

Double parton scatterings in high-energy proton-nucleus collisions and partonic correlations

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The joint study of double-parton scattering (DPS) in high-energy proton-proton and proton-nucleus collisions can provide a lot of information on multiparton correlations. The multiparton structure is in fact probed in different ways by DPS in p - p and in p - A collisions. In p - A collisions, the interpretation of the experimental results may be complicated, however, by the presence of interference terms, which are missing in p - p collisions. A suitable reaction channel, where interference terms are absent, is WJJ production. By studying WJJ production in p - Pb collisions, we estimate that the fraction of events due to DPS may be larger by a factor 3 or 4 as compared to p - p , while the amount of the increased fraction can give information on the importance of different correlation terms.

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I. INTRODUCTION

Multiple-parton interactions (MPIs) have been introduced in the dynamical description of hadronic collisions as a natural solution of the unitarity problem, originated at high energies by the rapid growth of the hard cross sections at small x [1–3]. The inclusive cross section is in fact proportional to the multiplicity of elementary partonic interactions, and the increasingly large values of the cross section are in this way understood as the result of an increasingly large average number of elementary interactions in an inelastic event. Each elementary partonic interaction is localized in transverse space, inside the much larger overlap region of the matter distribution of the colliding hadrons. A given final state can, of course, be produced by different MPI processes, which contribute to the cross section with different weights. At small x , the leading contribution is provided by the term that maximizes the number of interacting partons, which corresponds to the processes where the hard component of the interaction is maximally disconnected. In the simplest case, namely in double-parton scattering (DPS), the dominant contribution to the cross section at small x is thus given by the term where two different pairs of partons interact independently in two different points in transverse space.

As recently pointed out [4,5], the typical back-to-back configuration of four large- p_t partons, produced in a DPS by the leading contribution at small x and utilized as a

distinctive signature for the experimental search of DPS events, can be generated also by a hard interaction involving three partons in the initial state, all localized in the same point in transverse space. The initial partonic flux in $3 \rightarrow 4$ processes is very different as compared to the initial partonic flux of $(2 \rightarrow 2)^2$ processes and, as discussed in Ref. [4], the typical unbalance of the final-state parton pairs is rather different in the two cases. The initial-state partonic flux is a measurable quantity, and a careful study of the dependence of the cross section on the initial-state fractional momenta and on the momentum unbalance should be able to separate experimentally the contributions to the observed DPS cross section, due to $3 \rightarrow 4$ processes from the leading ones at small x . In the present paper, we will focus on the disconnected component of DPS in p - p and in p - A collisions, while the problem of the experimental identification and subtraction of a possible $3 \rightarrow 4$ background to the observed DPS cross section lies outside our scope and will not be discussed in the present paper. With DPS, we will thus refer specifically to the contribution to the inclusive cross section due to disconnected hard interactions.

When dealing with disconnected hard interactions, the simplest assumption, which leads to very compact results, is that the different partons with small x in the proton are uncorrelated with each other. On the other hand, several different types of correlations can be expected, and the topic of correlations has addressed a lot of attention [6–17]. In addition to the dependence on the kinematical variables, DPS amplitudes are in fact expected to depend on spin and color, which induce interference terms in the cross section. Color correlations are Sudakov suppressed, and thus small

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for double-parton scattering at high energies [6,13]. Probably more important are spin correlations, which are expected to affect the rate of double-parton scattering and the angular distribution of the final state in particular reaction channels, even if partons are not polarized [16].

The study of correlations in DPS is therefore a rather rich topic, and a proper approach to the problem requires the introduction of flavor- and spin-dependent double-parton distribution functions. The issue is still under theoretical investigation while present experimental results cannot yet provide indications on the relative importance of different spin and flavor contributions to the double-parton distributions. A complete description of DPS in p - A collisions, even without considering possible $3 \rightarrow 4$ contributions, including, however, all terms required to account for the dependence on flavor and spin, is therefore still premature. On the other hand, there are reasons to expect that, in p - A collisions, the basic features of DPS will change substantially as compared with the case of DPS in p - p [18–23]. It may therefore be instructive to work out explicitly the expectations of DPS in p - A collisions when assuming the simplest possible scenario still not in contradiction with present experimental evidence of DPS in p - p collisions.

The simplest possibility is to neglect the effects of spin and color in the disconnected component of the DPS cross section. The DPS cross section is thus factorized into functions which depend on the fractional momenta of the interacting partons, on the resolution of the hard processes, and on the relative transverse distance β between the two interaction points. The expression of the cross section for two parton processes A and B in a p - p collision is thus given by

$$\begin{aligned} \sigma_D^{pp(A,B)} &= \frac{m}{2} \sum_{i,j,k,l} \int \Gamma_{i,j}(x_1, x_2; \beta) \hat{\sigma}_{i,k}^A(x_1, x'_1) \\ &\quad \times \hat{\sigma}_{j,l}^B(x_2, x'_2) \Gamma_{k,l}(x'_1, x'_2; \beta) dx_1 dx'_1 dx_2 dx'_2 d^2\beta, \end{aligned} \quad (1)$$

where $\Gamma_{i,j}(x_1, x_2; \beta)$ are the double-parton distribution functions, and the dependence on the fractional momenta of the interacting partons $x_{1,2}$ and on their relative transverse distance β is explicitly indicated, while the dependence on the scales of the two hard processes A and B is understood. The indices i and j label parton flavors. For identical interactions $m = 1$, and $m = 2$ otherwise. $\hat{\sigma}^A$, $\hat{\sigma}^B$ are the two elementary cross sections.

Equation (1) may lead to a very simple expression, where the cross section is given by the product of the two single-scattering inclusive cross sections of the hard processes A and B :

$$\frac{\sigma_D^{pp(A,B)}}{dx_i dx'_i dp_{ti}} = \frac{m}{2} \frac{1}{\sigma_{\text{eff}}} \frac{d\sigma^A}{dx_1 dx'_1 dp_{t1}} \frac{d\sigma^B}{dx_2 dx'_2 dp_{t2}}, \quad (2)$$

which is the ‘‘pocket formula’’ utilized in all experimental analyses of DPS [24–28]. All unknowns in the process converge in this way in the value of a single quantity with the dimensions of a cross section, σ_{eff} , which is therefore expected to depend on fractional momenta, resolution, parton flavors, and on the two-body correlation parameters, which characterize the double-parton distributions.

Equation (2) has a transparent physical meaning. When hard interactions are rare, the probability of also having the process B in an inelastic interaction is given by the ratio $\sigma^B/\sigma_{\text{inel}}$. Once the process A takes place, the probability of having the process B in the same inelastic interaction is different. It can, anyway, always be written as $\sigma^B/\sigma_{\text{eff}}$, where σ_{eff} plays *effectively* the role which was that of the inelastic cross section in the unbiased case.

Notice that, although σ_{eff} is related to the transverse distance between the two hard interactions, it cannot be understood as the effective transverse interaction area, since σ_{eff} depends also on the multiparton distributions in multiplicity. While initial fractional momenta and resolution are measured in the final state and the dependence of σ_{eff} on parton flavors can be obtained, at least to a certain extent, by selecting different reaction channels, the effects of the dependence on the partonic distributions in multiplicity and on the correlation in the relative transverse distance cannot be disentangled by looking only at p - p collisions.

Even in the simplest scenario, σ_{eff} is thus expected to depend on flavor and on all kinematical variables. In spite of that, Eq. (2) has been shown to be able to describe the experimental results of the direct search of double-parton collisions in rather different reaction channels and kinematical regimes [24–28] with a value of σ_{eff} not incompatible with a universal constant, while the study of CDF [25], of the dependence of σ_{eff} on the fractional momenta of the incoming partons, is again not inconsistent with a value of σ_{eff} independent of x . When the DPS cross section is generalized by introducing parton distributions depending on transverse momenta and off-shell T-matrix elements, the same value of σ_{eff} allows describing DPS also in the regime of very small x , where the back-to-back kinematical configuration, typical of the large- p_t partons originated by DPS, is lost. The observed production rates of $(J/\psi, J/\psi)$ are thus understood [29], while the production of different combinations of charmed mesons, the differential distributions in the $D^0 D^0$ invariant mass, and the azimuthal correlation between two D^0 mesons, as worked out in Ref. [30], are not incompatible with the recent measurements of the LHCb Collaboration [31].

One should underline that the experimental indication of a value of σ_{eff} consistent with a universal constant represents a nontrivial test of the simple interaction mechanism leading to Eq. (2). The expression of σ_D in Eq. (2) depends, in fact, rather strongly on the kinematical conditions of the observed process. In particular, the dependence on the

incoming parton flux is much stronger as compared with the case of a single hard scattering process.

As already noticed, although σ_{eff} is directly related to parton correlations, even in the simplest scenario, by measuring DPS only in p - p collisions, one does not have enough information to decide how much the observed value of σ_{eff} is originated by the typical separation in transverse space between the two pairs of interacting partons and how much it is rather due to the actual distribution in the multiplicity of parton pairs in the hadronic structure. Additional information to discriminate between the two cases can nevertheless be obtained by studying DPS in p - A collisions. MPIs in p - A collisions introduce, in fact, novel features in the process. A relevant novel feature is that one may have an MPI where two or more target nucleons are active participants in the hard process [18–21,23]. A relevant consequence of having two or more active target nucleons is that, while in the simplest picture of the interaction considered here, in p - p collisions MPIs are described by the incoherent superposition of sets of elementary partonic interactions [1–3,32], in p - A collisions interference terms may, on the contrary, play an important role [20].

To have some quantitative indication on the impact of the different features of DPS in p - A collisions, we will study the simplest option, where in p - p collisions σ_{eff} is a universal constant, and it is completely determined by the typical transverse distance between the two pairs of interacting partons, and by the multiplicity of parton pairs in the hadronic structure. We will further simplify the problem by selecting a suitable reaction channel, where there are no contributions of interference terms in p - A collisions. One can then show that the multiplicity of pairs of partons and their typical transverse separation have rather different effects on the DPS cross section in p - p and in p - A collisions. The amount of increase in the cross section when going to p - A can be in fact linked in a rather direct way to the multiplicity of parton pairs in the projectile, while the effects of the typical separation of the parton pairs in transverse space are only of minor importance.

