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CP and *CPT* violating parameters determined from the joint decays of $C = +1$ entangled neutral pseudoscalar mesons

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Entangled pseudoscalar neutral meson pairs have been used in studying CP violation and searching for CPT violation, but almost all the previous works concern the $C = -1$ entangled state. Here we consider the $C = +1$ entangled state of pseudoscalar neutral mesons, which is quite different from the $C = -1$ entangled state and provides complementary information on symmetry violating parameters. After developing a general formalism, we consider three kinds of decay processes, namely, semileptonic-semileptonic, hadronic-hadronic, and semileptonic-hadronic processes. For each kind of processes, we calculate the integrated rates of joint decays with a fixed time interval, as well as asymmetries defined for these joint rates of different channels. In turn, these asymmetries can be used to determine the four real numbers of the two indirect symmetry violating parameters, based on a general relation between the symmetry violating parameters and the decay asymmetries presented here. Various discussions are provided on indirect and direct violations and the violation of the $\Delta\mathcal{F} = \Delta Q$ rule, with some results presented as theorems.

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I. INTRODUCTION

Pseudoscalar neutral mesons are important in studying CP violation, as well as CPT violation, which is important in the standard model extension [1]. Moreover, these particles in entangled or EPR correlated states have also been used in discussing these violations [2–5]. Several experimental groups have investigated the violations of the CP and CPT symmetries in the entangled pseudoscalar neutral mesons, such as $B_d\bar{B}_d$ pairs produced in $\Upsilon(4\mathbf{S})$ resonance and $B_s\bar{B}_s$ pairs produced in $\Upsilon(5\mathbf{S})$ resonance [6–13], as well as $K^0\bar{K}^0$ pairs produced in ϕ resonance [14–17]. Various theoretical studies have also been made [18–32]. However, most of them concern the $C = -1$ state $|\Psi_-\rangle$.

On the other hand, it is known that the $C = +1$ entangled state $|\Psi_+\rangle$ can also be produced, for example, for $B_s\bar{B}_s$ pairs produced in the $\Upsilon(5\mathbf{S})$ resonance with 10% branch ratio [11,12,29] and, most remarkably, for $B_d\bar{B}_d$ pairs in an energy range just above the $\Upsilon(4\mathbf{S})$ resonance with 100% branch ratio [33]. Hence it is very interesting to investigate the decay properties of the $C = +1$ entangled state, which is the purpose of this paper. Towards the end of the paper, we shall note some complementarities between the uses of $|\Psi_+\rangle$ and $|\Psi_-\rangle$.

After a review of various CP and CPT violating parameters and the relations among them in Sec. II, we calculate the rates of the joint decays of the $C = +1$ entangled meson pairs in Sec. III. Then we discuss various experimentally observable asymmetries between different joint decay rates in Sec. IV, giving the general expressions in subsection IV A, and considering the semileptonic-semileptonic decays in subsection IV B, the hadronic-hadronic decays in subsection IV C, and the semileptonic-hadronic decays in subsection IV D. Subsequently in Sec. V, we discuss how to obtain the four real numbers of CP and CPT symmetry violating parameters from the asymmetries of joint decays. In Sec. VI, we discuss some specific experimentally relevant cases and present some simple results in the form of theorems. A summary is given in Sec. VII.

II. INDIRECT SYMMETRY VIOLATING PARAMETERS AND TIME EVOLUTION

As usual, we denote the pseudoscalar neutral meson with the flavor eigenvalue $+1$ as $|M^0\rangle$, and its antiparticle with the flavor eigenvalue -1 as $|\bar{M}^0\rangle \equiv CP|M^0\rangle$. The time-dependent state of a single meson is

$$|M(t)\rangle = \alpha(t)|M^0\rangle + \beta(t)|\bar{M}^0\rangle, \tag{1}$$

where $\alpha(t)$ and $\beta(t)$ are determined by

$$i\frac{d}{dt}\begin{pmatrix}\alpha(t)\\ \beta(t)\end{pmatrix} = \begin{pmatrix}H_{11} & H_{12}\\ H_{21} & H_{22}\end{pmatrix}\begin{pmatrix}\alpha(t)\\ \beta(t)\end{pmatrix}. \tag{2}$$

The effective Hamiltonian H has the following properties:
(i) if CPT or CP is conserved, then $H_{11} = H_{22}$,
(ii) if T or CP is conserved, then $H_{12} = H_{21}$.

