

Using integral dispersion relations to extend the LHC reach for new physicsPeter B. Denton^{*} and Thomas J. Weiler[†]*Department of Physics and Astronomy, Vanderbilt University, Nashville, Tennessee 37235, USA*

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Many models of electroweak symmetry breaking predict new particles with masses at or just beyond LHC energies. Even if these particles are too massive to be produced on-shell at the LHC, it may be possible to see evidence of their existence through the use of integral dispersion relations (IDRs). Making use of Cauchy's integral formula and the analyticity of the scattering amplitude, IDRs are sensitive in principle to changes in the cross section at arbitrarily large energies. We investigate some models of new physics. We find that a sudden, order-one increase in the cross section above new particle mass thresholds can be inferred well below the threshold energy. On the other hand, for two more physical models of particle production, we show that the reach in energy and the signal strength of the IDR technique is greatly reduced. The peak sensitivity for the IDR technique is shown to occur when the new particle masses are near the machine energy, an energy where direct production of new particles is kinematically disallowed, phase-space suppressed, or, if applicable, suppressed by the soft parton distribution functions. Thus, IDRs do extend the reach of the LHC, but only to a window around $M_\chi \sim \sqrt{s_{\text{LHC}}}$.

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I. INTRODUCTION

Despite some predictions of a quick jump to new physics at the LHC [1], it seems distinctly possible that the next energy scale for new physics is out of the reach of direct observation (or does not manifest itself as missing energy) at the LHC. It is, however, still possible to constrain certain new physics models at energies beyond those accessible at the LHC, by using integral dispersion relations (IDRs) [2].

We briefly overview IDRs in Sec. II. In Sec. III we discuss the present state and future expectations of LHC experiments in relation to the IDR technique. We first solve these IDRs analytically in Sec. IV to understand their general behavior in certain limits. We state the present status of total cross section parametrizations in Sec. V, based on Standard Model (SM) assumptions. In Sec. VI, we model how new physics beyond the SM and beyond the direct reach of accelerators may increase the cross section. We consider one simple and two more physical model enhancements of the cross section. In Sec. VII, we discuss the reach of the IDR technique, as illustrated with the various new physics models. Section VII ends with our conclusions and a brief outlook to the future. Some details are presented in two appendices.

II. INTEGRAL DISPERSION RELATION THEORY

A brief introduction to elastic scattering and IDRs follows. For a more thorough introduction on elastic scattering see reference [3] and for IDRs see reference [2]. The mathematics behind IDRs is Cauchy's integral formula

$$f(z') = \frac{1}{2\pi i} \oint_{\partial A} \frac{f(z)}{z - z'} dz, \quad (1)$$

for an analytic function f . Here, the integration contour is around the boundary ∂A , $f(z)$ is analytic in the region A , and $z' \in A$. Next, we have the optical theorem

$$\sigma_{\text{tot}} = \frac{4\pi}{p} \Im f(\theta = 0), \quad (2)$$

which relates the total cross section σ_{tot} to the imaginary part $\Im f$ of the forward elastic scattering amplitude; p and θ are the center of mass (CoM) momentum and scattering angle respectively. Note that $\theta = 0$ for elastic scattering is the same as $t = 0$, where t is the usual Mandelstam variable for the square of the transfer energy. The ratio of the real to the imaginary parts of the forward scattering amplitude is conventionally given the symbol

$$\rho(E) = \frac{\Re f(E, t = 0)}{\Im f(E, t = 0)}, \quad (3)$$

where E is the laboratory energy for a fixed target experiment, related to the CoM energy squared (s) by $s = 2m_p E + 2m_p^2$. Next, we select a particular closed curve shown in Fig. 1 in the complex plane with $R \rightarrow \infty$. We then integrate \mathcal{F} , a complex valued function that is the analytic extension of the scattering amplitudes given by

$$f_{pp, p\bar{p}}(s, t = 0) = \lim_{\epsilon \rightarrow 0} \mathcal{F}(\pm(s + i\epsilon), t = 0) \quad (4)$$

around this contour. In order to deal with convergence as the linear portions go to infinity for the actual behavior of the pp , $p\bar{p}$ cross sections, one must also perform a subtraction

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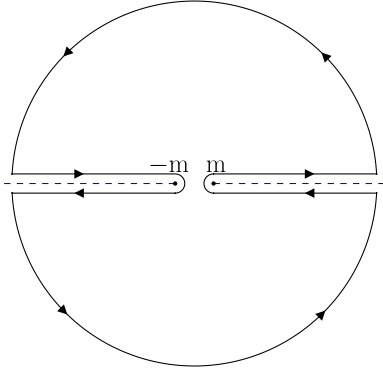


FIG. 1 (color online). The contour in the complex E plane for the integral of \mathcal{F} is shown. Note that the physical pp amplitude approaches the right-hand cut from above and the $p\bar{p}$ amplitude approaches the left-hand cut from below.

leading to an additional $f(E=0)$ constant. This gives the following equations for the forward scattering amplitudes $f_{pp}, f_{p\bar{p}}$ ($t=0$ now suppressed from our notation):

$$\Re f_{pp}(E) = \Re f_{pp}(0) + \frac{E}{4\pi^2} P \int_m^\infty dE' \frac{p'}{E'} \left[\frac{\sigma_{pp}(E')}{E' - E} - \frac{\sigma_{p\bar{p}}(E')}{E' + E} \right] \quad (5a)$$

$$\Re f_{p\bar{p}}(E) = \Re f_{p\bar{p}}(0) + \frac{E}{4\pi^2} P \int_m^\infty dE' \frac{p'}{E'} \left[\frac{\sigma_{p\bar{p}}(E')}{E' - E} - \frac{\sigma_{pp}(E')}{E' + E} \right], \quad (5b)$$

where σ is the total cross section, $(P \int)$ is the principal value integral, and $p' \equiv \sqrt{E'^2 - m_p^2}$. By the Pomeranchuk theorem [4] (and supported by QCD-parton considerations and experimental evidence) $\Delta\sigma = \sigma_{pp} - \sigma_{p\bar{p}} \rightarrow 0$ as $E \rightarrow \infty$, thus guaranteeing convergence of the integrals in Eqs. (5a), (5b); were these integrals still divergent, doubly subtracted IDRs would be necessary.

Finally, these formulas together with the optical theorem again give the useful IDRs

$$\rho_{pp}(E)\sigma_{pp}(E) = \frac{4\pi}{p} \Re f_{pp}(0) + \frac{E}{p\pi} P \int_m^\infty dE' \frac{p'}{E'} \left[\frac{\sigma_{pp}(E')}{E' - E} - \frac{\sigma_{p\bar{p}}(E')}{E' + E} \right], \quad (6a)$$

$$\rho_{p\bar{p}}(E)\sigma_{p\bar{p}}(E) = \frac{4\pi}{p} \Re f_{p\bar{p}}(0) + \frac{E}{p\pi} P \int_m^\infty dE' \frac{p'}{E'} \left[\frac{\sigma_{p\bar{p}}(E')}{E' - E} - \frac{\sigma_{pp}(E')}{E' + E} \right]. \quad (6b)$$

A built-in assumption from Eq. (1) is that the scattering amplitude is analytic. Based on the fact that current fits to the inelastic and total cross sections suggest that the proton asymptotically approaches a black disk, the evidence strongly suggests that the pp scattering amplitude is analytic [5].

In addition, the forward scattering amplitudes $f_{pp}(0) = f_{p\bar{p}}(0)$ are consistent with being purely imaginary at the nonphysical energy $E=0$ so the subtraction constant is ignorable [6].

In what follows, we exploit Eq. (6) to extend the reach of the LHC to energies beyond the LHC-equivalent fixed-target energy $E \equiv (s - 2m_p^2)/2m_p$. Formally, the IDRs relate contributions from new physics occurring all the way up in energy to infinity, to observables at present energies. In practice, the integral of the IDR falls off in energy away from the observation energy, so the reach beyond present energies will be limited.