The paper is organized as follows: In Sec. II, we recall some of the main features of DPS in p - p and in p - A collisions. In Sec. III, we discuss the case of W J J production. Section IV is devoted to illustrating, with some numerical estimates, the different effects on the p - A cross section of varying either the multiplicity of pairs of partons or their relative transverse distance. The last section is dedicated to summarizing remarks.

II. DPS IN p - p AND IN p - A COLLISIONS

It has been pointed out that, for sufficiently small values of β , the distributions $\Gamma(x_1, x_2; \beta)$ can be expressed in terms of known quantities [12]. For small β , $\Gamma(x_1, x_2; \beta)$ may be obtained from a single-parton distribution times a perturbative dynamics, which yields the splitting function for the longitudinal variables and a $1/\beta^2$ singularity in the

transverse relative distance. The divergent behavior of the DPS cross section at small β needs, therefore, to be properly subtracted, and the subtraction terms included in the single scattering contribution. The issue of the subtraction of the divergent contribution and of the correlation in fractional momenta, induced by perturbative splitting, has been discussed by several authors and is still a matter of debate, in particular for what concerns the QCD evolution of the double-parton distribution functions [4,5,12,33–35].

The common origin of the initial-state partons leaves anyway track in the DPS cross section. A main qualitative feature is the presence of additional contributions, which, however, cannot be considered anymore as disconnected in transverse space and cannot be expressed by Eq. (1). The importance of these contributions grows with the fractional momenta of the incoming partons and would thus induce a measurable dependence of σ_{eff} on the initial-state fractional momenta. Although there was no systematic study of the x dependence, the available experimental evidence does not seem to imply a sizable dependence of σ_{eff} on the initial-state fractional momenta. As stated in the Introduction, we will therefore take the simplified attitude of assuming that possible additional contributions to the measured DPS cross section can be identified and subtracted experimentally, and we will focus on the disconnected DPS interaction mechanism.

Disregarding for simplicity the dependence on flavor and on the resolution, one may introduce

$$\begin{aligned} G(x_1, x_2) &\equiv \int \Gamma(x_1, x_2; \beta) d^2\beta, \\ G(x_1, x_2) &\equiv K_{x_1 x_2} G(x_1) G(x_2), \end{aligned} \quad (3)$$

where $G(x)$ are the usual one-body distribution functions. Without any loss of generality, one may thus write

$$\Gamma(x_1, x_2; \beta) = K_{x_1 x_2} G(x_1) G(x_2) f_{x_1 x_2}(\beta), \quad (4)$$

with $\int f_{x_1 x_2}(\beta) d^2\beta = 1$. One has

$$\begin{aligned} \sigma_D^{pp(A,B)}(x_1, x'_1, x_2, x'_2) &= \frac{m}{2} K_{x_1 x_2} K_{x'_1 x'_2} G(x_1) \hat{\sigma}_A(x_1, x'_1) G(x'_1) \\ &\quad \times G(x_2) \hat{\sigma}_B(x_2, x'_2) G(x'_2) \\ &\quad \times \int f_{x_1 x_2}(\beta) f_{x'_1 x'_2}(\beta) d^2\beta \\ &= \frac{m}{2} \frac{K_{x_1 x_2} K_{x'_1 x'_2}}{\pi \Lambda^2(x_1, x'_1, x_2, x'_2)} \sigma_A(x_1, x'_1) \sigma_B(x_2, x'_2), \end{aligned}$$

where

$$\int f_{x_1 x_2}(\beta) f_{x'_1 x'_2}(\beta) d^2\beta = \frac{1}{\pi \Lambda^2(x_1, x'_1, x_2, x'_2)}. \quad (5)$$

The effective cross section is therefore given by

$$\sigma_{\text{eff}}(x_1, x'_1, x_2, x'_2) = \frac{\pi \Lambda^2(x_1, x'_1, x_2, x'_2)}{K_{x_1 x_2} K_{x'_1 x'_2}}, \quad (6)$$

and $\Lambda(x_1, x'_1, x_2, x'_2)$ measures the typical transverse distance between the pairs of interacting partons for given values of fractional momenta, while $K_{x_1 x_2}$ gives the second moment of the multiparton exclusive multiplicity distribution. More precisely, $K_{x_1 x_2} = \langle n(n-1) \rangle_{x_1 x_2} / (\langle n \rangle_{x_1} \langle n \rangle_{x_2})$ [19] in such a way that, in the simplest case of a Poissonian distribution in multiplicity, one would have $K_{x_1 x_2} = 1$. Present experimental indication is that the effective cross section depends only weakly on fractional momenta.

As apparent in Eq. (6), in nucleon-nucleon collisions all effects due to parton correlations are summarized in the value of a single quantity (the effective cross section), and nucleon-nucleon collisions alone do not allow one to measure Λ and K separately. To obtain additional information on multiparton correlations, one needs to study DPS p - A collisions.

Obviously, double- (and, more in general, multiple-) parton scatterings are more abundant in reactions with nuclei. DPS is thus more interesting in p - A collisions. When nonadditive corrections to the nuclear parton distributions are only a minor effect, in p - A collisions DPS originate either from interactions with a single active target nucleon or from interactions with two different active target nucleons. While the first contribution does not add much to the information already available from DPS on an isolated nucleon, the second contribution has the peculiar property of *enhancing the effects of longitudinal correlations in the proton*. In the latter case, the relative transverse distance between the interacting pairs does not play, in fact, any relevant role when compared to the much larger nuclear radius [18–23]. By selecting the contribution to DPS, with two active target nucleons, one will hence have direct access to the longitudinal correlations of the hadron structure. The cross section thus splits into two terms:

$$\sigma_D^{pA} = \sigma_D^{pA}|_1 + \sigma_D^{pA}|_2, \quad (7)$$

which correspond to the two different contributions, where the double hard interaction takes place with one or with two different target nucleons. In a simplest probabilistic picture of the interaction, one would write

$$\begin{aligned} \sigma_D^{pA}|_1 &= \frac{1}{2} \frac{\sigma_S^2}{\sigma_{\text{eff}}} \int d^2 B T(B) \propto A, \\ \sigma_D^{pA}|_2 &= \frac{1}{2} \sigma_S^2 \int d^2 B T^2(B) \propto A^{4/3}, \end{aligned} \quad (8)$$

where the case of two identical partonic interactions has been considered. Here σ_S is the inclusive single-scattering cross section, and $T(B)$ is the nuclear thickness as a function of the

impact parameter of the collisions B . The two terms have a transparent geometrical meaning and are distinguished by their different dependence on the atomic mass number A .

A closer look at the kinematics of the process [20,21] shows, however, that one needs to take into account an additional contribution to the cross section. The diagonal term $\sigma_D^{pA}|_2$ in Eq. (8) does not exhaust in fact all possibilities of interaction and, in the case of two different active target nucleons, one needs to add an interference term. The two terms are conveniently expressed as contributions to the discontinuity of the forward elastic amplitude. The two corresponding unitarity diagrams are shown in Fig. 1, representing the diagonal [Fig. 1(a)] and the off-diagonal [Fig. 1(b)] contributions. Notice that having focused on the disconnected component of the hard interaction, the description of the process is greatly simplified, with respect to the sizably more complex situation discussed in [21].

Following the lines described in full detail in Refs. [19,20], the contribution of the diagonal term to the cross section, with two active target nucleons, is given by

$$\begin{aligned} \sigma_D^{pA}|_{2,\text{diag}} &= \frac{1}{2(2\pi)^3} \int \Gamma(x_1, x_2; \beta_1 - \beta_2) \frac{d\hat{\sigma}(x_1, x'_1)}{d\Omega_1} \frac{d\hat{\sigma}(x_2, x'_2)}{d\Omega_2} \\ &\quad \times \Gamma(x'_1/Z_1; b_1) \Gamma(x'_2/Z_2; b_2) |\tilde{\Psi}_A(Z_i; B_i)|^2 \\ &\quad \times db_1 db_2 d(\beta_1 - \beta_2) \delta(B_1 - B_2 + b_1 - b_2 + \beta_1 - \beta_2) \\ &\quad \times \delta\left(\sum Z_i - A\right) dx_1 dx_2 dx'_1 dx'_2 d\Omega_1 d\Omega_2 \prod_i dB_i \frac{dZ_i}{Z_i}, \end{aligned} \quad (9)$$

where $\hat{\sigma}(x_i, x'_i)$ are the partonic cross sections, and the nuclear wave function $\Psi_A(Z_i; B_i)$ is in a mixed representation, with Z_i being the nucleons' fractional momenta and B_i the nucleons' transverse coordinates. The nuclear wave function is peaked at $Z_i = 1$, while MPIs are most important at $x \approx 10^{-2} \div 10^{-3}$. Keeping into account that the

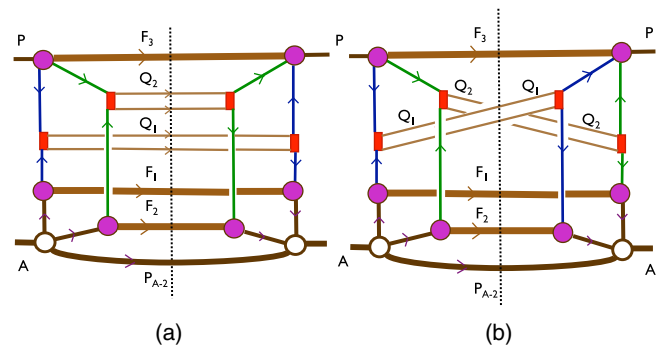


FIG. 1 (color online). Double-parton scattering contributions to the discontinuity of the forward p - A interaction amplitude: (a) diagonal term, (b) interference term.

scale of the nucleon's Fermi momentum is small as compared to the nucleon mass, a meaningful approximation is to integrate on Z_i while keeping $Z_1 = Z_2 = 1$ in the partonic distributions Γ . The nuclear dependence is thus expressed through the two-body nuclear density $\rho(B_1, z_1; B_2, z_2)$, where the quantities z_1 and z_2 are the longitudinal coordinates of the two interacting nucleons:

$$\begin{aligned} & \int \Gamma(x'_1/Z_1; b_1) \Gamma(x'_2/Z_2; b_2) |\tilde{\Psi}_A(Z_i; B_i)|^2 \prod_i \frac{dZ_i}{Z_i} \\ & \simeq \Gamma(x'_1; b_1) \Gamma(x'_2; b_2) \\ & \quad \times \int \rho(B_1, z_1; B_2, z_2) dz_1 dz_2. \end{aligned} \quad (10)$$

With the help of Eq. (4), in the case of two identical interactions, one thus obtains

$$\begin{aligned} \frac{d\sigma_D^{pA}|_{2,\text{diag}}}{dx_i dx'_1 d\Omega_i} &= K_{x_1 x_2} \frac{1}{2} \frac{d\sigma_S(x_1, x'_1)}{d\Omega_1} \frac{d\sigma_S(x_2, x'_2)}{d\Omega_2} \\ & \times \int f_{x_1 x_2}(\beta_1 - \beta_2) f_{x'_1}(b_1) f_{x'_2}(b_2) \\ & \times \rho(B_1, z_1; B_2, z_2) dz_1 dz_2 \\ & \times \delta(B_1 - B_2 + b_1 - b_2 + \beta_1 - \beta_2) \\ & \times db_1 db_2 d(\beta_1 - \beta_2) dB_1 dB_2, \end{aligned} \quad (11)$$

where σ_S is the usual single-scattering inclusive cross section on a nucleon, and we made the positions $\Gamma(x; b) \equiv G(x) f_x(b)$, $\int f_x(b) d^2b = 1$. The configuration in transverse space corresponding to the DPS cross section in Eq. (11) is illustrated in Fig. 2.

