Limits on the quartic couplings $Z\gamma\gamma\gamma$ and $ZZ\gamma\gamma$ from e^+e^- colliders

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We obtain limits on the quartic neutral gauge bosons couplings $Z\gamma\gamma\gamma$ and $ZZ\gamma\gamma$ using LEP 2 data published by the L3 Collaboration on the reactions $e^+e^- \to \gamma \gamma \gamma$, Z $\gamma \gamma$. We also obtain 95% C.L. limits on these couplings at the future linear colliders' energies. The LEP 2 data induce limits of order 10−⁵ GeV−⁴ for the Zγγγ couplings and of order 10^{-3} GeV⁻² for the ZZγγ couplings, which are still above the respective standard model predictions. Future e^+e^- linear colliders may improve these limits by one or two orders of magnitude.

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I. INTRODUCTION

Neutral gauge bosons self-couplings provide a window to study physics beyond the standard model (SM) [\[1](#page-4-0)*–*3]. While trilinear neutral gauge boson couplings (TNGC) $V_iV_iV_k$, with $V_i = Z$, γ , test the gauge structure of the SM [\[3\]](#page-4-1), it has been argued that quartic neutral gauge boson couplings (QNGC) $V_iV_iV_kV_l$ may provide a connection to the mechanism of electroweak symmetry breaking [\[1\]](#page-4-0). Since the longitudinal components of the Z gauge boson are Goldstone bosons associated to the electroweak symmetry breaking mechanism, these QNGC could represent then a connection with the scalar sector of the gauge theory that has become popular after the recent evidence of a new boson with a mass around 125 GeV [\[4\].](#page-4-2) However, it has been found recently in a detailed calculation of the oneloop induced decay mode $Z \rightarrow \gamma \gamma \gamma$, in both the SM and the 331 model, that the respective scalar contributions are suppressed with respect to the dominant virtual fermionic contributions [\[5\].](#page-4-3) This is also the case in the one-loop contributions to TNGC [\[3,6\]](#page-4-1). The QNGC are induced by effective operators of dimension greater or equal to six and, in the SM, the QNGC are highly suppressed, with the only exception of the ZZZZ vertex, because they arise at the one-loop level [\[6,7\].](#page-4-4) Any deviation from the SM predictions for the QNGC will be associated to a signal of new physics effects [\[1\].](#page-4-0)

While considerable effort has been devoted to the study of the TNGC, the QNGC are only starting to receive some attention. TNGC have been measured with an accuracy of the few percent level at LEP 2 [\[8\]](#page-4-5) and the Tevatron [\[9\]](#page-4-6), while QNGC are only loosely constrained at LEP 2 [\[8\].](#page-4-5) In fact, the Zγγγ couplings have not been bounded yet by direct measurements [\[8\]](#page-4-5). In the present paper, we are interested in obtaining limits on the quartic vertices Zγγγ and ZZγγ coming from the LEP 2 data on the reactions $e^+e^- \rightarrow \gamma \gamma \gamma$, Z $\gamma \gamma$ that were used to get limits on the anomalous $HZ\gamma$ coupling but not on the QNGC [\[10,11\]](#page-4-7). We will obtain also 95% C.L. limits on these quartic couplings at the future International Linear Collider (ILC) and the Compact Linear Collider (CLIC) [\[12,13\].](#page-5-0) Since there is not a published account, as far as we know, of the calculation of the $Z\gamma\gamma\gamma$ vertices in the SM, we present a brief analysis on the connection of the Zγγγ form factors to the analytical results obtained in Ref. [\[5\]](#page-4-3) for the branching ratio of the decay mode Zγγγ in both the SM and the 331 model. However, a similar calculation for the ZZγγ form factors in the SM is not available in the published literature.

