrightarrow p_2). \quad (1)$$

The hadronic tensor $J_{\mu\nu}$ corresponds to the product of two nuclear currents written in the impulse approximation [4]. Following the same procedure as in the conventional two-neutrino double beta decay, including the implementation of the long-wave and closure approximation for the hadronic tensor [4], we obtain

$$\sum_{spin} |\mathcal{M}|^2 = 64 G_F^4 |V_{ud}|^4 g_A^4 (p_1 \cdot p_2)(\tilde{q}_1 \cdot \tilde{q}_2)|M^{2\nu}|^2, \quad (2)$$

where the nuclear matrix element involves vector and axial couplings for Fermi and Gamow-Teller transitions in the form $g_A^2 M^{2\nu} = g_V^2 M_F^{2\nu} - g_A^2 M_{GT}^{2\nu}$ [4]. The two antineutrinos appear with an effective 4-momentum $\tilde{q}^\alpha = (\omega, \boldsymbol{q} + \boldsymbol{a}_{of}^{(3)} - \overset{\circ}{a}_{of}^{(3)}\hat{\boldsymbol{q}})$, where $\overset{\circ}{a}_{of}^{(3)}$ corresponds to the isotropic component of $(a_{of}^{(3)})^\alpha$. The decay rate is given by

$$d\Gamma = \frac{1}{4} \int \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} \frac{d^3 q_1}{(2\pi)^3 2\omega_1} \frac{d^3 q_2}{(2\pi)^3 2\omega_2}$$
$$\times F(Z, E_1)F(Z, E_2) \sum |\mathcal{M}|^2$$
$$\times 2\pi\delta(E_1 + E_2 + \omega_1 + \omega_2 - \Delta M), \quad (3)$$

where we have included the Fermi function to account for the Coulomb interaction of the two emitted electrons and the daughter nucleus of atomic number Z and the symmetry factor for the two pairs of outgoing leptons. Since the two antineutrinos are not measured in these types of experiments, the integration over all orientations leaves only the isotropic coefficient $\overset{\circ}{a}_{of}^{(3)}$, and the phase space takes the

form $d^3 q = 4\pi(\omega^2 + 2\omega \overset{\circ}{a}_{of}^{(3)})d\omega$. After a suitable change of integration variables and defining the sum of kinetic energies $K = T_1 + T_2$ for the two electrons, we obtain the electron sum spectrum

$$\frac{d\Gamma}{dK} = C(K^4 + 10K^3 + 40K^2 + 60K + 30)K$$
$$\times [(K_0 - K)^5 + 10\overset{\circ}{a}_{of}^{(3)}(K_0 - K)^4], \quad (4)$$

where K_0 is the maximum kinetic energy available in the decay. We have written the energy in units of the electron mass and used the Primakoff-Rosen limit for nonrelativistic electrons [29–33]. The overall constant factor is given by $C = G_F^4 g_A^4 |V_{ud}|^4 |M^{2\nu}|^2 F_{PR}^2(Z)m_e^{11}/7200\pi^7$, with $F_{PR}(Z) = 2\pi\alpha Z/(1 - e^{-2\pi\alpha Z})$. We find that the isotropic coefficient $\overset{\circ}{a}_{of}^{(3)}$ produces a distortion of the conventional electron sum spectrum. A similar effect is found for studies of the spectrum of neutron decay [7]. Since the Lorentz-violating modification of the spectrum appears with a well-defined energy dependence, a search for deviations from the conventional spectrum would allow for studying the effects of the isotropic coefficient $\overset{\circ}{a}_{of}^{(3)}$. In particular, the energy dependence of the modification allows for determining the energy K_m at which the effect of this coefficient is maximal and hence gives the best chance to observe its effect. This is the energy at which the residual spectrum reaches its maximum. Table I shows the value of this energy for some double-beta-decay emitters commonly studied. It should be noticed that the coefficient $\overset{\circ}{a}_{of}^{(3)}$ also controls a source of CP violation in the neutrino sector that remains experimentally unexplored.

Experimental studies require the search for deviations from the conventional spectrum, and the energy listed in Table I for a given element should serve as a guide indicating the region of the spectrum of highest sensitivity. We have also found that the anisotropic components of $(a_{of}^{(3)})^\alpha$ are unobservable in this kind of experiment; therefore, studies of neutron and tritium decay appear as important complementary techniques [7].

TABLE I. Sum of the kinetic energy of the two electrons at which the Lorentz-violating modification introduced by the coefficient $\overset{\circ}{a}_{of}^{(3)}$ is maximal.