The contribution of the interference term to the cross section is obtained in a similar way [20]. The expression is

$$\begin{aligned} \frac{\sigma_D^{pA}|_{2,\text{int}}}{dx_i dx'_i d\Omega_i} &= \frac{1}{(2\pi)^3} \int \Gamma(x_1, x_2; \beta_1 - \beta_2) \frac{d\hat{\sigma}(x_1, x'_1)}{d\Omega_1} \frac{d\hat{\sigma}(x_2, x'_2)}{d\Omega_2} W(Z_1, Z_2; Z'_1, Z'_2; x'_1, x'_2; b_1, b_2; B_1, B_2) \\ & \times \tilde{\Psi}_A(Z_i; B_i) \tilde{\Psi}_A^*(Z'_i; B_i) \delta(B_1 - B_2 - b_1 + b_2 - \beta_1 + \beta_2) \delta(Z_1 - Z'_1 - x'_1 + x'_2) \\ & \times \delta(Z_2 - Z'_2 + x'_1 - x'_2) \delta\left(\sum Z_i - A\right) \delta\left(\sum Z'_i - A\right) db_1 db_2 d(\beta_1 - \beta_2) \prod dB_i \frac{dZ_i}{Z_i} \frac{dZ'_i}{Z'_i}, \end{aligned} \quad (12)$$

where the off-diagonal parton amplitudes in the process are all included in the function W . Some details on the construction of the function W are presented in Appendix A.

The main features of the off-diagonal contribution originate from kinematics and are summarized in Fig. 3.

As far as the longitudinal variables are concerned, the interference term requires the nuclear wave function to be

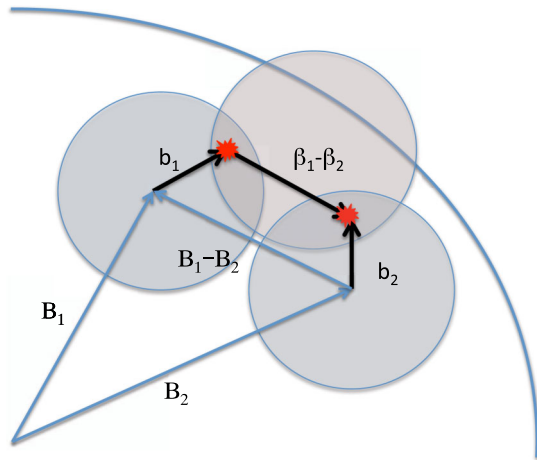


FIG. 2 (color online). Configuration in transverse space corresponding to the DPS cross section in Eq. (11).

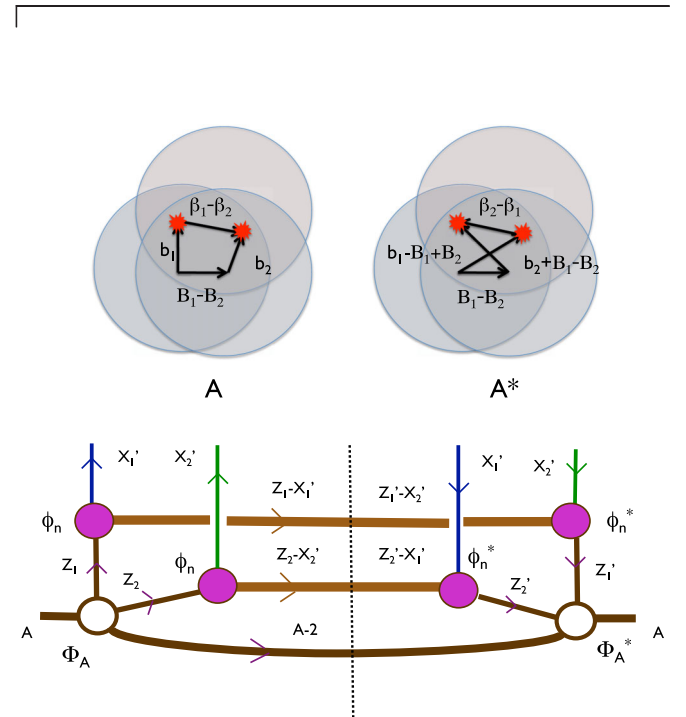


FIG. 3 (color online). Upper part of the figure: Configurations in transverse space of the left-hand (A) and right-hand (A*) site amplitudes in the off-diagonal contribution to the cross section in Eq. (12). Lower part of the figure: Nucleons' fractional momentum flow in the off-diagonal contribution to the cross section in Eq. (12).

taken at different values of Z . As is apparent in the lower part of Fig. 3, one must have, in fact, $Z_1 - Z'_1 = x'_1 - x'_2 = Z'_2 - Z_2$. When the differences $Z_i - Z'_i$ are not too small, the interference term is therefore depressed with respect to the diagonal term by the nuclear form factor. One should, however, keep in mind that at large energies the values of x can be rather small, still maintaining the process within the limits of perturbative dynamics, so the depression factor may not be strong.

When looking at transverse momenta, there are two different scales in the initial state: The typical transverse momentum of the order of a few GeV of the initial state partons, originated by QCD evolution, and the typical transverse momentum of the order of a hundred MeV of the bound nucleons, due to Fermi motion. One might therefore expect the interference term to be further depressed by the presence of the two different scales. As discussed in Appendix A, a closer look at kinematics shows, however, that this is not the case. The difference between the overall transverse momenta of the large- p_t partons, generated in the two hard collisions, actually $Q_{1,t} - Q_{2,t}$ in Fig. 9 in Appendix A, is in fact originated in the upper vertex φ_p in the figure and does not propagate to the target nucleons' momenta in the lower part of the diagram, in such a way that there are not transverse momenta of the GeV scale in the nucleons' lines in the interference diagram. Going to the coordinates space, one obtains a rather transparent picture of the interaction, as shown in the upper part of Fig. 3, where the configuration in transverse space of the amplitude in the left-hand side of the cut in Fig. 1(b) is labeled with A, and the configuration of the amplitude in the right-hand side of the cut, is labeled with A*. By exchanging $Q_{1,t}$ with $Q_{2,t}$ when moving from the left- to the right-hand side of the cut in Fig. 1(b), one produces a change in the sign of the difference between the transverse coordinates of the two projectile partons, β_1 and β_2 , in the nonperturbative vertex φ_p . The argument of φ_p is therefore $(\beta_1 - \beta_2)$ in the amplitude A and $(-\beta_1 + \beta_2)$ in the amplitude A*, while one does not expect any dependence of φ on the sign of its argument. For what concerns the remaining transverse variables, as illustrated in the upper part of Fig. 3, the nuclear configurations in A and in A* are the same, the two hard interactions remain localized in the same points, and the two interacting partons exchange their parent hadrons.

When the two active partons are identical, the initial partonic configuration of the DPS process can therefore be produced in two independent ways, since each of the two active nucleons can generate each of the two interacting partons. The two nuclear configurations therefore have to be added coherently in the cross section. The two active target nucleons have different longitudinal fractional momenta in the two configurations, and the difference is equal to the difference of the fractional

momenta of the two interacting partons, $x'_1 - x'_2$. The interference term is thus characterized by the peculiar dependence of the nuclear form factor as a function of $x'_1 - x'_2$. Notice that the interference term is directly proportional to the off-diagonal parton distributions, while the value of $x'_1 - x'_2$ can be controlled in the process by selecting the kinematical configuration of the final state produced by the hard interaction. There is thus the interesting possibility of obtaining additional information on off-diagonal parton distribution functions by using an inclusive process, actually DPS in p -A collisions, which, comparing with exclusive processes, would be able to provide larger rates of events and to access different kinematical regimes.

III. WJJ PRODUCTION BY DPS IN p -A COLLISIONS

The presence of an interference term in DPS in p -A collisions may have an interesting potential in the study of generalized parton distributions. On the other hand, our present purpose is to identify and work out a simple case, where the measured cross section has a straightforward relation with partonic correlations. We will thus consider a reaction channel where the interference term is strongly suppressed. To our aim, a particularly interesting channel is the inclusive production of WJJ . In WJJ production, the two initial-state partons, both from the side of the projectile and from the side of the target nucleus, are a quark and a gluon, since, in the kinematical regime of interest for DPS, JJ production is dominated by the gluonic channel. The two active target partons cannot therefore be identical, and the interference term is absent. A further reason of interest in WJJ is that DPS production is presently studied experimentally in p - p both by ATLAS and by CMS [27,28], while, after the recent experimental results and in view of the next runs planned at the LHC, there is increasing activity in the study of p -A collision, both experimental and theoretical [36]. The experimental study of DPS in WJJ production in p -Pb collisions might therefore represent a feasible option for the experimental groups in a not-too-distant future.

