PHYSICAL REVIEW D **89,** 034013 (2014)

Form factors for semileptonic D decays

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(Received 20 June 2013; published 10 February 2014)

We study transition form factors for decays of D mesons. That is, we consider matrix elements of the weak left-handed quark current for the transitions $D \to P$ and $D \to V$, where P and, V are light pseudoscalar or vector mesons, respectively. Our motivation to perform the present study of these form factors is future calculations of nonleptonic decay amplitudes. We consider the transition form factors within a class of chiral quark models. Especially, we study how the large energy effective theory limit works for D-meson decays. In this paper, we extend previous work on the case $B \to \pi$ to the case $D \to P = \pi, K$. Further, we extend our previous model based on the large energy effective theory to the entirely new case $D \to V = \rho, K^*, \ldots$ To determine some of the parameters in our model, we use existing data and results based on some other methods like lattice calculations, light-cone sum rules, and heavy-light chiral perturbation theory. We also obtain some new predictions for relations between form factors.

DOI: 10.1103/PhysRevD.89.034013 PACS numbers: 12.39.St, 12.39.Fe, 12.39.Hg

I. INTRODUCTION

In the present paper, we study transition form factors of D-meson decays, i.e., the $D \to P = \pi, K, \ldots$ and $D \to V = \rho, K^*, \ldots$ transition form factors within extended chiral quark models. Knowledge of the semileptonic form factors is, of course, necessary to calculate factorizable contributions to the nonleptonic decays of mesons. Further, knowledge about these form factors might determine or at least restrict some parameters of our models and thereby indirectly be of importance for our (model dependent) calculations for nonleptonic decays. We are, of course, aware of the technical challenges when calculating nonleptonic decays of D mesons [1], and we will come back to this in a future publication.

The $D \to P$ and $D \to V$ transition form factors have been calculated by various methods. These have their strength in different regions of the momentum transfer q squared, from q^2 near zero for light-cone sum rules (LCSR) [2–7] to $q^2 = q^2_{\text{max}}$ for the heavy-light chiral perturbation theory (HLχPT) [8]. For earlier work, see, for instance, Refs. [9,10]. In the region $q^2 \to 0$ where the momentum of the outgoing meson is high, one might study form factors within the large energy effective theory (LEET), invented in Ref. [11] and further elaborated in Ref. [12]. This theory was later developed into the soft collinear effective theory (SCET) [13].

In the region of large momentum transfer ($q^2 \to q^2_{\text{max}}$), lattice QCD has been used [14–17]. Form factors have been calculated [8,18–20] within HLχPT, which is based on the heavy quark effective theory (HQEFT). Calculations within HLχPT have also been supplemented [21] by calculations within the heavy-light chiral quark model (HLχQM) [21–25]. Within the heavy quark symmetry, there are corrections of the order $\mathcal{O}(1/m_c)$, which will be larger in the D sector than in the B sector. In any case, the form factors are influenced by nearby meson poles. Heavy ($H = B,\ D$) to light ($P = \pi,\ K,\ \eta$) transitions have also been treated in a mesonic picture [26] and in relativistic quark models [27–29].

Our intention is to find how well chiral quark models describe the form factors. Namely, in the next step, we want to calculate nonfactorizable contributions to nonleptonic decays of D mesons. Then we ought to know how well the chiral quark models work in various energy regions, and specifically we need to know the various form factors within the LEET. Some form factors are relatively well known. But for some cases, we perform additional model-dependent studies. Therefore, these models will be briefly presented. Compared to previous work, we will, in this paper, also include light vectors $V = (\rho, \omega, K^*)$. The transitions $H \to P$ and $H \to V$ are illustrated in Fig. 1.

II. DECOMPOSITION OF SEMILEPTONIC FORM FACTORS

For an heavy pseudoscalar meson $H = B,\ D$ decaying into a light pseudoscalar meson P, the vector current $J^\mu_V(H \to P)$ depends on the involved momenta p_H and p. This current can be decomposed into two form factors. There are two commonly used decompositions,

$$J^\mu_V(H \to P) = F_+(q^2)(p_H + p)^\mu + F_-(q^2)(p_H - p)^\mu \tag{1}$$

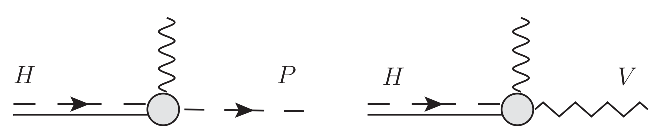

FIG. 1. Diagrams for $H \to P$ and $H \to V$ transitions at the mesonic level. The vertical line denotes a virtual electroweak boson (W, Z, γ).

and

$$J_V^\mu(H\to P) = F_1(q^2)\left[(p_H+p)^\mu - \frac{(M_H^2-m_P^2)}{q^2}q^\mu\right] + \frac{M_H^2-m_P^2}{q^2}F_0(q^2)q^\mu, \tag{2}$$

where $q = p_H - p$ is the momentum transfer and M_H and m_P are the masses of the heavy and light mesons, respectively. The relations between the form factors in Eqs. (1) and (2) are

$$F_1 = F_+; \qquad F_0 = F_+ + \frac{q^2}{M_H^2-m_P^2}F_-. \tag{3}$$

The heavy to light transitions $H\to V$, where $H=(B,D)$ and $V=(\rho,K^*,\omega,\phi)$, with mass m_V, can proceed through both vector and axial currents. These can be decomposed into (in total) four form factors. The vector current depends on only one form factor $V(q^2)$, and is commonly parametrized as

$$J_V^\mu(H\to V) = \langle V(p,\varepsilon)|\bar{q}\gamma^\mu Q|H(p_H)\rangle = \frac{2V(q^2)}{M_H+m_V}\varepsilon^{\mu\nu\rho\sigma}(\varepsilon_V^*)_\nu p_\rho (p_H)_\sigma, \tag{4}$$

where ε_V^* is the polarization vector for the outgoing vector meson V. The axial current includes three form factors, A_0, A_1, and A_2, and is written as

$$J_A^\mu(H\to V) = \langle V(p,\varepsilon)|\bar{q}\gamma^\mu\gamma_5 Q|H\rangle = (M_H+m_V)\left(\varepsilon_V^{*\mu} - \frac{(\varepsilon_V^*\cdot q)}{q^2}q^\mu\right)A_1(q^2) - \left((p+p_H)^\mu - \frac{M_H^2-m_V^2}{q^2}q^\mu\right)\frac{(\varepsilon_V^*\cdot q)}{M_H+m_V}A_2(q^2) + \frac{2m_V(\varepsilon_V^*\cdot q)}{q^2}q^\mu A_0(q^2). \tag{5}$$

For the light leptons ($l=\mu$, e), the amplitudes for $D\to Vl\nu$ are dominated by the form factors $V(q^2)$, $A_1(q^2)$, and $A_2(q^2)$. The vector form factor $V(q^2)$ is dominated by vector resonances, while the $A_1(q^2)$ and $A_2(q^2)$ are dominated by axial resonances, and the $A_0(q^2)$ form factor is dominated by the pseudoscalar resonances.

Bećirević and Kaidalov [30] proposed a double pole form for the $F_+(q^2)$ function. This includes the pole at a heavy vector meson H^* for the first pole and a term that includes contributions for higher mass resonances in an effective pole. The form factors, $F=F_+$, V, A_0, etc., can be written in the generic form

$$F(q^2) = \frac{F(0)}{[1-\frac{q^2}{m_{\rm pole}^2}][1-\frac{\alpha q^2}{m_{\rm pole}^2}]}, \tag{6}$$

where the parameter α parametrizes the contribution of the higher mass resonances into an effective pole.

