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Pseudo-Dirac neutrinos via a mirror world and depletion of ultrahigh energy neutrinos

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We propose a particle physics explanation of the nonobservation of muon neutrino events at IceCube coincident with gamma ray bursts (GRBs) at the rates predicted by the standard Bahcall-Waxman model, in terms of neutrino oscillations. Our model is based on assuming that (a) all neutrinos are pseudo-Dirac particles and (b) there exists a mirror world interacting gravitationally with the observed world. This scenario has three sterile neutrinos associated with each flavor of ordinary neutrinos. Very tiny mass splitting between these neutrinos is assumed to arise from lepton number violating dimension-five Planck scale suppressed operators. We show that if a mass splitting of $10^{-15}\ \mathrm{eV}^2$ is induced between the four mass eigenstates of a given species, then its flux will be suppressed at IceCube energies by a factor of 4 compared to GRB model predictions. Hierarchies in mass splitting among different flavors may result in different amounts of suppression of each flavor, and based on this we predict a difference in the flavor ratios of the observed neutrinos that is significantly different compared to the standard three-flavor prediction of 1 : 1 : 1, which could serve as a test for this model.

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I. INTRODUCTION

Gamma ray bursts (GRBs) are believed [1–3] to be responsible for accelerating the charged cosmic rays to very high energies and a sizable fluence of neutrinos is expected from GRBs through the interactions of protons with photons in the fireball [4]. The $p\gamma$ interaction produces charged pions, $p+\gamma \to \Delta^+ \to n+\pi^+$, and subsequently the decays $\pi^+ \to \mu^+\nu_\mu$ and $\mu^+ \to e^+\nu_e\bar{\nu}_\mu$ produce muon and electron neutrinos coincident in direction with γ rays. The energy spectrum of these neutrinos is expected to peak in the range 10^5–10^7 GeV, and in km^3-sized detectors like IceCube and ANTARES, about 10–100 neutrino events per year, coincident with GRB photons, are predicted [5–12]. So far, no muon neutrino events coincident with GRB photons have been detected in measurements at IceCube [13,14], which looks for the charged-current produced muons from ν_μ through the Cerenkov radiation of upward-going muons. Other experiments probing the ultrahigh energy regime, such as ANITA[15], have not seen any evidence of PeV energy neutrinos in association with GRB events either. A recent observation [16] of a GRB at a low redshift of $z \sim 0.34$ and an estimated isotropic electromagnetic energy of 10^{54} makes it the most energetic of GRB emissions seen at $z < 0.5$. No neutrinos coincident with this GRB event within 100 s and 3.5° were seen at IceCube [17]. A search for muon neutrinos associated with GRBs performed with the ANTARES detector [18] shows no events over the atmospheric neutrino background.

There are two PeV neutrino events which have been seen at IceCube [19]. They are expected to be of astrophysical origin [20]. There are a further 26 events seen at IceCube at energies between 0.02–0.3 PeV [21]. These 28 events may be initiated by neutrinos from dark matter decay [22,23], or the neutrinos may have a astrophysical origin [24]. There are seven track events consistent with the canonical 1 : 1 : 1 flavor ratio of neutrinos.

In the light of the IceCube nonobservation of the GRB neutrinos, the standard GRB fireball or internal shock model parameters have been revisited [25–29] in a full numerical calculation by taking into account in greater detail dilution of charged pions and kaons in the expanding fireball and due to multipion and kaon production. The new neutrino flux limits (and some models which predate the IceCube measurements [30]) are consistent with the upper bound put from IceCube [13,14,17]. Other ways of explaining the paucity of IceCube neutrinos without overthrowing the GRB models is to explain it by oscillations of the flavor neutrinos into sterile ones. There exist strong constraints on the possible oscillations of active to sterile neutrinos from the terrestrial experiments [31–33] and from nucleosynthesis [34–36]. But due to very high energy and long distances, the relevant mass scale probed through the active sterile oscillations of ultrahigh energy (UHE) are completely different and are as yet unconstrained. For example, consider the neutrinos from GRBs (at cosmological distances $L = 1000$–3000 Mpc) and in the range $E = 10^5$–10^7 GeV. These active UHE neutrinos

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(ν_α, $\alpha = e, \mu, \tau$) can significantly oscillate to sterile neutrinos (ν') if the $(\mathrm{mass})^2$ difference between them is

$$\Delta m^2_{\nu_\alpha \nu'} \geq 0.8 \times 10^{-15}\ \mathrm{eV}^2 \left(\frac{1000\ \mathrm{Mpc}}{L}\right)\left(\frac{E}{10^7\ \mathrm{GeV}}\right), \tag{1}$$

and mixing between them is significant. A pair of active and sterile neutrinos with the above $(\mathrm{mass})^2$ difference may be regarded as a pseudo-Dirac neutrino, which in some limit can be considered a Dirac particle respecting some unbroken Lepton number. Small violation of this symmetry not only splits them but also mixes them maximally. UHE neutrinos can thus be used to probe the pseudo-Dirac nature of neutrinos [37–43]. The maximal mixing within a pseudo-Dirac pair can bring a suppression in the original flux by a factor of $1/2$. Note that this is over and above the suppression in muon neutrino flux resulting from the active to active oscillation, which is already included in the Waxman-Bahcall prediction [5] of the neutrino flux. If $\Delta m^2_{\nu_\alpha \nu'}$ is in the range 10^{-18}–10^{-16} eV^2, the interference terms in the oscillation probability do not average to zero for the range of redshifts of the observed GRBs, and the suppression factor can be lower than $1/2$ [42]. Other explanations offered for the IceCube muon neutrino deficit are neutrino spin flip in a magnetic field [44] and neutrino decays over cosmological distances [43,45].

