Transitions of $B_c \to \psi(1S,2S)$ and the modified harmonic oscillator wave function in the light front quark model

Hong-Wei Ke,[1,*] Tan Liu,[1] and Xue-Qian Li[2,†]

[1]*School of Science, Tianjin University, Tianjin 300072, China*
[2]*China School of Physics, Nankai University, Tianjin 300071, China*

(Received 5 August 2013; published 21 January 2014)

The LHCb Collaboration has systematically measured the rates of $B_c \to J/\psi K$, $B_c \to J/\psi D_s$, $B_c \to J/\psi D_s^*$, and $B_c \to \psi(2S)\pi$. The new data enable us to study relevant theoretical models and further determine the model parameters. In this work, we calculate the form factors for the transitions $B_c \to J/\psi$ and $B_c \to \psi(2S)$ numerically and then determine the partial widths of the semileptonic and nonleptonic decays. The theoretical predictions on the ratios of $\Gamma(B_c \to J/\psi K)/\Gamma(B_c \to J/\psi\pi)$, $\Gamma(B_c \to J/\psi D_s)/\Gamma(B_c \to J/\psi\pi)$, and $\Gamma(B_c \to J/\psi D_s^*)/\Gamma(B_c \to J/\psi\pi)$ are consistent with data within only 1σ. Especially, for calculating $\Gamma(B_c \to \psi(2S)X)$ the modified harmonic oscillator wave function which we developed in early works is employed, and the results indicate that the modified harmonic oscillator wave function works well.

DOI: 10.1103/PhysRevD.89.017501 PACS numbers: 14.40.Nd, 12.39.Ki, 13.30.Ce

I. INTRODUCTION

Recently, the LHCb Collaboration has measured several decay modes of B_c and obtained $\Gamma(B_c \to \psi(2S)\pi)/\Gamma(B_c \to J/\psi\pi) = 0.25 \pm 0.068 \pm 0.014 \pm 0.006$ [1], $\Gamma(B_c \to J/\psi K)/\Gamma(B_c \to J/\psi\pi) = 0.069 \pm 0.019 \pm 0.005$ [2], $\Gamma(B_c \to J/\psi D_s)/\Gamma(B_c \to J/\psi\pi) = 2.9 \pm 0.57 \pm 0.24$, and $\Gamma(B_c \to J/\psi D_s{}^*)/\Gamma(B_c \to J/\psi D_s) = 2.37 \pm 0.56 \pm 0.10$ [3]. It would be a good time to carry out serious theoretical studies on those decay modes which may provide us more information about the structure of such two-heavy-flavor mesons and especially serve as a probe for our models which deal with the nonperturbative QCD. Though the typical $P \to V$ (P and V denote a pseudoscalar meson and a vector meson, respectively) transitions have been studied by various approaches [4–7], the theoretical predictions on B_c are few [8]. In Ref. [9], Cheng, Chua, and Hwang studied $P \to V$ transitions in the light front quark model (LFQM) [9–16]. In this work we will apply the formula derived by Cheng, Chua, and Hwang in Ref. [9] to study the semileptonic decay $B_c \to J/\psi(\psi(2S))e\bar{\nu}_e$ and nonleptonic decay $B_c \to J/\psi(\psi(2S)) + X$ (X can be π, K, K^*, D, D^*, D_s, and D_s^*). Hopefully, we can further test the validity degree of the LFQM and constrain the model parameter space.

In the LFQM, a phenomenological wave function is introduced to describe the momentum distribution amplitudes of the constituent quarks, and the harmonic oscillator wave functions may be the most convenient and applicable one among all possible forms. Most of the previous studies explored only the transitions between ground states. In our early work [17], we calculated the decay constants of $\Upsilon(nS)(n > 1)$ (excited states of bottomonia) with the traditional harmonic oscillator wave functions and found that the theoretical results obviously conflict with the data, so we proposed to choose a modified harmonic oscillator wave function instead for the radially excited states. With this change, the inconsistency between theoretical predictions and the data disappears. In this work we would like to further check the modified harmonic oscillator wave functions for a radially excited state in $B_c \to \psi(2S)$ weak decays.

After the introduction, we present the relevant formulas for $P \to V$ transition in Sec. II, where we introduce briefly our modified harmonic oscillator wave functions. Then we numerically evaluate the form factors and the decay widths for the available decay modes and predict the rates for some channels. Last, we make a brief summary.