III. ASYMPTOTIC BEHAVIOR OF FORM FACTORS

The HQET and LEET give constraints on the structure of the form factors. From the HQET one can estimate the behavior of the form factors in the limit of zero recoil (see Ref. [21] and references therein):

$$F_+ \sim \sqrt{M_H}; \qquad F_- \sim \frac{1}{\sqrt{M_H}}. \tag{7}$$

The form factors in the LEET limit, with $p_H^\mu = M_H v^\mu$ and $p = En^\mu$, can be parametrized as [12]

$$\langle P|\bar{q}\gamma^\mu Q_v|H\rangle = 2E(\zeta n^\mu + \zeta_1 v^\mu). \tag{8}$$

The 4-vectors v, n are given by $v=(1;\vec{0})$ and $n=(1;0,0,1)$ in the rest frame of the decaying heavy meson. Here, the ζ should scale with energy E as [12]

$$\zeta \equiv \zeta(M_H,E) = C\frac{\sqrt{M_H}}{E^2}, \qquad C\sim(\Lambda_{\rm QCD})^{3/2}, \qquad \frac{\zeta_1}{\zeta}\sim\frac{1}{E}. \tag{9}$$

In the limit $M_H\to\infty$ and $E\to\infty$, the ratio $\zeta_1/\zeta\to 0$. An explicit relation between ζ_1 and ζ will be given later in Sec. VI. The LEET may be used to estimate form factors at large recoil, where the momentum carried by the electroweak bosons (W,Z,γ) is at a minimum, that is, for $q^2\to 0$. Using Eq. (9) for small q^2, i.e., for $E\simeq M_H/2$, one obtains the behavior [31]

$$F_+ \sim F_0 \sim \frac{1}{M_H^{3/2}}. \tag{10}$$

We will need the following relations between the various form factors and the quantities ζ_i of the LEET formalism:

$$F_1 = F_+ = \zeta + \frac{E}{M_H}\zeta_1; \qquad F_- = -\zeta + \frac{E}{M_H}\zeta_1. \quad (11)$$

It should be noted that in Ref. [12] ζ_1 is neglected because it is suppressed by $1/E$, as seen in Eq. (9) and later in Eq. (64).

For transitions $H(0^-) \to V(1^-)$, one obtains in the LEET limit ($M_H \to \infty$ and $E \to \infty$) for the vector current:

$$\langle V|\bar{q}\gamma^\mu Q_v|H\rangle = 2iE\zeta_\perp \varepsilon^{\mu\nu\rho\sigma} v_\nu n_\rho (\varepsilon_V^*)_\sigma. \quad (12)$$

Here, the form factor $\zeta_\perp$ scales in the same way as ζ in Eq. (9) but with a different factor C:

$$\zeta_\perp = C_\perp \frac{\sqrt{M_H}}{E^2}. \quad (13)$$

For the axial current, the corresponding matrix element should have the form

$$\langle V|\bar{q}\gamma^\mu\gamma_5 Q_v|H\rangle = 2E\zeta_\perp^{(a)}[\varepsilon_V^{*\mu} - (\varepsilon_V^* \cdot v)n^\mu] + 2m_V \zeta_\parallel (\varepsilon_V^* \cdot v)n^\mu. \quad (14)$$

Here, the form factor $\zeta_\perp^{(a)}$ is equal to $\zeta_\perp$ to leading order, and $\zeta_\perp^{(a)}$ and $\zeta_\parallel$ scale in the same manner as $\zeta_\perp$ and ζ.

We will need the relations between the various form factors V, A_0, A_1, and A_2 and the quantities ζ_i in the LEET case [12],

$$V = \left(1+\frac{m_V}{M_H}\right)\zeta_\perp; \quad A_0 = \frac{m_V}{M_H}\zeta_\perp^{(a)} + \left(1-\frac{m_V^2}{M_H E}\right)\zeta_\parallel,$$
$$A_1 = \left(\frac{2E}{M_H+m_V}\right)\zeta_\perp^{(a)}; \quad A_2 = \left(1+\frac{m_V}{M_H}\right)\left[\zeta_\perp^{(a)} - \frac{m_V}{E}\zeta_\parallel\right], \quad (15)$$

which should be valid in the $q^2 \to 0$ limit. These form factors are plotted in Sec. VII.

IV. HEAVY-LIGHT CHIRAL PERTURBATION THEORY

The HLχPT is based on the HQEFT, where, to lowest (zero) order in $1/m_Q$, the 0^- and the 1^- heavy mesons are degenerate and described by a field H_v,

$$H_v = P_+(v)(\gamma \cdot P^* - i\gamma_5 P_5), \quad (16)$$

where $P_+(v) = (1 + \gamma \cdot v)/2$ is a projection operator and v is the velocity of the heavy quark. Further, P_μ^* is the 1^- field, and P_5 is the 0^- part of the heavy meson field. These mesonic fields enter the Lagrangian of the HLχPT,

$$\mathcal{L}_{\mathrm{HL}\chi\mathrm{PT}} = -\mathrm{Tr}(\bar{H}_v i v_\mu \partial^\mu H_v) + \mathrm{Tr}(\bar{H}_v{}^a H_v^b v_\mu \mathcal{V}_{ba}^\mu) - g_{\mathcal{A}}\mathrm{Tr}(\bar{H}_v{}^a H_v^b \gamma_\mu\gamma_5 \mathcal{A}_{ba}^\mu), \quad (17)$$

where a, b are $SU(3)$ flavor indices and $g_A = 0.59$ is the axial coupling. Further, $\mathcal{V}_\mu$ and $\mathcal{A}_\mu$ are vector and axial vector fields, for pseudoscalar mesons given by

$$\mathcal{V}_\mu \equiv \frac{i}{2}(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger), \qquad \mathcal{A}_\mu \equiv -\frac{i}{2}(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger), \quad (18)$$

where

$$\xi = \exp\{i\Pi/f\},$$
$$\Pi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}, \quad (19)$$

where $\eta \equiv \eta_8$. To calculate the form factors for the η and η', we use the η_8, η_0 basis,

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix}. \quad (20)$$

Here, we use the value of $\theta = 13.7^\circ$ from Ref. [32].

Based on the symmetry of the HQEFT, the bosonized current for decay of a system with one heavy quark and one light quark ($Q_v\bar{q}$) forming H_v is [8,33]

$$\bar{q}_L\gamma^\mu Q_v \longrightarrow \frac{\alpha_H}{2}\mathrm{Tr}[\xi^\dagger\gamma^\mu L H_v], \quad (21)$$

where Q_v is a reduced heavy quark field that is described in Sec. V, v is its velocity, and H_v is the corresponding heavy meson field. This bosonized current is compared with the matrix elements defining the meson decay constants f_H (where $H = B, D$). These currents are the same when QCD corrections below m_Q are neglected (see Refs. [25,34]). The $H \to P$ form factors obtained from HLχPT are illustrated in Fig. 2. Using the double pole parametrization, form factors were calculated in Ref. [19]:

$$F_+(q_{\mathrm{max}}^2) = \frac{\alpha_H}{2\sqrt{M_H}f} g_{\mathcal{A}} \frac{M_H}{m_P + \Delta_{H^*}} + \frac{\tilde{\alpha}}{2\sqrt{M_H}f}\tilde{g}\frac{M_H}{m_P + \Delta H'^*}. \quad (22)$$

Here, α_H is defined,

$$\alpha_H = f_H\sqrt{M_H}. \quad (23)$$

The term involving $\tilde{\alpha}$ and $\tilde{g}$ is the contribution from the higher resonances. (In Ref. [21], the higher resonance term was not included. Instead, some nonpole terms were included). One can also include light vectors with an effective coupling to heavy mesons, given by Ref. [20],

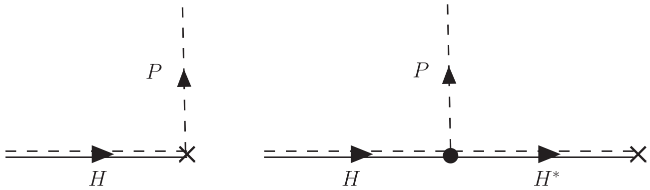

FIG. 2. Contributions to F_+ within the HLχPT. The single pole term is shown on the right.

$$\mathcal{L}_{\mathrm{HHV}} = i\frac{g_V}{\sqrt{2}}\lambda \mathrm{Tr}(\bar{H}_v H_v \sigma_{\mu\nu} F_V^{\mu\nu}), \tag{24}$$

where the coupling $g_V \simeq 5.9$ and

$$F_V^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu + [V^\mu, V^\nu]. \tag{25}$$

This term will give a dominating pole term in the $D \to V$ form factor similar to the one for $D \to P$ above. From Eq. (24), one obtains [20]

$$V(q^2_{\max}) = -\frac{\alpha_H}{2\sqrt{M_H}f}\frac{g_V\lambda}{\sqrt{2}}\frac{M_H}{m_V + \Delta_{H^*}} + \frac{\tilde{\alpha}}{2\sqrt{M_H}f}\tilde{\lambda}\frac{M_H}{m_V + \Delta H'^*}, \tag{26}$$

where the second term is coming from higher resonances. It might also be calculated in the HLχQM following closely the calculation for $D^* \to D\gamma$ [35]. The coupling $\tilde{\lambda}$ is a corresponding term for higher resonances.