Isotope	K_m (MeV)
^{48}Ca	1.98
^{76}Ge	0.81
^{82}Se	1.30
^{96}Zr	1.48
^{100}Mo	1.32
^{116}Cd	1.20
^{130}Te	1.05
^{136}Xe	1.02
^{150}Nd	1.49

III. NEUTRINOLESS DOUBLE BETA DECAY

The presence of Lorentz violation in the neutrino sector modifies both the neutrino dispersion relation and propagator. This means that Lorentz-violating effects arise in two independent manners involving different types of coefficients. As discussed before, the modified neutrino dispersion relation alters the phase space, which for the neutrinoless mode introduces a modification to the so-called neutrino potential [4]. It can be shown that current limits on the lifetime of the neutrinoless mode of different isotopes are less sensitive to the relevant SME coefficients than tritium decay experiments [7]. Additionally, the effects of Lorentz violation in the neutrino potential are weighted with the conventional nuclear matrix elements, which having magnitudes $\mathcal{O}(1)$ are unable to make these types of Lorentz-violating effects more noticeable. For these reasons, we neglect the Lorentz-violating modifications of the nuclear matrix elements introduced by the coefficient $\mathring{a}_{\text{of}}^{(3)}$ and study in detail the effects of Lorentz-violating Majorana couplings that arise in the neutrino propagator.

Using the SME Lagrangian [27] and neglecting Dirac couplings, the relevant propagator can be written at leading order as

$$S(q) = \frac{1}{q^2} \left[\slashed{q} - e_{Ma'\bar{a}'}^{(4)\lambda} q_\lambda - i f_{Ma'\bar{a}'}^{(4)\lambda} q_\lambda \gamma_5 \right.$$
$$\left. - \frac{1}{2} g_{Ma'\bar{a}'}^{(4)\lambda\rho\sigma} \sigma_{\lambda\rho} q_\sigma - \frac{1}{2} H_{Ma'\bar{a}'}^{(3)\lambda\rho} \sigma_{\lambda\rho} \right], \qquad (5)$$

where the coefficients $e_{Ma'\bar{a}'}^{(4)\lambda}$, $f_{Ma'\bar{a}'}^{(4)\lambda}$, $g_{Ma'\bar{a}'}^{(4)\lambda\rho\sigma}$, and $H_{Ma'\bar{a}'}^{(3)\lambda\rho}$ are Majorana couplings in the neutrino sector of the SME. The indices $a'\bar{a}'$ indicate that the coefficients are written in the basis of neutrino eigenstates, with $a' = 1, 2, 3$. The bar over the second index reveals the Majorana nature of these coefficients by connecting neutrino and antineutrino states. The numbers in parentheses denote the mass dimension of the associated operator. We have neglected the contribution from a possible Majorana mass in the propagator because we are interested in a pure Lorentz-violating mechanism for neutrinoless double beta decay. Inspection shows that the coefficient $g_{Ma'\bar{a}'}^{(4)\lambda\rho\sigma}$ is the only one in the propagator (5) that couples to an operator that preserves charge conjugation. This suggests that $g_{Ma'\bar{a}'}^{(4)\lambda\rho\sigma}$ is the only relevant coefficient in this decay mode because neutrinoless double beta decay requires the neutrino and the antineutrino to be the same particle.

Direct calculation shows that due to the Dirac-matrix structure, the transition matrix generated by the scalar $e_{Ma'\bar{a}'}^{(4)\lambda}$ and pseudoscalar $f_{Ma'\bar{a}'}^{(4)\lambda}$ couplings as well as the tensor coupling $H_{Ma'\bar{a}'}^{(3)\lambda\rho}$ is symmetric under the interchange of the two emitted electrons. For this reason, the total transition matrix vanishes due to its antisymmetry under the interchange of identical fermions. In contrast, the coefficient $g_{Ma'\bar{a}'}^{(4)\lambda\rho\sigma}$ produces a transition matrix that is antisymmetric under the interchange of the two electrons. This result verifies that Lorentz-violating neutrinoless double beta decay only depends on the coefficient $g_{Ma'\bar{a}'}^{(4)\lambda\rho\sigma}$, which also controls *CPT* violation. The result above can be shown to be valid for operators of arbitrary dimension; nevertheless, in this work we only describe the effects of operators of the lowest dimension, in this case $d = 4$ [27].

Integration over the neutrino energy shows that the effects of $g_{Ma'\bar{a}'}^{(4)\lambda\rho 0}$ are unobservable; hence, only $g_{Ma'\bar{a}'}^{(4)\lambda\rho k}$ is relevant. The Dirac matrix $\sigma_{\lambda\rho}$ introduces a coupling between the components of the coefficient $g_{Ma'\bar{a}'}^{(4)\lambda\rho k}$ and the phase space of the two emitted electrons, which leads to unique electron angular correlations. Other unconventional terms appear due to the coupling between $g_{Ma'\bar{a}'}^{(4)\lambda\rho k}$ and the components of the nuclear currents. It can be shown that the Dirac-matrix structure can be written as the sum of symmetric, antisymmetric, and mixed products of the Dirac matrices. The symmetric piece leads to the conventional Fermi and Gamow-Teller nuclear matrix elements, while the antisymmetric part produces no effects because the product of the nuclear currents is symmetric [4]. The mixed term leads to unconventional forms for the nuclear matrix elements. For a detailed study, a careful analysis of the nuclear matrix elements coupled to the $g_{Ma'\bar{a}'}^{(4)\lambda\rho k}$ coefficient is needed, which goes beyond the scope of this work. In what follows we will focus on the effects arising from the coupling between $g_{Ma'\bar{a}'}^{(4)\lambda\rho k}$ and the phase space of the two outgoing electrons, in which case the nuclear matrix elements remain unchanged.