The general strategy that we will use to explore new physics is to first use the IDR to calculate ρ at a particular energy (an LHC energy) without the inclusion of new physics. Then we calculate ρ at the same energy with the inclusion of the new physics cross section. Since ρ can be calculated without IDRs in a model-independent fashion, as briefly described in Appendix A, enhancements of the cross section can be either identified or ruled out by comparing theoretical and experimental values of $\rho(E)$.

III. EXPERIMENTAL STATUS

The total elastic and diffractive cross section measurement (TOTEM) [7] experiment at the LHC is designed to measure forward cross sections by probing very low $|t|$ regions. TOTEM places a series of Roman pot detectors very close to the beam and very far from the interaction point. With improved LHC optics, TOTEM should be able to provide an improved measurement of ρ independent of IDRs [8]. A comparison of TOTEM's ρ , so determined, with the IDR prediction of ρ , then provides the potential evidence for new physics.

Similarly, the absolute luminosity for the ATLAS (ALFA) [9] experiment, the LHC forward (LHCf)[10] experiment, along with a host of others will also make comparable measurements in an attempt to improve the precision of the luminosity calculation, which is necessary to infer σ_{tot} , and then to infer ρ without the use of IDRs [11] (see Appendix A below). Thus, there are several experiments that aim to measure the total cross section. These offer hope for smaller error bars on IDR-independent determinations of the crucial parameter $\rho(E)$.

A recent $\sqrt{s} = 7$ TeV TOTEM paper presented a state of the art value for the IDR-independent ρ , of $\rho = 0.145$ with error bars of $\sim 60\%$ [12]. TOTEM cited a 95% significance level (roughly speaking, a 2σ bound) that $\rho < 0.32$. Comparing this to the SM prediction of $\rho(\sqrt{s} = 7 \text{ TeV}) = 0.1345$ gives an upper limit of the fractional increase

$(\rho - \rho_{\text{SM}})/\rho_{\text{SM}} = 1.38$ at the 95% significance level. For brevity, in what follows we denote $(\rho - \rho_{\text{SM}})/\rho_{\text{SM}}$ as $\Delta\rho/\rho$. In addition, an early report from TOTEM gives their error estimate as ~ 0.04 from their $\sqrt{s} = 8$ TeV analysis [13] corresponding to a fractional error of 30%.

As an illustrative example of what a future determination of ρ might mean for the IDR technique, we investigate a definite value for ρ ; we choose as the definite value the experimentally-inferred mean value $\rho(\sqrt{s} = 7 \text{ TeV}) = 0.145$. For this example, the fractional increase in ρ is $\Delta\rho/\rho = 7.8\%$. This value for ρ is chosen for illustration only, as it offers insight into the merit of IDRs should experiments greatly reduce their errors in the inference of ρ .

Compared to the presently published TOTEM error of 63%, the “new physics increase” has negligible significance, $\sim 0.1\sigma$. With the ongoing analysis at TOTEM, the new significance is expected to be $\sim 0.25\sigma$ [13]. Clearly, further improvement in the inference of ρ is needed for the program constructed in this paper. Another reduction in error by about a factor of four (eight) would give a $1(2)\sigma$ significance to our illustrative example. One must hope that either the measured error is reduced significantly in the next LHC run, the new physics contribution to the cross section is larger than our chosen example, or both.

To get a feel for the reach and nature of this integral dispersion relation approach, we examine the integral under simplifying approximations in the next section. Then in the following sections, we examine the SM contribution to the IDRs and ρ , and the contributions from three constructed models of new physics.

IV. A SIMPLIFIED DISPERSION INTEGRAL TO SET EXPECTATIONS

In this section we make two assumptions to reduce the dispersion integrals in Eqs. (6a) and (6b) to a form that can be integrated analytically. While neither assumption is strictly valid, they are useful to reveal the gross features of the dispersion integral. The first assumption is to set m_p to zero. Besides replacing the lower limit of integration with zero, this assumption also sets p'/E' equal to one. The second assumption is to set σ_{pp} and $\sigma_{p\bar{p}}$ equal to each other, and to a constant which we call σ_0 . With these two assumptions, both dispersion integrals can be written as

$$(2\sigma_0)\mathcal{P} \int \frac{dx}{x^2 - 1} = (2\sigma_0) \log\left(\frac{|1-x|}{1+x}\right), \quad (7)$$

with $x \equiv E'/E$ and is valid everywhere except at $x = 1$, where the integral is singular. Blind evaluation of the definite integral over the range $[0, \infty]$ then gives zero. That this is correct can also be seen in the following way: By definition, the definite integral from Eq. (7) is

$$\lim_{\epsilon \rightarrow 0} \left[\int_0^{1-\epsilon} \frac{dx}{x^2 - 1} + \int_{1+\epsilon}^{\infty} \frac{dx}{x^2 - 1} \right]. \quad (8)$$

Replacing x by $u \equiv \frac{1}{x}$ in either integral, maps the integration region into that of the other integral, and reveals that the two integrals are equal but with opposite sign. Thus, the total integral vanishes. In particular, the singularity in the integrand vanishes in the principal value.

In Fig. 2, we plot the integrand $(x^2 - 1)^{-1}$ of our simplified dispersion integral. As the lower limit of integration x_{min} is moved up from zero, the cancellation above and below the singularity is no longer complete. However, the vanishing of the total integral when integrated from zero to infinity allows us replace the integration across the singularity with a simple, manifestly nonsingular integral as follows:

$$\mathcal{I}(x_{\text{min}}) \equiv \int_{x_{\text{min}}}^{\infty} \frac{dx}{x^2 - 1} = \int_0^{x_{\text{min}}} \frac{dx}{1 - x^2}. \quad (9)$$

For $x_{\text{min}} = 1$, the cancellation is maximally incomplete and the integral is infinite. We plot $\mathcal{I}(x_{\text{min}})$ in Fig. 3. As expected, the integral is everywhere positive, and diverges at $x_{\text{min}} = 1$. The divergence seems unphysical in that it corresponds to either $\Im f = 0 \Rightarrow \sigma_{\text{tot}} = 0$ by the optical theorem which shouldn't be the case or that $\Re f \rightarrow \infty \Rightarrow \sigma_{\text{tot}} \rightarrow \infty$ which is also unphysical. Since these particles have mass (which is ignored here), they have a finite lifetime and a finite width which would keep this integral finite at $x_{\text{min}} = 1$.

We may ask how the singularity is approached, from below and from above. Writing $x_{\text{min}} = 1 - \Delta$ and $1 + \Delta$, we have the two integrals $\int_{1-\Delta}^{\infty} \frac{dx}{x^2 - 1}$ and $\int_{1+\Delta}^{\infty} \frac{dx}{x^2 - 1}$. The first integral crosses the singularity and according to Eq. (9) is equal to the clearly finite integral $\int_0^{1-\Delta} \frac{dx}{1-x^2}$. With the replacement $x \rightarrow 1/x$, the second integral becomes $\int_0^{\frac{1}{1+\Delta}} \frac{dx}{1-x^2}$. Thus, the two integrations differ only in the upper limit of integration. At first order in Δ they are identical, as they must be to give a finite principal value integral. At higher order in Δ , the second integral exceeds

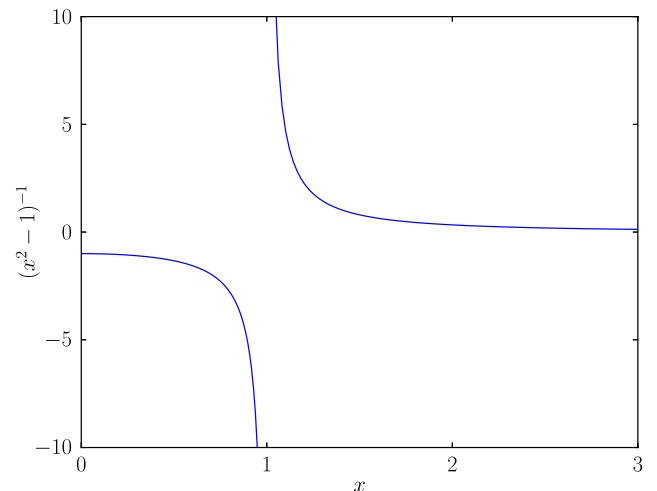


FIG. 2 (color online). The integrand of the IDRs with the $\sigma = \text{constant}$ and $m_p \rightarrow 0$ limits taken.