V. VARIOUS CHIRAL QUARK MODELS

Calculating the matrix elements of quark currents, we have used chiral quark models. Within such models, one splits the various quark fields into different categories according to their relevant energy and mass scale.

In Sec. V A, we consider the ordinary soft quark fields q and the related soft flavor rotated fields χ (representing soft constituent light quarks) at energies ranging from the constituent quark mass $m \sim 220$ MeV up to the chiral symmetry breaking scale Λ_χ of order 1 GeV. These are the quarks of the chiral quark model (χQM) [25,36–39]. where light quarks couple to light mesons.

In Sec. V B, we also indicate how the quark fields chiral quark model of Sec. V A might be connected to light vectors $V = \rho, K^*, \ldots$, in a model we call VχQM to be described in Sec. V B. In Sec. V C, we describe the HLχQM [21–25] based on the HQEFT [34]. Here, the motion of the heavy quark with mass m_Q ($= m_b$ or m_c) with momentum p_Q is split in the leading term $m_Q v$, where v is the velocity of the heavy quark, and the motion for the reduced quark field Q_v is corresponding to momenta k of order a few hundred MeV such that $p_Q = m_Q v + k$. The reduced heavy quark field Q_v (also called h_v in the literature) is together with a quark field of χQM coupled to heavy meson fields H_v.

In Sec. V D we describe the large energy chiral quark model (LEχQM) based on the LEET [11,12] and invented in Ref. [40], and later used in Ref. [41]. Here, the motion of the energetic light quark with energy E and 4-momentum $p_q = En + k$ (where n is a lightlike vector) is split off, and the reduced energetic quark fields q_n have momenta k analogous to the reduced heavy quark fields. Here, the reduced energetic quark fields q_n combine with the ordinary χQM to make energetic light pseudoscalar meson fields M_n. In the second part of Sec. V D, we describe how this LEχQM can be extended to light energetic vectors V_n^μ. This is an invention that is new in this paper.

A. χQM for low-energy light quarks

For the pure light sector, the chiral quark model gives the interactions between light quarks and light pseudoscalar mesons. The χQM Lagrangian can be written as

$$\mathcal{L}_{\chi\mathrm{QM}} = \bar{q}(i\gamma^\mu D_\mu - \mathcal{M}_q)q - m(\bar{q}_R \Sigma^\dagger q_L + \bar{q}_L \Sigma q_R), \tag{27}$$

where q is the light quark flavor triplet, $\mathcal{M}_q$ is the current mass matrix, and $\Sigma = \xi \cdot \xi$ contains the light pseudoscalar mesons. (The current mass term $\mathcal{M}_q$ will often be neglected). The covariant derivative D_μ contains soft gluons, which might form gluon condensates within the model. The quantity m is interpreted as the constituent light quark mass appearing after the spontaneous symmetry breaking $SU(3)_L \times SU(3)_R \to SU(3)_V$. The Lagrangian (27) can be transformed into a useful version in terms of the flavor-rotated fields $\chi_{L,R}$:

$$\chi_L = \xi^\dagger q_L, \qquad \chi_R = \xi q_R. \tag{28}$$

The Lagrangian in Eq. (27) is then rewritten in the form

$$\mathcal{L}_{\chi\mathrm{QM}} = \bar{\chi}[\gamma \cdot (iD + \mathcal{V}) + \gamma \cdot \mathcal{A}\gamma_5 - m]\chi - \bar{\chi}\tilde{M}_q \chi, \tag{29}$$

where the fields $\mathcal{V}$ and $\mathcal{A}$ are given in Eq. (18) and where the term including the current mass matrix $\mathcal{M}_q$ is given by

$$\tilde{M}_q = \tilde{M}_q^V + \tilde{M}_q^A \gamma_5, \tag{30}$$

where

$$\tilde{M}_q^V = \frac{1}{2}(\xi \mathcal{M}_q \xi + \xi^\dagger \mathcal{M}_q^\dagger \xi^\dagger) \quad \text{and}$$
$$\tilde{M}_q^A = \frac{1}{2}(\xi \mathcal{M}_q \xi - \xi^\dagger \mathcal{M}_q^\dagger \xi^\dagger). \tag{31}$$

This term has to be taken into account when calculating $SU(3)$-breaking effects.

B. χQM including light vector mesons (VχQM)

The VχQM adds light vector mesons to the χQM. The vector meson fields V_μ are given as Π in Eq. (19) with pseudoscalars $P = (\pi, K, \eta)$ replaced by vectors $V = (\rho, \omega, K^*, \phi)$:

$$V_\mu = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -\phi \end{pmatrix}. \tag{32}$$

These fields are coupled to the light quark fields by the interaction Lagrangian,

$$\mathcal{L}_{IV} = h_V \bar{\chi}\gamma^\mu V_\mu \chi. \tag{33}$$

The coupling constant h_V can be determined from the left-handed current for $vac \to V$, and we find the $SU(3)$ octet current

$$J^a_\mu(vac \to V) = \frac{1}{2} m_V f_V \mathrm{Tr}[\Lambda^a V_\mu], \tag{34}$$

where the quantity Λ^a is given by $\Lambda^a = \xi\lambda^a\xi^\dagger$, and λ^a is the relevant $SU(3)$ flavor matrix. For the currents, we obtain

$$m_V f_V = \frac{1}{2} h_V \left(-\frac{\langle \bar{q}q \rangle}{m} + f_\pi^2 - \frac{1}{8m^2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \right), \tag{35}$$

which can be used to determine h_V. We find, by using $f_\rho \simeq 216$ MeV, that $h_V \simeq 7$ for standard values of m, $\langle \bar{q}q \rangle$, and $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ [21,25,35].

C. HLχQM

The HLχQM adds heavy meson and heavy quark fields to the χQM. The (reduced) heavy quark field Q_v is related to the full-field $Q(x)$ as

$$Q_v(x) = P_+ e^{-im_Q v\cdot x} Q(x), \tag{36}$$

where $P_\pm$ are projection operators $P_\pm = (1 \pm \gamma \cdot v)/2$. The heavy quark propagator (corresponding the reduced field Q_v) is $S_v(p) = P_+/(v \cdot p)$. The Lagrangian for the reduced heavy quark fields is

$$\mathcal{L}_{\mathrm{HQEFT}} = \bar{Q}_v i v \cdot D Q_v + \mathcal{O}(m_Q^{-1}), \tag{37}$$

where D_μ is the covariant derivative containing the gluon fields.

To couple the heavy quarks to light pseudoscalar mesons, there are additional meson-quark couplings within the HLχQM [21],

$$\mathcal{L}_{\mathrm{int}} = -G_H [\bar{\chi}_a \bar{H}^a_v Q_v + \bar{Q}_v H^a_v \chi_a], \tag{38}$$

where a is an $SU(3)$ flavor index and Q_v is the reduced heavy quark field in Eq. (36). The quark-meson coupling G_H is determined within the HLχQM to be [21]

$$G_H^2 = \frac{2m}{f_\pi^2} \rho, \tag{39}$$

where ρ is a hadronic quantity of order 1 [21].

