High precision measurement of the masses of the D^0 and K_S mesons

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Using 580 pb^{-1} of e^+e^- annihilation data taken with the CLEO–c detector at $\psi(3770)$, the decay $D^0(\bar{D}^0) \to K^\pm \pi^\mp \pi^+ \pi^-$ has been studied to make the highest precision measurement of D^0 mass, $M(D^0) = 1864.845 \pm 0.025 \pm 0.022 \pm 0.053$ MeV, where the first error is statistical, the second error is systematic, and the third error is due to uncertainty in kaon masses. As an intermediate step of the present investigation, the mass of the K_S meson has been measured to be $M(K_S) = 497.607 \pm 0.007 \pm 0.015$ MeV. Both $M(D^0)$ and $M(K_S)$ are the most precise single measurements of the masses of these mesons.

The D^0 meson, the ground state of the charm meson family, and the K_S meson, the ground store of the strange meson family, occupy an important place in hadron spectroscopy, and precision determination of their masses is of particular importance. Not only do the masses of K_S and D^0 mesons provide precision calibration standards for masses and mass differences below 2.5 GeV as $M(J/\psi)$ and $M(\psi(2S))$ do in the 3–4 GeV mass region [1,2], but precision determination of $M(D^0)$ is of crucial importance in time-dependent analyses of $D^0 - \overline{D^0}$ mixing and CP violation [3–5]. Recently, many observations of mesons that do not conveniently fit in the conventional $|q\bar{q}\rangle$ meson families have been reported, and several of these are conjectured to be weakly bound hadronic molecules of D and D_S mesons [6]. The most famous of these "exotics" is the X(3872) meson which can be modeled as a $\overline{D^0}D^{*0}$ molecule. The small binding energy of X(3872) requires a precision determination of the masses of D^0 and D^{*0} mesons [7,8]. In this paper we present results for the highest precision measurement of $M(D^0)$. As an intermediate step in our analysis, we have also made a precision measurement of $M(K_S)$.

We had earlier [8] reported the measurement of $M(D^0)$ using 280 pb^{-1} of CLEO–c data taken at the $\psi(3770)$. We reported $M(D^0) = 1864.847 \pm 0.150 \pm 0.095$ MeV (throughout this paper, the first error is statistical and the second error is systematic), using the decay $D^0 \to K_S\phi$, $\phi \to K^+K^-$, $K_S \to \pi^+\pi^-$, which has the overall branching fraction $\mathcal{B} = 1.4 \times 10^{-3}$. The measurement was based on 319 ± 18 events. Recently, the LHCb Collaboration has reported $M(D^0) = 1864.75 \pm 0.15 \pm 0.11$ MeV [9] based on 4608 ± 89 events in the decay $D^0 \to K^+K^-\pi^+\pi^-$, which has $\mathcal{B} = 2.4 \times 10^{-3}$ [10], and 849 ± 36 events in the decay $D^0 \to K^+K^-K^-\pi^+$, which has $\mathcal{B} = 2.2 \times 10^{-4}$ [10]. Also, BABAR has reported $M(D^0) = 1864.841 \pm 0.048 \pm 0.063$ MeV [11] based on 4345 ± 70 events observed in the $D^0 \to K^+K^-K^-\pi^+$ decay. The goal of our present measurement is to determine the mass of D^0 with an overall precision three times better than our previous measurement, i.e., $\sim$60 keV. To minimize statistical errors we choose to study the most prolific charged particle decay, $D^0 \to K^-\pi^+\pi^-\pi^+$ ($K3\pi$) (throughout this paper inclusion of charge conjugate decays is implied), which has a branching fraction $\mathcal{B} = 8.1 \times 10^{-2}$ [10], sixty times that in our previous measurement, and $\sim$370 times larger than that for the $D^0 \to 3K\pi$ decay used by *BaBar* and LHCb. To obtain the best energy calibration for charged hadrons, we analyze the decay $\psi(2S) \to J/\psi\pi^+\pi^-$, and anchor our energy calibration to the high precision measurement of the mass of J/ψ, $M(J/\psi) = 3096.917 \pm 0.010 \pm 0.007$ MeV [1], and mass of $\psi(2S)$, $M(\psi(2S)) = 3686.114 \pm 0.007 \pm 0.011^{+0.002}_{-0.012}$ MeV [2], made by the KEDR Collaboration at Novosibirsk using the resonance depolarization technique.