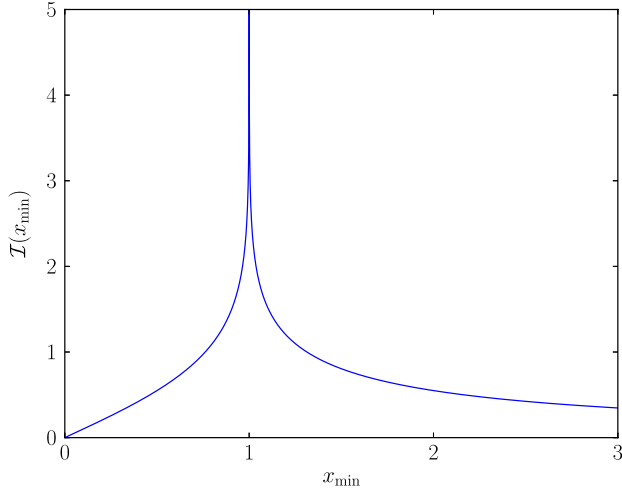


FIG. 3 (color online). The integral of the IDRs with the $\sigma = \text{constant}$ and $m_p \rightarrow 0$ limits taken. We see the expected singularity at $x_{\min} = 1$. For the new physics contribution, $x_{\min} = E_{\text{thr}}/E$.

the first integral. So we expect $\mathcal{I}(x_{\min})$ to show symmetry about the singular value $x_{\min} = 1$ for small deviations, but a larger value above $x_{\min} = 1$ than below for larger deviations. This expectation is visible in Fig. 3.

Why do we investigate x_{\min} values other than zero? When new physics enters at a threshold energy E_{thr} , the contribution of the new physics to the dispersion integral begins at $x_{\min} = \frac{E_{\text{thr}}}{E}$, where E is the energy of the accelerator. Thus, Fig. 3 gives the shape of the new physics contribution as a function of the new physics threshold. In what follows, our much more realistic parametrizations of new physics will present curves that qualitative have the form given by the simplistic model discussed in this section.

We may summarize this section by saying that the SM cross section is expected to give a modest contribution to the dispersion integral (zero in our simplistic model of constant and equal pp and $p\bar{p}$ cross sections with vanishing proton mass). On the other hand, new physics enters at a nonzero threshold which implies an incomplete cancellation in the dispersion integral, and thus a possibly significant contribution to the dispersion integral. Therefore, the ratio of new physics to total physics as revealed in the IDR potentially offers an observable window to new physics even with threshold energy above the direct reach of the LHC.

V. $\rho(E)$ IN THE STANDARD MODEL

Before venturing into speculative constructions of new physics contributions, we present the SM contribution.

In the real world, the exact cancellation of the integral presented in the previous section does not happen because our assumptions are (slightly) violated. In the real world, the proton mass m_p is not zero, and the pp and $p\bar{p}$ cross sections are neither equal nor constant in energy. Fits to data suggest that the cross sections decrease with E until

$E \sim 60$ GeV ($\sqrt{s} = 10.6$ GeV) and $E \sim 250$ GeV ($\sqrt{s} = 21.8$ GeV) for pp , $p\bar{p}$, respectively, before increasing. Furthermore, Froissart theory tells us that a $\log^2 s$ growth eventually dominates the energy dependence, a fact that has been confirmed with fits to present experimental data.

The SM total pp , $p\bar{p}$ cross section σ_{SM} is typically parametrized as

$$\begin{aligned} \sigma_{\text{SM}}(E) = & c_0 + c_1 \log\left(\frac{E}{m}\right) + c_2 \log^2\left(\frac{E}{m}\right) \\ & + c_3 \left(\frac{E}{m}\right)^{-\frac{1}{2}} \pm c_4 \left(\frac{E}{m}\right)^{\alpha-1}, \end{aligned} \quad (10)$$

where m_p , the proton mass, is used as the energy scale. c_i , α are fit parameters with $\alpha < 1$. The $E^{-\frac{1}{2}}$ term is a result of invoking Regge behavior. The upper sign refers to pp scattering and the lower to $p\bar{p}$ scattering. This form is motivated by being the most general and fastest rising form allowed by the Froissart bound. The values of the c_i and α from [6] are shown in Table I. The total pp cross section for the SM is included in Fig. 4 (solid line, labeled as the h_0 case).

Note that different fits to the pp , $p\bar{p}$ cross section do not substantively change the results of this paper. The current limits on the pp total cross section are predominately derived from data at and below the LHC. Fits to functions that behave differently than $\log^2(s)$ such as $\log(s)$ and s^ϵ have been essentially ruled out [5,15]. Auger does quote a value for the pp total cross section at 57 TeV [16], but the precision is low (a fractional error of 0.20) and depends on specifics built into the Glauber model. It does not severely limit the high energy behavior of the cross section.

Concerning the first approximation of the previous section, namely $m_p = 0$, we find that returning the physical, nonzero m_p to the integral (including $p'/E' \neq 1$) gives nonzero but negligible integral values of 2.649×10^{-8} and 5.966×10^{-9} at LHC energies $\sqrt{s} = 7$ and 14 TeV. On the other hand, keeping m_p zero but returning σ_{pp} and $\sigma_{p\bar{p}}$ to their realistic energy dependences yields nonzero integral values of 0.1345 and 0.1309 at $\sqrt{s} = 7$ and 14 TeV. And finally, using nonzero m_p and realistic pp and $p\bar{p}$ cross sections returns the values 0.1345 and 0.1309 at $\sqrt{s} = 7$ and 14 TeV. The final two integration sets (realistic σ 's and

TABLE I. Fit parameters [6] with various analyticity constraints.

c_0 (mb)	36.95
c_1 (mb)	-1.350
c_2 (mb)	0.2782
c_3 (mb)	37.17
c_4 (mb)	-24.42
α	0.453

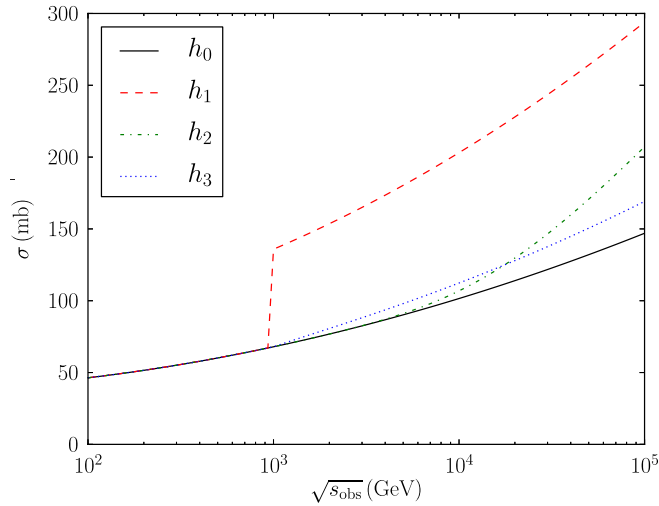


FIG. 4 (color online). The pp total cross sections $\sigma_{\text{SM}} + \sigma_{\text{BSM}}$ are plotted, for the choice $\sqrt{s_{\text{thr}}} = M_X = 10^3$ GeV. Doubling of the cross section at threshold is assumed for the h_1 model, i.e., $D = 1$. The SM cross section uses the parameters [6] shown in Table I, and leads to an error in the SM value of ρ of less than 1% [14]. The slow initial rise in h_2 is a result of the parton distribution functions. While the high energy behavior of h_3 is small compared to other models, it rises quickly at threshold, in contrast to h_2 .

zero or nonzero m_p) agree to about seven to eight decimal places, respectively (on the order of m_p^2/s). The conclusion is that the $m_p \rightarrow 0$ approximation is generally a valid one, whereas the constant and equal SM cross section approximation in the previous section is not. However, the integral contributions of the SM to the IDRs [the solid lines in Fig. 5] are not large, and we are encouraged to pursue

further the contributions that might arise due to physics beyond the SM.