In the inclusive cross section for WJJ production, $\sigma^{pA}(WJJ)$, one identifies three different contributions:

$$\sigma^{pA}(WJJ) = \sigma_S^{pA}(WJJ) + \sigma_D^{pA}(WJJ), \quad \text{where}$$

$$\sigma_D^{pA}(WJJ) = \sigma_D^{pA}(WJJ)|_1 + \sigma_D^{pA}(WJJ)|_2.$$

The first term, $\sigma_S^{pA}(WJJ)$, represents the processes where WJJ is produced by a single-parton collision, while the contribution due to DPS, $\sigma_D^{pA}(WJJ)$, is expressed, according with Eq. (7), by the sum of two terms, which distinguish whether the DPS takes place against a single nucleon, $\sigma_D^{pA}(WJJ)|_1$, or against two different target nucleons, $\sigma_D^{pA}(WJJ)|_2$.

The contribution due to single-parton collisions, $\sigma_S^{pA}(WJJ)$, can be evaluated according to the standard rules, again accounting for the different contributions due to the interaction with a target proton or neutron and making use of the parton distributions of the bound nucleons [37,38]. This term does not provide much additional information on hadron structure, and it can be considered as a known quantity.

The explicit expression of the contribution due to a double-parton scattering in a collision with a single target nucleon, $\sigma_D^{pA}(WJJ)|_1$, is

$$\begin{aligned} \sigma_D^{pA}(WJJ)|_1 &= \frac{1}{\sigma_{\text{eff}}} [Z\sigma_S^{p[p]}(W)\sigma_S^{p[p]}(JJ) \\ &+ (A-Z)\sigma_S^{p[n]}(W)\sigma_S^{p[n]}(JJ)], \end{aligned} \quad (13)$$

where $\sigma_S^{p[p],p[n]}(W)$ are the single-scattering cross sections for inclusive production of a W in a collision of a proton with a bound proton or with a bound neutron, while $\sigma_S^{p[p],p[n]}(JJ)$ is, analogously, the single-scattering cross section to produce a pair of jets. The effective cross section, σ_{eff} , has been assumed to be a universal constant. A is the atomic mass number and Z the nuclear charge. Analogously to the term due to single parton collisions, $\sigma_S^{pA}(WJJ)$, therefore $\sigma_D^{pA}(WJJ)|_1$ is also expressed fully explicitly in terms of known quantities and is evaluated with the standard rules of the QCD-parton model, with the help of the parton distributions of the bound nucleons. The contribution due to a double-parton scattering, in hard collisions with a single target nucleon, does not have much to add to the information on the hadron structure already available from double-parton interactions in proton-proton collisions, and this term can thus be regarded as a known contribution to the cross section.

All novel information on hadron structure provided by DPS in p - A collisions is to be found in the last term, $\sigma_D^{pA}(WJJ)|_2$, where two different nucleons participate in the double-parton interaction. According with the discussion in the previous section, the corresponding contribution to the cross section is

$$\begin{aligned} \sigma_D^{pA}(WJJ)|_2 &= K_{x_1x_2}\sigma_S(W)\sigma_S(JJ) \\ &\times \int f_{x_1x_2}(\beta_1 - \beta_2)f_{x'_1}(b_1)f_{x'_2}(b_2) \\ &\times \rho(B_1, z_1; B_2, z_2)dz_1dz_2 \\ &\times \delta(B_1 - B_2 + b_1 - b_2 + \beta_1 - \beta_2) \\ &\times db_1db_2d(\beta_1 - \beta_2)dB_1dB_2. \end{aligned} \quad (14)$$

The expression in Eq. (14) has been obtained by disregarding the dependence of $\Gamma(x'_i/Z_i; b_i)$ on Z_i in Eq. (10). The lower limit of the integration on Z_i in Eq. (10) is x'_i , and one has therefore implicitly assumed that $x'_i \ll Z_i$, which limits the validity of Eq. (14) to the region of small x'_i .

The produced spectrum is directly proportional to the overlap integral in the transverse coordinates. The corresponding configuration is shown in Fig. 2. The overlap integral depends on the three different transverse scales, which characterize $f_{x'_i}(b_i)$, $f_{x_1x_2}(\beta_1 - \beta_2)$, and $\int \rho(B_1, z_1; B_2, z_2)dz_1dz_2$. When comparing hadronic and nuclear scales, a sensible approximation is to neglect the hadronic scale when compared to the nuclear scale. On the other hand, DPS forces the two target nucleons to be very close in transverse space. The contribution of short-range correlations in the two-body nuclear density may therefore give non-negligible effects, considering that the value of the scale of the short range nuclear correlation is $r_c \simeq 0.5$ fm [39,40].

To the present purposes, a relevant property is that short-range nuclear correlations are universal [41,42]. As discussed in Appendix B, by treating the correlation term as a perturbation, one may write

$$\begin{aligned} \rho^{(C,2)}(r_1, r_2) &\approx \rho^{(2)}(r_1, r_2)[1 - C(r_1 - r_2)]^2 \\ C(w) &= e^{-(w^2/2r_c^2)}, \end{aligned} \quad (15)$$

where $\rho^{(2)}(r_1, r_2)$ is the two-body nuclear density in the single-particle model, and for the correlation term $C(w)$ we used a Gaussian shape. For small relative distances, one can approximate

$$\begin{aligned} \rho^{(C,2)}(r_1, r_2)|_{r_1 \approx r_2} &\approx [\rho^{(1)}(r_1)]^2[1 - C(w)]^2, \\ w &= r_1 - r_2. \end{aligned} \quad (16)$$

Taking into account that the functions $f_{x'_i}(b_i)$, $f_{x_1x_2}(\beta_1 - \beta_2)$ are normalized to 1, and that $\rho^{(2)}(r_1, r_2)$ is smooth as a function of $r_1 - r_2$, the contribution to the overlap integral, in absence of short-range nuclear correlations, is equal to $\int T(B)^2 d^2B$.

To evaluate the terms in the overlap integral with the nuclear correlation $C(w)$, one needs to use explicit expressions for $f_{x'_i}(b_i)$ and $f_{x_1x_2}(\beta_1 - \beta_2)$. The overlap integral is most conveniently evaluated in momentum space. The term linear in C is

$$\begin{aligned} &\int f_{x_1x_2}(\beta)f_{x'_1}(b_1)f_{x'_2}(b_2)[\rho^{(1)}(B_1, z_1)]^2 \\ &\times (-2)C(B_1 - B_2, z_1 - z_2)\delta(B_1 - B_2 + b_1 - b_2 + \beta) \\ &\times dz_1dz_2db_1db_2d\beta dB_1dB_2 \\ &= -2 \int [\rho^{(1)}(B, z)]^2 dBdz \\ &\times \frac{1}{(2\pi)^2} \int \tilde{f}_{x_1x_2}(q)\tilde{f}_{x'_1}(q)\tilde{f}_{x'_2}(q)\tilde{C}(q)d^2q, \end{aligned} \quad (17)$$

where the functions with the tilde are the two-dimensional Fourier transforms in the transverse momentum space. The generalized parton distributions are known quantities. Following Ref. [43], we use the expression

$$\tilde{f}_{x'}(q) = \left(1 + \frac{q^2}{m_g^2}\right)^{-2}, \quad (18)$$

with $m_g^2 \approx 1.1 \text{ GeV}^2$ for $x' \approx .03$ and small q^2 .

If the multiparton distribution in multiplicity were a Poissonian ($K = 1$) and in absence of transverse correlations, one would have

$$\frac{1}{\sigma_{\text{eff}}} = \int [\tilde{f}_{x_1 x_2}(q)]^2 \frac{d^2 q}{(2\pi)^2},$$

$$\tilde{f}_{x_1 x_2}(q) = \tilde{f}_{x_1}(q) \times \tilde{f}_{x_2}(q) = \left(1 + \frac{q^2}{m_g^2}\right)^{-4}. \quad (19)$$

In such a case, one would obtain for the effective cross section

$$\sigma_{\text{eff}} = \frac{28\pi}{m_g^2} = 31.36 \text{ mb},$$

while ATLAS measures $\sigma_{\text{eff}} = 15 \text{ mb}$, which implies that the uncorrelated option gives an effective cross section too large by roughly a factor of 2. According with Eq. (6), $\sigma_{\text{eff}} = \pi\Lambda^2/K^2$. The uncorrelated case corresponds to the values $K = 1$ and $\Lambda^2 = 28/m_g^2$. Correlations may thus be introduced by allowing different values for K and Λ , keeping fixed, however, their ratio in order to reproduce the measured value of the effective cross section. To have an indication on how different values of K and Λ can affect the DPS cross section in p - A collisions, we have considered two extreme options:

Option (a): $\Lambda^2 = 28/m_g^2$ and $K^2 = 31.36/15 \approx 2$, which corresponds to the case where the actual value of σ_{eff} is solely due to the multiplicity of parton pairs in the hadronic structure. In such a case the multiplicity of parton pairs would be about a factor $K \approx 1.45$ times larger than expected if the distribution in multiplicity were a Poissonian, while transverse correlation between parton pairs would be completely absent, in such a way that the distribution of pairs in transverse space would be obtained by the convolution of two one-body distributions.

Option (b): $K^2 = 1$ and $\pi\Lambda^2 = 15 \text{ mb}$, which corresponds to assuming a Poissonian for the multiparton distribution in multiplicity and introducing a smaller typical transverse distance between partons, in comparison with the uncorrelated case. The functional form of the correlated distribution of parton pairs in transverse space is unknown. One would, however, expect that the main features will be determined by the value of the actual scale characterizing the typical transverse distance. To proceed, we will thus consider the simplest option, where the functional form of $\tilde{f}_{x_1 x_2}(q)$ is the same as in the uncorrelated case, and the only modification is in the value of the scale m_g , which we replace with the relevant scale for the transverse separation between the parton pairs,

which we denote with h_c . When $K^2 = 2$, one thus has $h_c = m_g$, while to reproduce the observed value of σ_{eff} when $K^2 = 1$, one has $h_c \approx 1.52 \text{ GeV}$.

The two options correspond, therefore, to the following values:

- (a) $h_c^2 = 1.1 \text{ GeV}^2$, $K^2 = 2$.
- (b) $h_c^2 = 2.3 \text{ GeV}^2$, $K^2 = 1$.