We use data taken with the CLEO–c detector, 580 pb^{-1} of e^+e^- annihilation at $\psi(3770)$, $\sqrt{s} = 3770$ MeV, twice as much as in our previous measurements to determine D^0 mass, and 49 pb^{-1} of data taken at $\psi(2S)$, $\sqrt{s} = 3686$ MeV to fine tune the CLEO–c solenoid magnetic field. The CLEO–c detector has been described in detail elsewhere [12]. Briefly, it consists of a CsI(Tl) electromagnetic calorimeter, an inner vertex drift chamber, a central drift chamber, and a ring imaging Cherenkov (RICH) detector, all inside a superconducting solenoid magnet providing a nominal 1.0 Tesla magnetic field. For the present measurements, the important components are the drift chambers, which provide a coverage of 93% of 4π for the charged particles. The detector response was studied using a GEANT-based Monte Carlo (MC) simulation including radiation corrections [13].

The $\psi(2S)$ data are analyzed for the exclusive decay, $\psi(2S) \to \pi^+\pi^-J/\psi$, $J/\psi \to \mu^+\mu^-$ and for the inclusive decay, $\psi(2S) \to K_S + X$, $K_S \to \pi^+\pi^-$. We select events with well-measured tracks by requiring that they be fully contained in the barrel region of the detector,

$|\cos\theta(\text{polar})| < 0.8$, and have transverse momenta >120 MeV. For the pions from K_S decay, we make the additional requirement that they originate from a common vertex displaced from the interaction point by more than 10 mm. We require a K_S flight distance significance of more than three standard deviations. We accept K_S candidates with mass in the range 497.7 ± 12.0 MeV. We identify muons from J/ψ decays as having momenta more than 1 GeV, and $E_{CC}/p < 0.25$ for at least one muon candidate, and $E_{CC}/p < 0.5$ for the other muon, where E_{CC} is the energy deposited in electromagnetic calorimeter associated with the track of momenta p.

We require that there should be only two identified pions and two identified muons with opposite charges in the event. The momenta of $\mu^+\mu^-$ pairs is kinematically fitted to the KEDR J/ψ mass, $M(J/\psi)_{\text{KEDR}} = 3096.917$ MeV, and only events with $\chi^2 < 20$ are accepted. We also require that there should not be any isolated shower with energy more than 50 MeV in the event.

The $\psi(3770)$ data are analyzed for the decays $\psi(3770) \to D^0\overline{D^0}$, $D^0/\overline{D^0} \to K3\pi$. We select D^0 candidates using the standard CLEO D-tagging criteria, which impose a very loose requirement on the beam energy constrained D^0 mass, as described in Ref. [14]. We again select well-measured tracks as described above, and in addition require that they have energy loss, dE/dx, in the drift chamber consistent with the pion or kaon hypothesis within three standard deviations.

There are three distinct steps involved in our analysis:

(1) Determination of the improved energy calibration of the CLEO–c detector for charged particles by using the exclusive decay, $\psi(2S) \to J/\psi\pi^+\pi^-$, and the precision masses of $\psi(2S)$ and J/ψ.

(2) Precision measurement of the mass of K_S in the inclusive decay, $\psi(2S) \to K_S + X$, $K_S \to \pi^+\pi^-$ using the improved calibration.

(3) Precision measurement of the mass of D^0 in the exclusive decay, $D^0 \to K3\pi$ by monitoring and correcting for small changes in calibration as revealed by $M(K_S)$ determined for individual "subruns."

The first step consists of determining the new calibration for the momenta of charged particles with the highest possible precision. The charged particle energy calibration generally used in the analyses of CLEO–c data is based on tuning of the nominal magnetic field of the CLEO III detector done in 2003. By requiring that in the decays $\psi(2S) \to \mu^+\mu^-$, and $J/\psi \to \mu^+\mu^-$ the reconstructed $\psi(2S)$ and J/ψ masses be equal to their then known average PDG2002 values of $M(J/\psi) = 3096.87 \pm 0.04$ MeV and $M(\psi(2S)) = 3685.96 \pm 0.09$ MeV, it was determined that the nominal B-field of the solenoid needed to be multiplied by a default correction factor $\text{B}_{\text{COR}}(\text{default}) = 0.9952$. With the improved values of $M(\psi(2S))$ and $M(J/\psi)$ now available, and with our required level of high

precision, it is necessary to determine the new value of the B-field correction factor appropriate for our present measurements.