For the subtraction constant, we will take $f(0) = 0$, since the value from the fits above is $f(0) = -0.073 \pm 0.67$ mb GeV. We note that even at the value 1σ away from zero, the term $4\pi f(0)/(p\sigma_{pp})$ at LHC energies contributes less than one part in 10^5 to ρ .

VI. NEW PHYSICS CONTRIBUTIONS

We turn now to the construction of three models for new physics beyond the Standard Model (BSM). We consider a class of modified cross sections of the general form

$$\sigma(s) = \sigma_{\text{SM}}(s)[1 + h_i(s)], \quad (11)$$

where the $h_i = (\sigma_{\text{BSM}}/\sigma_{\text{SM}})_i$ are cross section ratios; they vanish below the threshold s_{thr} for new physics. We apply the same enhancement to both σ_{pp} and $\sigma_{p\bar{p}}$ since, by the Pomanchuk theorem discussed above, each cross section should respond to new physics in the same way at energies well above the proton mass.

The first model we present is a simple step function at s_{thr} . This model results in an especially close analogy to the idealized IDRs we discussed in Sec. IV. In particular, due to its nonzero new cross section at $E = E_{\text{thr}}$, this model yields a singularity in the IDR integrand at $E = E_{\text{thr}}$ and therefore a singular value for $\rho(E = E_{\text{thr}})$.

More realistically, we expect phase space to present a cross section for new physics that has no jump discontinuity at threshold. For example, two-body phase space is $\beta/8\pi$, where β is either particle velocity in the CoM frame; at threshold, β is identically zero. Furthermore, including parton distribution functions to the model also yields a zero

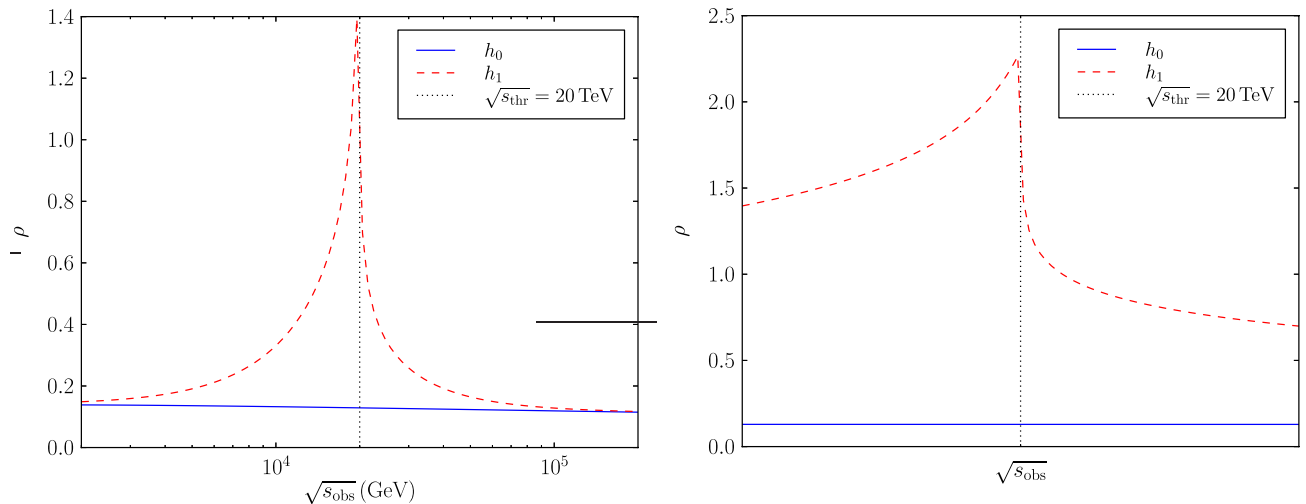


FIG. 5 (color online). At observational energies around LHC energies, the SM ρ (solid line) remains roughly constant. Using the step function enhancement h_1 with $D = 1$ and $\sqrt{s_{\text{thr}}} = 20$ TeV, we find a dramatic increase in ρ well below the new particle threshold. The right panel is an expanded piece of the left panel with a width of ~ 500 GeV on each side of the threshold energy, better showing the asymmetry of ρ about its singular peak value.

cross section at threshold. The new physics matrix elements may also vanish right at threshold. So we are led to the next two models of BSM physics. The second model we present involves hard-scattering parton production of new particles, while the third model is constructed from diffractive phenomenology. The second and third models provide cross sections that vanish at threshold, leading to finite values for $\rho(E = E_{\text{thr}})$.

Since only the first model, the step function, yields a nonzero change in the cross section at threshold, the ρ -value resulting from model h_1 should be considered an upper bound to the contribution of new physics BSM. The bounding of cross sections by the h_1 step function model is evident in Fig. 4, where we show the SM cross section (given by zero enhancement and labeled by $h_0 = 0$) and its enhancements ($h_i, i = 1, 2, 3$) by the three new physics models that are presented in detail below.

No new conserved quantum number is assumed in our models (valid, e.g., for broken R -parity SUSY models). Thus, energy is the only impediment to production of heavy new single particles, and the heavy single mass value M_χ determines E_{thr} . Without a new quantum number, the new particle would decay to SM particles, and due to its large mass, decay very quickly. Consequently, other than invariant mass combinatorics, there is no good signature of the new particle's production. One may have to rely on IDRs and/or an anomalous $\Delta\sigma$ for new particle identification. Thus, we plot $\Delta\rho/\rho$ and $\Delta\sigma/\sigma$ versus M_χ ($M_\chi = \sqrt{s_{\text{thr}}} - 2m_p \approx \sqrt{s_{\text{thr}}}$), to see if the IDR technique can identify new physics via an anomalous ρ measurement, before the new physics would be directly noticeable in the cross section increase.

A. A simple BSM enhancement—a step function

A simple example function for BSM physics is

$$h_1(s) = D\Theta(s - s_{\text{thr}}), \quad (12)$$

a stepwise jump in cross section at the threshold CoM energy $\sqrt{s_{\text{thr}}}$. The parameter D is a measure of the size of the new cross section relative to the SM. For example, in the event that new physics exactly doubles the cross section at $s = s_{\text{thr}}$, then we take $D = 1$. Modifying σ_{SM} in the form of Eq. (12) guarantees that the new total cross section σ continues to grow as fast as $\sigma_{\text{SM}} \propto \log^2 s$ (but not faster), and that the new physics contribution remains large over a sizable energy range beyond the threshold energy.

As mentioned above, an unphysical aspect of the step function enhancement is a nonvanishing cross section at threshold, which leads to the uncanceled singularity in the IDR integrand at $E = E_{\text{thr}}$ and a singular value for $\rho(E = E_{\text{thr}})$. However, the model has redeemable value in that the width of the singularity is small. Thus, the model offers a meaningful upper bound to new particle production away from the singularity.