We discuss a scenario where there can be a larger suppression, by a factor of $\sim 1/4$, compared to the predictions of the astrophysical models, in the flux of UHE neutrinos (a) if all neutrinos are pseudo-Dirac and (b) if there exists a mirror world [46–48] interacting gravitationally with the observed world. Global lepton number breaking through the gravitational interactions provides a source in this scenario, which can split the mass eigenstates of all the Dirac neutrinos and make them pseudo-Dirac.

II. PSEUDO-DIRAC NEUTRINOS VIA A MIRROR WORLD

First we consider the case of a single flavor, say, ν_μ. Assume that ν_μ is a Dirac particle and is accompanied by a mirror "muon neutrino," also a Dirac particle. They together consist of four, two-component, left-handed states labeled as $\nu'_{\mu a}$ with $a = 1\ldots4$. Of these, $\nu'_{\mu 1}$ is active and the others are sterile states. Their mass matrix to the leading order is given by

$$\mathcal{M}^0_\mu = m_{\nu_\mu}\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}. \tag{2}$$

We have assumed the same mass for the both the Dirac states. All the zeros in the above mass matrix are protected by Lepton numbers in our and the mirror worlds. Breaking of these symmetries is assumed to induce small entries in places of zeros. Thus for example, nonzero values for 11 and its mirror symmetric 33 elements are induced by the conventional dimension-five Weinberg operator in our and the mirror world, respectively, and may arise from the seesaw mechanism or gravity-induced effects. Following the mirror world scenario [46], we assume that ordinary and mirror worlds communicate only gravitationally with each other, thus $(\nu'_{\mu 1}, \nu'_{\mu 3})$ get coupled only gravitationally through the dimension-five operator [46–48],

$$\mathcal{L}_{\mathrm{comm}} = \frac{\lambda_{13}}{M_P}(\nu'_{\mu 1}\phi)(\nu'_{\mu 3}\phi'), \tag{3}$$

where ϕ and ϕ' are the neutral components of the Higgses in our and the mirror world, respectively. The contribution of this dimension-five operator to the mass matrix is

$$\epsilon_{13} \equiv \frac{\lambda_{13} v^2}{M_P}, \tag{4}$$

where we have taken $v = \langle\phi\rangle \simeq \langle\phi'\rangle \simeq 174$ GeV. The sterile partners ($\nu'_{\mu 2}$ and $\nu'_{\mu 4}$) may couple to a different set of Higgs η and η' and assuming that these Higgs VEVs are at the TeV scale, one can have a gravitational mixing term from all the sterile pairs. For example $(\nu'_{\mu 2}, \nu'_{\mu 4})$ will mix via the operator

$$\mathcal{L}'_{\mathrm{comm}} = \frac{\lambda_{24}}{M_P}(\nu'_{\mu 2}\eta)(\nu'_{\mu 4}\eta'), \tag{5}$$

where η and η' are the neutral components of the Higgses in our and the mirror world, respectively. The contribution of this dimension-five operator to the mass matrix is

$$\epsilon_{24} = \frac{\lambda_{24}}{M_P}\langle\eta\rangle\langle\eta'\rangle. \tag{6}$$

Taking similar terms for mixing of all sterile pairs, we can write the mass matrix in the $\nu'_{\mu a}$ basis as

$$\mathcal{M}_\mu = \begin{pmatrix} \epsilon_{11} & m_{\nu_\mu} & \epsilon_{13} & \epsilon_{14} \\ m_{\nu_\mu} & \epsilon_{22} & \epsilon_{23} & \epsilon_{24} \\ \epsilon_{13} & \epsilon_{23} & \epsilon_{11} & m_{\nu_\mu} \\ \epsilon_{14} & \epsilon_{24} & m_{\nu_\mu} & \epsilon_{22} \end{pmatrix}. \tag{7}$$

We have assumed that the ordinary and mirror worlds are symmetric if their mixing is neglected, and thus have assumed $\epsilon_{33} = \epsilon_{11}$ and $\epsilon_{22} = \epsilon_{44}$ in Eq. (7). We also assume that ε parameters are $\ll m_{\nu_\mu}$ and neutrinos remain pseudo-Dirac. Let

$$\mathcal{V}_4^T \mathcal{M}_\mu \mathcal{V}_4 \equiv \mathrm{diag}(m_{\mu 1}, m_{\mu 2}, m_{\mu 3}, m_{\mu 4}) . \tag{8}$$