The KEDR determined precision values of the masses of the $M(J/\psi)$ and $M(\psi(2S))$ provide us the opportunity to determine precision calibration for charged pion momenta in the decays $\psi(2S) \to \pi^+\pi^-J/\psi$. The pions in this decay have momenta up to ~ 400 MeV, and the calibration obtained for them can be reliably used in the study of $D^0 \to K3\pi$ decays which contain charged pions and kaons in a similar range of momenta. By analyzing our data for the exclusive reaction, $\psi(2S) \to \pi^+\pi^-J/\psi$, with $M(J/\psi)$ fixed at the precision value, $M(J/\psi)_{\text{KEDR}} = 3096.917$ MeV we determine the new value of the solenoid B-field correction factor, $\text{B}_{\text{COR}}(\text{new})$, which corrects the pion momenta such the mass of $\psi(2S)$ we measure, $M(\psi(2S))_{\text{PRESENT}}$ equals the precision value $M(\psi(2S))_{\text{KEDR}} = 3686.114$ MeV. It is found that the CLEO–c default value $\text{B}_{\text{COR}}(\text{default})$ has to be increased by 0.0289%, or 2.89×10^{-4}, so that $\text{B}_{\text{COR}}(\text{new}) = 0.995488$. The $\psi(2S)$ mass spectrum obtained with this corrected B-field is shown in Fig. 1 in terms of $\Delta M(\psi(2S)) \equiv M(\psi(2S))_{\text{PRESENT}} - M(\psi(2S))_{\text{KEDR}}$. The unbinned spectrum is fitted with a constant linear background (~ 2 counts/0.1 MeV bin) and a peak which is the sum of a simple Gaussian function (54%), and a bifurcated Gaussian function (46%) with the same mean. The fit leads to $N(\psi(2S)) = 125,299 \pm 354$ events, FWHM $= 4.4$ MeV, $\Delta M(\psi(2S)) = 0.0 \pm 6.7$ keV(stat), and $\chi^2/\text{d.o.f.} = 0.85$. The normalized residuals for the fit defined as
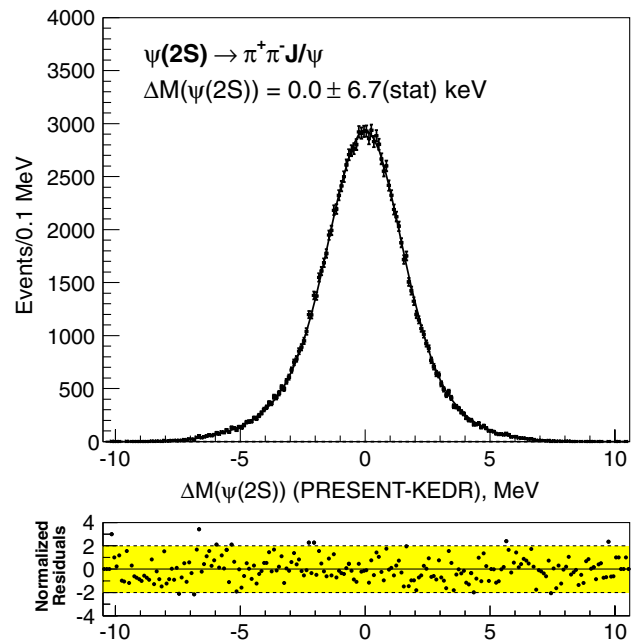

FIG. 1 (color online). Results of the unbinned maximum likelihood fit to the distribution $\Delta M(\psi(2S)) \equiv M(\psi(2S))_{\text{PRESENT}} - M(\psi(2S))_{\text{KEDR}}$ for the exclusive decays $\psi(2S) \to \pi^+\pi^-J/\psi$, $J/\psi \to \mu^+\mu^-$ using the corrected magnetic field.