The VχQM can be combined with the HLχQM to give a reasonable description of the weak current for D-meson decays $D \to V$ [20]. A coupling of V^μ to heavy mesons might be given by Eq. (17) with $\mathcal{V}^\mu \to h_V V^\mu$ or by the tensor coupling in Eq. (24). In Ref. [20] the factor λ is found to be $\lambda = -0.53\ \mathrm{GeV}^{-1}$. It might also be calculated in the HLχQM following closely the calculation for $D^* \to D\gamma$ [35]. Using the results of Ref. [35], we obtain

$$\lambda = -\frac{\sqrt{2} h_V \beta}{4 g_V}, \tag{40}$$

where β is defined in Ref. [35]. The value of β obtained there gives $\lambda \simeq -0.4\ \mathrm{GeV}^{-1}$, in agreement with the value $\lambda \simeq -0.41\ \mathrm{GeV}^{-1}$ in Ref. [8].

The current $J^\mu(H \to V)$, obtained from a quark loop diagram such as in Fig. 4, has the form

$$J^\mu_{\mathrm{tot}}(H_v \to V) = \mathrm{Tr}\{\xi^\dagger \gamma^\mu L H_v [A\gamma \cdot V + B v \cdot V]\}, \tag{41}$$

where A and B are hadronic parameters containing the couplings G_H and h_V, gluon condensates, and the constituent quark mass. This expression is analogous to Eq. (28) in Ref. [21] for the case $H \to P$. However, the $D \to V$ form factor will be dominated by the pole term shown on the right in Fig. 3, and we will not go further into the detailed structure of the nonleading terms A and B.

D. LEχQM

The LEχQM adds high-energy light mesons and quarks to the χQM. Unfortunately, the combination of the standard version of the LEET [11,12] with the χQM will lead to infrared-divergent loop integrals for $n^2 = 0$. Therefore, the following formalism is modified and instead of $n^2 = 0$: we use $n^2 = \delta^2$, with $\delta = \nu/E$, where $\nu \sim \Lambda_{\mathrm{QCD}}$, such that $\delta \ll 1$. In the following, we derive a modified LEET in which we keep $\delta \neq 0$ with $\delta \ll 1$. We call this construction LEETδ [40] and define the *almost* lightlike vectors

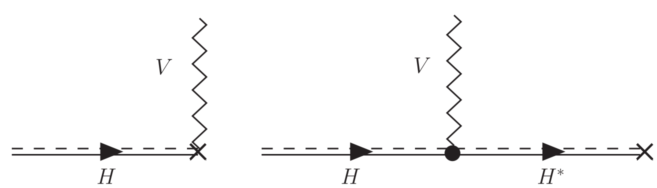

FIG. 3. Contributions to $H \to V$ form factors within the HLχPT. The single pole term is shown on the right.

$$n = (1, 0, 0, +\eta); \qquad \tilde{n} = (1, 0, 0, -\eta), \tag{42}$$

where $\eta = \sqrt{1 - \delta^2}$. This gives

$$n^\mu + \tilde{n}^\mu = 2v^\mu, \qquad n^2 = \tilde{n}^2 = \delta^2,$$
$$v \cdot n = v \cdot \tilde{n} = 1, \qquad n \cdot \tilde{n} = 2 - \delta^2. \tag{43}$$

For the LEET, the reduced quark field is defined by

$$q_n(x) = e^{-iEn\cdot x}\mathcal{P}_+ q(x), \tag{44}$$

corresponding to Eq. (36) and where the projection operators are

$$\mathcal{P}_+ = \frac{1}{N^2}\gamma \cdot n(\gamma \cdot \tilde{n} + \delta), \qquad \mathcal{P}_- = \frac{1}{N^2}(\gamma \cdot \tilde{n} - \delta)\gamma \cdot n, \tag{45}$$

where $N^2 = n \cdot \tilde{n}$. The LEETδ Lagrangian corresponding to the HQEFT Lagrangian in Eq. (37) is [40]

$$\mathcal{L}_{\mathrm{LEET}\delta} = \bar{q}_n\left(\frac{\gamma \cdot \tilde{n} + \delta}{N}\right)(in \cdot D)q_n + \mathcal{O}(E^{-1}). \tag{46}$$

For $\delta \to 0$, this is the first part of the SCET Lagrangian. The quark propagator is

$$S_n(k) = \frac{\gamma \cdot n}{N(n \cdot k)}, \tag{47}$$

which reduces to the LEET propagator in the limit $\delta \to 0$ (which also means $N \to 2$). For further details, we refer to Ref. [40].

The term $\mathcal{O}(E^{-1})$ in Eq. (46) contains a term originating from the current mass m_q for the light energetic quark(s). We have found that a further development beyond Ref. [40] gives the $SU(3)$-breaking current mass term m_q:

$$\Delta\mathcal{L}_{\mathrm{LEET}\delta}(m_q) = \frac{m_q}{E}\bar{q}_n\left(i\tilde{n} \cdot D - \frac{m_q}{2}\gamma \cdot \tilde{n}\right)q_n. \tag{48}$$

For hard light quarks and chiral quarks coupling to a hard light meson multiplet field M, the χQM and HLχQM were extended [40], and it was assumed that the energetic light mesons couple to light quarks with a derivative coupling to an axial current,

$$\mathcal{L}_{\mathrm{int}q} \sim \bar{q}\gamma_\mu\gamma_5(i\partial^\mu M)q. \tag{49}$$

The outgoing light energetic mesons are described by an octet 3×3 matrix field $M = \exp{(+iEn \cdot x)}M_n$, where M_n has the same form as Π in Eq. (19):

$$M_n = \begin{pmatrix} \frac{\pi_n^0}{\sqrt{2}} + \frac{\eta_n}{\sqrt{6}} & \pi_n^+ & K_n^+ \\ \pi_n^- & -\frac{\pi_n^0}{\sqrt{2}} + \frac{\eta_n}{\sqrt{6}} & K_n^0 \\ K_n^- & \bar{K}_n^0 & -\frac{2\eta_n}{\sqrt{6}} \end{pmatrix}, \tag{50}$$

where π_n^0, π_n^+, K_n^+, etc., are the fields for the hard mesons. Furthermore, q_n is related to M_n in the same manner as Q_v is related to H_v.

Combining the interaction (49) with the rotated soft quark fields in Eq. (28), and using $\partial^\mu \to iEn^\mu$, yields the LEχQM interaction Lagrangian [40]

$$\mathcal{L}_{\mathrm{LE}\chi\mathrm{QM}} = G_A E\bar{\chi}(\gamma \cdot n)Z_n q_n + \mathrm{H.c.} \tag{51}$$

Here, q_n is the reduced field corresponding to an energetic light quark having a momentum fraction close to 1 [see Eq. (46)], and χ represents a soft quark [see Eq. (28)]. Further, G_A is an unknown coupling to be determined by relating a current calculation to measured data. Further,

$$Z_n = \xi M_R R - \xi^\dagger M_L L. \tag{52}$$

Here, M_L and M_R are both equal to M_n, but they have formally different transformation properties. This is in analogy with chiral perturbation theory, in which the quark mass matrices $\mathcal{M}_q$ and its Hermitian conjugate $\mathcal{M}_q^\dagger$ are equal but have formally different transformation properties under $SU(3)_L \times SU(3)_R$). Equation (51) for the LEχQM is the analog of Eq. (38) in the HLχQM case.