Eigenvalues $m_{\mu a}$ are given to leading orders in ϵ as

$$m_{\mu 1} \simeq \frac{1}{2}(2m_{\nu_\mu} + \epsilon_{11} + \epsilon_{13} + \epsilon_{14} + \epsilon_{22} + \epsilon_{23} + \epsilon_{24}) ,$$
$$m_{\mu 2} \simeq \frac{1}{2}(-2m_{\nu_\mu} + \epsilon_{11} + \epsilon_{13} - \epsilon_{14} + \epsilon_{22} - \epsilon_{23} + \epsilon_{24}) ,$$
$$m_{\mu 3} \simeq \frac{1}{2}(2m_{\nu_\mu} + \epsilon_{11} - \epsilon_{13} - \epsilon_{14} + \varepsilon_{22} - \epsilon_{23} - \epsilon_{24}) ,$$
$$m_{\mu 4} \simeq \frac{1}{2}(-2m_{\nu_\mu} + \epsilon_{11} - \epsilon_{13} + \varepsilon_{14} + \epsilon_{22} + \epsilon_{23} - \epsilon_{24}) . \quad (9)$$

The diagonalizing matrix is given to the same order by $V_4 \equiv \mathcal{V}_4^0 \bar{\mathcal{V}}_4$

$$\simeq \frac{1}{2}\begin{pmatrix} 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & y_1 & 0 & y_2 \\ -y_1 & 1 & y_2 & 0 \\ 0 & -y_2 & 1 & y_3 \\ -y_2 & 0 & -y_3 & 1 \end{pmatrix}, \quad (10)$$

where

$$y_1 \equiv \frac{1}{4m_{\nu_\mu}}(\epsilon_{11} + \epsilon_{13} - \epsilon_{22} - \epsilon_{24}),$$
$$y_2 \equiv \frac{1}{4m_{\nu_\mu}}(\epsilon_{23} - \epsilon_{14}),$$
$$y_3 \equiv \frac{1}{4m_{\nu_\mu}}(\epsilon_{11} - \epsilon_{13} - \epsilon_{22} + \epsilon_{24}). \quad (11)$$

The role of ϵ_{ab} is essentially to split all four degenerate states, and mixing between them is essentially determined by the ϵ_{ab} independent matrix $\mathcal{V}_4^0$ in Eq (10). In the following, we shall assume that all the parameters ϵ_{ab} have the same typical magnitude given by

$$\epsilon_{ab} \equiv \epsilon \sim \frac{\lambda}{M_P} v^2 \approx 2.4 \times 10^{-6} \lambda \text{ eV}, \quad (12)$$

where $v \sim 174$ GeV. Then it follows from Eq. (9) that a typical scale responsible for the long-wavelength oscillations of muon neutrinos is given by

$$\Delta_2 \approx 2m_{\nu_\mu}\epsilon \approx 4.5 \times 10^{-8} \text{ eV}^2 \lambda \left(\frac{m_{\nu_\mu}}{0.009 \text{ eV}}\right). \quad (13)$$

The oscillation length associated with this scale and energy $E = 10^5$–10^7 GeV is smaller than the typical distance of the UHE sources [see Eq. (1)], and the effect of Δ_2 gets averaged out resulting in suppression of the muon neutrino flux. The averaged survival probability is essentially determined by $\mathcal{V}_4^0$ in Eq. (10) and is given by

$$P_{\mu\mu} = 4\left(\frac{1}{2}\right)^4 = \frac{1}{4}. \quad (14)$$

This reduction is over and above the flux reduction which takes place due to averaged oscillations between active flavors and one needs to generalize the above formulation to take this effect into account. We do this in the next section.

III. THREE GENERATIONS

In the following, we shall assume a straightforward generalization of the above scenario and require that all three active neutrinos and their mirror partners are pseudo-Dirac. We are thus dealing with 12 left-handed states $\nu'_{\alpha a}$, $\alpha = e, \mu, \tau$, $a = 1 \ldots 4$ in this case, and mixing among them would now be governed by a 12×12 matrix. The mass matrix (7) for a single flavor is generalized to a 12×12 matrix for the three flavors as follows. The mass m_{ν_μ} is replaced by a 3×3 mass matrix $m^{\alpha\beta}$ in the flavor space α, $\beta = e, \mu, \tau$,

$$m_{\nu_\mu} \to m^{\alpha\beta} \equiv \mathbf{m}. \quad (15)$$

This matrix can be diagonalized by the usual biunitary transformation:

$$\mathbf{U_L}^T \mathbf{m} \mathbf{U_R} = \text{diagonal}(m_1, m_2, m_3) \equiv \mathbf{d}_\nu. \quad (16)$$