One can define [34]

$$\delta_M \equiv \frac{H_{22} - H_{11}}{\sqrt{H_{12}H_{21}}}, \qquad \epsilon_M \equiv \frac{\sqrt{H_{12}} - \sqrt{H_{21}}}{\sqrt{H_{12}} + \sqrt{H_{21}}}. \tag{3}$$

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Indirect CPT conservation implies $\delta_M = 0$, while indirect CP conservation implies $\epsilon_M = 0$ and $\delta_M = 0$.

The eigenvalues of H are

$$\begin{aligned}\lambda_L &\equiv m_L - \frac{i}{2}\Gamma_L \\ &= \frac{1}{2}[H_{11} + H_{22} - \sqrt{(H_{11} - H_{22})^2 + 4H_{12}H_{21}}], \\ \lambda_S &\equiv m_S - \frac{i}{2}\Gamma_S \\ &= \frac{1}{2}[H_{11} + H_{22} + \sqrt{(H_{11} - H_{22})^2 + 4H_{12}H_{21}}], \end{aligned} \tag{4}$$

and the corresponding eigenstates are

$$\begin{aligned}|M_L\rangle &= \frac{1}{\sqrt{|p_L|^2 + |q_L|^2}}(p_L|M^0\rangle - q_L|\bar{M}^0\rangle), \\ |M_S\rangle &= \frac{1}{\sqrt{|p_S|^2 + |q_S|^2}}(p_S|M^0\rangle + q_S|\bar{M}^0\rangle). \end{aligned} \tag{5}$$

Defining

$$\frac{1+\Delta_M}{1-\Delta_M} \equiv \frac{\delta_M}{2} + \sqrt{1 + \frac{\delta_M^2}{4}}, \tag{6}$$

we have

$$\frac{p_L}{q_L} = \frac{(1+\epsilon_M)(1+\Delta_M)}{(1-\epsilon_M)(1-\Delta_M)}, \qquad \frac{p_S}{q_S} = \frac{(1+\epsilon_M)(1-\Delta_M)}{(1-\epsilon_M)(1+\Delta_M)}.$$

One also defines

$$\frac{p_L}{q_L} \equiv \frac{1+\epsilon_L}{1-\epsilon_L}, \qquad \frac{p_S}{q_S} \equiv \frac{1+\epsilon_S}{1-\epsilon_S}, \tag{7}$$

and

$$\begin{aligned}\delta &= \frac{1}{2}(\epsilon_S - \epsilon_L) = \frac{H_{11} - H_{22}}{H_{12} + H_{21} + \sqrt{(H_{11} - H_{22})^2 + 4H_{12}H_{21}}}, \\ \epsilon &= \frac{1}{2}(\epsilon_S + \epsilon_L) = \frac{H_{12} - H_{21}}{H_{12} + H_{21} + \sqrt{(H_{11} - H_{22})^2 + 4H_{12}H_{21}}}. \end{aligned} \tag{8}$$

Hence $\delta = 0$ corresponds to $H_{11} = H_{22}$, while $\epsilon = 0$ corresponds to $H_{12} = H_{21}$. (δ_M, ϵ_M) and (δ, ϵ) are related as

$$\begin{aligned}\delta &= -\frac{1}{2}\frac{\delta_M(1-\epsilon_M^2)}{1 + \epsilon_M^2 + (1-\epsilon_M^2)\sqrt{1 + \frac{\delta_M^2}{4}}}, \\ \epsilon &= \frac{2\epsilon_M}{1 + \epsilon_M^2 + (1-\epsilon_M^2)\sqrt{1 + \frac{\delta_M^2}{4}}}. \end{aligned} \tag{9}$$

We would like to emphasize that δ and ϵ are each dependent on both δ_M and ϵ_M; that is, a nonzero value of δ or ϵ corresponds to mixing of CP and CPT violations. Moreover, as seen in (6), $\delta_M = 0$ is equivalent to $\Delta_M = 0$, and using Δ_M can avoid square roots in the calculations.