Calculating the matrix elements of quark currents for the $H_v \to M_n$ transition in the LEχQM, we obtain an expression for the form factor ζ in terms of model parameters [40],

$$\zeta = \frac{1}{4}m^2 G_H G_A F\sqrt{\frac{M_H}{E}}, \tag{53}$$

where the quantity F coming from loop integration in Fig. 4 (with soft gluons forming gluon condensates added) is [40]

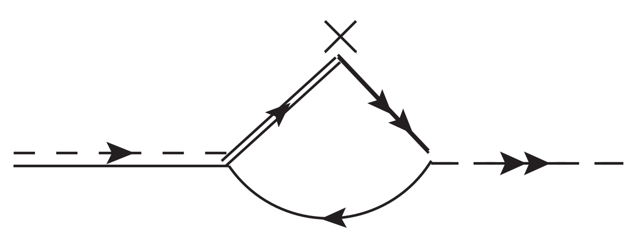

FIG. 4. Current matrix element in the LEχQM. The double dashed line is the (external) heavy meson H_v, and the dashed line with two arrows is the external energetic light meson. The internal lines are double for heavy quark Q_v, single with two arrows for the energetic light quark q_n, and with one arrow for the soft light quark χ.

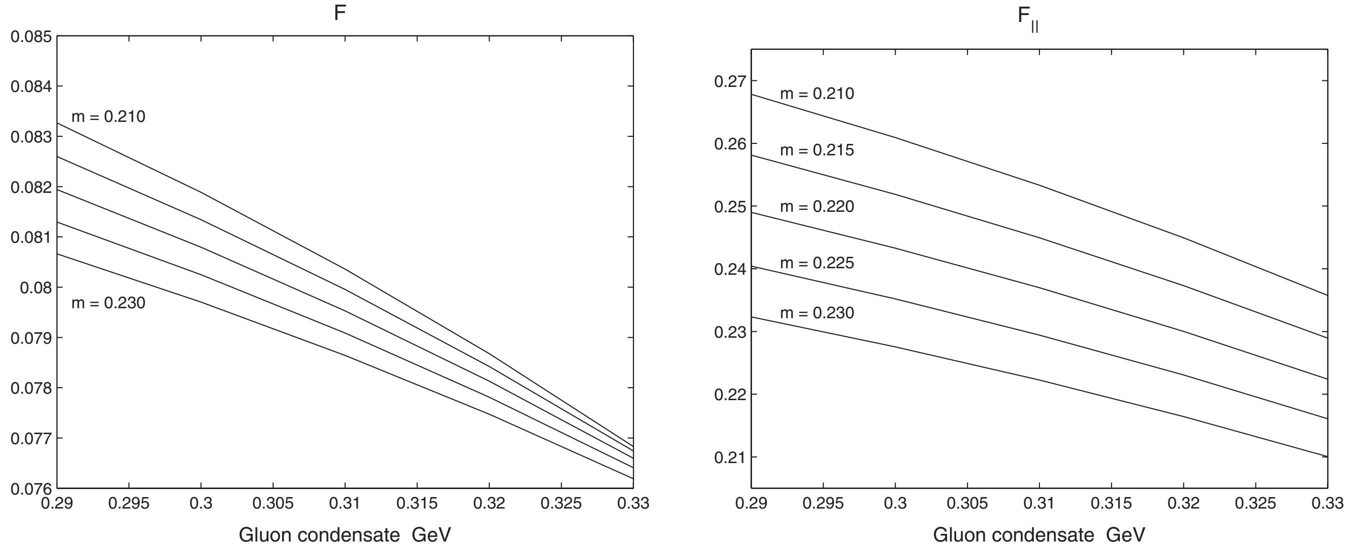

FIG. 5. F and $F_{\parallel}$ as a function of $\langle\frac{\alpha_s}{\pi}G^2\rangle^{1/4}$ for values of the constituent quark mass from $m = 0.210$ to $m = 0.230$ GeV.

$$F = \frac{N_c}{16\pi} + \frac{3f_\pi^2}{8m^2\rho}(1-g_A) - \frac{(24-7\pi)}{768m^4}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle, \quad (54)$$

which is numerically $F \simeq 0.08$. In Fig. 5, the quantity F is plotted as function of the quark condensate for typical values of the constituent quark mass. We obtain the expression for the coupling constant

$$G_A = \frac{4\zeta}{m^2 G_H F}\sqrt{\frac{E}{M_H}}, \quad (55)$$

where ζ is numerically known [2,7,42] to be $\simeq 0.3$ for the transition $B \to \pi$ but is larger, $\zeta \simeq 0.6$ for $D \to \pi$ [6]. We can use the data for ζ in the $D \to \pi$ and $D \to K$ transitions to determine the value of the coupling G_A. We then use this to calculate the ζ form factors for the transitions $D \to \eta$ and $D \to \eta'$.

Within our model, the constituent light quark mass m is the analog of $\Lambda_{\rm QCD}$. To see the behavior of G_A in terms of the energy E, we therefore write C in Eq. (9) as $C \equiv \hat{c}m^{\frac{3}{2}}$ and obtain

$$G_A = \left(\frac{4\hat{c}f_\pi}{mF\sqrt{2\rho}}\right)\frac{1}{E^{\frac{3}{2}}}, \quad (56)$$

which explicitly displays the behavior $G_A \sim E^{-3/2}$. In terms of the number N_c of colors, $f_\pi \sim \sqrt{N_c}$ and $F \sim N_c$, which gives the behavior $G_A \sim 1/\sqrt{N_c}$, i.e., the same behavior as for the coupling G_H in Eq. (38). The coupling G_A is an auxiliary quantity that can be used in place of the quantity ζ.

In this paper, we will extend the LEχQM further to include energetic vector mesons, V_n^μ in analogy with M_n in Eq. (50). In this model, we will use a derivative coupling, as was used for the coupling of light energetic mesons to quarks through an axial vector field in Eq. (49). This is in analogy with light mesons coupling to quarks in Eq. (29). We will therefore begin from the ansatz with the tensor field $F_V^{\mu\nu}$ in Eq. (25) [49]:

$$\mathcal{L}_{{\rm LE}\chi{\rm V}} \sim \bar{\chi}\sigma\cdot F_V\chi. \quad (57)$$

It was found in Ref. [40] that derivative coupling gave the best description of the $H \to P$ high-energy current. Using $V \to \exp\,(iEn\cdot x)V_n$, we obtain the interaction (remember that $\partial^\mu V_\mu = 0$ implies $n\cdot V_n = 0$)

$$\mathcal{L}_{{\rm LE}\chi{\rm V}} = EG_V\bar{\chi}(\gamma\cdot n\gamma\cdot Z_n)q_n + {\rm H.c.}, \quad (58)$$

where

$$Z_n^\mu = V_n^\mu(\xi R + \xi^\dagger L) \quad (59)$$

and

$$V_n^\mu = \begin{pmatrix} \frac{\rho_n^0}{\sqrt{2}} + \frac{\omega_n}{\sqrt{2}} & \rho_n^+ & K_n^{*+} \\ \rho_n^- & -\frac{\rho_n^0}{\sqrt{2}} + \frac{\omega_n}{\sqrt{2}} & K_n^{*0} \\ K_n^{*-} & \bar{K}_n^{*0} & -\Phi_n \end{pmatrix}^\mu. \quad (60)$$

Here, ρ_n^0, ρ_n^+, K_n^{*+}, etc., are the (reduced) vector meson fields corresponding to energetic light vector mesons. The coupling G_V is determined by the experimental value for the form factors for $B \to \rho$ (for B decays) or the $D \to \rho$ (for D decays) at $q^2 = 0$, obtained by considering experiment and lattice calculations when available or LCSR calculations.

In our case, where no extra soft pions are going out, we set $\xi \to 1$, and for the momentum space, we set $V_n^\mu \to k_M\sqrt{E}(\epsilon_V^*)^\mu$. The isospin factor is $k_M = 1/\sqrt{2}$ for ρ^0 and $k_M = 1$ for charged ρ's. For the D meson with spin parity 0^-, we have $H_v^{(+)} \to P_+(v)(-i\gamma_5)\sqrt{M_H}$. Then, the involved traces are calculated, and we obtain $J^\mu_{\rm tot}(H_v \to V_n)$ for the $H_v \to V_n$ transition.