Constraints on the anomalous quartic gauge couplings ZZγγ have been studied in $γγ$ and $Zγ$ fusion processes at the LHC [\[14\]](#page-5-1), in $ZZ\gamma$, $Z\gamma\gamma$ production processes at future e^+e^- linear colliders [\[6\]](#page-4-4) and from the nonobservation of the rare decay $Z \rightarrow \nu \bar{\nu} \gamma \gamma$ at LEP 1 [\[7\]](#page-4-8). However, constraints on the anomalous $Z\gamma\gamma\gamma$ vertex are more difficult to get. In the present paper we find that the negative search for the reactions $e^+e^- \rightarrow \gamma \gamma \gamma$, Z $\gamma \gamma$ at LEP 2 by the L3 Collaboration may be translated into limits of order 10−⁵ GeV−⁴ on the Zγγγ couplings and of order 10^{-3} GeV⁻² on the ZZ_{$\gamma\gamma$} couplings. We also find that sensitivity studies on these couplings at future e^+e^- colliders may improve these limits by one or two orders of magnitude.

The paper is organized as follows. In Sec. [II](#page-0-2) we present the calculation of the respective cross sections for the processes $e^+e^- \rightarrow \gamma \gamma \gamma$, $Z \gamma \gamma$ and in Sec. [III](#page-2-0) we include our results and conclusions. In particular, we present the connection among our quartic couplings $G_{1,2}$ and the results obtained in Ref. [\[5\]](#page-4-3) for the branching ratio of the decay mode $Z \rightarrow \gamma \gamma \gamma$ in the SM and the 331 model.

II. CROSS SECTIONS

We will use the following parametrizations for the QNGC [\[2,15\]](#page-4-9),

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$$
\mathcal{L}_{Z\gamma\gamma\gamma} = \frac{G_1}{\Lambda^4} F_{\mu\nu} F^{\mu\nu} F_{\rho\sigma} Z^{\rho\sigma} + \frac{G_2}{\Lambda^4} F_{\mu\nu} F^{\mu\rho} F_{\rho\sigma} Z^{\sigma\nu}, \quad (1)
$$

$$
\mathcal{L}_{ZZ\gamma\gamma} = -\frac{e^2}{16\Lambda^2 c_W^2} a_0 F_{\mu\nu} F^{\mu\nu} Z^{\alpha} Z_{\alpha} \n- \frac{e^2}{16\Lambda^2 c_W^2} a_c F_{\mu\nu} F^{\mu\alpha} Z^{\nu} Z_{\alpha},
$$
\n(2)

where $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$ and $Z_{\mu\nu} = \partial_{\mu}Z_{\nu} - \partial_{\nu}Z_{\mu}$ are the respective gauge tensor fields for the photon and the Z boson. Λ represents the energy scale at which new physics interactions may appear. The respective Feynman rules for these effective vertices are thus given by

$$
\sum_{a=1}^{6} iP_{a} \Big\{ \frac{G_{1}}{\Lambda^{4}} [(p_{1} \cdot p_{2})(p_{2} \cdot p_{3})g_{\alpha\rho}g_{\mu\nu} - (p_{1} \cdot p_{3})p_{1\nu}p_{2\mu}g_{\alpha\rho} - (p_{1} \cdot p_{3})p_{1\nu}p_{2\alpha}g_{\rho\mu} + p_{1\nu}p_{1\rho}p_{2\mu}p_{3\alpha}] + \frac{G_{2}}{\Lambda^{4}} [-(p_{1} \cdot p_{2})(p_{1} \cdot p_{3})g_{\alpha\mu}g_{\rho\nu} + (p_{2} \cdot p_{3})p_{1\alpha}p_{1\nu}g_{\rho\mu} - (p_{2} \cdot p_{3})p_{1\nu}p_{1\rho}g_{\alpha\mu} + (p_{2} \cdot p_{3})p_{1\nu}p_{2\alpha}g_{\rho\mu} + 2(p_{2} \cdot p_{3})p_{1\rho}p_{2\mu}g_{\alpha\nu} - (p_{1} \cdot p_{3})p_{1\alpha}p_{2\rho}g_{\mu\nu} - p_{1\alpha}p_{1\nu}p_{2\rho}p_{3\mu}] \Big\},
$$
(3)