$[N(\text{observed}) - N(\text{fit})]/\sqrt{N(\text{fit})}$, are also shown. All subsequent spectra in this paper are fitted in the same manner.

The second step of analysis consists of a precision determination of $M(K_S)$, the mass of the K_S meson which we use to monitor the stability of the magnetic field for the different $\psi(3770) \to D\bar{D}$ data subruns. We analyze the inclusive reaction $\psi(2S) \to K_S + X$ to determine $M(K_S)$. We use the precision calibration of the B-field as determined in the first step for this purpose. The pions in $\psi(2S) \to \pi^+\pi^- J/\psi$ calibration have momenta up to 400 MeV. For determining $M(K_S)$ we only use K_S with momenta $p(K_S) < 400$ MeV for which 95% of pions from $K_S \to \pi^+\pi^-$ decay have momenta < 360 MeV. Figure 2 shows that for K_S of momenta < 400 MeV, the π^+ and π^- from $\psi(2S)$ decay and from $K_S \to \pi^+\pi^-$ decay have nearly identical pion momentum distributions and angular distributions of the pions with respect to beam.

Figure 3 shows the $M(\pi^+\pi^-)$ distribution corresponding to $B_{COR}(\text{new}) = 0.995488$ for events from the decay $K_S \to \pi^+\pi^-$. The distribution is fitted as described before, with the fraction of the simple Gaussian and bifurcated Gaussian being 52% and 48%, respectively. It leads to $N(K_S) = 261,394 \pm 752$, FWHM $= 4.1$ MeV, $\chi^2/\text{d.o.f.} = 1.2$, and

$$M(K_S)_{\text{PRESENT}} = 497.607 \pm 0.007(\text{stat}) \text{ MeV}. \quad (1)$$

Although we have used $M(\psi(2S))$ based energy calibration obtained for pions with $p(\pi) < 400$ MeV to determine $M(K_S)$ for K_S decays with $p(K_S) < 400$ MeV, we find that the calibration is good for higher momenta. For example, we find that if decays with $p(K_S)$ up to 650 MeV are included, $M(K_S)$ varies by less than 1σ, or < 10 keV.

The third step of analysis consists of the determination of the mass of the D^0 meson using the $\psi(3770) \to D^0\bar{D^0}$ data taken at $\psi(3770)$, $\sqrt{s} = 3770$ MeV, and reconstructing D^0 in the decay $D^0 \to K3\pi$. The data which we analyze were taken in four subruns totaling 580 pb^{-1}. These data were taken after a three months shut–down after $\psi(2S)$ running of CLEO/CESR. Before analyzing the $D^0 \to K3\pi$ decays, it is necessary to determine the appropriate B_{COR} values for the D^0 subruns. We do so by analyzing each individual subrun for the inclusive decay, $D \to K_S + X$, $K_S \to \pi^+\pi^-$, with $p(K_S) < 650$ MeV and determining individual values of B_{COR} required to make $M(K_S)$ equal to $M(K_S)_{\text{PRESENT}}$, as determined in the second step. More than 99% of the pions in the inclusive decay $D \to K_S + X$, $K_S \to \pi^+\pi^-$ have momenta < 650 MeV for which our calibration of pion momenta is appropriate. The correction factors for $B_{COR}(\text{default})$ for individual subruns so determined are found to be $(0.79, 0.49, 0.68, 0.26) \times 10^{-4}$. These are smaller than 2.89×10^{-4} determined in the first step by fitting $M(\psi(2S))$ data taken before the three months shut–down. Using the above individual B_{COR} values the invariant
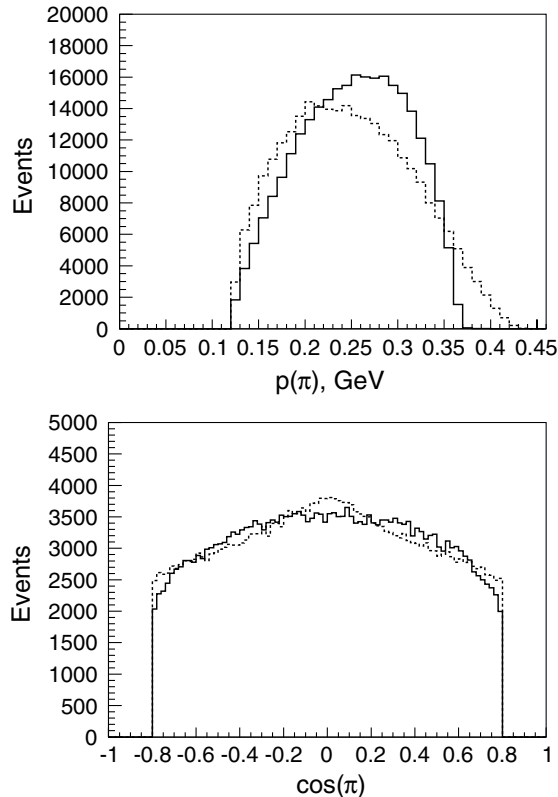