From the current calculation, we obtain a relation between the coupling G_V and the form factor $\zeta_\perp$. The formula relating $\zeta_\perp$ and G_V will be similar to that relating ζ and G_A,

$$\zeta_\perp = \frac{1}{4} m^2 G_H G_V F \sqrt{\frac{M_H}{E}}, \tag{61}$$

that is obtained by the replacing $\zeta \to \zeta_\perp$ and $G_A \to G_V$ in Eqs. (55) and (56). The loop integration is the same for both cases; therefore, the loop factor will also be F in this case as in Ref. [40]. Here, $\zeta_\perp$ is numerically known for $B \to \rho$, where it is $\zeta_\perp \simeq 0.3$ [3,7], and for $D \to \rho$, it is $\zeta_\perp \simeq 0.59$ from CLEO data [46].

VI. RESULTS FROM THE LEχQM

Within the LEχQM, and in the limit $\zeta_1/\zeta \sim \delta \to 0$, the bosonized current for the $H_v \to M_n$ transition can be written as

$$J^\mu_{\rm tot}(H_v \to M_n) = -2i\zeta\sqrt{\frac{E}{M_H}}{\rm Tr}\{\gamma^\mu L H_v[\gamma\cdot n]\xi^\dagger M_L\}. \tag{62}$$

Simlarly, in the LEχQM, the bosonized current for the vector case $H_v \to V_n$ can be written as

$$J^\mu_{\rm tot}(H_v \to V_n) = -2i\sqrt{\frac{E}{M_H}}{\rm Tr}\left\{\gamma^\mu L H_v\left(\zeta_\perp\gamma\cdot n - \frac{m_V}{m}\zeta_\parallel\right)\sigma\cdot F_n\xi^\dagger[\gamma\cdot n]\right\}, \tag{63}$$

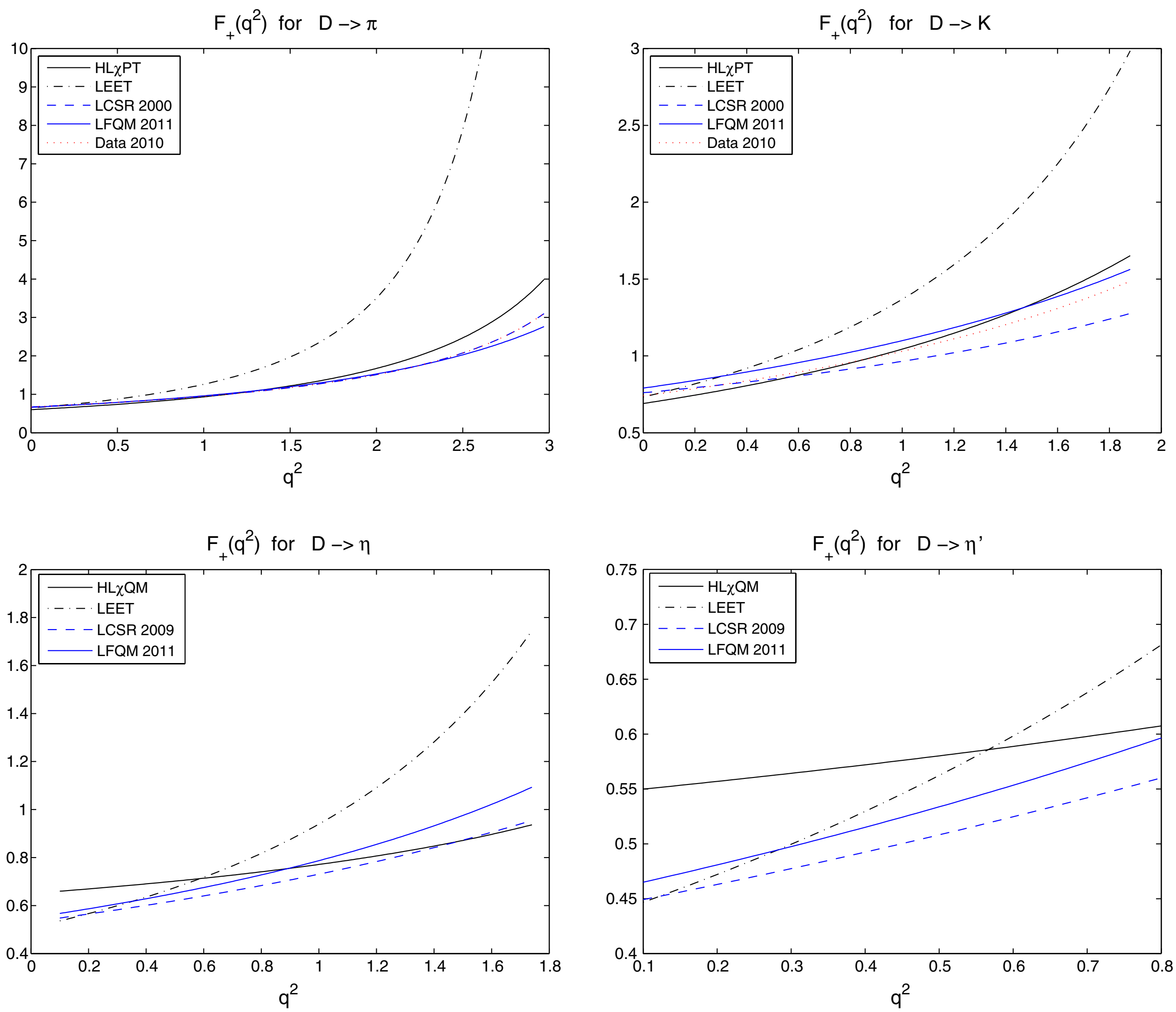

FIG. 6 (color online). $D \to P$ form factors F_+ comparing frameworks used: the HLχPT is from Ref. [19], LCSR 2000 is from Ref. [4], LCSR 2009 is from Ref. [43], the LEET is from Ref. [12], the LFQM from Ref. [44], and the "Data" are from Ref. [45].