Here, the matrix $\mathbf{U_L}$ governing the left-handed mixing can be identified with the usual Maki-Nagakawa-Sakata-Pontecorvo (MNSP) matrix $\equiv \mathbf{U}$. Each of the parameters ε_{ab} appearing in Eq. (10) now gets replaced by a 3×3 matrix in flavor space,

$$\epsilon_{ab} \to \epsilon_{ab}^{\alpha\beta} \equiv \epsilon_{ab}. \quad (17)$$

These matrices are generated by dimension-five operators as before, e.g. $\epsilon_{13}^{\alpha\beta}$ arise from gravitational mixing between neutrinos in our universe and the mirror universe,

$$\frac{\lambda_{13}^{\alpha\beta}}{M_P}(\nu'_{\alpha 1}\phi)(\nu'_{\beta 3}\phi'), \quad (18)$$

which gives the 3×3 mixing matrix in flavor space,

$$\epsilon_{13}^{\alpha\beta} = \frac{\lambda_{13}^{\alpha\beta} v^2}{M_P} \approx 2.5 \times 10^{-6} \lambda_{13}^{\alpha\beta} \text{ eV}. \quad (19)$$

Taking similar terms for mixing of all pairs, we write the 12×12 mass matrix in the $(\nu'_{\alpha,1}, \nu'_{\alpha,2}, \nu'_{\alpha,3}, \nu'_{\alpha,4})$ basis as

$$\mathcal{M} = \begin{pmatrix} \epsilon_{11} & \mathbf{m} & \epsilon_{13} & \epsilon_{14} \\ \mathbf{m} & \epsilon_{22} & \epsilon_{23} & \boldsymbol{\varepsilon}_{24} \\ \epsilon_{13}^T & \epsilon_{23}^T & \epsilon_{11} & \mathbf{m} \\ \epsilon_{14}^T & \epsilon_{24}^T & \mathbf{m} & \epsilon_{22} \end{pmatrix}. \quad (20)$$

Note that each entry above is a 3×3 matrix in the generation space. This matrix can be diagonalized by the following steps. We first diagonalize the $\mathbf{m}$ blocks by the matrix,

$$\mathcal{U}' = \begin{pmatrix} \mathbf{U_L} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{U_R} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{U_L} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{U_R} \end{pmatrix}, \tag{21}$$

with the transformation,

$$\mathcal{M}' = \mathcal{U}'^T M U' = \begin{pmatrix} \tilde{\epsilon}_{11} & \mathbf{d}_\nu & \tilde{\epsilon}_{13} & \tilde{\epsilon}_{14} \\ \mathbf{d}_\nu & \tilde{\epsilon}_{22} & \tilde{\epsilon}_{23} & \tilde{\epsilon}_{24} \\ \tilde{\epsilon}_{13}^T & \tilde{\epsilon}_{23}^T & \tilde{\epsilon}_{11} & \mathbf{d}_\nu \\ \tilde{\epsilon}_{14}^T & \tilde{\epsilon}_{24}^T & \mathbf{d}_\nu & \tilde{\epsilon}_{22} \end{pmatrix}, \tag{22}$$

where

$$\begin{aligned} \tilde{\epsilon}_{ab} &\equiv U_L^T \varepsilon_{ab} U_L \quad \text{for} \quad ab = 11, 13, \\ \tilde{\epsilon}_{ab} &\equiv U_L^T \varepsilon_{ab} U_R \quad \text{for} \quad ab = 14, \\ \tilde{\epsilon}_{ab} &\equiv U_R^T \varepsilon_{ab} U_L \quad \text{for} \quad ab = 23, \\ \tilde{\epsilon}_{ab} &\equiv U_R^T \varepsilon_{ab} U_R \quad \text{for} \quad ab = 22, 24. \end{aligned} \tag{23}$$

The diagonal elements $(\tilde{\epsilon}_{ab})^{ii}$ in flavor space serve to split the masses m_{ν_i} of the ith flavor. The of-diagonal entries give corrections to them and also mix sterile states of different flavors. Since mixing of an active neutrino of one flavor with a sterile neutrino associated with a different flavor is strongly constrained from experiments, we shall assume that of-diagonal entries of each of the matrices $\tilde{\epsilon}_{ab}$ are small compared to the diagonal ones and take these matrices as diagonal,

$$\tilde{\epsilon}_{ab}^D \equiv \text{diagonal}(\epsilon_{ab}^1, \epsilon_{ab}^2, \epsilon_{ab}^3) , \tag{24}$$

where $\tilde{\epsilon}_{ab}^D$ are now a diagonal 3×3 matrix for each ab. In this approximation, Eq. (22) can be written as

$$\mathcal{M}' \approx \begin{pmatrix} \tilde{\epsilon}_{11}^D & d_\nu & \tilde{\epsilon}_{13}^D & \tilde{\epsilon}_{14}^D \\ d_\nu & \tilde{\epsilon}_{22}^D & \tilde{\epsilon}_{23}^D & \tilde{\epsilon}_{24} \\ \tilde{\epsilon}_{13}^D & \tilde{\epsilon}_{23}^D & \tilde{\epsilon}_{11}^D & d_\nu \\ \tilde{\epsilon}_{14}^D & \tilde{\epsilon}_{24}^D & d_\nu & \tilde{\epsilon}_{22}^D \end{pmatrix}. \tag{25}$$