II. FORMULAS

A. $P \to V$ transition in the LFQM

The form factors for $B_c \to J/\psi$ and $B_c \to \psi(2S)$ which are the typical $P \to V$ transitions are defined as

$$\begin{aligned}\langle V(p'', \varepsilon'')|V_\mu|P(p')\rangle &= i\Big\{(M' + M'')\varepsilon''^*_\mu A_1^{PV}(q^2) \\ &\quad - \frac{\varepsilon''^* \cdot p'}{M' + M''} p_\mu A_2^{PV}(q^2) \\ &\quad - 2M'' \frac{\varepsilon''^* \cdot p'}{q^2} q_\mu [A_3^{PV}(q^2) \\ &\quad - A_0^{PV}(q^2)]\Big\}, \\ \langle V(p'', \varepsilon'')|A_\mu|P(p')\rangle &= -\frac{1}{M' + M''} \epsilon''_{\mu\nu\rho\sigma}\varepsilon^{*\nu} p^\rho q^\sigma V^{PV}(q^2),\end{aligned} \tag{1}$$

*khw020056@hotmail.com
†lixq@nankai.edu.cn

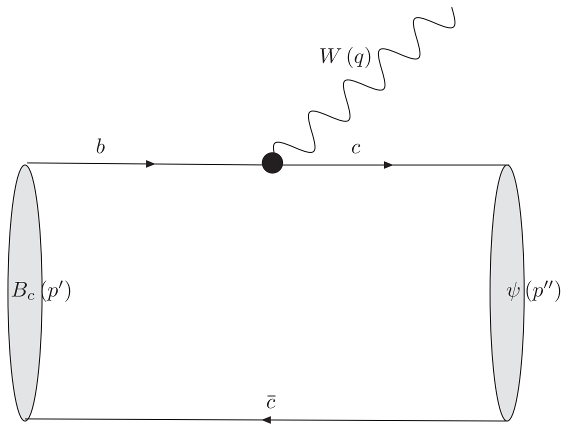

FIG. 1. The $B_c \to \psi$ transition.

with

$$A_3^{PV}(q^2) = \frac{M' + M''}{2M''} A_1^{PV}(q^2) - \frac{M' - M''}{2M''} A_2^{PV}(q^2), \quad (2)$$

where M' (M'') and p' (p'') are the mass and momentum, respectively, of the vector (pseudoscalar) state. We also define $p = p' + p''$ and $q = p' - p''$.

As discussed in Ref. [9], these form factors are calculated in the spacelike region with $q^+ = 0$; thus, to obtain the physical amplitudes, an extension to the timelike region is needed. To make the extension, one may write out analytical expressions for the form factors, and in Ref. [9] a three-parameter form was suggested:

$$F(q^2) = \frac{F(0)}{\left[1 - a\left(\frac{q^2}{M^2_{\Lambda_b}}\right) + b\left(\frac{q^2}{M^2_{\Lambda_b}}\right)^2\right]}. \quad (3)$$

The relevant Feynman diagrams for the transitions are shown in Fig. 1. In Ref. [9], the authors deduce all the detailed expressions for the form factors A_0, A_1, A_2, and V in the covariant LFQM. One can refer to Eqs. (32) and (B4) of Ref. [9] to find their explicit expressions.

B. The modified harmonic oscillator wave functions

For calculating the form factors A_0, A_1, A_2, and V, the light front momentum distribution amplitudes need to be specified. In most such works, the harmonic oscillator wave function is employed because of its obvious advantages. In our previous work [17], we found that predictions on the rates of the processes where radially excited states are involved do not coincide with the data as long as the transitional harmonic oscillator wave function was employed; thus, we suggested to use a modified harmonic oscillator wave function to replace the traditional one for the radially excited states. It is found that the modified wave function indeed works well when we calculate the radiative decays of $\Upsilon(nS)(n > 1)$.

The decay of $B_c \to \psi(2S)$, where $\psi(2S)$ is a radially excited state, would serve as an alternative probe for testing the modified wave function. Thus we use both the traditional and modified harmonic oscillator wave functions to calculate the rates of $B_c \to \psi(2S) + X$, where X denotes some relevant mesons. Through comparing the results obtained in terms of the two kinds of $\psi(2S)$ wave function with the data, we can determine their reasonability. The relevant modified wave function is