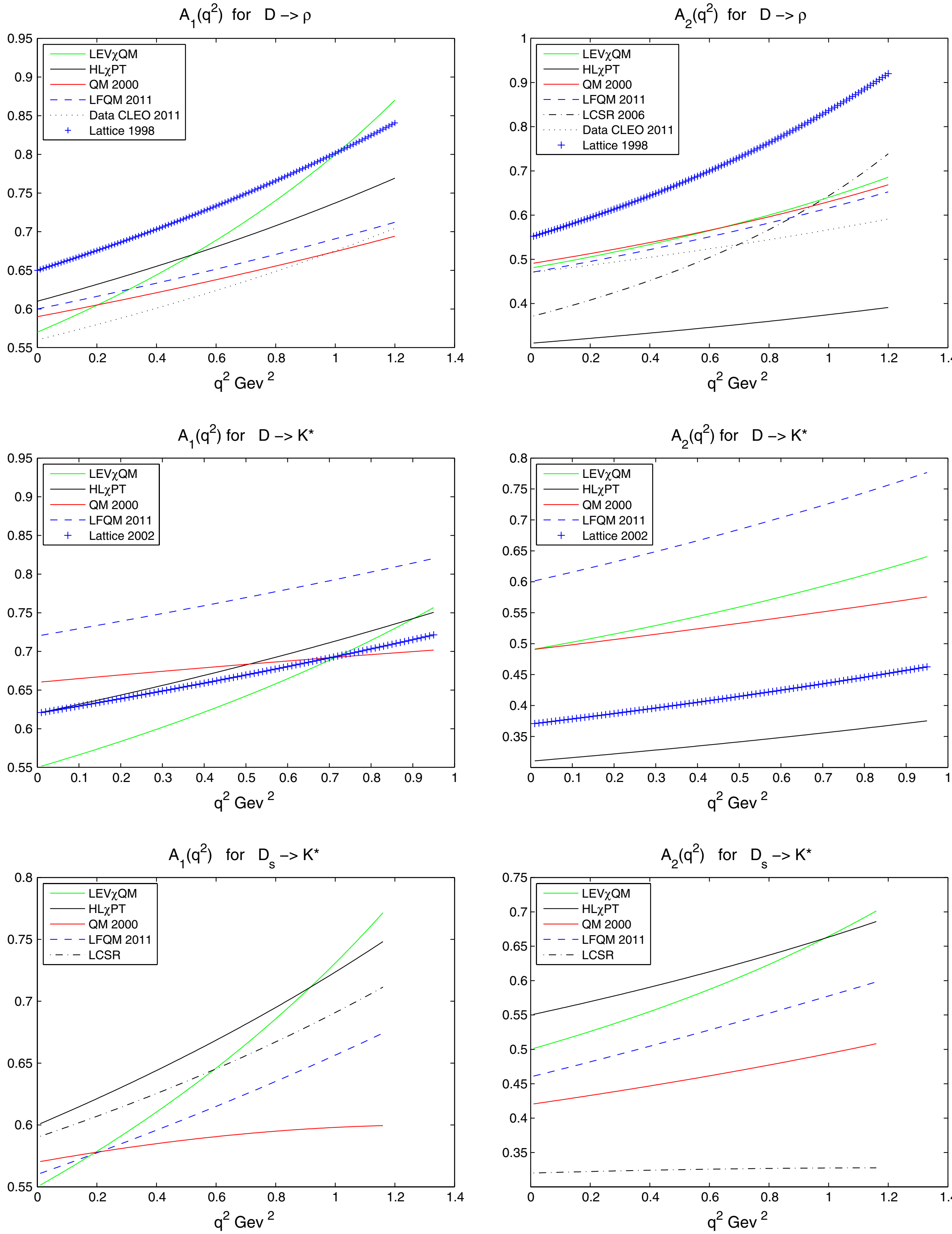

FIG. 7 (color online). $D \to V$ form factors A_1 and A_2: Data CLEO are from Ref. [46], LCSR 2006 is from Ref. [47], the LFQM is from Ref. [44], Lattice 1998 is from Ref. [14], Lattice 2002 is from Ref. [15], the HLχPT is from Ref. [20], and the QM 2000 is from Ref. [48]. The LEχQM is from the calculation in this paper.

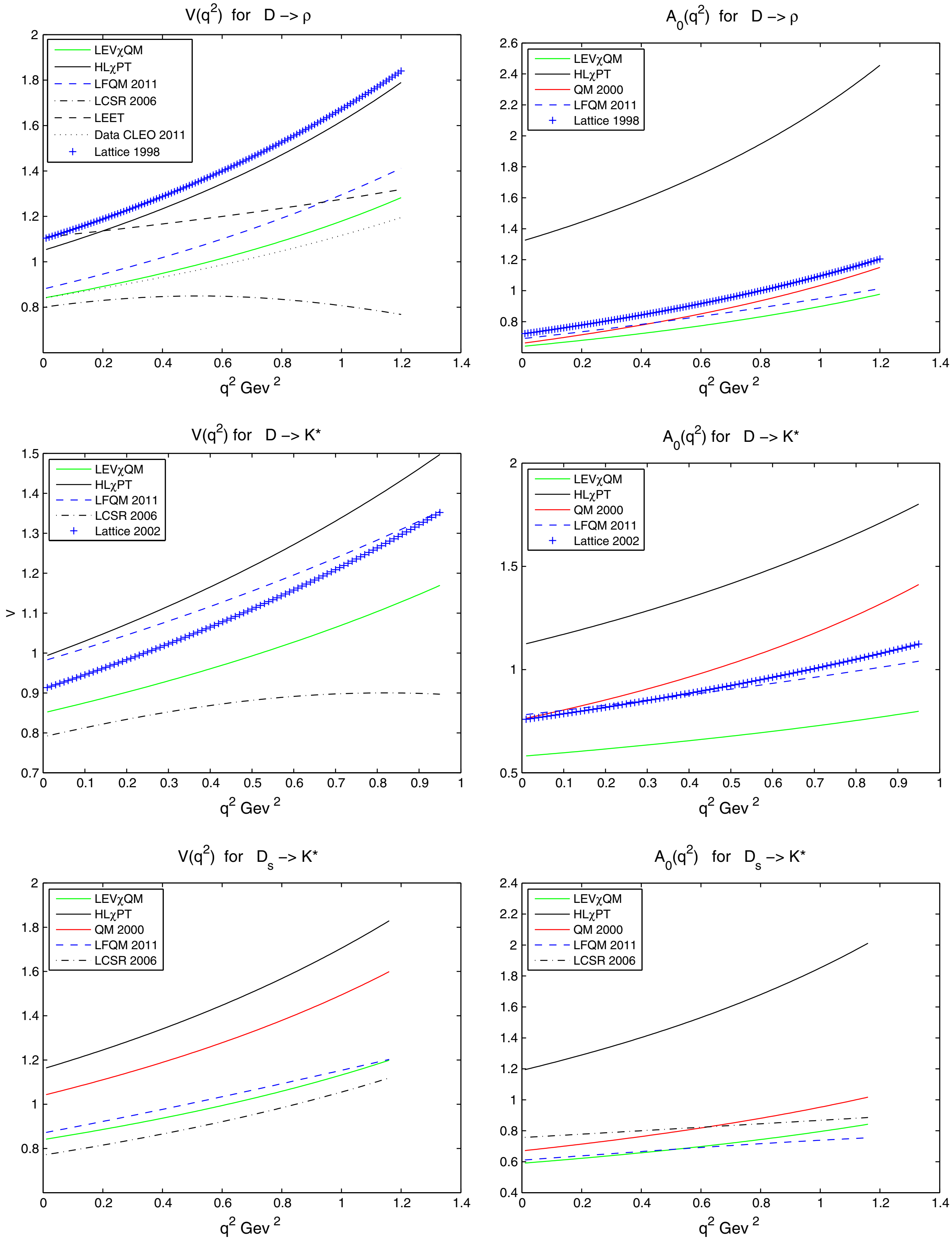

FIG. 8 (color online). $D \to V$ form factors A_0 and V: Data CLEO are from Ref. [46], LCSR 2006 is from Ref. [47], the LFQM is from Ref. [44], Lattice 1998 is from Ref. [14], Lattice 2002 is from Ref. [15], the HLχPT is from Ref. [20], and the QM 2000 is from Ref. [48]. The LEχQM is from the calculation in this paper.

where the tensor F_n is given by Eq. (25) with V_n given as in Eq. (60). Here, we assume that $\delta = m/E \ll 1$, which implies that $\zeta_\perp^{(a)} \to \zeta_\perp$.

We find the following new predictions within the LEχQM:

$$\zeta_1 = \frac{mF_\parallel}{EF}\zeta, \qquad \zeta_\perp^{(a)} = \zeta_\perp + \frac{m}{E}\zeta_\parallel, \qquad \zeta_\parallel = \frac{mF_\parallel}{m_V F}\zeta_\perp, \tag{64}$$

where

$$F_\parallel = \frac{N_c}{16\pi} + \frac{3f_\pi^2}{8m^2\rho}(1-g_A) + \frac{f_\pi^2}{2m^2}\ell + \frac{1}{48m^4}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\left(\frac{7\pi}{16}-2\right) \tag{65}$$

is a loop function analogous to F in Eq. (54), which arises from the loop integrals of the current in Eq. (63), and plotted in Fig. 5, right. Here, the appearance of $\ell = \ln(2/\delta)$ is due to the infrared behavior of some of the loop integrals. With the model parameters, we find $F_\parallel \simeq 0.24 \simeq 3F$.