By evaluating the overlap integral, one obtains

$$\frac{1}{(2\pi)^2} \int \tilde{f}_{x_1 x_2}(q) \tilde{f}_{x'_1}(q) \tilde{f}_{x'_2}(q) \tilde{C}(q) d^2 q = C_K r_c \quad (20)$$

with

$$C_K = 0.82, \quad \text{in case (a).}$$

$$C_K = 0.99, \quad \text{in case (b).}$$

The cross section is given by

$$\sigma_D^{pA}(WJJ)|_2 = K \left[\frac{Z}{A} \sigma_S^{pp}(W) + \frac{A-Z}{A} \sigma_S^{pn}(W) \right] \sigma_S^{pp}(JJ)$$

$$\times \left[\int T(B)^2 d^2 B \right.$$

$$\left. - 2 \int \rho(B, z)^2 d^2 B dz \times r_c C_K \right], \quad (21)$$

where we made the approximation $\sigma_S^{pp}(JJ) \approx \sigma_S^{pn}(JJ)$. The ratio

$$\mathcal{R} = \frac{\sigma_D^{pA}(WJJ)}{\sigma_D^{pA}(WJJ)|_1} \quad (22)$$

is thus independent on the final-state phase space:

$$\mathcal{R} = 1 + K \frac{\sigma_{\text{eff}}}{A} \left[\int T(B)^2 d^2 B \right.$$

$$\left. - 2 \int \rho(B, z)^2 d^2 B dz \times r_c C_K \right]. \quad (23)$$

For lead, using the Woods-Saxon nuclear density in the two cases (a) and (b), one obtains

- (a) $K^2 = 2$, $\pi\Lambda^2 = 31.36 \text{ mb}$: $\mathcal{R} = 1 + 2.94 \approx 4$,
 - (b) $K^2 = 1$, $\pi\Lambda^2 = 15 \text{ mb}$: $\mathcal{R} = 1 + 2.03 \approx 3$,
- (24)

and the correction induced by short-range nuclear correlations to the term $\int T(B)^2 d^2 B$ is about 8% in case (a) and about 10% in case (b). The ratio \mathcal{R} therefore depends weakly on nuclear correlations and is rather sensitive to the different options for the values of K and Λ . Notice also the strong dependence of $\sigma_D^{pA}(WJJ)|_2$ [Eq. (21)] on K and its weak dependence on Λ (only through C_K).

IV. NUMERICAL ESTIMATES

To obtain a rough estimate of the different effects of the DPS in p - Pb and in p - p collisions, one may compare the production rates in the same kinematical region where DPS has been measured by ATLAS in p - p collisions. According with ATLAS, the fraction of events with DPS is about 7%, and one would not expect that, in interactions with a nucleus, the ratio $\sigma_D^{pA}(WJJ)|_1/\sigma_S^{pA}(WJJ)$ will be much different. Taking the ratio

$$\frac{\sigma^{pA}(WJJ)}{\sigma_S^{pA}(WJJ)} = 1 + \frac{\sigma_D^{pA}(WJJ)|_1}{\sigma_S^{pA}(WJJ)} \times \mathcal{R}, \quad (25)$$

one thus obtains that the fraction of events with DPI will grow to about 27.3%, if there are no transverse correlations [case (a)], and to about 22.5%, if the distribution in multiplicity is Poissonian [case (b)].

Nuclear effects and the different roles of parton correlations are, of course, made more transparent by looking at differential distributions.

To have some indication of the differential distributions, we have evaluated the DPS differential cross section in p - p and p - Pb collisions, in accordance with the different options discussed above. The elementary cross section is evaluated at the leading order in perturbation theory. For the numerical integration, we used two different sets of PDF, provided by the LHAPDF interface [44]: the LO MSTW (MSTW2008lo68cl) and the CTEQ6 LO. The leading-order matrix elements are generated by means of the MadGraph 5 package [45], in the framework of the Standard Model with the CKM matrix. We chose its C++ output and introduced a namespace characterizing every subprocess.

For the multidimensional integration we used VEGAS [46]. More specifically, we used Suave (SUBregion-Adaptive VEGAS), an algorithm implemented in the CUBA library [47], which combines the advantages of VEGAS and subregion sampling. The division into subregions allows us to overcome the VEGAS problem to adapt its weight function to structures not aligned with the coordinate axes.

For a more direct comparison with available results in p - p , we worked out the differential distributions, both in p - p and in p - Pb collisions, in the same kinematical conditions as the ATLAS DPS measurements [27]. Namely, the beam energy is $\sqrt{s} = 7$ TeV, $|\eta_{l^+}| < 2.47$, $p_{t^+} > 20$ GeV, $E_t > 25$ GeV, $m_{lW} > 40$ GeV. We did not implement any fragmentation and, to reproduce the observed cross section, we slightly increased (by 10%–15%) the lower cutoff in the transverse momentum of the large- p_t partons [48]. Jets are thus identified with large- p_t final-state partons and, considering that in p - A collisions the transverse spectra are not modified substantially by the presence of the nucleus [49,50], the effects of the nuclear modification factors are not taken into account. To simulate the process, during the integration we took traces of the final particle configurations and of the value of the integrand and plotted them with an analysis program such as ROOT [51]. In the case of DPS, the transverse momentum of the boson W^+ was obtained following the prescriptions of Refs. [52–54].

In Fig. 4, we plot the distribution in p_t of the leading jet in p - p collisions. In the DPS contribution (in green), we used $\sigma_{\text{eff}} = 15$ mb (left panel). The same distribution is shown in p - Pb collisions in the right panel. The pink histograms refer to the single-scattering contribution, the green ones to the DPS contribution, and the black histograms to the sum of the two contributions. The histograms in the figures have been computed with the MSTW parton distribution functions.

While in p - p collisions, DPS represents a barely noticeable contribution to the p_t spectrum of the leading jet produced in the process, DPS has a much stronger effect in the p_t spectrum of the leading jet in p - Pb collisions, where the shape of the distribution is very different for p_t smaller than 50 GeV.

The dependence of the transverse spectrum of the leading jet, as a function of the values of the σ_{eff} and K , is shown in Fig. 5. By looking at the green histograms, one may see that, after subtracting the single-scattering contribution, which can be considered as a known quantity, once DPS has been measured in p - p collisions in

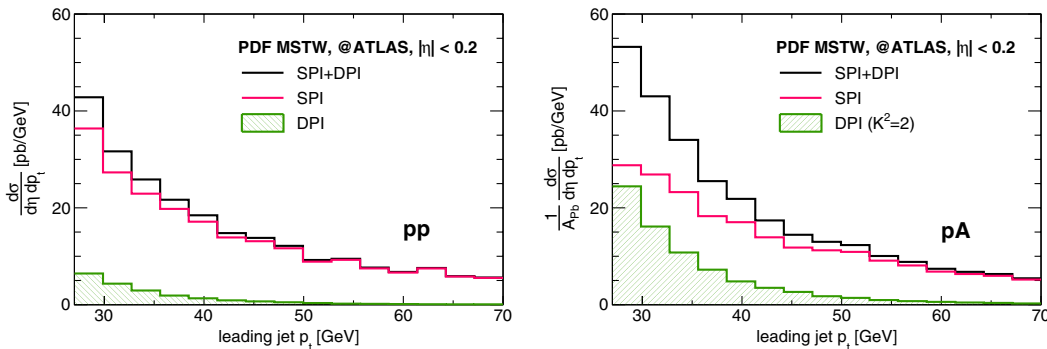


FIG. 4 (color online). Distribution in p_t of the leading jet in p - p collisions (left panel) and in p - Pb collisions (right panel). The pink histograms refer to the single scattering contribution, the green ones to the DPS contribution, and the black histograms to the sum of the two contributions. We used $\sigma_{\text{eff}} = 15$ mb and $K^2 = 2$.

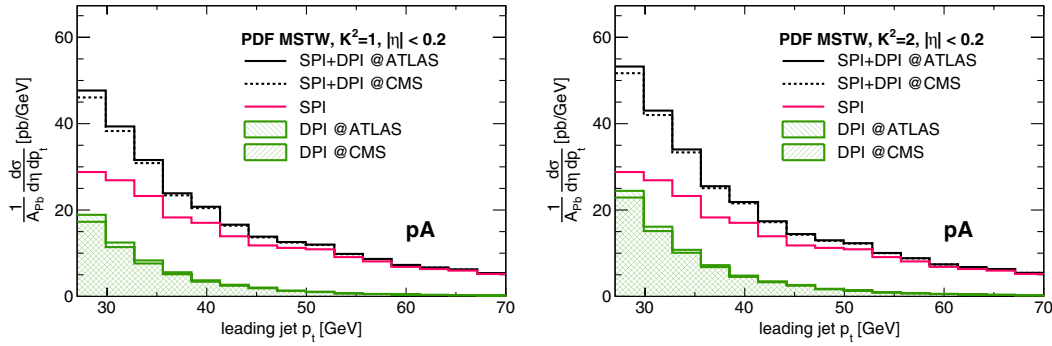


FIG. 5 (color online). Distribution in p_t of the leading jet in p -Pb collisions in the cases $\sigma_{\text{eff}} = 15$ mb (ATLAS) and $\sigma_{\text{eff}} = 20.7$ mb (CMS [28]) for $K^2 = 1$ (left panel) and $K^2 = 2$ (right panel).

the same kinematical conditions, the shape in p_t shows an appreciable dependence on the value of K .

A more suitable observable to learn concerning the distribution in multiplicity of the multiparton distribution is probably the p_t spectrum of the charged lepton, produced by the decay of the W^+ . In a single scattering collision, W bosons recoil against the produced jets and are typically characterized by a transverse momentum of the order of the lower cutoff in p_t of the observed accompanying jets. In the case of a DPS, the jets and the W are produced in different partonic interactions. The transverse momentum of the W is therefore typically rather small, and the spectrum of the decay lepton is thus rather different in single and double-parton scattering. In the former case, when the lower cutoff for the produced jets is 20 GeV, the transverse momentum of the produced lepton can easily exceed 60–70 GeV. In the latter case, the lepton is produced by a W boson with a rather small transverse momentum, and its transverse spectrum is thus limited to values close to 1/2 of the W mass. In Fig. 6, we plot the distribution in p_t of the charged lepton from the W^+ decay. The left panel refers to the case of p - p collisions; the right panel to the case of p -Pb collisions. The enhancement of the spectrum at $p_t < 40$ GeV, due to the contribution of DPS, is not a big effect in p - p collisions. It is, on the contrary, a rather strong effect

in p -Pb collisions, where the difference with respect to the contribution to the spectrum due to single-parton scattering (pink histograms in Fig. 6) is quite noticeable.