$$\begin{aligned}
\phi(1S) &= 4\left(\frac{\pi}{\beta^2}\right)^{3/4}\sqrt{\frac{\partial k_z}{\partial x}}\exp\left(-\frac{k_z^2 + k_\perp^2}{2\beta^2}\right),\\
\phi(2S) &= 4\left(\frac{\pi}{\beta^2}\right)^{3/4}\sqrt{\frac{\partial k_z}{\partial x}}\exp\left(-\frac{1}{2}\frac{k_z^2 + k_\perp^2}{\beta^2}\right)\\
&\quad\times\frac{1}{\sqrt{6}}\left(3 - 2\frac{k_z^2 + k_\perp^2}{\beta^2}\right),\\
\phi_M(2S) &= 4\left(\frac{\pi}{\beta^2}\right)^{3/4}\sqrt{\frac{\partial k_z}{\partial x}}\exp\left(-\frac{2^\delta}{2}\frac{k_z^2 + k_\perp^2}{\beta^2}\right)\\
&\quad\times\left(a - b\frac{k_z^2 + k_\perp^2}{\beta^2}\right),
\end{aligned} \quad (4)$$

where β is a phenomenological parameter and needs to be fixed by fitting the data. k is the relative momentum of the constituents, and x is the momentum fraction of the quark while $1 - x$ is for the antiquark. More details can be found in Refs. [7,17]. In Ref. [17], we fixed $a = 1.89$, $b = 1.55$, and $\delta = 1/1.82$ for $\Upsilon(2S)$, and by the heavy quark effective theory it is reasonable to suppose that they are the same for $\psi(2S)$.

C. Rates of the semileptonic and nonleptonic decays

Since there is no strong interaction in the final states to contaminate the processes, semileptonic decays can shed more light for understanding the meson structure which is associated with nonperturbative QCD and help to fix the model parameters. The amplitude for the semileptonic decay is

$$\langle\psi l\bar{\nu}_l|\mathcal{H}|B_c\rangle = \frac{G_F}{\sqrt{2}} V_{cb}\langle V|V_\mu - A_\mu|P\rangle\bar{l}\gamma^\mu(1-\gamma_5)\nu_l. \quad (5)$$

For evaluating the rates of nonleptonic decays $P \to V + X$, generally factorization is assumed; i.e., the hadronic transition matrix element can be factorized into a product of two independent matrix elements: the transition matrix $\langle P|J'_\mu|V\rangle$ and $\langle 0|J^\mu|X\rangle$ which is determined by a decay constant. For the nonleptonic decays $B_c \to J/\psi(\psi(2S))X$, the effective interaction at the quark level $b \to c\bar{q}_1 q_2$ is

$$\mathcal{H}_W = \frac{G_F}{\sqrt{2}} V_{cb} V^*_{q_1 q_2}(c_1 O_1 + c_2 O_2), \quad (6)$$

where c_i denote the Wilson coefficients and O_i are four-quark operators. The hadronic transition matrix elements are

$$\langle\psi M|\mathcal{H}_W|B_c\rangle = \frac{G_F}{\sqrt{2}}V_{cb}V^*_{q_1q_2}a_1\langle V|V_\mu - A_\mu|P\rangle f_M q^\mu, \quad M \text{ is a pseudoscalar}, \tag{7}$$

$$= \frac{G_F}{\sqrt{2}}V_{cb}V^*_{q_1q_2}a_1\langle V|V_\mu - A_\mu|P\rangle m_M f_M \epsilon^\mu_M, \quad M \text{ is a vector}, \tag{8}$$

where the Wilson coefficient $a_1 = c_1 + c_2/N_c$ with N_c being an effective color number which is 3 when the color-octet contributions are not taken into account [18].

III. NUMERICAL RESULTS

In this section, we will calculate the form factors for these $P \to V$ transitions. The masses $m_{B_c} = 6.277$ GeV, $m_{J/\psi} = 3.096$ GeV, and $m_{\psi(2S)} = 3.686$ GeV are taken from the data book [19]. The parameter β in the wave function of J/ψ is fixed to be 0.631 GeV when $m_c = 1.4$ GeV [20]. However, until now there were no available data to fix the model parameter β in the wave function of B_c, so we will make an estimate based on reasonable arguments. In Ref. [17], we fixed $\beta = 1.257$ GeV for Υ where $m_b = 5.2$ GeV was set; accordingly, we take an average of 0.631 and 1.257 GeV as the value of β in the wave function of B_c, which is fixed to be 0.944 GeV. In our calculation we set $m_c = 1.4$ GeV and $m_b = 5.2$ GeV. The Cabibbo-Kobayashi-Maskawa (CKM) matrix elements take values $V_{cb} = 0.0406$, $V_{cd} = 0.2252$, $V_{ud} = 0.97425$, $V_{us} = 0.2252$, $V_{cs} = 1.006$ [19], and $a_1 = 1$ [12]. The decay constants and masses for the relevant mesons are listed in Table I.