The implementation of the results discussed above leads to the transition amplitude

$$i\mathcal{M} = iG_F^2 V_{ud}^2 \langle g_M^{\lambda\rho k}\rangle \hat{r}_k \bar{u}(p_1)\gamma^\mu \sigma_{\lambda\rho}\gamma^\nu(1+\gamma_5)u^C(p_2)$$
$$\times \frac{H(\boldsymbol{r},\Delta)J_{\mu\nu}}{4\pi R^2}, \qquad (6)$$

where $H(\boldsymbol{r},\Delta)$ is the conventional neutrino potential from the integration over the neutrino momentum [4], the nuclear radius is given by $R = 6.1 \times A^{1/3}$ GeV^{-1}, and $\hat{r}$ is the direction of emission of the virtual neutrino. The coefficients for Lorentz violation appear weighted by the electron elements of the Pontecorvo-Maki-Nakagawa-Sakata matrix in the form of an effective coefficient

$$\langle g_{\beta\beta}^{\lambda\rho k}\rangle = \sum_{a'} U_{a'e}^2 g_{Ma'\bar{a}'}^{(4)\lambda\rho k}, \qquad (7)$$

similar to the conventional effective Majorana-mass parameter $\langle m_{\beta\beta}\rangle$ [4]. From direct calculation we obtain

$$\sum_{\text{spin}} |\mathcal{M}|^2 = \frac{G_F^4 |V_{ud}|^4 g_A^4}{8\pi^2 R^4} |M^{0\nu}|^2 |g_{\beta\beta}^{\lambda\rho}|^2 [(p_1 \cdot p_2)\eta^{\lambda\lambda}\eta^{\rho\rho}$$
$$- 2p_1^\lambda p_2^\lambda \eta^{\rho\rho} - 2p_1^\rho p_2^\rho \eta^{\lambda\lambda}], \qquad (8)$$

where $M^{0\nu}$ is the conventional nuclear matrix element for neutrinoless double beta decay [4]. In this expression we have written the vector $\hat{r}$ in the Sun-centered frame [34] and averaged over all possible orientations, which produces the effective coefficient

$$|g_{\beta\beta}^{\lambda\rho}|^2 = \frac{1}{3}(|\langle g_{\beta\beta}^{\lambda\rho X}\rangle|^2 + |\langle g_{\beta\beta}^{\lambda\rho Y}\rangle|^2 + |\langle g_{\beta\beta}^{\lambda\rho Z}\rangle|^2). \qquad (9)$$

It must be noted that in the transition amplitude (8) there is no sum over repeated indices. Instead, this expression is valid individually for the six possible pairs of indices $\lambda\rho$. The decay rate is given by

$$d\Gamma = \frac{1}{2}\int \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} F(Z, E_1)F(Z, E_2)$$
$$\times \sum |\mathcal{M}|^2 2\pi\delta(E_1 + E_2 + \Delta M), \qquad (10)$$

where Lorentz-violating effects only appear at the level of the transition amplitude because the physics of the two outgoing electrons is conventional.

A. Angular correlations

For experiments with tracking systems that allow the determination of the direction of the two emitted electrons, it is useful to identify the way the angular correlation of the electrons gets modified. In the Sun-centered frame [34] the momentum of each electron satisfies

$$\frac{\boldsymbol{p}}{E} = \beta \begin{pmatrix} \cos\omega_\oplus T_\oplus & -\sin\omega_\oplus T_\oplus & 0 \\ \sin\omega_\oplus T_\oplus & \cos\omega_\oplus T_\oplus & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} N^X \\ N^Y \\ N^Z \end{pmatrix}, \qquad (11)$$

where β is the speed of the electron and $\omega_\oplus \simeq 2\pi/(23\,\text{h}\,56\,\text{min})$ is the sidereal frequency that accounts for the rotation of the Earth. The directional factors N^X, N^Y, and N^Z indicate the direction of each electron in the Sun-centered frame during the 2000 vernal equinox, which defines $T_\oplus = 0$ [34]. Using polar coordinates in the laboratory frame with the z axis directed towards the zenith, the x axis pointing south, and the y axis pointing east, the directional factors are functions of (θ, ϕ) and the colatitude of the laboratory χ [17]. Since we will integrate over the azimuthal angles ϕ_j, we only keep the elements with azimuthal symmetry, in which case the momentum components of each electron ($j = 1, 2$) become

$$p_j^X = \cos\omega_\oplus T_\oplus \sin\chi \cos\theta_j,$$
$$p_j^Y = \sin\omega_\oplus T_\oplus \sin\chi \cos\theta_j,$$
$$p_j^Z = \cos\chi \cos\theta_j. \qquad (12)$$