FIG. 2 (color online). Comparison of momentum distributions (upper) and angular distributions (lower) for pions in $\psi(2S) \to \pi^+\pi^- J/\psi$ decays (full histogram) and for pions in $K_S \to \pi^+\pi^-$ decays (dashed histogram).
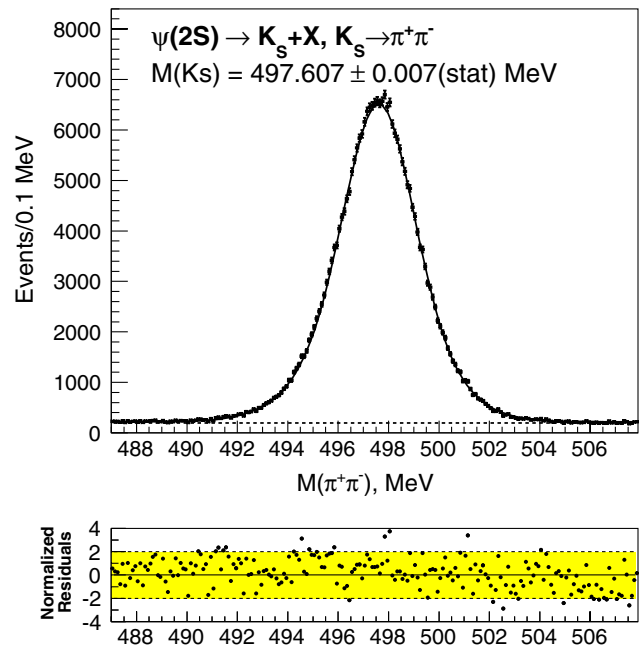

FIG. 3 (color online). Results of the unbinned maximum likelihood fit to the invariant mass distribution $M(\pi^+\pi^-)$ for the inclusive reaction $\psi(2S) \to K_S + X$, $K_S \to \pi^+\pi^-$, using the corrected magnetic field.
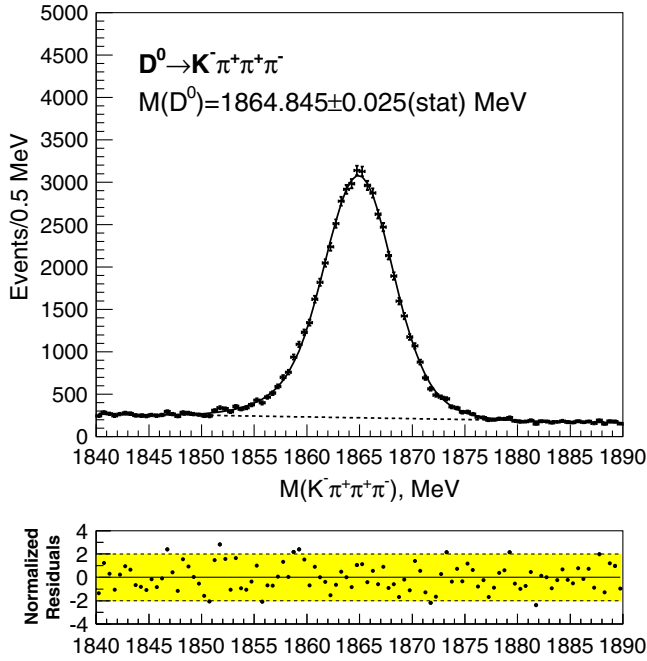
FIG. 4 (color online). Invariant mass spectra for the decays $D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-$ (plus charge conjugation decays).