and

$$
\frac{ie^2}{8c_W^2\Lambda^2} \{4a_0 g^{\alpha\beta}[(p_1 \cdot p_2)g^{\mu\nu} - p_1^{\nu} p_2^{\mu}] + a_c [(p_1^{\alpha} p_2^{\beta} + p_1^{\beta} p_2^{\alpha})g^{\mu\nu} + (p_1 \cdot p_2)(g^{\mu\alpha} g^{\nu\beta} + g^{\nu\alpha} g^{\mu\beta}) - p_1^{\nu} (p_2^{\beta} g^{\mu\alpha} + p_2^{\alpha} g^{\mu\beta}) - p_2^{\mu} (p_1^{\beta} g^{\nu\alpha} + p_1^{\alpha} g^{\nu\beta})]\},
$$
\n(4)

where the four momenta $p_{1,2,3}$ correspond to the emitted photons and P_a denotes possible permutations $(p_1, \mu) \leftrightarrow (p_2, \nu) \leftrightarrow (p_3, \rho).$

All the couplings $G_{1,2}$ and $a_{0,c}$ are CP conserving and within the SM all of them vanish at tree level. As far as we know, the $a_{0,c}$ have not been computed explicitly in the SM, whereas the couplings $G_{1,2}$ can be extracted directly from the recent calculation performed in the SM and the 331 model [\[16\]](#page-5-2) for the branching ratio of the rare decay mode $Z \rightarrow \gamma \gamma \gamma$ [\[5\]](#page-4-3). Since these authors did not use explicitly the parametrization given in Eqs. [\(1\)](#page-0-4) and [\(2\),](#page-1-0) we include the connection of the $G_{1,2}$ couplings to the results obtained for the $Z \rightarrow \gamma \gamma \gamma$ decay in Ref. [\[5\]](#page-4-3). These form factors are dominated by the fermionic virtual contributions and they are essentially the same in both the SM and the 331 model, but unfortunately with rather low values, 1.63×10^{-10} and 1.33×10^{-10} , respectively.

In Figs. [1](#page-1-1) and [2](#page-1-2) we present the contributions of the effective interactions given in Eqs. [\(3\)](#page-1-3) and [\(4\)](#page-1-4) to the

couplings $HZ\gamma$ and $H\gamma\gamma$. However, using their data we are able to get also limits on the $G_{1,2}$ and $a_{0,c}$ couplings: r.z Smr

processes $e^+e^- \rightarrow \gamma \gamma \gamma$ and $e^+e^- \rightarrow Z \gamma \gamma$. The SM contributions to these processes occur via t-channel diagrams involving initial-state radiation [\[2,6\]](#page-4-9). The SM cross section for the process $e^+e^- \rightarrow Z\gamma\gamma$ has been computed by Stirling and Werthenbach [\[6\]](#page-4-4) for energies greater than 200 GeV. In Fig. [3](#page-2-1) we present the SM results for the cross sections of both processes, they are of order of few femtobarns as it was obtained in Ref. [\[6\]](#page-4-4) for the $Z\gamma\gamma$ case. According to this reference, in order to reduce the contributions due to initialstate radiation in these reactions, the L3 Collaboration introduced cuts on the photon energies and their polar angles, $E_{\gamma} > 5$ GeV and $|\cos \theta_{\gamma}| < 0.97$ [\[10\]](#page-4-7). Events from $e^+e^- \rightarrow \gamma \gamma \gamma$, $Z \gamma \gamma$ processes were selected by requiring the photon candidates to lay in the central region of the detector with $|\cos \theta_y|$ < 0.8 and a total CM electromagnetic energy large than $\sqrt{s}/2$. In this case, the L3 Collaboration was interested in getting limits on the anomalous Higgs

FIG. 1. Feynman diagram for the process $e^+e^- \rightarrow \gamma \gamma \gamma$ induced by the effective vertex Zγγγ.

FIG. 2. Feynman diagrams for the process $e^+e^- \rightarrow Z\gamma\gamma$ induced by the effective vertices $ZZ\gamma\gamma$ and $Z\gamma\gamma\gamma$.

FIG. 3. Cross sections for the processes $e^+e^- \rightarrow \gamma \gamma \gamma$, Z $\gamma \gamma$ as functions of the CM energy in the SM. We have used cuts on the photon energies and their polar angles, $E_y > 20$ GeV and $|\cos \theta_{\gamma}| < 0.8$.