With these parameters we calculate the form factors for the transitions $B_c \to J/\psi$ and $B_c \to \psi(2S)$ numerically, and an analytical form [Eq. (4)] is eventually obtained. The three parameters for the different cases are listed in Table II. For the $B_c \to \psi(2S)$ transition, since $\psi(2S)$ is a radially excited state, two different momentum distribution amplitudes defined in Eq. (5) are employed in our numerical calculations.

With these form factors, we calculate the rates for several decay modes. The theoretical predictions are listed in Table III, where the theoretical uncertainties are estimated by varying the parameters m_b, m_c, and β within a 10% range. The predictions of the ratios $\Gamma(B_c \to J/\psi K)/\Gamma(B_c \to J/\psi\pi)$, $\Gamma(B_c \to J/\psi D_s)/\Gamma(B_c \to J/\psi\pi)$, and $\Gamma(B_c \to J/\psi D_s^*)/\Gamma(B_c \to J/\psi D_s)$ are 0.079 ± 0.033, 2.06 ± 0.86, and 3.01 ± 1.23, respectively, which are consistent with the data $0.069 \pm 0.019 \pm 0.005$, $2.9 \pm 0.57 \pm 0.24$, and $2.37 \pm 0.56 \pm 0.10$ within 1σ.

As for the transition $B_c \to \psi(2S)$, by using the two different harmonic oscillator wave functions we obtain $\Gamma(B_c \to \psi(2S)\pi)/\Gamma(B_c \to J/\psi\pi) = 0.45 \pm 0.14$ and $\Gamma(B_c \to \psi_M(2S)\pi)/\Gamma(B_c \to J/\psi\pi) = 0.23 \pm 0.08$, where the subscript M refers to the modified harmonic oscillator wave function. The result with the modified harmonic oscillator wave function is obviously closer to the data $0.25 \pm 0.068 \pm 0.014$ than using the traditional one. This fact indicates that the modified harmonic oscillator wave functions for radially excited states are reasonable and applicable. More theoretical predictions on the channels which have not been yet measured so far are made and presented in Table III. All the predictions will be tested by future experiments at LHCb or other facilities such as the planned ILC or Z_0, Higgs factories, etc. Since the parameter β in the wave function of B_c is obtained by an interpolation between the values for J/ψ and Υ, it is not accurate, and thus the obtained values of the widths listed in Table III may change for different β values; however, the ratio between two widths would not vary much, because the effect caused by the uncertainty of β is partly compensated in the ratios.

TABLE I. Meson decay constants and masses (in units of MeV).

Meson	π	K	K^*	D	D^*	D_s	D_s^*
m [19]	139.6	493.7	891.7	1869.6	2010.3	1968.5	2112.3
f [9]	131	160	210	200	220	230	230

TABLE II. The form factors given in the three-parameter form.

F	$F(0)$	a	b
$A_0^{B_cJ/\psi}$	0.502	1.66	2.04
$A_2^{B_cJ/\psi}$	0.398	1.97	1.84
$A_0^{B_c\psi(2S)}$	0.452	0.92	0.50
$A_2^{B_c\psi(2S)}$	0.102	−2.73	4.63
$A_0^{B_c\psi_M(2S)}$	0.300	1.15	0.60
$A_2^{B_c\psi_M(2S)}$	0.109	−1.93	3.71
$A_1^{B_cJ/\psi}$	0.467	1.51	0.95
$V^{B_cJ/\psi}$	0.638	2.15	2.21
$A_1^{B_c\psi(2S)}$	0.335	−0.21	0.88
$V^{B_c\psi(2S)}$	0.525	0.53	0.96
$A_1^{B_c\psi_M(2S)}$	0.251	−0.058	0.98
$V^{B_c\psi_M(2S)}$	0.388	0.68	1.16

IV. SUMMARY

In this paper, we calculate the weak decays $B_c \to J/\psi + X$ and $B_c \to \psi(2S) + X$ within the light front quark model. Though there is uncertainty for the value of β in the wave function of B_c, the theoretically evaluated ratios $\Gamma(B_c \to J/\psi K)/\Gamma(B_c \to J/\psi\pi) = 0.079 \pm 0.033$,

TABLE III. The decay widths of some modes.