B. A partonic model of new particle production

The most popular model of new physics at the electro-weak symmetry breaking scale is R -parity-preserving supersymmetry (SUSY), with masses tuned to the EW-scale to stabilize the ratio m_h/M_{Planck} (the ‘‘hierarchy problem’’). Unfortunately, R -parity conservation requires s , t , and u to have EW-scale values, which severely suppresses the SUSY cross section to about 10^{-10} times the SM cross section. However, as LHC limits on R -parity-conserving SUSY are becoming more constraining, R -parity-violating (RPV) models are getting a closer look [17–21]. If R parity is violated, we can replace one final state particle from a SM process with an effectively identical heavier counterpart for each possible final state. Then the only difference between the modified cross section and the SM cross section comes in the form of the reduced final state phase space and the threshold parton energy. Importantly, the fast growing $\log^2 s$ contribution to the SM σ_{tot} , which arises from soft and collinear gluon divergences, may be maintained. Also, other exotic models with extra dimensions [22] and a nonconserved KK number might grow a large cross section as a power law instead of the Froissart $\log^2 s$ limit.

Let $\sigma_i(s) = \sigma_i(pp \rightarrow \dots)$ be the SM cross section and $\sigma_i^{\text{BSM}}(s) = \sigma_i(pp \rightarrow \chi + \dots)$ be the new physics contribution, where $i = \{\text{el, inel, tot}\}$, and dots denote additional SM particles in the final states. Hats will denote parton cross sections instead of pp cross sections. We note that since $\sigma_{\text{el}}^{\text{BSM}} = 0$, then $\sigma_{\text{inel}}^{\text{BSM}}$ must equal $\sigma_{\text{tot}}^{\text{BSM}}$. Then the physical total pp cross section is $\sigma_{\text{tot}} + D\sigma_{\text{inel}}^{\text{BSM}} = \sigma_{\text{tot}}(1 + Dh_2(s))$ where $h_2 = \sigma_{\text{inel}}^{\text{BSM}}/\sigma_{\text{tot}}$ in the form of Eq. (11).

We start with an expression of the conservation of momentum for the new physics contribution,

$$\sigma_{\text{tot}}^{\text{BSM}}(s) = \sum_{i,j} \int_{(\hat{s} > M_\chi^2)} dx_1 dx_2 f_i(x_1) f_j(x_2) \hat{\sigma}_{\text{tot}}^{\text{BSM}}(\hat{s}), \quad (13)$$

where $\hat{s} \approx x_1 x_2 s$ is the parton CoM energy and the f_i are the various parton distribution functions (pdfs). Let the SM final state masses be zero. The summations are over parton types and the integrals are over the accessible x_1, x_2 space: $\hat{s} > M_\chi^2$.

If we assume that for each SM particle in the final state, there is an analogous new particle χ produced with the same coupling, then there is little t - or u -channel propagator suppression (see Appendix B), and so the matrix elements will be similar. The new, heavier final state masses suppress only the available phase space. So we can set parameter $D = 1$, and write

$$\frac{\hat{\sigma}_{\text{tot}}^{\text{BSM}}(\hat{s})}{\sqrt{\lambda(\hat{s}, M_\chi^2, 0)}} = \frac{\hat{\sigma}_{\text{inel}}(\hat{s})}{\sqrt{\lambda(\hat{s}, 0, 0)}} \quad (14)$$

where the triangle function (symmetric in its arguments) is defined as

$$\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ca. \quad (15)$$

The inelastic cross section shows up in the SM-equivalent case since the related new particle cross sections must be inelastic.

It is easy to see that the relevant ratio can be simplified to

$$\sqrt{\frac{\lambda(\hat{s}, M_\chi^2, 0)}{\lambda(\hat{s}, 0, 0)}} = 1 - \frac{M_\chi^2}{\hat{s}}. \quad (16)$$

Then, combining Eqs. (13,14,16), and integrating out the internal $\sigma_{\text{tot}}(\hat{s})$ which leaves behind a factor of $\hat{s}/s = x_1 x_2$, we obtain

$$\sigma_{\text{inel}}^{\text{BSM}}(s) = \sigma_{\text{inel}}(s) \sum_{i,j} \int dx_1 dx_2 f_i(x_1) f_j(x_2) x_1 x_2 \left(1 - \frac{M_\chi^2}{\hat{s}}\right). \quad (17)$$

Consistency of this model derivation gains support by noting that as either $M_\chi \rightarrow 0$ or $s \rightarrow \infty$, we recover the SM cross section (recalling that $\sum_i \int dx f_i(x) x = 1$ expresses conservation of momentum when the momentum of the parent nucleon is partitioned among partons).

Let us introduce the ratio $z \equiv \sigma_{\text{inel}}/\sigma_{\text{tot}}$. As suggested by the black disk limit, $z \rightarrow \frac{1}{2}$ as $s \rightarrow \infty$. However, data for the LHC $\sqrt{s} = 7$ TeV run, and CR data in the vicinity of $\sqrt{s} = 57$ TeV suggest that z is well (and conservatively) approximated as a constant $z \approx 0.7$ [5]. Our interest is the upcoming $\sqrt{s} = 14$ TeV LHC run, for which $z \approx 0.7$ is the appropriate value. Finally, we arrive at our model for new physics:

$$h_2(s, M_\chi) = z \sum_{i,j} \int_{x_1 x_2 > M_\chi^2/s} dx_1 dx_2 f_i(x_1, M_\chi) \times f_j(x_2, M_\chi) x_1 x_2 \left(1 - \frac{M_\chi^2}{\hat{s}}\right). \quad (18)$$

Of course, the parton distribution functions f_i also depend on the transfer energy Q , which we take to be M_χ . For our numerical work with pdfs, we use the CT10 parton distribution functions [23].

Note that this model has a vanishing cross section right at threshold, (at $\hat{s} = M_\chi^2$), due to the $(1 - M_\chi^2/\hat{s})$ factor, and due to the vanishing parton distributions at threshold. Thus, ρ is finite for all E values, including the peak at $E = E_{\text{thr}}$. Furthermore, the rise from threshold is the very slow, a notable feature of the h_2 model. This slow rise in h_2 is evident in Fig. 4. We find that the slow rise is due to the suppressed pdfs near threshold; the phase-space reduction factor contributes a negligible suppression to the rise. We conclude that any deep inelastic model with partons as initial state particles will experience a similar slow rise from threshold. Finally, we note from Eq. (18) that h_2 has a

finite asymptotic value. In particular, $\lim_{s \rightarrow \infty} h_2 = z$. For our purposes we have fixed z at 0.7, yet extrapolating z as $s \rightarrow \infty$ likely gives $z = \frac{1}{2}$. This discrepancy in the constant value of z does not affect our results as IDRs are only sensitive in the vicinity of the machine energy. In addition, we note that for comparison with Auger results at 57 TeV, for $M_\chi = 1, 5, 10$ TeV we have $h_2(57 \text{ TeV}) = 0.32, 0.053, 0.0099$, respectively.

C. A diffractive model of new particle production

An alternative to the partonic approach just presented is to consider general descriptions of pp inelastic cross sections without reference to partonic substructure. Inelastic cross sections can be described by the parameter $\xi \equiv M_X^2/s$. M_X is defined by first making a pseudorapidity (η) cut at the mean η of the two tracks with the greatest difference in η . M_X is then taken as the larger invariant mass of the two halves. Ref. [24] provides a model form for the inelastic cross section. It is

$$\frac{d\sigma}{d\xi} \propto \frac{1 + \xi}{\xi^{1+\epsilon}}, \quad (19)$$

where $\epsilon = \alpha(0) - 1$ and $\alpha(0)$ is the Pomeron trajectory intercept at $t = 0$. Values for ϵ are typically in the $[0.06, 0.1]$ range. We take the mean of this range, $\epsilon = 0.08$, in this paper. Next, we note that $1 \geq \xi > m_p^2/s \equiv \xi_p$, since $\xi_{\text{min}} = \xi_p$ describes elastic scattering. To find the total cross section, we integrate Eq. (19) across $\xi \in [\xi_p, 1]$ and get

$$\sigma \propto \frac{(1 - 2\epsilon) + (\epsilon - 1)\xi_p^{-\epsilon} + \epsilon\xi_p^{1-\epsilon}}{\epsilon(\epsilon - 1)}. \quad (20)$$

As an interesting aside, we note that to order ϵ^1 in Eq. (20), the leading energy behavior grows like $\log^2(s/m_p^2)$, thereby providing the expected asymptotic Froissart growth [25]. However, higher order terms in ϵ lead to higher logarithmic orders, indicating that Eq. (20) is preasymptotic.