mass of D^0 was reconstructed for the decay $D^0 \rightarrow K3\pi$ for each of the four subruns. Their weighted average is

$$\langle M(D^0) \rangle = 1864.833 \pm 0.024 \text{(stat) MeV.} \qquad (2)$$

For our final result we sum the corrected spectra for the four subruns and fit the summed spectrum as described before. The fractions of the simple Gaussian function and the bifurcated Gaussian function are 67% and 33%, respectively. The results of the fit shown in Fig. 4 are $N = 62,557 \pm 361$ events, FWHM $= 8.9$ MeV, $\chi^2/\text{d.o.f.} = 0.91$, and

$$M(D^0) = 1864.845 \pm 0.025 \text{(stat) MeV.} \qquad (3)$$

Table I illustrates that $M(D^0)$ is stable to within ± 5 keV even for π and K momenta up to 800 MeV.

The systematic uncertainties in $M(K_S)$ and $M(D^0)$ were obtained as follows.

For $M(K_S)$ measurement, we have corrected the magnetic field using KEDR measured $M(\psi(2S))$ and $M(J/\psi)$,

TABLE I. Illustrating stability of D^0 mass for different ranges of kaon and pion momenta.

$p(K, \pi's)$, MeV	$N(D^0)$	$M(D^0)$, MeV
< 600	$50,964 \pm 316$	1864.849 ± 0.027
< 650	$62,557 \pm 361$	1864.845 ± 0.025
< 700	$69,461 \pm 383$	1864.849 ± 0.024
< 750	$73,046 \pm 404$	1864.847 ± 0.023
< 800	$74,728 \pm 412$	1864.846 ± 0.022

TABLE II. Systematic uncertainties in $M(K_S)$.

Source: variation	Uncertainty in $M(K_S)$, keV
$\psi(2S)$ mass: $-18 + 13$ keV	$-12.3 + 8.9$
J/ψ mass: ± 12 keV	8.2
Fit Range width, ± 2 MeV	4
Background polynomial, 1,2 order	1
$\psi(2S)$ formation energy	5
Total	15

which have the total errors of $-18 + 13$ and ± 12 keV, respectively, [1,2]. The change in $M(K_S)$ due to the change in the magnetic field is found to vary linearly with the change in $M(\psi(2S))$ and $M(J/\psi)$, and is a factor 1.46 smaller. We therefore assign $\sim^{+8.9}_{-12.3}$ and $\sim \pm 8.2$ keV, as the uncertainties in $M(K_S)$ due to the uncertainties in $M(\psi(2S))$ and $M(J/\psi)$, respectively. The variation of the fit range by ± 2 MeV yields a change of ± 4 keV in $M(K_S)$. Changing the fits to the background from polynomials of order one to polynomials of order two changes $M(K_S)$ by < 1 keV. The effect of the possible formation of $\psi(2S)$ at an energy different from $M(\psi(2S))_{\text{KEDR}}$ was investigated in detail. The uncertainty in the formation energy was estimated by fitting the $\psi(2S)$ mass distribution with MC shape using different beam energies, and was found to be ± 7 keV. It contributes ± 5 keV to the systematic uncertainty in $M(K_S)$. The systematic uncertainties in $M(K_S)$ are listed in Table II. The sum in quadrature of all the above B_{COR} contributions is a total systematic uncertainty of ± 15 keV.

We have studied K_S mass dependence on momenta, polar angle θ and azimutal angle ϕ of K_S with respect to the positron beam. The K_S mass values in all cases are seen to be statistically in agreement with the average value, with $\chi^2/\text{d.o.f.}$ equal to 0.76, 0.79, 0.96 for momenta, $\cos\theta$, and ϕ.

Our final result for $M(K_S)$ is thus

$$M(K_S)_{\text{PRESENT}} = 497.607 \pm 0.007 \text{(stat)}$$
$$\pm 0.015 \text{(syst) MeV.} \qquad (4)$$

TABLE III. Systematic errors in $M(D^0)$ for the range of variation of different parameters.