Using these expressions, we can write the angular distribution as

$$\rho_{\lambda\rho}(\boldsymbol{p}_1, \boldsymbol{p}_2) = 1 - k_{\lambda\rho}\beta_1\beta_2 \cos\theta_1 \cos\theta_2, \qquad (13)$$

with the factor $k_{\lambda\rho}$ given for each component in the form

$$k_{TX} = k_{YZ} = 2\sin^2\chi\cos^2\omega_\oplus T_\oplus - 1,$$
$$k_{TY} = k_{XZ} = 2\sin^2\chi\sin^2\omega_\oplus T_\oplus - 1,$$
$$k_{TZ} = k_{XY} = \cos 2\chi. \qquad (14)$$

We have found that all the factors $k_{\lambda\rho}$ depend on the location of the experiment, and four of them change with sidereal time. For comparison, in neutrinoless double beta decay triggered by a Majorana mass, this factor is $k = 1$ [4,35]. In order to construct a useful observable, we define the quantity

$$K_{\lambda\rho} = k_{\lambda\rho}k_\theta(Z), \qquad (15)$$

which includes the factors $k_{\lambda\rho}$ obtained above and also the parameter $k_\theta(Z)$ that accounts for the integration over the electron energies and depends on the isotope used through the atomic number Z and the Q value. These parameters must be determined by numerical integration of the decay rate (10) over the allowed electron energies. Table II shows the value of this parameter for some double beta decays. The definition of factor (15) can be used to write

$$\frac{d\Gamma}{dx_1 dx_2} = \frac{\Gamma}{4}(1 - K_{\lambda\rho}x_1 x_2), \qquad (16)$$

with Γ the total decay width and $x_j = \cos\theta_j$. In the absence of Lorentz violation, the local coordinates can be taken with any orientation. In our case, we do not have this freedom because the coordinates of the laboratory

TABLE II. Relevant parameters for some double beta decays.

Process	k_θ
$^{48}\text{Ca} \rightarrow {}^{48}\text{Ti}$	0.93
$^{76}\text{Ge} \rightarrow {}^{76}\text{Se}$	0.81
$^{82}\text{Se} \rightarrow {}^{82}\text{Kr}$	0.88
$^{96}\text{Zr} \rightarrow {}^{96}\text{Mo}$	0.90
$^{100}\text{Mo} \rightarrow {}^{100}\text{Ru}$	0.88
$^{116}\text{Cd} \rightarrow {}^{116}\text{Sn}$	0.87
$^{136}\text{Xe} \rightarrow {}^{136}\text{Ba}$	0.84
$^{130}\text{Te} \rightarrow {}^{130}\text{Xe}$	0.85
$^{150}\text{Nd} \rightarrow {}^{150}\text{Sm}$	0.89

frame are defined according to the rotations used to write the two electron momenta in the Sun-centered frame (12); therefore, the integration over the two electron orientations must be performed carefully. Let us define the number of events N^- (N^+) emitted with a relative angle smaller (greater) than $90°$ in the form

$$N^- = \int_{x_{\min}}^{x_{\max}} dx_1 \int_{-\sqrt{1-x_1^2}}^{x_1} dx_2 \left(\frac{d\Gamma}{dx_1 dx_2}\right),$$

$$N^+ = \int_{x_{\min}}^{x_{\max}} dx_1 \int_{-x_1}^{-\sqrt{1-x_1^2}} dx_2 \left(\frac{d\Gamma}{dx_1 dx_2}\right). \quad (17)$$

The forward-backward asymmetry of the decay distribution can be constructed by properly choosing the range $[x_{\min}, x_{\max}]$. Given the form of the angular distribution, if the integration range is too symmetric then the terms of interest cancel. For this reason, we take the range $[-\frac{1}{\sqrt{2}}, 1]$, which gives the asymmetry

$$\mathcal{A} = \frac{N^+ - N^-}{N^+ + N^-} = \frac{K_{\lambda\rho}}{4} - \frac{3\pi}{2} - 1. \quad (18)$$

This asymmetry corresponds to the counting of all the events between $\theta = 0°$ measured from the vertical at the laboratory frame a nd $\theta = 135°$. Notice that the above asymmetry depends on the location of the experiment and can also oscillate with sidereal time, in which case the amplitude of the oscillation is independent of the size of the coefficient for Lorentz violation. The asymmetry in the conventional Majorana-mass driven decay is constant and only depends on the element used in the form $\mathcal{A}_0(Z) = k_\theta(Z)/2$. We find that the asymmetry defined above allows a clear separation of the effects due to $|g_{\beta\beta}^{\lambda\rho}|$ from the conventional neutrinoless double beta decay triggered by a Majorana-mass parameter.