We now consider a rapidity cluster containing a new particle of mass M_χ . With the substitution $\xi_p \rightarrow \xi_\chi \equiv M_\chi^2/s$ in Eq. (20), divided by the SM case, we arrive at the useful ratio

$$R(M_\chi, s) \equiv \frac{\sigma_{\text{diff}}^{\text{BSM}}}{\sigma_{\text{diff}}^{\text{SM}}} = \frac{1 - 2\epsilon + (\epsilon - 1)\xi_\chi^{-\epsilon} + \epsilon\xi_\chi^{1-\epsilon}}{1 - 2\epsilon + (\epsilon - 1)\xi_p^{-\epsilon} + \epsilon\xi_p^{1-\epsilon}}. \quad (21)$$

Next we note the relation in Eq. (19) describes single dissociative processes, which constitute only 15% of the inelastic cross section. We make the model assumption that the remaining 85% of the inelastic cross section, including double dissociative and nondiffractive processes, are also governed by the form in Eq. (19). Finally, we include the factor $z = \sigma_{\text{inel}}/\sigma_{\text{tot}} \approx 0.7$ described in the previous subsection, and make explicit the on-shell requirement $M_\chi^2 \leq s$

with a Heaviside function, to arrive at our final model expression

$$h_3(s) = z \frac{1 - 2\epsilon + (\epsilon - 1)\xi_\chi^{-\epsilon} + \epsilon\xi_\chi^{1-\epsilon}}{1 - 2\epsilon + (\epsilon - 1)\xi_p^{-\epsilon} + \epsilon\xi_p^{1-\epsilon}} \Theta(1 - \xi_\chi). \quad (22)$$

As with model h_2 , model h_3 has the desirable feature that the BSM cross section vanishes at threshold (here, $\xi_\chi = 1$). Thus, ρ is finite all energies, including the peak at $E = E_{\text{thr}}$.

In Fig. 4 we see that the h_3 model rises more quickly at threshold than the h_2 model, but attains a smaller asymptotic value,

$$\lim_{s \rightarrow \infty} h_3(s) = z \left(\frac{m_p}{M_\chi} \right)^{2\epsilon} \approx 0.23 \left(\frac{1 \text{ TeV}}{M_\chi} \right)^{2\epsilon}, \quad (23)$$

i.e., about a 25% increase beyond the SM cross section. This faster rise but lower asymptotic value for h_3 compared to h_2 is evident in Fig. 4. Note that with the appropriate $z \rightarrow \frac{1}{2}$ correction discussed in the previous subsection, the coefficient drops from $0.23 \rightarrow 0.16$. In addition, we note that for comparison with Auger results at 57 TeV, for $M_\chi = 1, 5, 10$ TeV we have $h_3(57 \text{ TeV}) = 0.14, 0.080, 0.058$, respectively.

VII. RESULTS

For each of the three models discussed in the previous section, we calculate the effect they have on ρ . The parameter considered is the fractional increase in ρ , given as $\Delta\rho/\rho \equiv (\rho - \rho_{\text{SM}})/\rho_{\text{SM}}$. This is then related to the TOTEM results at $\sqrt{s} = 7$ TeV. We consider the mean value from their experiment as an example signal: $\rho = 0.145 (\pm 0.091, 1\sigma \text{ confidence level})$ is compared to the SM prediction of $\rho = 0.1345$, a value which implies a fractional increase of $\Delta\rho/\rho = 0.0781$. The error in the theoretical calculation of ρ from existing measurements is $< 1\%$ [14]. This error is negligibly small compared to the effects presented here; we are justified in neglecting this error. We also look at the TOTEM upper limit, given as $\rho < 0.32$ at the 2σ confidence level, leading to a maximum fractional increase of $\Delta\rho/\rho < 1.38$ (2σ).

In what follows we conservatively neglect the increase in the elastic cross section due to new physics. This increase is model dependent. With the simplest assumptions, unitarity yields

$$\frac{\sigma_{\text{el}}}{\sigma_{\text{inel}}} = \frac{\eta}{2 - \eta}, \quad (24)$$

where $0 < \eta \leq 1$ is a measure of the absorption [26]. In general η may vary with the partial wave value J . We also note that $\eta = 1$ corresponds to the black disk limit, but the LHC is preasymptotic.

A. Results from the step function model

The step function enhancement of ρ is shown in Fig. 5. As expected, a shape very similar to that of Fig. 3 results. For comparison, the SM behavior of ρ is also shown. Here we have taken $D = 1$ and $\sqrt{s_{\text{thr}}} = 20$ TeV. We see for a doubling of the cross section at $\sqrt{s_{\text{thr}}} = 20$ TeV, a small increase in ρ is evident already at an energy an order of magnitude below $\sqrt{s_{\text{thr}}} = 20$ TeV, and that ρ increases by nearly a factor of four at $\sqrt{s_{\text{obs}}} = 14$ TeV.

Next, we look at what range of $\sqrt{s_{\text{thr}}}$ and D values will give an large increase in ρ . The left and right panels of Fig. 6 show contours of $\Delta\rho/\rho$ in the ranges $D \in [0, 1]$ and $D \in [0, 10]$. The $D \in [0, 1]$ range of the left panel may be relevant to broken R -parity-violating SUSY-like models, in which some or all of the SM particles might be doubled. The larger D range is plotted in the right panel, to show the increased reach of IDRs for still larger cross sections, as might be the case with extra-dimensional models. For the simple case of a step function with a significant increase in cross section, we see that IDRs offer a very powerful window to physics BSM.

Also displayed in Fig. 6 are the regions of the generous h_1 step function model that are ruled out at 95% significance by these TOTEM results. The IDR technique is sensitive to a large range of $(\sqrt{s_{\text{thr}}}, D)$ parameter space of the h_1 step function model, even with the currently large TOTEM errors on the IDR-independent ρ . In particular, the IDR technique is sensitive to new energy thresholds well beyond the direct energy reach of the LHC. A minimal inference to be drawn from the 95% confidence level exclusion in the figure is that the cross section cannot increase particularly quickly near the LHC energy $\sqrt{s} = 7$ TeV.

Each higher energy probed by the LHC will rule out an additional region of h_1 parameter space. Going forward, improvements are planned for the TOTEM optics, which will reduce the errors on ρ and thereby increase the sensitivity of the IDR toolkit to BSM physics.

Shown also in Fig. 6 is the contour corresponding to our $\rho = 0.145$ example signal. Our example value of ρ is taken from the TOTEM experiment's inferred mean value. If such a signal were statistically and systematically significant, we would expect new physics to show up as an increase in the pp cross section of height D and threshold $\sqrt{s_{\text{thr}}}$ somewhere on this contour. (We don't consider a signal of new physics at energies much below the machine energy, as direct detection of new event topologies or increased cross section would likely provide a better signal than a change in ρ as inferred through IDRs.)