 $G_1/\Lambda^4 < 1.2 \times 10^{-5} \,\text{GeV}^{-4}$, $G_2/\Lambda^4 < 9.4 \times 10^{-6} \,\text{GeV}^{-4}$, a_0/Λ^2 < 5.9 × 10⁻³ GeV⁻² and a_c/Λ^2 < 1.6 × 10⁻² GeV⁻². The latter limits are close to the bounds obtained by the L3 Collaboration from a direct search of Zγγ events at LEP 2 energies: $[-0.008, 0.021] \text{GeV}^{-2}$ and $[-0.029, 0.039] \text{GeV}^{-2}$, respectively [\[8\].](#page-4-5)

The expressions for the respective cross sections induced by the effective vertices given in Eqs. [\(3\)](#page-1-3) and [\(4\)](#page-1-4) are given by

$$
\sigma(e^+e^- \to \gamma\gamma\gamma) = \frac{\alpha M_Z^{10}}{1105920\pi^2} \left[\frac{1 - 4x_W + 8x_W^2}{x_W(1 - x_W)} \right] \times \left[\frac{2(\frac{G_1}{\Lambda^4})^2 + 3(\frac{G_2}{\Lambda^4})^2 - 3(\frac{G_1}{\Lambda^4})(\frac{G_2}{\Lambda^4})}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \right], \quad (5)
$$

and

$$
\sigma(e^+e^- \to Z\gamma\gamma) = \frac{\alpha M_Z^6 (s - M_Z^2)^4}{5308416\pi^2 s^4} \left[71 \left(\frac{G_1}{\Lambda^4} \right)^2 + 138 \left(\frac{G_1}{\Lambda^4} \right) \left(\frac{G_2}{\Lambda^4} \right) + 96 \left(\frac{G_2}{\Lambda^4} \right)^2 \right] + \frac{5\alpha M_Z^6 (s - M_Z^2)^4}{4608\pi^2 s^4 [(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2]} \times \left[\frac{1 - 4x_W + 8x_W^2}{x_W (1 - x_W)} \right] \times \left[\frac{a_0^2}{\Lambda^4} + \frac{1}{8} \frac{a_c^2}{\Lambda^4} + \frac{1}{2} \frac{a_c}{\Lambda^2} \frac{a_0}{\Lambda^2} \right].
$$
 (6)

In Eq. [\(6\),](#page-2-2) the first term comes from the Feynman diagrams shown in Fig. [2](#page-1-2) for the exchanged photon, and the second one comes from the exchanged Z boson. We did not include the contribution coming from the interference of the two diagrams because we will get limits on the form factors one at a time.

III. RESULTS AND CONCLUSIONS

In order to get limits on the $G_{1,2}$ quartic couplings, we shall use the bound obtained by the L3 Collaboration on the cross section $\sigma(e^+e^- \to Z \to \gamma\gamma\gamma)$ at LEP 2 energies [\[10\]](#page-4-7) and our expression for this cross section in terms of the quartic couplings given in Eq. (5) . Similarly, we have used the bounds on the cross section $\sigma(e^+e^- \to Z\gamma\gamma)$ obtained by the L3 Collaboration in order to get limits on the $a_{0,c}$ couplings specified in Eq. [\(6\).](#page-2-2) Since this expression does not improve the limits on the quartic couplings $G_{1,2}$, in this case we have set them to zero in order to obtain the limits on the $a_{0,c}$ couplings. We obtain in this approach $G_1/\Lambda^4 < 1.2 \times 10^{-5}$ GeV⁻⁴, $G_2/\Lambda^4 < 9.4 \times 10^{-6}$ GeV⁻⁴, $a_0/\Lambda^2 < 5.9 \times 10^{-3}$ GeV⁻², and $a_c/\Lambda^2 < 1.6 \times 10^{-2} \text{ GeV}^{-2}$. The respective 95% sensitivity limits for the $a_{0,c}$ couplings will be obtained for CM energies of 500 and 1000 GeV and the luminosity expected at the ILC/CLIC accelerators: 500 fb⁻¹.