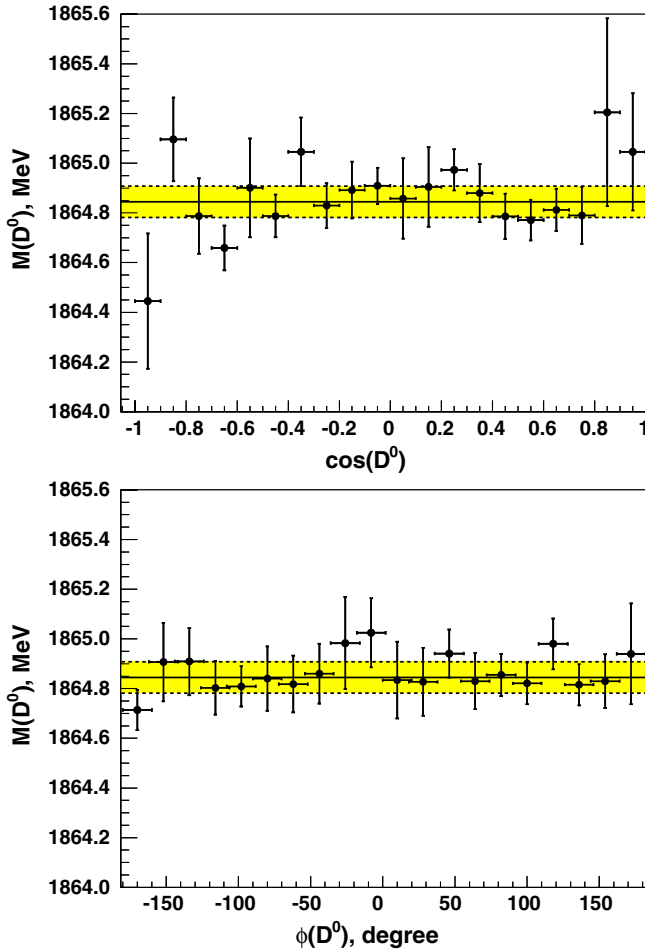| Source: variation	Uncertainty in $M(D^0)$, keV
$\lvert \cos\theta(\text{polar}) \rvert_{\text{max}}$: 0.8, 0.75	6
p_{min} (trans): 120, 135 MeV	6
p_{max} (total): 650, 550 MeV	15
Fit Range width, ± 5 MeV	12
Background polynomial 1,2 order	4
MC Input/Output of $M(D^0)$	7
Total: event selection and fit	22
Error in K_S mass: ± 16 keV	52
Error in $K^{\pm}$ mass: ± 16 keV	12
Total: kaon masses	53

FIG. 5 (color online).   Values of $M(D^0)$ obtained from fits to data divided into subsets in $\cos\theta$ and ϕ of D^0 mesons. Solid and dashed lines correspond to the central value and total error band corresponding to our present measurement $M(D^0) = 1864.845 \pm 0.063$ MeV.

The systematic errors in $M(D^0)$ are listed in Table III. They are dominated by uncertainties in the masses of the kaons. The ±16 keV uncertainty in the mass of K_S leads to the largest uncertainty, ±52 keV in $M(D^0)$.

The PDG(2012) mass of $K^\pm$ has an error of ±16 keV [10]. It leads to ±12 keV uncertainty in $M(D^0)$, which is calculated by changing of $M(K^\pm)$ by ±16 keV. Added in quadrature, the total systematic uncertainty due to uncertainties in kaon masses is ±53 keV.

Other contributions to systematic error in $M(D^0)$ due to event selection and peak fitting procedure are all smaller, as shown in Table III. They include variation of maximum value of $|\cos\theta|$ for decay particles, variation of minimum value of transverse momenta and maximum value of total momenta of all particles, and variation of the fit range and background shape. We estimate the uncertainty in our analysis procedure as the difference between MC input and output values of $M(D^0)$. The difference is found to be $\Delta M(D^0)(\text{output} - \text{input}) = 7 \pm 1$ keV and we assign a systematic uncertainty of ±7 keV. Added in quadrature, the total systematic uncertainty due to event selections and fit procedure is ±22 keV.