To the first order in $\tilde{\epsilon}_{ab}^D \mathbf{d}_\nu^{-1}$, the matrix $\mathcal{M}'$ is now diagonalized by

$$\mathcal{V} \equiv \mathcal{V}^0 \bar{\mathcal{V}} \simeq \frac{1}{2} \begin{pmatrix} \mathbf{I} & -\mathbf{I} & -\mathbf{I} & \mathbf{I} \\ \mathbf{I} & \mathbf{I} & -\mathbf{I} & -\mathbf{I} \\ \mathbf{I} & -\mathbf{I} & \mathbf{I} & -\mathbf{I} \\ \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{I} & \mathbf{y}_1 & \mathbf{0} & \mathbf{y}_2 \\ -\mathbf{y}_1 & \mathbf{I} & \mathbf{y}_2 & \mathbf{0} \\ \mathbf{0} & -\mathbf{y}_2 & \mathbf{I} & \mathbf{y}_3 \\ -\mathbf{y}_2 & \mathbf{0} & -\mathbf{y}_3 & \mathbf{I} \end{pmatrix}, \tag{26}$$

where $\mathbf{I}$ ($\mathbf{0}$)denotes 3×3 identity (null) matrix. The diagonal 3×3 matrices $\mathbf{y}_{1,2,3}$ are given by expressions analogous to Eq. (11):

$$\begin{aligned} \mathbf{y}_1 &\equiv \frac{1}{4}(\tilde{\epsilon}_{11}^D + \tilde{\epsilon}_{13}^D - \tilde{\epsilon}_{22}^D - \tilde{\epsilon}_{24}^D) d_\nu^{-1}, \\ \mathbf{y}_2 &\equiv \frac{1}{4}(\tilde{\epsilon}_{23}^D - \tilde{\epsilon}_{14}^D) d_\nu^{-1}, \\ \mathbf{y}_3 &\equiv \frac{1}{4}(\tilde{\epsilon}_{11}^D - \tilde{\epsilon}_{13}^D - \tilde{\epsilon}_{22}^D + \tilde{\epsilon}_{24}^D) d_\nu^{-1} . \end{aligned} \tag{27}$$

The 12 eigenvalues of $\mathcal{M}$ are given to leading order in $\tilde{\boldsymbol{\varepsilon}}_{ab}$ by equations analogous to (9):

$$\begin{aligned} \mathbf{m}_1 &\simeq \frac{1}{2}(2\mathbf{d}_\nu + \tilde{\epsilon}_{11}^D + \tilde{\epsilon}_{13}^D + \tilde{\epsilon}_{14}^D + \tilde{\epsilon}_{22}^D + \tilde{\epsilon}_{23}^D + \tilde{\epsilon}_{24}^D) \\ \mathbf{m}_2 &\simeq \frac{1}{2}(-2\mathbf{d}_\nu + \tilde{\epsilon}_{11}^D + \tilde{\epsilon}_{13}^D - \tilde{\epsilon}_{14}^D + \tilde{\epsilon}_{22}^D - \tilde{\epsilon}_{23}^D + \tilde{\epsilon}_{24}^D) \\ \mathbf{m}_3 &\simeq \frac{1}{2}(2\mathbf{d}_\nu + \tilde{\epsilon}_{11}^D - \tilde{\epsilon}_{13}^D - \tilde{\epsilon}_{14}^D + \tilde{\epsilon}_{22}^D - \tilde{\epsilon}_{23}^D - \tilde{\epsilon}_{24}^D) \\ \mathbf{m}_4 &\simeq \frac{1}{2}(-2\mathbf{d}_\nu + \tilde{\epsilon}_{11}^D - \tilde{\epsilon}_{13}^D + \tilde{\epsilon}_{14}^D + \tilde{\epsilon}_{22}^D + \tilde{\epsilon}_{23}^D - \tilde{\epsilon}_{24}^D) . \end{aligned} \tag{28}$$

The mixing matrix $\mathcal{U}$ between the 12 gauge eigenstates $(\nu'_{\alpha 1}, \nu'_{\alpha 2}, \nu'_{\alpha 3}, \nu'_{\alpha 4})$, $(\alpha = e, \mu, \tau)$ and the mass eigenstates $(\nu_{i1}, \nu_{i2}, \nu_{i3}, \nu_{i4})$, $i = 1 \ldots 3$ is given by the product of $\mathcal{U}'$, Eq. (21), and $\mathcal{V}$, Eq. (26). To zeroth order in ϵ_{ab}, one can approximate $\mathcal{V}$ by $\mathcal{V}^0$ and $\mathcal{U}$ is approximately given by

$$\mathcal{U} \equiv \mathcal{U}'\mathcal{V} \simeq \frac{1}{2} \begin{pmatrix} \mathbf{U_L} & -\mathbf{U_L} & -\mathbf{U_L} & \mathbf{U_L} \\ \mathbf{U_R} & \mathbf{U_R} & -\mathbf{U_R} & -\mathbf{U_R} \\ \mathbf{U_L} & -\mathbf{U_L} & \mathbf{U_L} & -\mathbf{U_L} \\ \mathbf{U_R} & \mathbf{U_R} & \mathbf{U_R} & \mathbf{U_R} \end{pmatrix}. \tag{29}$$