B. Half-life measurements

The complete integration of the decay rate (10) allows us to identify the signature of the effective coefficients $|g_{\beta\beta}^{\lambda\rho}|$ in the half-life of neutrinoless double beta decay. The decay constant can be written in conventional form as the product of a phase-space factor $G^{0\nu}(Z, Q)$ that depends on the double beta emitter, the nuclear matrix element, and a particle physics quantity in the form

$$(T_{1/2}^{0\nu})^{-1} = \Gamma = G^{0\nu}(Z, Q)|M^{0\nu}|^2 \frac{|g_{\beta\beta}^{\lambda\rho}|^2}{4R^2}. \quad (19)$$

Comparing this expression with the conventional form of the decay constant [4], we find that effective coefficients for *CPT*-odd Lorentz violation $|g_{\beta\beta}^{\lambda\rho}|$ play the role of the Majorana mass parameter $|\langle m_{\beta\beta}\rangle|$ in the form [7]

TABLE III. Conservative upper limits on the effective coefficients $|g_{\beta\beta}^{\lambda\rho}|$ from the corresponding upper bound on the effective Majorana mass at the 90% C.L. in different experiments. Lower rows show attainable limits based on the expected sensitivities in future experiments.

Process	Estimated upper limit	Experiment	Reference
^{48}Ca $\to$ ^{48}Ti	1×10^{-8}	IGEX	[36]
^{48}Ca $\to$ ^{48}Ti	4×10^{-7}	ELEGANT VI	[37]
^{48}Ca $\to$ ^{48}Ti	3×10^{-7}	NEMO-3	[38]
^{76}Ge $\to$ ^{76}Se	4×10^{-9}	GERDA	[39]
^{82}Se $\to$ ^{82}Kr	3×10^{-8}	NEMO-3	[40]
^{96}Zr $\to$ ^{96}Mo	2×10^{-7}	NEMO-3	[41]
^{100}Mo $\to$ ^{100}Ru	1×10^{-8}	NEMO-3	[40]
^{116}Cd $\to$ ^{116}Sn	2×10^{-8}	Solotvina	[42]
^{130}Te $\to$ ^{130}Xe	9×10^{-9}	CUORICINO	[43]
^{136}Xe $\to$ ^{136}Ba	5×10^{-9}	EXO-200	[44]
^{136}Xe $\to$ ^{136}Ba	3×10^{-9}	KamLAND-Zen	[45]
^{150}Nd $\to$ ^{150}Sm	8×10^{-8}	NEMO-3	[46]
^{48}Ca $\to$ ^{48}Ti	4×10^{-9}	CANDLES	[47]
^{76}Ge $\to$ ^{76}Se	4×10^{-10}	GERDA Phase II	[39]
^{76}Ge $\to$ ^{76}Se	4×10^{-10}	MAJORANA	[48]
^{82}Se $\to$ ^{82}Kr	6×10^{-9}	SuperNEMO	[40]
^{130}Te $\to$ ^{130}Xe	5×10^{-9}	CUORE-0	[49]
^{130}Te $\to$ ^{130}Xe	1×10^{-9}	CUORE	[50]
^{130}Te $\to$ ^{130}Xe	8×10^{-10}	SNO+	[51]
^{136}Xe $\to$ ^{136}Ba	4×10^{-10}	nEXO	[52]
^{136}Xe $\to$ ^{136}Ba	1×10^{-9}	NEXT	[53]

$$|\langle m_{\beta\beta}\rangle| \to \frac{|g_{\beta\beta}^{\lambda\rho}|}{2R}. \quad (20)$$

Since neutrinoless double beta decay remains unobserved to date, limits on the half-life of this decay mode in different isotopes can be used to set upper bounds on the effective coefficients $|g_{\beta\beta}^{\lambda\rho}|$. Table III presents the estimated limits on these coefficients based on published results by different experiments as well as the projected sensitivity of several experiments under construction or in the process of upgrade.

The limits presented in Table III are conservative in the sense that the lowest value of the corresponding nuclear matrix element has been used. Limits 2–3 times better can be obtained when using the largest value of the nuclear matrix elements. Since the definition of $|g_{\beta\beta}^{\lambda\rho}|$ involves the sum of three positive quantities, we can take each of the components $|\langle g_{\beta\beta}^{\lambda\rho K}\rangle|$ in definition (9) to be nonzero at the time. From the values in Table III, we can write 90% C.L. upper limits in 18 effective coefficients in the form

$$|\langle g_{\beta\beta}^{\lambda\rho K}\rangle| < 5 \times 10^{-9}, \quad (21)$$

with $\lambda\rho = TX, TY, TZ, XY, XZ, YZ$, and $K = X, Y, Z$. The definition of the effective coefficients $\langle g_{\beta\beta}^{\lambda\rho K}\rangle$ shows that