In p -Pb collisions, the sensitivity of the spectrum to K is shown in Fig. 7. In the left panel, we show the spectrum in the case $K^2 = 1$ for $\sigma_{\text{eff}} = 15$ mb and $\sigma_{\text{eff}} = 20.7$ mb. In the right panel, we show the case $K^2 = 2$. The enhancement of the spectrum due to the DPS contribution at $p_t < 40$ GeV is rather substantial, and the amount of the increase is significantly different as a function of K .

By selecting events with a charged lepton in the 40 GeV p_t region, one will thus obtain a sample where the contribution of DPS is about 50% of the total, and one will thus be able to obtain rather direct information on the second moment of the multiparton distribution in multiplicity in the proton structure.

A final observation is that, due to the different production mechanism as compared to the case of p - p collisions, nuclear spectra do not depend much on the value of σ_{eff} measured in p - p collisions. A change from 20 to 15 mb implies an increase of the DPS cross section of more than 30% in p - p collisions and of only 5%–6% in p -Pb collisions, as apparent in the figures above by comparing the dotted and continuous histograms. In the latter case, the production rate is in fact proportional, to a large extent, to the multiplicity of pairs of partons in the

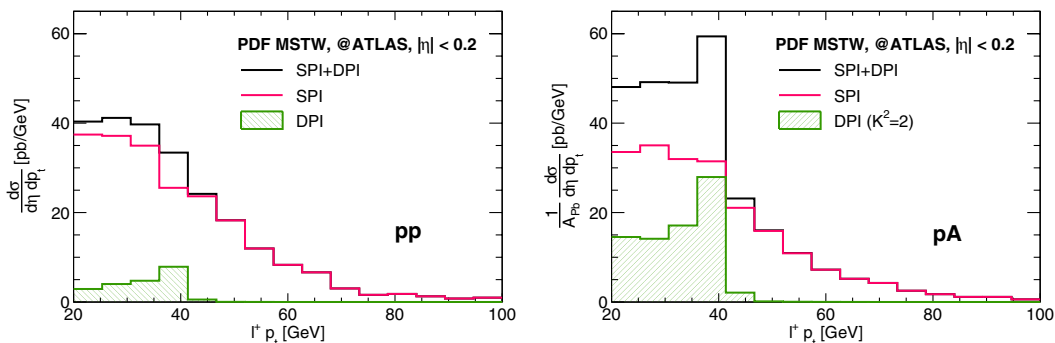


FIG. 6 (color online). Distribution in p_t of the charged lepton from the W^+ decay, in p - p collisions (left panel) and in p -Pb collisions in the case of $\sigma_{\text{eff}} = 15$ mb and $K^2 = 2$ (right panel).

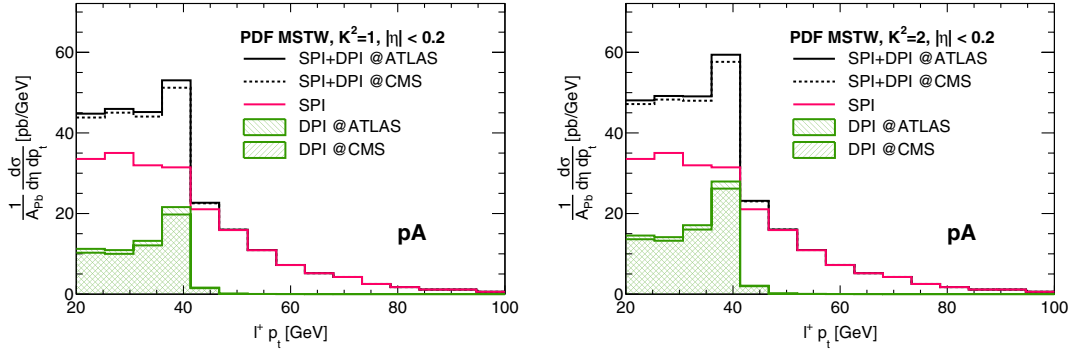


FIG. 7 (color online). Rapidity distribution of the charged lepton from the W^+ decay, in p - Pb collisions in the case of $\sigma_{\text{eff}} = 15$ mb (ATLAS) and $\sigma_{\text{eff}} = 20.7$ mb (CMS) for $K^2 = 1$ (left panel) and $K^2 = 2$ (right panel).

projectile, while the typical transverse distance between the interacting parton pairs does not play a relevant role. Which is precisely the reason why DPS in p - Pb collisions has the potential to provide a lot of information on parton correlations.

V. CONCLUDING SUMMARY

Double-parton scattering processes are directly related to unknown nonperturbative properties of the hadron structure, which in p - p collisions converge in the value of a single quantity, the effective cross section. The interaction mechanism is more complex in p - A collisions, where one may have either one or two different target nucleons interacting with large momentum transfer. In the case of two different active target nucleons, in addition to the diagonal contribution, which has a direct probabilistic interpretation, one may need to take into account also the contribution of an interference term. When the two active partons in the initial state are identical, the nucleus can in fact generate the initial partonic configuration in two different ways. The description of the interaction is simpler when the interference term is absent, which is the case with reaction channels where the pair of initial state active partons are a quark and a gluon, like in WJJ , $Wb\bar{b}$, and $Wc\bar{c}$ production at relatively low fractional momenta. The increased complexity of the interaction in p - A collisions can thus provide an additional handle to obtain information on the nonperturbative hadron structure not accessible by other means.

To gain some insight into the actual possibilities of learning about parton correlations by studying DPS in p - p and p - A collisions, we have considered a particularly simple case, still not inconsistent with present experimental evidence, where DPS is described by the dominant term at small x , while in p - p collisions, the effective cross section can be approximated with a universal constant. The effective cross section is thus fully determined by the typical transverse distance between the interacting partons Λ [Eq. (5)] and by the multiplicity of parton pairs, which here is characterized by the value of K [(Eq. (3)]. To keep

the interaction with the nucleus as simple as possible, we have looked at a reaction channel where the interference term is absent. Specifically, we have studied WJJ production, which is of particular interest, since DPS in WJJ production is presently studied experimentally in p - p collisions both by ATLAS and by CMS.

In this simplified scheme, the DPS cross section in p - p collisions depends only on the ratio between K and Λ . In p - A collisions, the contribution to the DPS cross section with two active target nucleons depends, on the contrary, (almost) only on K . To have an indication on the possibility of determining Λ and K by measuring DPS in p - p and p - Pb collisions and to allow a direct comparison of the two cases, we have evaluated the W^+JJ production cross sections in the kinematical conditions of the ATLAS experiment.

In p - A collisions, the contribution to the DPS cross section, due to the processes where two different target nucleons interact with large momentum transfer, is proportional to the factor K and grows with $A^{4/3}$. Depending on the value of K , in p - Pb collisions this contribution may be 2 or 3 times as big as the contribution to DPS, where only a single target nucleon interacts with large momentum transfer, while short-range nuclear correlations can produce at most a reduction of 10%. The effect of varying the value of the typical distance in transverse space between the pairs of interacting partons has in this case only a minor effect, which we estimate to be of the order of 5%–6% of the cross section. Considering also the contribution due to single hard collisions, we expect that, while in p - p the observed fraction of events with DPS was about 7%, with the same cuts used by ATLAS, in p - Pb the fraction of events with DPS will range between 22.5% (in the absence of longitudinal correlations) and 27.3% (in the absence of transverse correlations). CMS reports a smaller fraction of events with DPS, about 5%. In such a case, and with the same cuts, in p - Pb collisions the fraction of events with DPS will range between 21.0% and 26.0%.

In summary, a main feature of DPS in p - A collisions is that, for large atomic mass numbers, the most important contribution to the DPS cross section is due to the

interactions with two active target nucleons, rather than to the interactions with a single target nucleon [18,55]. In other terms, DPS in p - Pb collisions is characterized by a very strong antishadowing, which may represent a 200%–300% correction to the DPS cross section on a single nucleon. The value of this antishadowing term is proportional to the flux of incoming pairs of partons, and, by measuring the amount of antishadowing, one has thus a direct indication of the number of pairs of incoming partons in the projectile. Different properties of the incoming pair of partons, which may have important effects in DPS in p - p collisions, are likely to be much less important in p - A . In the actual case, we have discussed the effect of the distribution of partons in transverse space, which in p - p may be even more important than the multiplicity of parton pairs to determining the observed value of the DPS cross section. In p - Pb , we estimate that different values of the typical separation between partons in transverse space can, on the contrary, affect the DPS cross section only by about 5%–6%.

We think that, although there are still several open problems to understanding DPS in p - p collisions, the study of DPS in p - A collisions has a great potential for a deeper insight into the problem. In fact, DPS in p - A collisions allows us to single out an important feature in the process, the value of the incoming flux of parton pairs,

which is directly proportional to the amount of antishadowing observed in the DPS cross section. In the present paper, we have worked out the amount of antishadowing to be expected in the simplest conceivable scheme and the corresponding value of the incoming flux of parton pairs. A comparison with an experimental study of DPS in p - Pb at the LHC would thus be very instructive, providing for the first time a direct indication on a property of the correlated parton structure of the hadron, not achievable to our knowledge by other means, and allowing us at the same time to make a quantitative test or even to disprove the simplest conceivable description of DPS.

APPENDIX A: DIAGONAL AND INTERFERENCE TERMS

The construction of the amplitudes and of the corresponding cross sections, Eqs. (11) and (12) in the main text, is performed by following strictly the procedure used in Ref. [20], in particular in the discussion of the tritium case. We will not reproduce all details of the procedure. Rather, we will try to point out some of the main differences between the two cases.