In this approximation, the three flavor eigenstates $\nu_\alpha \equiv \nu'_{\alpha 1}$ are given in terms of 12 mass eignestates ν_{ia} from the above equation by

$$\nu_\alpha \equiv \nu'_{\alpha 1} = \mathcal{U}_{1a}^{\alpha i} \nu_{ia} \equiv \frac{1}{2} \mathbf{U}^{\alpha i} (\nu_{i1} - \nu_{i2} - \nu_{i3} + \nu_{i4}), \tag{30}$$

with $\mathbf{U_L} \equiv \mathbf{U}$ denoting the MNSP matrix. The mass m_{ia} of each component ν_{ia} is given by $(m_a)_{ii}$ from Eq. (28). The splitting among the mirror partners of a given mass eigenstate $\nu_{i,a}$ is then given by $\Delta_{ab}^i \equiv m_{ia}^2 - m_{ib}^2$. The corresponding oscillation probabilities follow from the time evolution of the state ν_i defined in Eq. (30),

$$\begin{aligned} P_{\alpha\beta}(L) = \frac{1}{16} \sum_{ij} & \mathbf{U}_{\alpha i}^* \mathbf{U}_{\beta i} \mathbf{U}_{\beta j} \mathbf{U}_{\alpha j}^* e^{-i(m_{j1}^2 - m_{i1}^2)\frac{L}{2E}} \\ & \times (1 + e^{i\chi_{12}^i} + e^{i\chi_{13}^i} + e^{i\chi_{14}^i}) \\ & \times (1 + e^{-i\chi_{12}^j} + e^{-i\chi_{13}^j} + e^{-i\chi_{14}^j}), \end{aligned} \tag{31}$$

where $\chi^i_{ab} \equiv \Delta^i_{ab}\frac{L}{2E}$. The four states ν_{ia} for a given i are degenerate when Δ^i_{ab} are small and χ^i_{ab} can be neglected as in typical short baseline experiments. In this limit, ν_i defined in Eq. (30) behave as a single mass eigenstate and one recovers the standard mixing and oscillations of the flavor states. The Δ^i_{ab} induce observable long-wavelength oscillations between active and sterile states. For long baselines with ($E/L > 10^{-12}$ eV^2) the exponential factor $e^{-i(m^2_{j1}-m^2_{i1})\frac{L}{2E}}$ in (31)) averages to zero if $i \neq j$. The oscillation probability in the long baseline experiments then can be written as

$$P_{\alpha\beta}(L) = \frac{1}{8}\sum_i |\mathbf{U}_{\alpha i}|^2 |\mathbf{U}_{\beta i}|^2 (2 + \cos\chi^i_{12} + \cos\chi^i_{13} + \cos\chi^i_{14} + \cos\chi^i_{23} + \cos\chi^i_{24} + \cos\chi^i_{34}). \quad (32)$$

Typical scales associated with these splittings can be written assuming normal mass hierarchy as

$$\Delta^1_{ab} \simeq 4\lambda_1 m_1 \frac{v^2}{M_p^2} \ll 9\times 10^{-8}\lambda_1 \text{ eV}^2,$$
$$\Delta^2_{ab} \simeq 4\lambda_2 \sqrt{\Delta_\odot}\frac{v^2}{M_p^2} = 9\times 10^{-8}\lambda_2 \text{ eV}^2,$$
$$\Delta^3_{ab} \simeq 4\lambda_3 \sqrt{\Delta_{\text{atm}}}\frac{v^2}{M_p^2} = 4.7\times 10^{-7}\lambda_3 \text{ eV}^2, \quad (33)$$

where $\lambda_{1,2,3}$ denote the gravitational couplings in three sectors controlling the strength of the dimension-five operators. These are constrained from two major considerations. The number of neutrino species in equilibrium at the time of Big Bang nucleosynthesis (BBN) is severely constrained. The requirement that a sterile neutrino does not equilibrate at that time through large-angle oscillations to an active one implies that their (mass)^2 difference must obey [34–36] $\Delta m^2_{\nu_a\nu'} \leq 10^{-9}$ eV^2. A stronger constraint exists on Δ^1_{ab}. In the approximation of neglecting mixing between active and sterile partners of different generations, Δ^1_{ab} controls the solar neutrino oscillations. The corresponding oscillation length for MeV neutrinos is of the order of the Sun-Earth distance for $\Delta^1_{ab} \sim 10^{-12}$ eV^2. Such Δ^1_{ab} can modify the large mixing angle solution, and detailed fits in case of pseudo-Dirac neutrinos [48] imply a bound $\Delta^1_{ab} < 1.8\times 10^{-12}$ eV^2 at 3σ. One expects a similar but somewhat stronger bound when mirror partners are also present. This bound can be satisfied either by choosing $m_1 \ll \sqrt{\Delta_\odot}$ or in the case of $m_1 \sim O(\sqrt{\Delta_\odot})$, by choosing $\lambda_1 \leq 10^{-5}$.