In both cases we shall rely on the respective SM predictions for their cross sections. In Fig. [3](#page-2-1) we depict the dependence of these cross sections with respect to the CM energy. We have used cuts on the photon energies and their polar angles: $E_y > 20$ GeV and $|\cos \theta_y| < 0.8$. In the case of the production of the three photons, our result agrees with that obtained by Stirling and Werthenbach [\[6\]](#page-4-4). On the other hand, the Feynman diagrams shown in Figs. [1](#page-1-1) and [2](#page-1-2) for the effective vertices contributions generate an increase for the cross sections with respect to the CM energy that may dominate the SM contributions. We will use this tendency in order to get 95% C.L. limits on these effective vertices for future e^+e^- colliders.

Using the numerical values $\sin^2 \theta_W = x_W = 0.2314$, $M_Z = 91.18$ GeV, and $\Gamma_Z = 2.49$ GeV [\[17\]](#page-5-3), we obtain the cross sections for the processes $e^+e^- \rightarrow \gamma \gamma \gamma$, $Z \gamma \gamma$ as functions of the CM energy and the $G_{1,2}$, $a_{0,c}$ couplings. We have also implemented in our calculation the cut used by the L3 Collaboration on the photons energy and their polar angle in order to suppress the SM contributions associated to initial-state radiation. In Fig. [4](#page-3-0) we depict the sensitivity limits at 95% C.L. for the $G_{1,2}$ couplings for CM energies of 500 GeV and 1000 GeV and we have taken the $G_{1,2}$ couplings one at the time. The respective combined limits contours are shown in Fig. [5](#page-3-1). On the other hand, in order to get sensitivity limits and the respective limits contours for the $a_{0,c}$ couplings we have set to zero the contribution associated to the $G_{1,2}$ couplings in Eq. [\(6\)](#page-2-2). The respective limits are given in Figs. [6](#page-3-2) and [7](#page-3-3) also for two different values of the energy of the ILC and CLIC accelerators.

FIG. 4. Sensitivity limits at 95% C.L. for the couplings $G_{1,2}/\Lambda^4$ [GeV⁻⁴] as functions of the integrated luminosity for two ILC/CLIC CM energies. We have taken the $G_{1,2}$ couplings one at the time.

In Figs. [4](#page-3-0) and [6](#page-3-2) we have used the statistical significance expression given in terms of the expected number of signal and background events in the reactions $e^+e^- \rightarrow \gamma \gamma \gamma$, $Z \gamma \gamma$ [\[18\]](#page-5-4). We have assumed that the background events arise

FIG. 5. Contour limits at 95% C. L. in the G_1-G_2 plane for the process $e^+e^- \rightarrow \gamma \gamma \gamma$ for $\sqrt{s} = 500$, 1000 GeV and $\mathcal{L} = 500$ fb⁻¹.
We have taken the G_{ree} couplings simultaneously We have taken the $G_{1,2}$ couplings simultaneously.

FIG. 7. Contour limits at 95% C. L. in the a_0 - a_c plane for the process $e^+e^- \to Z\gamma\gamma$ for $\sqrt{s} = 500, 1000$ GeV and $\mathcal{L} = 500$ fb⁻¹.
We have taken the a₀₀ couplings simultaneously We have taken the $a_{0,c}$ couplings simultaneously.

0.0000

 $a_0/\Lambda^2[GeV^{-2}]$

0.0001

0.0002

 -0.0001

 -0.0002

 30_C

300

 $500 GeV$

 $1000\ GeV$

 400

400

500

500

 $500\,GeV$

 $1000\ GeV$

 \sqrt{s} =

 10^c

100

 \sqrt{s}

 0.0005

0.0004

 0.0003

0.0002

0.0001

 0.0000 Ω

 0.0014

0.0012

 0.0010

 0.0008

 0.0006

 0.0004 0.0002

 $0.000($

 $a_0/\Lambda^2[GeV^{-2}]$

 $a_c/\Lambda^2 [GeV^{-2}]$

TABLE I. Values of $G_{1,2}/\Lambda^4$ [GeV⁻⁴] as function of BR according to Eq. (16) of Refs. [\[15,19\]](#page-5-7). We include the PDG 2012 limit for $BR(Z \rightarrow \gamma \gamma \gamma)$ [\[17\]](#page-5-3).