The diagonal term (Fig. 8) is given by

$$\begin{aligned}
\text{Disc } \mathcal{A}_d = & \frac{1}{(2\pi)^{21}} \int \frac{\hat{\varphi}_p}{l_1^2 l_2^2} \frac{\hat{\varphi}_p^*}{l_1'^2 l_2'^2} \frac{\varphi_1}{a_1^2} \frac{\varphi_1^*}{a_1'^2} \frac{\varphi_2}{a_2^2} \frac{\varphi_2^*}{a_2'^2} \\
& \times T_1(l_1, a_1 \rightarrow q_1, q_1') T_1^*(l_1', a_1' \rightarrow q_1, q_1') T_2(l_2, a_2 \rightarrow q_2, q_2') T_2^*(l_2', a_2' \rightarrow q_2, q_2') \\
& \times \frac{\Phi_A(N_1; N_2 | N_k)}{[N_1^2 - m^2][N_2^2 - m^2]} \frac{\Phi_A^*(N_1'; N_2' | N_k)}{[N_1'^2 - m^2][N_2'^2 - m^2]} \\
& \times \delta(L - l_1 - l_2 - F_p) \delta(L - l_1' - l_2' - F_p) \\
& \times \delta(N_1 - a_1 - F_1) \delta(N_1' - a_1' - F_1) \delta(N_2 - a_2 - F_2) \delta(N_2' - a_2' - F_2) \\
& \times \delta(l_1 + a_1 - Q_1) \delta(l_1' + a_1' - Q_1) \delta(l_2 + a_2 - Q_2) \delta(l_2' + a_2' - Q_2) \\
& \times \prod_{i,j} d^4 a_i d^4 a_i' d^4 l_i d^4 l_i' d^4 F_j \delta(F_j^2 - M_j^2) \delta(P_A - N_1 - N_2 - P_{A-2}) \delta(P_A - N_1' - N_2' - P_{A-2}) \\
& \times \delta \left(\sum_{k=3}^A N_k - P_{A-2} \right) \prod_{k=3}^A d^4 N_k d^4 P_{A-2} d^4 N_1 d^4 N_2 d^4 N_1' d^4 N_2' d^4 Q_i d(\Omega_i/8) dM_j^2. \tag{A1}
\end{aligned}$$

The interference term (Fig. 9) is given by

$$\begin{aligned}
\text{Disc } \mathcal{A}_i = & \frac{1}{(2\pi)^{21}} \int \frac{\hat{\varphi}_p}{l_1^2 l_2^2} \frac{\hat{\varphi}_p^*}{l_1'^2 l_2'^2} \frac{\varphi_1}{a_1^2} \frac{\varphi_2}{a_1'^2} \frac{\varphi_2}{a_2^2} \frac{\varphi_1^*}{a_2'^2} T_1(l_1, a_1 \rightarrow q_1, q_1') \\
& \times T_1^*(l_1', a_1' \rightarrow q_1, q_1') T_2(l_2, a_2 \rightarrow q_2, q_2') T_2^*(l_2', a_2' \rightarrow q_2, q_2') \\
& \times \frac{\Phi_A(N_1; N_2 | N_k)}{[N_1^2 - m^2][N_2^2 - m^2]} \frac{\Phi_A^*(N_1'; N_2' | N_k)}{[N_1'^2 - m^2][N_2'^2 - m^2]} \delta(L - l_1 - l_2 - F_p) \delta(L - l_1' - l_2' - F_p)
\end{aligned}$$

$$\begin{aligned}
 & \times \delta(N_1 - a_1 - F_1) \delta(N'_1 - a'_2 - F_1) \delta(N_2 - a_2 - F_2) \delta(N'_2 - a'_1 - F_2) \\
 & \times \delta(l_1 + a_1 - Q_1) \delta(l'_1 + a'_1 - Q_1) \delta(l_2 + a_2 - Q_2) \delta(l'_2 + a'_2 - Q_2) \\
 & \times \prod_{i,j} d^4 a_i d^4 a'_i d^4 l_i d^4 l'_i d^4 F_j \delta(F_j^2 - M_j^2) \delta(P_A - N_1 - N_2 - P_{A-2}) \delta(P_A - N'_1 - N'_2 - P_{A-2}) \\
 & \times \delta \left(\sum_{k=3}^A N_k - P_{A-2} \right) \prod_{k=3}^A d^4 N_k d^4 P_{A-2} d^4 N_1 d^4 N_2 d^4 N'_1 d^4 N'_2 d^4 Q_i d(\Omega_i/8) dM_j^2.
 \end{aligned} \tag{A2}$$

Here, P_A is the four-momentum of the incoming nucleus and L the four-momentum of the incoming proton; $\hat{\varphi}$ and $\hat{\varphi}^*$ are the effective vertices for emission of one or two partons by a nucleon; the integration variables Q and Ω come in through the transformation $d^3 q/2q_0 d^3 q'/2q'_0 = d^4 Q d\Omega/8$, where q and q' are the momenta of the massless particles produced in the hard scattering T_i , so $Q^2 > 0$, $Q_0 > 0$, and Ω gives the scattering angles in the center-of-momentum frame of the pair. The sum is over every possible final state compatible with the conservation laws, identified by the four-vectors N_k , ($k = 3, \dots, A$). We recall [20] the amplitude for finding one or two partons in the projectile when the remnant of the parent nucleon has mass M_j , where the possible values of the index are $j = 1, 2, p$ (cf. Figs. 8 and 9); setting $\lambda_- = \frac{1}{2}(l_1 - l_2)_-$, we get

$$\begin{aligned}
 \psi_{M_{j=1,2}} &= \frac{\varphi_j}{a_j^2} \frac{\varphi_j}{\bar{x}_j [m_j^2 - M_{j\perp}^2 / (1 - \bar{x}_j)] - a_{j\perp}^2} \hat{\psi}_{M_p} \\
 &= \frac{1}{\sqrt{2}} \int \frac{\hat{\varphi}_p d\lambda_-}{l_1^2 l_2^2 2\pi i} \\
 &= \frac{1}{\sqrt{2} L_- l_{1\perp}^2 x_2 + l_{2\perp}^2 x_1 - x_1 x_2 [m^2 - M_{p\perp}^2 / (1 - x_1 - x_2)]} \hat{\varphi}_p.
 \end{aligned} \tag{A3}$$

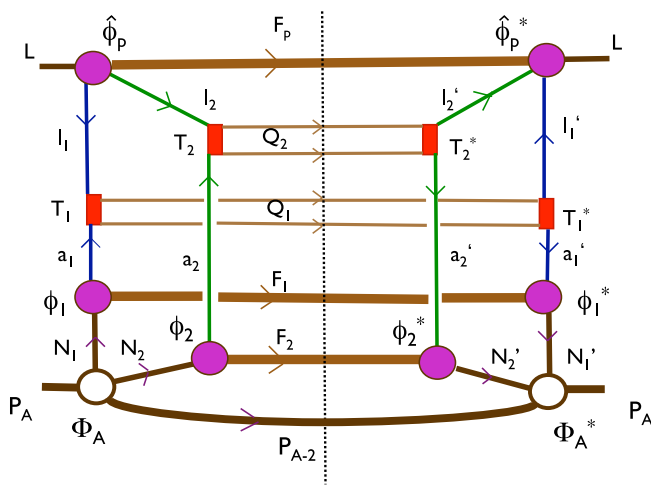


FIG. 8 (color online). Discontinuity of the forward amplitude in Eq. (A1).

Here, the light-cone components that grow with the total energy in the c.m. of the interacting nucleon pair are the plus components in the projectile proton and the minus components in the target nucleon. The dependence on the transverse mass of the remnant, $M_{j\perp}^2 \equiv M_j^2 + F_{j\perp}^2$, comes in through the conservation of the plus components, when $j = 1$ or 2 , and of the minus components, when $j = p$. In the same way, one defines the one-parton and the two-parton amplitudes in the bound nucleon. (In this case, since N has also transverse components, the initial state mass m^2 has been replaced by $m_j^2 \equiv m^2 + N_{j\perp}^2$.) Then the Fourier transformation on the transverse variables is performed.

The same procedure is applied to the nucleus side: one defines $\nu_+ = \frac{1}{2}(N_1 - N_2)_+$, and the covariant amplitude for finding two nucleons in the nucleus has the formal expression

$$\Psi_A(N_{1-}, N_{2-}) = \frac{1}{\sqrt{2}} \int \frac{d\nu_+}{2\pi i} \frac{\Phi_A}{[N_1^2 - m^2] \cdot [N_2^2 - m^2]}. \tag{A4}$$

This amplitude depends also on the configuration of the residual $(A - 2)$ nucleons:

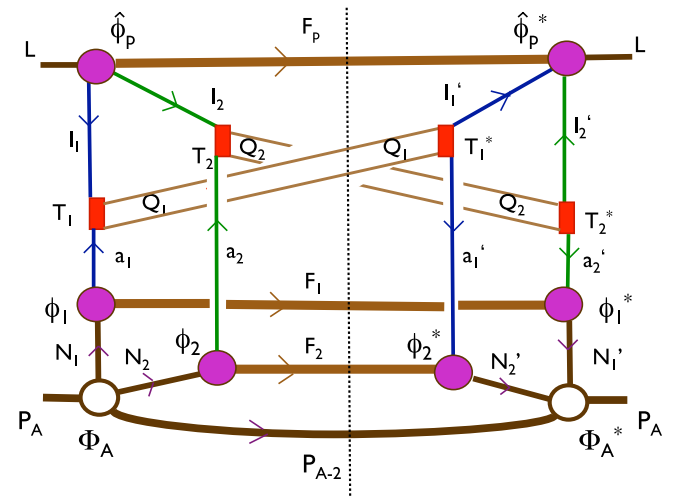


FIG. 9 (color online). Discontinuity of the forward amplitude in Eq. (A2).

$$\Psi_A(N_{1-}, N_{2-}) = \frac{1}{\sqrt{2}P_{A-}} \frac{\Phi_A}{Z_1 Z_2 \{(M_A/A)^2 - \mu_{\perp}^2/[A(A - Z_1 - Z_2)]\} - (Z_1 m_2^2 + Z_2 m_1^2)/A}. \quad (\text{A5})$$

μ_{\perp} is the overall transverse mass of the remnant nuclear spectators, M_A is the mass of the incoming nucleus, and $N_{i-} = Z_i P_{A-}/A$. In the dominant configurations, Z_i is close to 1.

We try now to point out the differences between the diagonal term and the interference term. In the diagonal term, the conservation of the large momentum components implies the following relations for the initial-state partons on the two sides of the diagram: $l_{i+} = l'_{i+}$, $a_{i-} = a'_{i-}$, $N_{i-} = N'_{i-}$, while the corresponding transverse variables become diagonal through Fourier transformation. The whole expression of the cross section can thus be expressed in terms of densities, i.e. the square of partonic wave functions and of the wave function of the bound nucleons.