We now turn to our more realistic models, h_2 and h_3 , describing the onset of new physics. The h_1 model contains two parameters, D and E_{thr} , and so for this model we showed the prediction for $\Delta\rho/\rho$ as a contour plot. With the h_2 and h_3 models, there is no analog of D , and the only parameter is E_{thr} . Thus, we may show $\Delta\rho/\rho$ and $\Delta\sigma/\sigma$ for

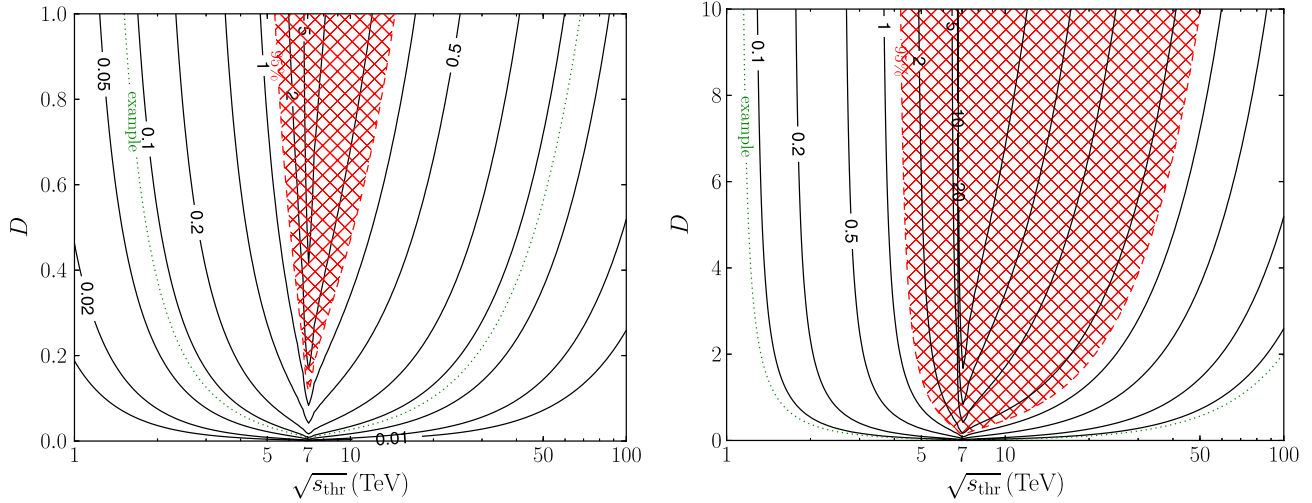


FIG. 6 (color online). At $\sqrt{s_{\text{obs}}} = 7$ TeV, the contours are parametrized by $\Delta\rho/\rho$. The form of the enhancement that includes new physics is h_1 —the step function. The left panel considers D in the range $[0,1]$, which is the relevant parameter space for SUSY-type models. The right panel considers D values up to 10, relevant for extra-dimensional theories that have arbitrarily large increases in the cross section. The shaded regions have already been ruled out by TOTEM's $\sqrt{s} = 7$ TeV preliminary results. The dotted green contours correspond to the $\rho = 0.145$ example signal.

these models as simple ordinates versus the mass M_χ of the new particle, and we do so. With these models, our enthusiasm for the IDR approach will be somewhat tempered.

B. Results from the partonic model

Results for the partonic h_2 model are displayed in Fig. 7, for the next-run LHC energy of $\sqrt{s} = 14$ TeV. For a small range of M_χ values, it is seen that $\Delta\rho/\rho$ is significantly larger than $\Delta\sigma/\sigma$. However, at present, the errors on the IDR-independent ρ measurement ($\Delta\rho/\rho \lesssim 1.38$ at 2σ) are much larger than the accuracy ($\sim 5\%$) with which energy-dependent changes in the total cross section can be inferred, so care is warranted here. From Fig. 7 we can estimate a region of energy in the 2–5 TeV range for which $\Delta\rho/\rho \gtrsim 0.1$ and $\Delta\sigma/\sigma \lesssim 0.05$. Our inference is that for new particle masses in the ~ 2 –5 TeV energy range, IDR-independent measurements of ρ to an accuracy of one part in ten could reveal new physics of the type described by h_2 in Sec. VI B at $\sqrt{s_{\text{obs}}} = 7$ TeV.

One may wonder why the peak in ρ occurs so far below the machine energy of 14 TeV. The reason is the slow rise of the BSM cross section due to suppression from the pdfs: a peak at energy $E_{\text{thr}} \sim \sqrt{s}$ weighted by the mean value of the parton momenta product, $\langle x_1 x_2 \rangle$, gives a peak at roughly an order of magnitude below the machine energy. A second inference is that models with new physics arising from initial state partons will enhance the value of ρ mainly below the machine energy. Of course, such models will also enhance the cross section below the machine energy, as seen in Fig. 7.

There is still a small increase in ρ at the machine energy of $\sqrt{s_{\text{thr}}} > 7$ TeV due to particle masses beyond 7 TeV. Beyond the machine energy, it is impossible for direct production to occur, so an inference of nonzero $\Delta\rho/\rho > 0$ due to particle masses beyond $\sqrt{s_{\text{thr}}} > 7$ TeV would present a unique, and striking, discovery. Unfortunately, in the h_2 model, such an inference does not seem possible, as $\Delta\rho/\rho$ is $\lesssim 0.01$ for new particle masses just beyond

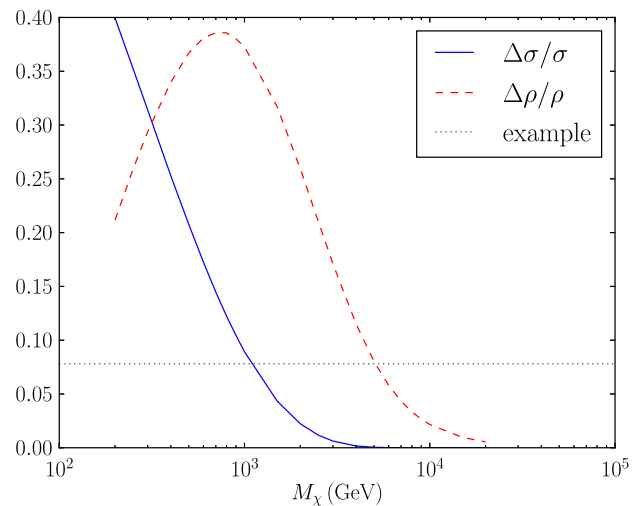


FIG. 7 (color online). The fractional increases in ρ and σ using $h_2(s)$ at $\sqrt{s_{\text{obs}}} = 7$ TeV, versus M_χ . With the present significance of ρ data, the exclusion region is well above the top of the graph. The dotted green line presents the value of $\Delta\rho/\rho$ corresponding to the $\rho = 0.145$ example signature; from intersecting lines, a new $M_\chi = 5.1$ TeV threshold is predicted.

7 TeV. A more optimistic inference is that, if the cross section were to rise much more quickly than that of the h_2 model, as happens with a Kaluza-Klein tower of new particles, it may be possible to infer such new physics even if the threshold energies/new masses exceed the LHC energy.

We see that our example signal, plotted in Fig. 7, implicates a new mass-scale $M_\chi = 5.1$ TeV. (The example $\Delta\rho/\rho$ also crosses the continuous curve at an energy below the machine energy; we assume that any new physics at this lower energy would be detected through more direct means.)

C. Results from the diffractive model

Finally, the h_3 model is plotted in Fig. 8 at $\sqrt{s_{\text{obs}}} = 7$ TeV. We see a modest contribution to ρ from the h_3 modification, as compared to that of the h_2 model. The larger contribution is due to the faster rise of the h_3 model from threshold ($s_{\text{thr}} = M_\chi^2$). The nonpartonic nature of model h_3 is at the heart of the larger, higher-energy peak. On the other hand, the effect of the smaller increase as $s \rightarrow \infty$ as described by Eq. (23) can be seen in Fig. 8 by the fast fall off in $\Delta\rho/\rho$ beyond $\sqrt{s} = M_\chi$.