BR	$ G_1/\Lambda^4 \text{GeV}^{-4} $	$ G_2/\Lambda^4[\rm GeV^{-4}] $
10^{-5} (PDG)	2.22×10^{-8}	1.81×10^{-8}
5.41×10^{-10} (SM)	1.63×10^{-10}	1.33×10^{-10}
5.26×10^{-10} (331)	1.61×10^{-10}	1.31×10^{-10}

than the new physics contributions. The number of expected events then are given by the integrated luminosity and the respective cross section. In Figs. [5](#page-3-1) and [7](#page-3-3) we used two CM energies planned for the ILC/CLIC accelerators in order to get 95% C.L. contour limits for the Zγγγ and ZZγγ effective couplings and the planned luminosity of 500 fb⁻¹. We can appreciate that these limits are about two orders of magnitude lower with respect to those obtained from the data obtained by the L3 Collaboration.

In order to compare our results for the quartic couplings with the SM predictions, we use the results obtained in Ref. [\[5\]](#page-4-3) for the branching ratio of the decay $Z \rightarrow \gamma \gamma \gamma$. A complete one-loop calculation of the Feynman diagrams for this decay mode was presented for the SM and the 331 model [\[16\]](#page-5-2). The relation of our quartic couplings $G_{1,2}$ to the F_{Z_i} form factors used in this reference is given by

$$
\left(\frac{G_{1,2}}{\Lambda^4}\right)^2 = 2\left[\frac{8\alpha^2(M_Z)}{s_W c_W}\right]^2
$$

$$
\times \int_0^1 \int_{1-x}^1 |F_{ZG_{1,2}}^{\frac{1}{2}} + F_{ZG_{1,2}}^1 + F_{ZG_{1,2}}^0|^2 dy dx,
$$
 (7)

where x and y are kinematical variables associated to the $Z \rightarrow \gamma \gamma \gamma$ decay mode [\[5\]](#page-4-3) and $g = e/s_W = \sqrt{4\pi \alpha}/s_W$. Each F_{α} form factor identifies the fermionic vectorial and F_{Z_i} form factor identifies the fermionic, vectorial, and scalar contributions to the one-loop diagrams. It was found that the dominant contribution comes from the fermionic amplitudes. In Table [1](#page-4-10) we include the predictions expected for the $G_{1,2}$ quartic couplings for the SM, 331 model, and from the PDG limits for the respective branching ratio using the expression for the decay width

$$
\Gamma(Z \to \gamma \gamma \gamma) = \frac{M_Z^9}{552960\pi^3} \frac{(2G_1^2 + 3G_2^2 - 3G_1G_2)}{\Lambda^8}.
$$
 (8)

In conclusion, we have obtained limits on the quartic couplings Zγγγ and ZZγγ at LEP 2 energies by using published L3 data for the reactions $e^+e^- \rightarrow \gamma \gamma \gamma$, Z $\gamma \gamma$. Our limits obtained from the LEP 2 data on the reaction $e^+e^- \rightarrow Z\gamma\gamma$ are close to the best limits obtained in the LEP collider [\[8\]](#page-4-5). In this case, SM predictions for the $a_{0,c}$ couplings are not available in the literature. Our 95% sensitivity limits expected for these couplings at ILC/CLIC energies are of order 10−⁴ GeV−² for a luminosity of 500 fb⁻¹. These limits are close to those obtained by Stirling and Werthenbach for a 300 fb⁻¹ luminosity [\[6\]](#page-4-4). Similar limits have been obtained from the process $e^+e^- \rightarrow$ $\nu \bar{\nu} \gamma$ [\[20\]](#page-5-5) and through effects induced by the polarization of the Z gauge boson and initial state radiation in the process $e^+e^- \rightarrow Z\gamma\gamma$ [\[21\].](#page-5-6)

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