Therefore, in the following, we use the parameters (Δ_M,ϵ_M) in characterizing indirect symmetry violations. $\epsilon_M \neq 0$ implies indirect CP violation, while $\Delta_M \neq 0$ implies indirect CPT violation and indirect CP violation.

With $|M^0(t=0)\rangle \equiv |M^0\rangle$ and $|\bar{M}^0(t=0)\rangle \equiv |\bar{M}^0\rangle$, we have

$$\begin{aligned}|M^0(t)\rangle = &\frac{1}{2}[(1-\xi)e^{-i\lambda_S t} + (1+\xi)e^{-i\lambda_L t}]|M^0\rangle \\ &+ \frac{1}{2}\eta_1(e^{-i\lambda_S t} - e^{-i\lambda_L t})|\bar{M}^0\rangle, \end{aligned} \tag{10}$$

$$\begin{aligned}|\bar{M}^0(t)\rangle = &\frac{1}{2}\eta_2(e^{-i\lambda_S t} - e^{-i\lambda_L t})|M^0\rangle + \frac{1}{2}[(1+\xi)e^{-i\lambda_S t} \\ &+ (1-\xi)e^{-i\lambda_L t}]|\bar{M}^0\rangle, \end{aligned} \tag{11}$$

where

$$\begin{aligned}&\xi \equiv \frac{2\Delta_M}{1+\Delta_M^2}, \qquad \eta_1 \equiv \frac{(1-\epsilon_M)(1-\Delta_M^2)}{(1+\epsilon_M)(1+\Delta_M^2)}, \\ &\eta_2 \equiv \frac{(1+\epsilon_M)(1-\Delta_M^2)}{(1-\epsilon_M)(1+\Delta_M^2)}. \end{aligned} \tag{12}$$

Now we consider the $C = +1$ entangled state, shared by particles a and b,

$$|\Psi_+\rangle = \frac{1}{\sqrt{2}}[|M^0\rangle|\bar{M}^0\rangle + |\bar{M}^0\rangle|M^0\rangle], \tag{13}$$

where each term in the form of $|x\rangle|y\rangle$ apparently means a direct product of $|x\rangle$ of a particle and $|y\rangle$ of b particle.

The joint probability, or the joint decay rate, that particle a decays to ψ^a at time t_a while particle b decays to ψ^b at time t_b, can be obtained as

$$I(\psi^a, t_a; \psi^b, t_b) = |\langle\psi^a\psi^b|\mathcal{H}_a\mathcal{H}_b|\Psi(t_a, t_b)\rangle|^2, \tag{14}$$

where $\mathcal{H}_\alpha$ is the weak interaction Hamiltonian governing the decay of particle $\alpha = a$, b, the time-dependent state $|\Psi_+(t_a, t_b)\rangle$ is given by

$$|\Psi_+(t_a, t_b)\rangle = \frac{1}{\sqrt{2}}[|M^0(t_a)\rangle|\bar{M}^0(t_b)\rangle + |\bar{M}^0(t_a)\rangle|M^0(t_b)\rangle], \tag{15}$$

where $|M^0(t_\alpha)\rangle$ and $|\bar{M}^0(t_\alpha)\rangle$, with $\alpha = a$, b, as given in Eqs. (10) and (11). This standard treatment using the decay times t_a and t_b of the two entangled mesons [3–5] gives the same result as that of the approach taking account of the two measurements at t_a and t_b.