We note that while no regions of M_χ parameter space can yet be excluded, our example signal implicates a new $M_\chi = 9.1$ TeV mass-scale. (We again ignore the lower energy crossing, where any new physics can be probed in a more direct manner.) This ~ 9 TeV mass-scale has not been

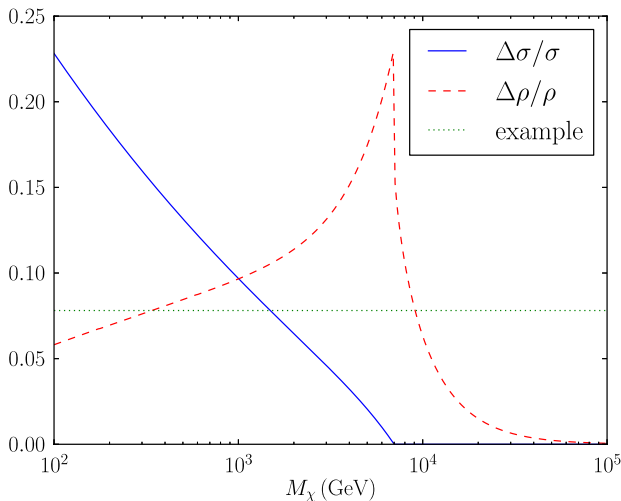


FIG. 8 (color online). The fractional increase in ρ and σ for the h_3 model, at $\sqrt{s} = 7$ TeV. The increase in ρ compared to its SM value shows a peak of 0.23 at $M_\chi = \sqrt{s_{\text{obs}}}$. We also see that ρ has increased compared to its SM value slightly across a range of energies beyond the observation energy. With the present significance of ρ data, the exclusion region is well above the top of the graph. The dotted green line presents the value of $\Delta\rho/\rho$ corresponding to the $\rho = 0.145$ example signature; from intersecting lines, a new $M_\chi = 9.1$ TeV threshold is predicted.

directly probed at the LHC, and likely will be only weakly probed even at the 14 TeV run.

D. Model conclusions

In the h_1 and h_3 models, the peak sensitivity of $\Delta\rho/\rho$ occurs when the new mass or new physics threshold is right at the machine energy. The sensitivity then falls off rapidly with increasing mass or threshold. However, the phase space for new particle production with mass at the machine energy is zero. Thus, a cross section measurement will not show an increase for such a mass value. However, the ρ -parameter will show a peak increase. Thus, the IDR technique primarily extends the reach of the LHC, to particle masses at the very end point of the machine energy. The LHC discovery potential is also extended beyond the machine energy, but with less sensitivity. In the h_2 model, the parton fractional momenta move the peak sensitivity to lower energies (by about an order of magnitude), thereby lessening the utility of the IDR technique for extending the LHC discovery potential to the machine energy and beyond.

It appears that this IDR technique may be sensitive to some reasonable models with large changes to the pp cross section, which have thresholds exceeding the reach of more direct detection.

The outlook for the near future is dependent on new measurements of ρ from experiments like TOTEM. The $\sqrt{s} = 8$ TeV data from TOTEM is in the process of being analyzed [27], and we eagerly await the next LHC run at $\sqrt{s} = 14$ TeV, which should begin in 2015.

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APPENDIX A: IDR INDEPENDENT CALCULATION OF ρ

To extract a value for ρ in an IDR-independent fashion, one invokes the optical theorem and extrapolates $d\sigma/dt$ to $t = 0$, as shown below. The cross section is related to the scattering amplitude by a simple exponential at low $|t|$. The differential cross section is

$$\frac{d\sigma}{dt} = \frac{\pi}{k^2} |f|^2.$$

At $t = 0$ one has

$$\begin{aligned} \left. \frac{d\sigma}{dt} \right|_{t=0} &= \frac{\pi}{k^2} |\Re f(t=0) + i\Im f(t=0)|^2 \\ &= \frac{\pi}{k^2} |(\rho + i)\Im f(t=0)|^2. \end{aligned}$$

Making use of the optical theorem, $\sigma_{\text{tot}} = (4\pi/k)\Im f(t=0)$, one arrives at

$$16\pi \frac{d\sigma}{dt} \Big|_{t=0} = (\rho^2 + 1)\sigma_{\text{tot}}^2,$$

where the desired ρ is the ratio of the real and imaginary parts of $f(t=0)$. From [2], the pp differential cross section in the low t limit is well approximated by

$$\frac{d\sigma}{dt} \propto e^{Bt},$$

where B is the ‘‘slope parameter,’’ assumed and measured to be very nearly constant. Thus, a measurement or estimate of σ_{tot} and an extrapolation of $d\sigma/dt$ to $t=0$ via the measured slope parameter are sufficient to determine ρ independently from the IDRs. While σ_{tot} is often evaluated in the ‘‘luminosity-independent’’ sense which includes an estimation of ρ , it can also be evaluated (although, less precisely) using a luminosity calculated through particle counting or beam sweeping techniques.

We also note that since the determination of ρ actually gives a value for ρ^2 there is an additional sign ambiguity. There are two approaches to dealing with this. The first is to compare results from modified cross sections in IDRs to either the positive or negative values, treating each equally. The second is to note that the IDR results for ρ from all of the fits done to the pp , $p\bar{p}$ cross sections (regardless of whether or not they follow the Froissart bound) yield a positive value for ρ . In practice we use TOTEM’s quoted upper limit on ρ statistically calculated from ρ^2 which accounts for the possibility that ρ could be negative and only places an upper limit on ρ .

APPENDIX B: MINIMUM TRANSFER ENERGY IN LIGHT TO LIGHT PLUS ONE HEAVY PROCESSES

We need $|t|$ small in h_2 to avoid amplitude suppression by propagators. Here we calculate the kinematic range of t

in the $2 \rightarrow 2$ process $p_1 + p_2 \rightarrow k_1 + k_2$, with $p_1^2 = p_2^2 = k_2^2 = 0$ all labeling SM particles and $k_1^2 = M_\chi^2$ labeling a new heavy particle. We will see that $t=0$ is allowed, leading to an unsuppressed amplitude for massless particle exchange.

Let θ be the angle between p_1 and k_1 in the CoM frame. Then the transfer energy squared is

$$\begin{aligned} t &= (p_1 - k_1)^2 \\ &= M_\chi^2 - 2(p_1^0 k_1^0 - |\vec{p}_1||\vec{k}_1| \cos \theta), \end{aligned}$$

where $k_1^0 = \vec{k}_1^2 + M_\chi^2$. So, $t_{\text{max/min}}$ are given by

$$t_{\text{max/min}} = M_\chi^2 - 2p_1^0 k_1^0 \pm 2|\vec{p}_1||\vec{k}_1|$$

at $\theta = 0, \pi$, respectively. Then we have

$$\begin{aligned} p_1^0 &= \frac{\sqrt{\hat{s}}}{2}, & k_1^0 &= \frac{\hat{s} + M_\chi^2}{2\sqrt{\hat{s}}}, & \text{and} \\ |\vec{p}_1| &= p_1^0, & |\vec{k}_1| &= \frac{\hat{s} - M_\chi^2}{2\sqrt{\hat{s}}}, & \text{and so} \\ p_1^0 k_1^0 &= \frac{\hat{s} + M_\chi^2}{4}, & |\vec{p}_1||\vec{k}_1| &= \frac{\hat{s} - M_\chi^2}{4}. \end{aligned}$$

Then, the maximum/minimum values of t are

$$t = M_\chi^2 - \frac{\hat{s} + M_\chi^2}{2} \pm \frac{\hat{s} - M_\chi^2}{2} = \begin{pmatrix} 0 \\ M_\chi^2 - \hat{s} \end{pmatrix},$$

where $t=0$ occurs for the forward scattering $\theta=0$ case, and the maximum $|t|$ transfer occurs for the backward scattering $\theta=\pi$ case. The $t_{\text{min}}=0$ result confirms that $pp \rightarrow \chi + \text{light particles}$ will favor small $|t|$.

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