The values for F and $F_\parallel$ are obtained with the simplified LEET propagator in Eq. (47). For the $B \to D$ case, an extra Δ of order 20 MeV was used in the heavy quark propagator [50]. A similar assumption (which is closer to the SCET propagator) might be used here, leading to modified values of F and $F_\parallel$. However, as we are already considering model-dependent predictions, we do not go into further details here. We observe that, although $\zeta_1/\zeta \sim \frac{m}{E}$ as it should, the numerical suppression is not strong because $F_\parallel \simeq 3F$, and $\delta = \frac{m}{E} \simeq 0.24$ is not as small for D-meson decays as it is for B decays (where $\delta \simeq 0.08$).

TABLE I. Form factors for $D \to P$ at $q^2 = 0$. The values for $F_+(0)$ are taken from data when available and from sum rules for $D \to \eta$, η'. The values for $F_+(0)_\chi$ are determined using the LEχQM.

Decay	$F_+(0)$	$F_+(0)_\chi$	ζ	ζ_1
$D \to \pi$	0.67	0.96	0.65	0.46
$D \to K$	0.74	1.06	0.65	0.44
$D \to \eta$	0.55	0.66	0.65	0.34
$D \to \eta'$	0.45	0.55	0.60	0.37

So far, we have considered the $SU(3)$ limit $m_q \to 0$. One may also calculate $SU(3)$ corrections from the mass correction Lagrangian in Eq. (48), for hard outgoing s quarks. We find that the first-order term does not contribute within the LEχQM. The second-order term in Eq. (48) contributes and gives terms suppressed by $m_s^2/(mE)$ compared to terms already calculated. These will therefore be discarded in this work. For decaying B_s and D_s, there will be first-order m_s corrections from the ordinary light sector χQM, through mass terms in Eq. (31). However, these corrections must be considered together with meson loops. Some of these loops might be calculated as in chiral perturbation theory, while others are formally suppressed and problematic to handle within our formalism. Therefore, we do not go further into these details.

VII. PLOTTING THE FORM FACTORS

In this section, we plot transition form factors for $D \to P$ in Fig. 6 and $D \to V$ in Figs. 7 and 8 as a function of the squared momentum transfer q^2. We have plotted the curves from experimental data [45,46], lattice gauge calculations [14,15], LCSRs [2–7], and the light front quark model (LFQM) [44]. The plots do not include error bars because these would make them difficult to read. For plots based on the LEET, $q^2 = 0$ is the reference point that is determined by data, and the shape is determined by a single pole.

To obtain the curves for our LEχQM for a generic form factor $F(q^2)$ (F_+, V, A_i), we use data (CLEO) for $D \to \pi$ and $D \to \rho$ for the $F(0)$'s. We then combine these $F(0)$'s with the theoretical relations in Eqs. (11), (15), and (64) to find the best numerical fit for the ζ_i's (see Tables I and II). Using the relations (15) and (64), we will obtain a reasonable overall fit for the following ζ's:

$$\zeta \simeq 0.6, \quad \zeta_1 \simeq 0.4, \quad \zeta_\perp \simeq 0.6, \quad \zeta_\parallel \simeq 0.5, \quad \zeta_\perp^{(a)} \simeq 0.7. \tag{66}$$

We have then plugged these values for the ζ_i's back in Eqs. (11) and (15) to produce values $F(0)_\chi$ for our model. We then use the single pole assumption in Eq. (13) to produce the curves for $F(q^2)_\chi$. As a biproduct, we predict the curves for other cases with no data (say with K or K^* in the final state) in the $SU(3)$ limit.

For the HLχPT, the no-recoil point $[q^2 = (q^2)_{\max}]$ is the reference point for plots that is determined by Eqs. (22) and

TABLE II. Form factors for $D \to V$ at $q^2 = 0$. The values for $V(0)$ and $A_0(0)$ are taken from LCSRs for $D_s \to K^*$ and lattice calculations for $D \to \rho$, K^*. The fitted values for $V(0)_\chi$, $A_0(0)_\chi$, $A_1(0)_\chi$, and $A_2(0)_\chi$ are determined from the ζ's, which are calculated using the LEχQM. $V(0)_\chi$ for $D \to \rho$ is the input value from CLEO data.

Decay	$V(0)$	$V(0)_\chi$	$A_0(0)$	$A_0(0)_\chi$	$\zeta_\perp$	$\zeta_\perp^{(a)}$	$\zeta_\parallel$	$A_1(0)$	$A_1(0)_\chi$	$A_2(0)$	$A_2(0)_\chi$
$D \to \rho$	0.84	0.84	0.65	0.64	0.59	0.69	0.50	0.56	0.58	0.47	0.48
$D \to K^*$	0.91	0.87	0.76	0.64	0.58	0.68	0.50	0.62	0.57	0.37	0.43
$D_s \to K^*$	0.77	0.86	0.76	0.64	0.58	0.67	0.43	0.59	0.55	0.32	0.44

(26). The plots for $D \to P$ with $P = \pi$, K, η are different because of the different masses. However, we have not explicitly calculated $SU(3)$-breaking effects, and Eq. (66) should be valid in the $SU(3)$ limit $m_s \to 0$. This means the plots for $D \to \pi$ and $D \to \rho$ are the most relevant. The other plots are included for comparison. According to our model [see Eq. (48)], $SU(3)$ corrections due to hard s quarks (as in $D \to K$ and $D \to K^*$ transitions) should be small, while $SU(3)$ corrections due to soft s quarks (as in decays of D_s) should be larger, as pointed out at the end of Sec. VI.

VIII. CONCLUSIONS

We have collected present information on various form factors for the transitions $D \to P$ and $D \to V$ ($P =$ pseudoscalar, $V =$ vector) obtained from various methods and sources such as data, lattice gauge theory, LCSRs, etc. From the plots, we have as far as possible determined the values of relevant form factors at $q^2 = 0$ and then extracted values for the LEET form factors ζ_i. The LEχQM gives relations between the ζ_i's. We have previously found [40] $\zeta_1/\zeta \sim m/E$. Here, we have in addition found relations between the ζ's and have shown that $\zeta_\perp^{(a)} \to \zeta_\perp$ for $m/E \to 0$ as it should.

We observe that the curves for the form factor F_+ for the case $D \to \pi$ show a remarkable agreement for $q^2 \to 0$ (for the LEET, this is done by construction). This is in contrast to the values of $V(0)$ for which the plots show a large variation among the various methods used. This makes $\zeta_\perp$ uncertain. However, $\zeta_\perp$ is also related to $A_0(0)$ such that we obtain a reasonable fit using Eqs. (13) and (66). We observe what we expected, namely, that the LEET and LEχQM work best for q^2 close to zero, while the HLχQM (eventually supplemented by the HLχPM) works best close to the no-recoil point.

The LEχQM gives a good fit to the V and A_0 form factors for the $D \to \rho$ and $D_s \to K^*$ transitions. However, for the $D \to K^*$ transition, the V and A_0 curves lie below the curves for the lattice data. For the $D \to K^*$ transition calculation, the hard quark in the loop is an s quark. We did not include the correction, which is on the order of the mass of the s quark, m_s. This is a source of small error for this transition. We observe that the LEχQM values for the axial form factor A_1, being transverse to the momentum [see Eqs. (5) and (15)], do not match well for any of the transitions.

The LEET form factors ζ and $\zeta_\perp$, together with data for the $D \to \pi$ and $D \to \rho$ transitions, will determine the coupling constants G_A and G_V, which may be used in the calculation of nonfactorizable (color suppressed) non-leptonic D-meson decays, in the same manner as has previously been done for $K \to \pi\pi$ [39,51], $D \to K^0\bar{K}^0$ [52], $B \to D\bar{D}$ [53,54], $B \to D\pi$ [40], and $B \to \pi^0\pi^0$ [41]. Then nonleptonic decay amplitudes can be written in terms of the LEET form factors ζ_i, both for the factorized and the color-suppressed cases. We are, of course, aware that the LEET expansion might have relatively large corrections beyond the order considered here. Still, we think that our results will be helpful for further studies of nonfactorizable nonleptonic decay amplitudes for D mesons.

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