We will assume that all the splittings among a given flavor Δ^{ii}_{ab} for different pairs of ab are equal, and in this case the oscillation probability (32) reduces to the simple form

$$P_{\alpha\beta}(L) = \frac{1}{4}\sum_i |\mathbf{U}_{\alpha i}|^2 |\mathbf{U}_{\beta i}|^2 (1 + 3\cos\chi_i), \quad (34)$$

where $\chi_i \equiv \chi^i_{ab}$ $\forall\, a,\, b$. One can now work out the observed flux ratios of UHE neutrinos using this $P_{\alpha\beta}$. The flux $\Phi_\beta = (\phi_e, \phi_\mu, \phi_\tau)$ in a flavor β is given by

$$\Phi_\beta = P_{\beta\alpha}\Phi^0_\alpha, \quad (35)$$

where Φ^0_α denotes the initial flux. For $\Phi^0_\alpha \sim \frac{\phi_0}{3}(1,2,0)$ one obtains

$$\Phi_\beta \sim \frac{\phi_0}{12}\sum_i |\mathbf{U}_{\beta i}|^2 (|\mathbf{U}_{ei}|^2 + 2|\mathbf{U}_{\mu i}|^2)(1 + 3\cos\chi_i)\ . \quad (36)$$

One recovers the standard value $\Phi^S_\beta = \frac{\phi_0}{3}\sum_i |\mathbf{U}_{\beta i}|^2(|\mathbf{U}_{ei}|^2 + 2|\mathbf{U}_{\mu i}|^2)$ with only three Dirac neutrinos in the limit $\chi_i = 0$. The deviation in flux compared to the standard value is thus given by

$$\delta\Phi_\beta \equiv \Phi_\beta - \Phi^S_\beta = -\frac{\phi_0}{2}\sum_i |\mathbf{U}_{\beta i}|^2 (|\mathbf{U}_{ei}|^2 + 2|\mathbf{U}_{\mu i}|^2)\sin^2\frac{\chi_i}{2}\ . \quad (37)$$

For maximal atmospheric mixing and $\theta_{13} \simeq 0$, $|\mathbf{U}_{ei}|^2 + 2|\mathbf{U}_{\mu i}|^2 = 1$ for every i and the above simplifies to

$$\delta\Phi_\beta = -\frac{\phi_0}{2}\sum_i |\mathbf{U}_{\beta i}|^2 \sin^2\frac{\chi_i}{2}\ . \quad (38)$$

This is to be compared with the corresponding formula [38,43] obtained for the pseudo-Dirac neutrinos in the absence of the mirror neutrinos:

$$\delta\Phi_\beta = -\frac{\phi_0}{3}\sum_i |\mathbf{U}_{\beta i}|^2 \sin^2\frac{\chi_i}{2}\ . \quad (39)$$

The presence of pseudo-Dirac mirror partners now leads to stronger deviation in Φ_β from the canonical value 1/3. The values of $\delta\Phi_\beta$ depend both on the values of $|\mathbf{U}_{\beta i}|$, which are now reasonably well known, and on the hierarchies in Δ_i given typically by Eq. (33). There exist two interesting ranges of Δ_i that can effect the oscillations of UHE in physically different ways: (a) strong mass hierarchies among neutrinos $m_1 \ll m_2 \simeq \sqrt{\Delta_{\text{odot}}} < m_3 \simeq \sqrt{\Delta_{\text{atm}}}$ [or equivalently $\lambda_1 \ll \lambda_2 \simeq \lambda_{2,3}$ in Eq. (33))] such that Δ_1 in Eq. (33)) is $< 10^{-16}$ eV^2 but $\Delta_{2,3} > 10^{-16}$ eV^2. The Δ_1 in this case does not induce the appreciable oscillations of the UHE neutrinos while effects of $\Delta_{2,3}$ can be averaged out. This corresponds to taking $\chi_1 = \cos\chi_2 = \cos\chi_3 = 0$ in Eq. (36) and one obtains

$$\phi_\beta \approx \frac{\phi_0}{12}(1 + 3|U_{\beta 1}|^2),$$

which translates to

$$\phi_e : \phi_\mu : \phi_\tau \approx 2:1:1 \tag{40}$$

for the tribimaximal mixing. The corresponding number for the current best-fit values [49] of mixing angles is 2.12∶1∶1.09. While fluxes in all three flavors are suppressed compared to the canonical value 1/3, the suppression of the electron neutrinos flux is less.