they are linear combinations of the eigenvalues of the original coefficients $g_M^{(4)\lambda\rho\sigma}$ in the action. It should be noticed that different linear combinations of these original coefficients can trigger neutrino-antineutrino oscillations [15,17], whose effects have recently been studied using accelerator neutrinos [54] and reactor antineutrinos [55]. The interferometric nature of oscillations between neutrinos and antineutrinos [17] makes them sensitive probes of the Majorana couplings in the SME, which appear as combinations of the original coefficients in the form $\tilde{g}_{a\bar{b}}^{\nu\sigma} = g_{a\bar{b}}^{T\nu\sigma} + \frac{i}{2}\varepsilon^{T\nu}{}_{\gamma\rho}g_{a\bar{b}}^{\gamma\rho\sigma}$ $(a = e, \mu, \tau, \;\; \bar{b} = \bar{e}, \bar{\mu}, \bar{\tau})$ [15]. The experimental signature in neutrinoless double beta decay allows for constraining independent combinations of these coefficients [56], which provides complementary tests of Lorentz invariance.

Even though neutrinoless double beta decay can access different combinations of coefficients from those of oscillations, under mild assumptions these combinations can be related. Below we present the relationships between the effective coefficient $|g_{\beta\beta}^{\lambda\rho}|$ observable in neutrinoless double beta decay and some components of the coefficients measured in oscillations, which are obtained by keeping a single coefficient while setting the others to zero [57]. This procedure is widely used in the literature, and it could hide some effects due to fortuitous cancellations between different coefficients; nonetheless, this method provides meaningful information for comparison with other experiments. The relations between coefficients are

$$|\tilde{g}_{e\bar{e}}^{JK}| < 3.9|\langle g_{\beta\beta}^{TJ}\rangle|, \qquad |\mathrm{Re}\tilde{g}_{e\bar{\mu}}^{JK}| < 3.2|\langle g_{\beta\beta}^{TJ}\rangle|,$$
$$|\tilde{g}_{\mu\bar{\mu}}^{JK}| < 10.8|\langle g_{\beta\beta}^{TJ}\rangle|, \qquad |\mathrm{Re}\tilde{g}_{e\bar{\tau}}^{JK}| < 5.3|\langle g_{\beta\beta}^{TJ}\rangle|,$$
$$|\tilde{g}_{\tau\bar{\tau}}^{JK}| < 28.7|\langle g_{\beta\beta}^{TJ}\rangle|, \qquad |\mathrm{Re}\tilde{g}_{\mu\bar{\tau}}^{JK}| < 8.9|\langle g_{\beta\beta}^{TJ}\rangle|, \qquad (22)$$

with $J, K = X, Y, Z$. The constraint (21) leads to the following limits for 54 coefficients in flavor space:

$$|\tilde{g}_{e\bar{e}}^{JK}| < 1 \times 10^{-8}, \qquad |\mathrm{Re}\tilde{g}_{e\bar{\mu}}^{JK}| < 1 \times 10^{-8},$$
$$|\tilde{g}_{\mu\bar{\mu}}^{JK}| < 3 \times 10^{-8}, \qquad |\mathrm{Re}\tilde{g}_{e\bar{\tau}}^{JK}| < 2 \times 10^{-8},$$
$$|\tilde{g}_{\tau\bar{\tau}}^{JK}| < 9 \times 10^{-8}, \qquad |\mathrm{Re}\tilde{g}_{\mu\bar{\tau}}^{JK}| < 3 \times 10^{-8}. \qquad (23)$$

IV. CONCLUSION

In this work, we have presented the effects of deviations from exact Lorentz invariance in experiments studying double beta decay in the context of the Standard-Model Extension. Observable signatures include a modification of the electron sum spectrum in the two-neutrino mode of the decay, which can be studied by searching for departures from the conventional spectrum. The countershaded coefficient responsible for this effect controls Lorentz and *CPT* violation in the neutrino sector and also controls a new source of *CP* violation. We also find that neutrinoless double beta decay could occur even if the Majorana neutrino mass is negligible; nevertheless, interference between the mass mechanism and the Lorentz-violating effect could also appear [7,58]. The corresponding angular correlations for the two emitted electrons have been determined for the relevant SME coefficients. Although there are other mechanisms that can trigger neutrinoless double beta decay without the conventional Majorana mass [4,59], the one presented here does not require new particles or forces.

In addition to limits on the Majorana mass parameter, the identification (20) will allow experiments to derive upper limits on coefficients for Lorentz violation from the lower bounds on the lifetime of neutrinoless double beta decay. Notice that if neutrinoless double beta decay is observed in the future, the unique Lorentz-violating signature that would allow us to separate the effects from a Majorana-mass mechanism is the dependence of the particle physics parameter in the half-life (19) on the nuclear radius R of the isotope used.

The absence of compelling positive signals of neutrinoless double beta decay in numerous experiments is used to determine the first limits on some components of the relevant SME coefficient up to the 10^{-9} level. The expected sensitivity of upcoming experiments could improve these limits by at least 1 order of magnitude.