In Fig. 5 we show the D^0 mass difference dependence on $\cos\theta$ and azimuthal angle ϕ. All $M(D^0)$ values are found to be statistically in agreement with the average value, with $\chi^2/\text{d.o.f.}$ of 0.96 and 0.47 for $\cos\theta$ and ϕ, respectively.

Thus our final result for $M(D^0)$ is

$$M(D^0)_{\text{PRESENT}} = 1864.845 \pm 0.025(\text{stat}) \pm 0.022(\text{syst})$$
$$\pm 0.053(\text{kaon masses}) \text{ MeV}. \qquad (5)$$

With all uncertainties added in quadrature, our present results are

$$M(K_S)_{\text{PRESENT}} = 497.607 \pm 0.016 \text{ MeV}, \qquad (6)$$
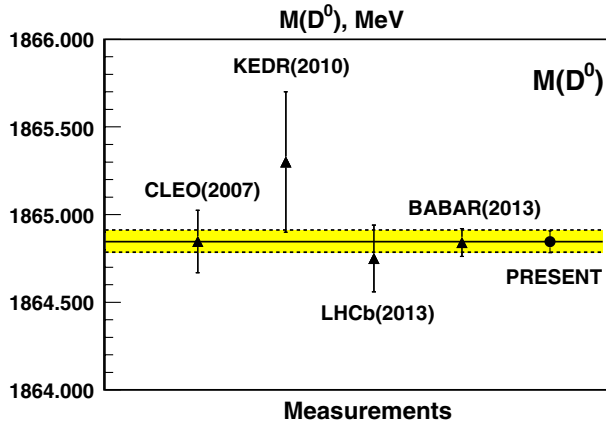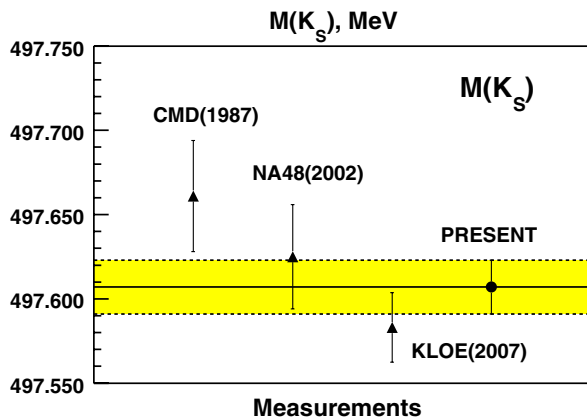

FIG. 6 (color online).   Summary of $M(K_S)$ and $M(D^0)$ from individual measurements from different experiments. The statistical and systematic errors of the measurements were added in quadrature. Solid and dashed lines correspond to the central value and total error band corresponding to our present measurement $M(K_S) = 497.607 \pm 0.016$ MeV, $M(D^0) = 1864.845 \pm 0.063$ MeV.

$$M(D^0)_{\text{PRESENT}} = 1864.845 \pm 0.063 \text{ MeV}. \qquad (7)$$

Both $M(K_S)$ and $M(D^0)$ are presently the world's most precise single measurements of these masses. Our $M(D^0)$ agrees with our previous measurement [8], and has a factor three smaller uncertainty. It also agrees with the recent the *BABAR* result [11], and is based on fourteen times larger number of events, has factor two smaller statistical error, and ∼20% smaller overall error. Figure 6 shows these results together with results of previous mass measurements [8,9,11,15–20]. The world average of all measurements, determined mainly by our results in Eq. (7), and

the *BABAR* results $M(D^0) = 1864.841 \pm 0.079$ MeV [11], is $M(D^0) = 1864.843 \pm 0.044$ MeV. The 1992 CLEO measurement [21], adopted by PDG [10], gives $M(D^{*0}) - M(D^0) = 142.12 \pm 0.07$ MeV. Thus, $M(D^{*0}) = 2006.963 \pm 0.083$ and $M(D^0) + M(D^{*0}) = 3871.806 \pm 0.112$ MeV. This leads to the binding energy of X (3872), B.E. $X(3872) = (3871.806 \pm 0.112) - (3871.68 \pm 0.17) = 0.126 \pm 0.204$ MeV.

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