(b) The alternative possibility corresponds to a milder hierarchy characterized by $m_1 \approx \sqrt{\Delta_\odot}$ and all λ_i similar in magnitude such that Δ_1 is suppressed compared to $\Delta_{2,3}$ to satisfy the solar bound but all of them still are bigger than the oscillation scale $\sim 10^{-16}$ eV^2 of the UHE neutrinos. This case corresponds to taking $\cos\chi_i = 0$ for all i in Eq. (36)) and all the flavors are suppressed by a factor of 4 compared to the canonical value of 1/3.

In our model we have introduced nine extra neutrinos, which can potentially be in conflict with the BBN constraints on the effective number of species of light particles during nucleosynthesis. Of these extra neutrinos, ν_2^α, ν_4^α ($\alpha = e, \mu, \tau$) are sterile and can decouple much before the time of BBN, so their temperatures will be smaller than the radiation bath and they will not contribute to the Hubble expansion at the time of BBN. In our model intergenerational mixing between an active neutrino of one flavor with sterile neutrinos associated with other is negligible. As a result, ν_2, ν_3 will not equilibriate with the active ν_1, ν_3 species by oscillation. There is no equilibrium attained by $\nu_1 \leftrightarrow \nu_{2,3,4}$ oscillations of the same flavor as the mass splittings is $\le 10^{-9}$ eV^2 [34–36]. However ν_3^α are "active" in the mirror world and they could count as three extra neutrino degrees of freedom if their temperature were to be identical to the temperature of the active neutrinos in our world. One way to avoid this doubling of neutrino degrees is to assume that the mirror world couplings to the inflaton are slightly different and the reheating temperature of the mirror world following inflation is lower than reheat temperature of our universe [50]. The effective neutrino degrees of freedom observed by Planck [51] at the time of matter-radiation equality is $N_{\rm eff} = 3.30 \pm 0.27$ at 68% C.L. If there are N_m species of mirror neutrinos with temperature T_m, then they will count as

$$N_{\rm eff} = 3.046 + N_m \left(\frac{T_m}{T_\nu}\right)^4 \tag{41}$$

neutrinos. We see that in order that $\Delta N_{\rm eff} < 0.3$ with $N_m = 9$ extra mirror neutrino species, the mirror neutrino temperature is $T_m < 0.43 T_\nu$ at the time of matter-radiation equality to evade the Planck bound. Another way in which sterile neutrinos, including mirror ones, can evade the stringent Planck bound is if there is an annihilation of MeV dark matter preferentially into photons, such that the photon temperature relative to the neutrino temperature is raised after neutrino decoupling and prior to $z_{\rm eq}$. Scenarios for evading the Planck constraint on sterile neutrinos via dark matter models have been discussed in Ref. [52,53]

IV. CONCLUSIONS

In this paper we have studied the neutrino fluxes and flavor ratios of 10^5–10^7 GeV neutrinos originating from the GRB in a scenario with three sterile neutrinos for each flavor having tiny splitting as given in Eq. (1). It is shown that in this scenario GRB neutrinos of all flavors or muon and tau flavor can get suppressed by factor of 1/4 as required by the IceCube result. This suppression can result in the presence of maximal mixing among a neutrino and three sterile partners as given in Eq. (10). Such mixing can arise if all the neutrinos are pseudo-Dirac and there is a mirror world replicating our own interactions. Assuming the most favorable parameters in GRB models, the complete IceCube is expected to observe GRB neutrinos in a couple of years of operation. The idea of pseudo-Dirac neutrinos as discussed in this paper may be testable in the near future.

As far as Δ_e is concerned, it is required to be $< 10^{-12}$ eV^2 [48]. This leads to an interesting possibility. UHE neutrinos are also expected at energies of the Greisen-Zatsepin-Kuzmin limit cosmic rays with energies of 10^9 GeV and their sources are closer at distances of 100 Mpc [54] . Thus the (mass)^2 difference required for significant conversion of these neutrinos should be $\ge 10^{-12}$ eV^2. Thus it is possible that electron neutrinos from GRB get depleted, but ones from the nearby sources and high energy remain undepleted. Similar things can happen for other flavors also if $\lambda_\mu, \lambda_\tau$ in Eq. (33) are such that $\Delta_{\mu,\tau}$ also fall in the range 10^{-12}–10^{-15} eV^2. These could serve as discriminating tests of models of pseudo-Dirac neutrinos like the one discussed in this paper.

A well-known characteristic of pseudo-Dirac structure is the almost vanishing of the amplitude m_{ee} for the neutrinoless double beta decay. This follows in the present case from Eqs. (30) and (28),

$$m_{ee} = \frac{1}{4}\left|\sum_i U_{ei}^2 \, (m_{1i} - m_{2i} + m_{3i} - m_{4i})\right|, \tag{42}$$

where m_{ai} are (positive) masses of the members of each pseudo-Dirac pair in flavor i. This amplitude is thus vanishingly small due to degeneracy in these masses, $\Delta_{ab}^i \sim 10^{-15}$ eV^2. Thus observation of neutrinoless beta decay in future experiment can rule out the model completely.

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