ACKNOWLEDGMENTS

[1] E. Fermi, Z. Phys. **88**, 161 (1934).

[2] M. Goeppert-Mayer, Phys. Rev. **48**, 512 (1935).

[3] J. Beringer *et al.* (Particle Data Group), Phys. Rev. D **86**, 010001 (2012).

[4] For reviews see S. R. Elliott and P. Vogel, Annu. Rev. Nucl. Part. Sci. **52**, 115 (2002); F. T. Avignone, III, S. Elliott, and J. Engel, Rev. Mod. Phys. **80**, 481 (2008); S. M. Bilenky, Phys. Part. Nucl. **41**, 690 (2010); W. Rodejohann, Int. J. Mod. Phys. E **20**, 1833 (2011).

[5] E. Majorana, Nuovo Cimento **14**, 171 (1937); G. Racah, Nuovo Cimento **14**, 322 (1937); W. H. Furry, Phys. Rev. **56**, 1184 (1939).

[6] V. A. Kostelecký and S. Samuel, Phys. Rev. D **39**, 683 (1989); V. A. Kostelecký and R. Potting, Nucl.

Phys. **B359**, 545 (1991); Phys. Rev. D **51**, 3923 (1995).

[7] J. S. Díaz, V. A. Kostelecký, and R. Lehnert, Phys. Rev. D **88**, 071902(R) (2013).

[8] J. P. Noordmans, H. W. Wilschut, and R. G. E. Timmermans, Phys. Rev. C **87**, 055502 (2013); Phys. Rev. Lett. **111**, 171601 (2013); H. W. Wilschut, H. Müller, and N. Severijns, Ann. Phys. (Berlin) **525**, 653 (2013); S. E. Müller *et al.*, Phys. Rev. D **88**, 071901(R) (2013); B. Altschul, Phys. Rev. D **88**, 076015 (2013).

[9] N. Severijns and O. Naviliat-Cuncic, Phys. Scr. **T152**, 014018 (2013).

[10] H. V. Klapdor-Kleingrothaus, H. Päs, and U. Sarkar, Eur. Phys. J. A **5**, 3 (1999).

[11] G. Barenboim, J. F. Beacom, L. Borissov, and B. Kayser, Phys. Lett. B **537**, 227 (2002).

[12] D. Colladay and V. A. Kostelecký, Phys. Rev. D **55**, 6760 (1997); Phys. Rev. D **58**, 116002 (1998); V. A. Kostelecký, Phys. Rev. D **69**, 105009 (2004).

[13] O. W. Greenberg, Phys. Rev. Lett. **89**, 231602 (2002); arXiv:1105.0927.

[14] V. A. Kostelecký and N. Russell, Rev. Mod. Phys. **83**, 11 (2011).

[15] V. A. Kostelecký and M. Mewes, Phys. Rev. D **69**, 016005 (2004).

[16] V. A. Kostelecký and M. Mewes, Phys. Rev. D **70**, 076002 (R) (2004).

[17] J. S. Díaz, V. A. Kostelecký, and M. Mewes, Phys. Rev. D **80**, 076007 (2009).

[18] Y. Abe *et al.* (Double Chooz Collaboration), Phys. Rev. D **86**, 112009 (2012).

[19] R. Abbasi *et al.* (IceCube Collaboration), Phys. Rev. D **82**, 112003 (2010).

[20] L. B. Auerbach *et al.* (LSND Collaboration), Phys. Rev. D **72**, 076004 (2005).

[21] A. A. Aguilar-Arevalo *et al.* (MiniBooNE Collaboration), Phys. Lett. B **718**, 1303 (2013); T. Katori (MiniBooNE Collaboration), Mod. Phys. Lett. **27A**, 1230024 (2012).

[22] P. Adamson *et al.* (MINOS Collaboration), Phys. Rev. Lett. **101**, 151601 (2008); (MINOS Collaboration)Phys. Rev. Lett.**105**, 151601 (2010); (MINOS Collaboration)Phys. Rev. D**85**, 031101 (2012).

[23] T. Akiri (Super-Kamiokande Collaboration), arXiv:1308.2210.

[24] V. A. Kostelecký and M. Mewes, Phys. Rev. D **70**, 031902 (2004); T. Katori, V. Alan Kostelecký, and R. Tayloe, Phys. Rev. D **74**, 105009 (2006); V. Barger, D. Marfatia, and K. Whisnant, Phys. Lett. B **653**, 267 (2007); V. Barger, J. Liao, D. Marfatia, and K. Whisnant, Phys. Rev. D **84**, 056014 (2011); J. S. Díaz and V. A. Kostelecký, Phys. Lett. B **700**, 25 (2011); Phys. Rev. D **85**, 016013 (2012); S.-J. Rong and Q.-Y. Liu, Chin. Phys. Lett. **29**, 041402 (2012).