	Width (GeV)	Branching ratio
$B_c \to J/\psi\pi$	$(9.64 \pm 2.82) \times 10^{-16}$	$(6.64 \pm 2.05) \times 10^{-4}$
$B_c \to J/\psi K$	$(7.66 \pm 2.23) \times 10^{-17}$	$(5.27 \pm 1.62) \times 10^{-5}$
$B_c \to J/\psi K^*$	$(1.58 \pm 0.46) \times 10^{-16}$	$(1.09 \pm 0.33) \times 10^{-4}$
$B_c \to J/\psi D$	$(8.02 \pm 2.33) \times 10^{-17}$	$(5.52 \pm 1.69) \times 10^{-5}$
$B_c \to J/\psi D^*$	$(2.65 \pm 0.76) \times 10^{-16}$	$(1.82 \pm 0.55) \times 10^{-4}$
$B_c \to J/\psi D_s$	$(1.99 \pm 0.58) \times 10^{-15}$	$(1.37 \pm 0.42) \times 10^{-3}$
$B_c \to J/\psi D_s^*$	$(5.98 \pm 1.72) \times 10^{-15}$	$(4.12 \pm 1.23) \times 10^{-3}$
$B_c \to J/\psi e\bar{\nu}_e$	$(1.67 \pm 0.49) \times 10^{-14}$	$(1.15 \pm 0.36)\%$
$B_c \to \psi(2S)\pi$	$(4.31 \pm 0.42) \times 10^{-16}$	$(2.97 \pm 0.41) \times 10^{-4}$
$B_c \to \psi(2S)K$	$(3.34 \pm 0.33) \times 10^{-17}$	$(2.30 \pm 0.32) \times 10^{-5}$
$B_c \to \psi(2S)K^*$	$(6.37 \pm 0.83) \times 10^{-17}$	$(4.39 \pm 0.71) \times 10^{-5}$
$B_c \to \psi(2S)D$	$(2.01 \pm 0.27) \times 10^{-17}$	$(1.38 \pm 0.23) \times 10^{-5}$
$B_c \to \psi(2S)D^*$	$(6.27 \pm 1.60) \times 10^{-17}$	$(4.32 \pm 1.17) \times 10^{-5}$
$B_c \to \psi(2S)D_s$	$(4.48 \pm 0.61) \times 10^{-16}$	$(3.08 \pm 0.52) \times 10^{-4}$
$B_c \to \psi(2S)D_s^*$	$(1.29 \pm 0.35) \times 10^{-15}$	$(8.85 \pm 2.54) \times 10^{-4}$
$B_c \to \psi(2S)e\bar{\nu}_e$	$(2.73 \pm 0.58) \times 10^{-15}$	$(1.88 \pm 0.44) \times 10^{-3}$
$B_c \to \psi_M(2S)\pi$	$(2.24 \pm 0.19) \times 10^{-16}$	$(1.54 \pm 0.20) \times 10^{-4}$
$B_c \to \psi_M(2S)K$	$(1.74 \pm 0.14) \times 10^{-17}$	$(1.20 \pm 0.15) \times 10^{-5}$
$B_c \to \psi_M(2S)K^*$	$(3.39 \pm 0.24) \times 10^{-17}$	$(2.33 \pm 0.28) \times 10^{-5}$
$B_c \to \psi_M(2S)D$	$(1.10 \pm 0.07) \times 10^{-17}$	$(7.57 \pm 0.87) \times 10^{-6}$
$B_c \to \psi_M(2S)D^*$	$(3.55 \pm 0.58) \times 10^{-17}$	$(2.44 \pm 0.47) \times 10^{-5}$
$B_c \to \psi_M(2S)D_s$	$(2.44 \pm 0.14) \times 10^{-16}$	$(1.68 \pm 0.19) \times 10^{-4}$
$B_c \to \psi_M(2S)D_s^*$	$(7.32 \pm 1.29) \times 10^{-16}$	$(5.04 \pm 1.02) \times 10^{-4}$
$B_c \to \psi_M(2S)e\bar{\nu}_e$	$(1.51 \pm 0.19) \times 10^{-15}$	$(1.04 \pm 0.17) \times 10^{-3}$

$\Gamma(B_c \to J/\psi D_s)/\Gamma(B_c \to J/\psi\pi) = 2.06 \pm 0.86$, and $\Gamma(B_c \to J/\psi D_s^*)/\Gamma(B_c \to J/\psi D_s) = 3.01 \pm 1.23$ are consistent with the data within only 1σ. The rates of other decays of $B_c \to J/\psi + X$ and $B_c \to \psi(2S) + X$ are also calculated which will be experimentally measured soon, and by then we can fix or extract some parameters including the value of β for B_c.