In the interference term, the conservation of the large momentum components implies different relations for the initial-state partons on the two sides of the diagram. Actually, $l_{i+} = l'_{i+}$, $a_{1-} = a'_{2-}$, $a_{2-} = a'_{1-}$. For the nuclear variables, one obtains: $(N_1 - N'_1)_- = (N'_2 - N_2)_- = (a_1 - a_2)_-$. Concerning the transverse components, the variables $\beta_{i\perp}$ (conjugated to $l_{i\perp}$) become diagonal through Fourier transformation, while, differently from the case of the diagonal contribution to the cross section, the variables $b_{i\perp}$ (conjugated to $a_{i\perp}$) are not diagonalized by the Fourier transformation. When looking to the nuclear part, the overall conservation of the fractional minus component implies $Z_1 + Z_2 = Z'_1 + Z'_2 = A - Z_{A-2}$, where Z_{A-2} is the minus fractional momentum of the incoming nucleus. Notice that the relation for the nuclear fractional momenta can be written also as $Z_1 - Z'_1 = Z'_2 - Z_2 = \bar{x}_2 - \bar{x}_1$, which shows that such differences can be actually measured. While \mathcal{A}_d can be expressed through the diagonal terms of the two-body nuclear density matrix, \mathcal{A}_i requires the off-diagonal two-body density matrix. In this latter case, the nonperturbative partonic input of the projectile proton is given again by the partonic densities $\Gamma(x_1, x_2; \beta_1, \beta_2)$; the nonperturbative input of the target nucleons, however, depends explicitly on the nondiagonal one-body parton densities, which, in the final expression of the cross section, are gathered into the function W , whose expression is shown below:

$$\begin{aligned} W(Z_1, Z_2; \bar{x}_1; \bar{x}_2; b_1, b_2) &= \frac{1}{4(2\pi)^6} \int dM_1^2 dM_2^2 \frac{\bar{x}_1 \bar{x}_2}{(Z_1 - \bar{x}_1)(Z_2 - \bar{x}_2)} \\ &\times \psi_{M_1}(\bar{x}_1/Z_1, b_1) \psi_{M_2}(\bar{x}_2/Z_2, b_2) \\ &\times \psi_{M_2}^*(\bar{x}_2/Z'_1, b_1 - B_1) \psi_{M_1}^*(\bar{x}_1/Z'_2, b_2 - B_2). \quad (\text{A6}) \end{aligned}$$

APPENDIX B: TWO-BODY NUCLEAR DENSITY

In this appendix, we describe the approach used to derive the two-body nuclear density in the main text. Nuclear states are normalized to 1: $\int \psi_m(u) \psi_n^*(u) du = \delta_{mn}$, and with u we mean all nucleon's degrees of freedom, spin, isospin, and space coordinates. The one-body nuclear density $\rho^{(1)}(u) = \sum_n |\psi_n(u_1)|^2$ is normalized to the atomic mass number, $\int \rho^{(1)}(u) du = A$. The antisymmetric two-body wave function, neglecting interactions between the two nucleons, is

$$\frac{1}{\sqrt{2}} [\psi_m(u_1) \psi_n(u_2) - \psi_m(u_2) \psi_n(u_1)].$$

Correspondingly, the two-body density for the states m, n is

$$\begin{aligned} g_{mn}(u_1, u_2) &= \frac{1}{2} [|\psi_m(u_1) \psi_n(u_2)|^2 + |\psi_m(u_2) \psi_n(u_1)|^2] \\ &\quad - \Re[\psi_m(u_1) \psi_n(u_2) \psi_m(u_2)^* \psi_n(u_1)^*]. \end{aligned}$$

By integrating on u_2 , one obtains

$$\begin{aligned} \int g_{mn}(u_1, u_2) du_2 &= \frac{1}{2} [|\psi_m(u_1)|^2 + |\psi_n(u_1)|^2] - \delta_{mn} |\psi_m(u_1)|^2, \end{aligned}$$

and by further integrating on u_1 , the result is $1 - \delta_{mn}$, which implies that by summing the two-body density over $m, n = 1 \dots A$, one obtains $A(A - 1)$. The two-body density is thus given by

$$\rho^{(2)}(u_1, u_2) = \sum_n |\psi_n(u_1)|^2 \sum_n |\psi_n(u_2)|^2 - |\Delta(u_1, u_2)|^2,$$

where $\Delta(u_1, u_2) = \sum_n \psi_n(u_1) \psi_n^*(u_2)$. Notice that, once \sum_n is done over a complete set of states in Hilbert space, one obtains $\Delta \rightarrow \delta$. As a direct consequence of its definition, one has $\int |\Delta(u_1, u_2)|^2 du_2 = \rho^{(1)}(u_1)$, and thus $\int \rho^{(2)}(u_1, u_2) du_2 = (A - 1) \rho^{(1)}(u_1)$.

We are interested in short-range nuclear correlations, which exhibit a universal behavior [41]. We introduce, therefore, an m - and n -independent correlation term in the two-body wave function:

$$\frac{1}{\sqrt{2}} [\psi_m(u_1) \psi_n(u_2) - \psi_m(u_2) \psi_n(u_1)] \times [1 - C(u_1, u_2)].$$

The corresponding two-body density of the states m, n is

$$f_{mn}(u_1, u_2) = g_{mn}(u_1, u_2) \times [1 - C(u_1, u_2)]^2.$$

By summing over states, one obtains the correlated two-body density:

$$\rho^{(C,2)}(u_1, u_2) = \rho^{(2)}(u_1, u_2)[1 - C(u_1, u_2)]^2.$$

At first order in C , one has

$$\begin{aligned} & \int \rho^{(C,2)}(u_1, u_2) du_2 \\ &= (A-1)\rho^{(1)}(u_1) - 2 \int \rho^{(2)}(u_1, u_2)C(u_1, u_2)du_2. \end{aligned}$$

One should now recall that u also includes the spin variables $\int du \equiv \int d^3r \sum_s$. When the two nucleons are in a spin triplet state, their space wave function is antisymmetric, and therefore it vanishes for $r_1 \rightarrow r_2$ irrespectively of the presence of the correlation term. When the two nucleons are in a spin singlet state, their space wave function is symmetric, and the effect of the short-range correlation term in this case is particularly important. In the spin singlet case and without interaction, the space wave function is

$$\Psi = \frac{1}{\sqrt{2}}[\psi_m(r_1)\psi_n(r_2) + \psi_m(r_2)\psi_n(r_1)].$$

For $r_1 \rightarrow r_2$, which is the region where short-range correlations are important, one may write $r_1 = r + w/2$, $r_2 = r - w/2$, and $C = C(w)$; for small w , one has $\Psi = \psi_m(r)\psi_n(r)\sqrt{2} + \mathcal{O}(w^2)$ and, as a first approximation,

$$\begin{aligned} & \int \rho^{(C,2)}(r_1, r_2) dr_2 \\ &= (A-1)\rho^{(1)}(r_1) + 2 \int \rho^{(2)}(r_1, r_2)C(r_1, r_2)dr_2 \\ &\approx (A-1)\rho^{(1)}(r_1) + 2[\rho^{(1)}(r_1)]^2 \int C(w)dw. \end{aligned} \quad (\text{B1})$$

One can argue similarly for the three-body wave function. The space components are

$$\Psi_A = \frac{1}{\sqrt{6}}\text{Det}[\psi_m(r_1)\psi_n(r_2)\psi_l(r_3)],$$

which is completely antisymmetric and corresponds to the spin quadruplet (wholly symmetric), and

$$\begin{aligned} \Psi_1 &= \psi_l(r_3) \frac{1}{\sqrt{2}}[\psi_m(r_1)\psi_n(r_2) - \psi_m(r_2)\psi_n(r_1)], \\ \Psi_2 &= \psi_l(r_1) \frac{1}{\sqrt{2}}[\psi_m(r_2)\psi_n(r_3) - \psi_m(r_3)\psi_n(r_2)], \end{aligned} \quad (\text{B2})$$

which have mixed symmetry and correspond to the two possible spin doublets (the third possible option is a linear combination of the two above). The correlation term is not of great importance for the completely antisymmetric case, which, as a first approximation, may not need to be corrected. For Ψ_1, Ψ_2 , one introduces the correlation terms C in non-antisymmetric products. For example,

$$\begin{aligned} \Psi_1^C &= \psi_l(r_3) \frac{1}{\sqrt{2}}[\psi_m(r_1)\psi_n(r_2) \\ &\quad - \psi_m(r_2)\psi_n(r_1)][1 - C(u_1, u_3) - C(u_3, u_2)]. \end{aligned}$$

Unless an explicit three-body correlation term is introduced, the correlation is thus again of the same kind as the two-body wave function case.

An interesting possibility is that of a totally symmetric space wave function, where antisymmetry is due to the spin-isospin variables. In such a case, the space wave function is

$$\frac{1}{\sqrt{6}} \sum_P \psi_l(r_1)\psi_m(r_2)\psi_n(r_3),$$

and correspondingly, the density is

$$\begin{aligned} & \rho^{(3)}(r_1, r_2, r_3) \\ &= \frac{1}{6} \left\{ \sum_l |\psi_l(r_1)|^2 \sum_m |\psi_m(r_2)|^2 \sum_n |\psi_n(r_3)|^2 \right. \\ &\quad + \sum_l |\psi_l(r_1)|^2 |\Delta(r_2, r_3)|^2 + \sum_l |\psi_l(r_2)|^2 |\Delta(r_3, r_1)|^2 \\ &\quad + \sum_l |\psi_l(r_3)|^2 |\Delta(r_1, r_2)|^2 \\ &\quad \left. + 2\Re[\Delta(r_1, r_2)\Delta(r_2, r_3)\Delta(r_3, r_1)] \right\}. \end{aligned} \quad (\text{B3})$$

After introducing correlations, at the first order, one has

$$\begin{aligned} \rho^{(C,3)}(r_1, r_2, r_3) &= \rho^{(3)}(r_1, r_2, r_3)[1 - 2C(r_1, r_2) \\ &\quad - 2C(r_2, r_3) - 2C(r_3, r_1)]. \end{aligned}$$

As in the previous case, one may introduce $r_1 + r_2 + r_3 = 3r$ and $r_1 = r + w$, $r_2 = r + w'$, $r_3 = r + w''$, where $w = w' + w'' = 0$. By expanding ψ near r , after summing over all permutations one obtains

$$\psi \approx \psi(r) + (w + w' + w'') \cdot \partial\psi/\partial r + \mathcal{O}(w^2),$$

and for small w , one may thus write

$$\Psi = \psi_l(r)\psi_m(r)\psi_n(r)\sqrt{6} + \mathcal{O}(w^2),$$

which allows one to treat this case on the same footing as the previous ones.

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