[25] M. G. Aartsen *et al.* (IceCube Collaboration), Phys. Rev. Lett. **111**, 021103 (2013); M. G. Aartsen *et al.*, (IceCube Collaboration), Science **342**, 1242856 (2013).

[26] J. S. Díaz *et al.*, arXiv:1308.6344.

[27] V. A. Kostelecký and M. Mewes, Phys. Rev. D **85**, 096005 (2012); Phys. Rev. D **88**, 096006 (2013).

[28] V. A. Kostelecký and J. Tasson, Phys. Rev. Lett. **102**, 010402 (2009); V. A. Kostelecký and J. Tasson, Phys. Rev. D **83**, 016013 (2011).

[29] H. Primakoff and S. P. Rosen, Rep. Prog. Phys. **22**, 121 (1959).

[30] E. J. Konopinski, *Theory of Beta Radioactivity* (Oxford University Press, London, England, 1966).

[31] M. Doi, T. Kotani, H. Nishiura, K. Okuda, and E. Takasugi, Prog. Theor. Phys. **66**, 1739 (1981); M. Doi, T. Kotani, H. Nishiura, and E. Takasugi *ibid.* **69**, 602 (1983).

[32] W. C. Haxton and G. J. Stephenson, Jr., Prog. Part. Nucl. Phys. **12**, 409 (1984).

[33] T. Tomoda, Rep. Prog. Phys. **54**, 53 (1991).

[34] R. Bluhm, V. Kostelecký, C. Lane, and N. Russell, Phys. Rev. Lett.**88**,090801(2002);Phys. Rev. D**68**,125008(2003); V. A. Kostelecký and M. Mewes, Phys. Rev. D **66**, 056005 (2002).

[35] H. Primakoff, Phys. Rev. **85**, 888 (1952).

[36] C. E. Aalseth *et al.* (IGEX Collaboration), Phys. Rev. D **65**, 092007 (2002).

[37] I. Ogawa *et al.*, Nucl. Phys. **A730**, 215 (2004).

[38] R. L. Flack (NEMO-3 Collaboration), J. Phys. Conf. Ser. **136**, 022032 (2008).

[39] M. Agostini *et al.* (GERDA Collaboration), Phys. Rev. Lett. **111**, 122503 (2013).

[40] L. Simard (NEMO-3 and SuperNEMO Collaboration), Prog. Part. Nucl. Phys. **64**, 270 (2010).

[41] J. Argyriades *et al.* (NEMO-3 Collaboration), Nucl. Phys. **A847**, 168 (2010).

[42] F. A. Danevich *et al.*, Phys. Rev. C **78**, 035502 (2008).

[43] E. Andreotti *et al.* (CUORICINO Collaboration), Astropart. Phys. **34**, 822 (2011).

[44] M. Auger *et al.* (EXO Collaboration), Phys. Rev. Lett. **109**, 032505 (2012).

[45] A. Gando *et al.* (KamLAND-Zen Collaboration), Phys. Rev. Lett. **110**, 062502 (2013).

[46] J. Argyriades *et al.* (NEMO-3 Collaboration), Phys. Rev. C **80**, 032501 (2009).

[47] S. Umehara *et al.* (CANDLES Collaboration), J. Phys. Conf. Ser. **39**, 356 (2006).

[48] C. E. Aalsetha *et al.* (MAJORANA Collaboration), Nucl. Phys. B Proc. Suppl. **217**, 44 (2011).

[49] P. Gorla (CUORE Collaboration), arXiv:1109.0494.

[50] L. Pattavina (CUORE Collaboration), J. Phys. Conf. Ser. **447**, 012066 (2013).

[51] S. Biller (SNO+ Collaboration), presented at TAUP 2013, 3-8 September, Asilomar, CA, USA; O. Cremonesi and M. Pavan, arXiv:1310.4692.

[52] G. Gratta (EXO Collaboration), Intensity Frontier Whitepaper, Mar 2013.

[53] V. Alvarez *et al.* (NEXT Collaboration), arXiv:1106.3630.

[54] B. Rebel and S. Mufson, Astropart. Phys. **48**, 78 (2013).

[55] J. S. Díaz, T. Katori, J. Spitz, and J. M. Conrad, Phys. Lett. B **727**, 412 (2013).

[56] B. Altschul, Phys. Rev. D **79**, 061702 (2009); Y. Bonder, Phys. Rev. D **88**, 105011 (2013).

[57] The Pontecorvo-Maki-Nakagawa-Sakata matrix is used to diagonalize the 3×3 Majorana block at leading order. The values $\sin^2 2\theta_{12} = 0.855, \sin^2 2\theta_{13} = 0.099, \sin^2 2\theta_{23} = 1.000$ have been used.

[58] S. Pascoli, M. Mitra, and S. Wong, arXiv:1310.6218.

[59] G. Feinberg and M. Goldhaber, Proc. Natl. Acad. Sci. U.S.A. **45**, 1301 (1959); B. Pontecorvo, Phys. Lett. **26B**, 630 (1968).