In Ref. [17], we suggested a modified harmonic oscillator wave function for the radially excited states in the LFQM. By using these modified wave functions, the obtained decay constants of $\Upsilon(nS)$ are in good agreement with the data, and we also checked the applicability of these wave functions in the radiative decays of $\Upsilon(nS)$. In this work we calculate the transition $B_c \to \psi(2S)\pi$ with the traditional and modified wave functions for $\psi(2S)$. The theoretical results are quite different when the two wave functions are employed, as the ratios are $\Gamma(B_c \to \psi(2S)\pi)/\Gamma(B_c \to J/\psi\pi) = 0.45 \pm 0.14$ and $\Gamma(B_c \to \psi_M(2S)\pi)/\Gamma(B_c \to J/\psi\pi) = 0.23 \pm 0.08$, and the result using the modified wave function is closer to the data $0.25 \pm 0.068 \pm 0.014 \pm 0.006$. Namely, our numerical results, which are satisfactorily consistent with the data of $B_c \to \psi(2S) + X$, indicate that the modified wave function works well not only for the radially excited bottomonia, but also for radially excited charmonia. The consistency degree of other predictions for $B_c \to \psi(2S) + X$ with the future experimental data will provide a further test to the modified wave function.

ACKNOWLEDGMENT

This work is supported by the National Natural Science Foundation of China (NNSFC) under Contracts No. 11075079, No. 11005079, and No. 11375128 and the Special Grant for the Ph.D. program of Ministry of Education of the People's Republic of China No. 20100032120065.

[1] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. D **87**, 071103 (2013).

[2] R. Aaij *et al.* (LHCb Collaboration), J. High Energy Phys. 09 (**2013**) 075.

[3] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. D **87**, 112012 (2013).

[4] H.-Y. Cheng and S. Oh, J. High Energy Phys. 09 (**2011**) 024.

[5] B. Bhattacharya and J. L. Rosner, Phys. Rev. D **79**, 034016 (2009); B. Bhattacharya and J. L. Rosner, Phys. Rev. D **81**, 099903(E) (2010).

[6] A. Ali, G. Kramer, Y. Li, C.-D. Lu, Y.-L. Shen, W. Wang, and Y.-M. Wang, Phys. Rev. D **76**, 074018 (2007).

[7] H.-N. Li and S. Mishima, Phys. Rev. D **74**, 094020 (2006).

[8] D. Ebert, R. N. Faustov, and V. O. Galkin, Phys. Rev. D **68**, 094020 (2003); Y.-M. Wang and C.-D. Lu, Phys. Rev. D **77**, 054003 (2008).

[9] H. Y. Cheng, C. K. Chua, and C. W. Hwang, Phys. Rev. D **69**, 074025 (2004).

[10] W. Jaus, Phys. Rev. D **41**, 3394 (1990); W. Jaus, Phys. Rev. D **44**, 2851 (1991); W. Jaus, Phys. Rev. D **60**, 054026 (1999).

[11] C. R. Ji, P. L. Chung, and S. R. Cotanch, Phys. Rev. D **45**, 4214 (1992).

[12] H.-Y. Cheng, C.-K. Chua, and C.-W. Hwang, Phys. Rev. D **70**, 034007 (2004).

[13] H. Y. Cheng, C. Y. Cheung, and C. W. Hwang, Phys. Rev. D **55**, 1559 (1997).

[14] G. Li, F. L. Shao, and W. Wang, Phys. Rev. D **82**, 094031 (2010).

[15] C. W. Hwang and Z. T. Wei, J. Phys. G **34**, 687 (2007); C. D. Lu, W. Wang, and Z. T. Wei, Phys. Rev. D **76**, 014013 (2007).

[16] H. M. Choi, Phys. Rev. D **75**, 073016 (2007).

[17] H. W. Ke, X. Q. Li, Z. T. Wei, and X. Liu, Phys. Rev. D **82**, 034023 (2010).

[18] A. J. Buras and J. M. Gerard, Nucl. Phys. **B264**, 371 (1986); H.-Y. Cheng, Z. Phys. C **32**, 237 (1986); X.-Q. Li, T. Huang, and Z.-C. Zhang, Z. Phys. C **42**, 99 (1989).

[19] J. Beringer *et al.* (Particle Data Group Collaboration), Phys. Rev. D **86**, 010001 (2012).

[20] H. W. Ke and X. Q. Li, Phys. Rev. D **84**, 114026 (2011).