By substituting $|M^0(t_\alpha)\rangle$ and $|\bar{M}^0(t_\alpha)\rangle$, one obtains

$$\begin{aligned}|\Psi_+(t_a,t_b)\rangle = \frac{1}{2\sqrt{2}}\{&[\eta_2(1-\xi)e^{-i\lambda_S(t_a+t_b)} + \eta_2\xi e^{-i(\lambda_S t_a+\lambda_L t_b)} + \eta_2\xi e^{-i(\lambda_L t_a+\lambda_S t_b)} - \eta_2(1+\xi)e^{-i\lambda_L(t_a+t_b)}]|M^0M^0\rangle \\ &+ [(1-\xi^2)e^{-i\lambda_S(t_a+t_b)} - \xi(1-\xi)e^{-i(\lambda_S t_a+\lambda_L t_b)} + \xi(1+\xi)e^{-i(\lambda_L t_a+\lambda_S t_b)} + (1-\xi^2)e^{-i\lambda_L(t_a+t_b)}]|M^0\bar{M}^0\rangle \\ &+ [(1-\xi^2)e^{-i\lambda_S(t_a+t_b)} + \xi(1+\xi)e^{-i(\lambda_S t_a+\lambda_L t_b)} - \xi(1-\xi)e^{-i(\lambda_L t_a+\lambda_S t_b)} + (1-\xi^2)e^{-i\lambda_L(t_a+t_b)}]|\bar{M}^0M^0\rangle \\ &+ [\eta_1(1+\xi)e^{-i\lambda_S(t_a+t_b)} - \eta_1\xi e^{-i(\lambda_S t_a+\lambda_L t_b)} - \eta_1\xi e^{-i(\lambda_L t_a+\lambda_S t_b)} - \eta_1(1-\xi)e^{-i\lambda_L(t_a+t_b)}]|\bar{M}^0\bar{M}^0\rangle\}. \end{aligned} \tag{16}$$

Note that the initial entangled state $|\Psi_+\rangle$ can be rewritten in *CP* basis *exactly* as

$$|\Psi_+\rangle = \frac{1}{\sqrt{2}}[|M_+\rangle|M_+\rangle - |M_-\rangle|M_-\rangle], \tag{17}$$

where

$$|M_\pm\rangle = \frac{1}{\sqrt{2}}(|M^0\rangle \pm |\bar{M}^0\rangle)$$

is the *CP* eigenstate of eigenvalue ±1. If needed, $|\Psi_+(t_a,t_b)\rangle$ can also be rewritten as

$$|\Psi_+(t_a,t_b)\rangle = \frac{1}{\sqrt{2}}[|M_+(t_a)\rangle|M_+(t_b)\rangle - |M_-(t_a)\rangle|M_-(t_b)\rangle], \tag{18}$$

where

$$\begin{aligned}|M_+(t)\rangle &= \frac{1}{4}\{[(2+\eta_1+\eta_2)e^{-i\lambda_S t} + (2-\eta_1-\eta_2)e^{-i\lambda_L t}]|M_+\rangle - (2\xi-\eta_2+\eta_1)(e^{-i\lambda_S t} - e^{-i\lambda_L t})|M_-\rangle\}, \\ |M_-(t)\rangle &= \frac{1}{4}\{[(2-\eta_1-\eta_2)e^{-i\lambda_S t} + (2+\eta_1+\eta_2)e^{-i\lambda_L t}]|M_-\rangle - (2\xi+\eta_2-\eta_1)(e^{-i\lambda_S t} - e^{-i\lambda_L t})|M_+\rangle\}. \end{aligned} \tag{19}$$

In comparison, the $C=-1$ state is

$$|\Psi_-\rangle = \frac{1}{\sqrt{2}}[|M^0\rangle|\bar{M}^0\rangle - |\bar{M}^0\rangle|M^0\rangle], \tag{20}$$

which can be rewritten in *CP* basis *exactly* as

$$|\Psi_-\rangle = \frac{1}{\sqrt{2}}(|M_-\rangle|M_+\rangle - |M_+\rangle|M_-\rangle). \tag{21}$$

Note that in the *CP* basis, $|\Psi_+\rangle$ is a superposition of equal-*CP* products $|M_+\rangle|M_+\rangle$ and $|M_-\rangle|M_-\rangle$, while $|\Psi_-\rangle$ is a superposition of unequal-*CP* products $|M_-\rangle|M_+\rangle$ and $|M_+\rangle|M_-\rangle$.

The time-dependent state $|\Psi_-(t_a,t_b)\rangle$ is given by

$$\begin{aligned}|\Psi_-(t_a,t_b)\rangle &= \frac{1}{\sqrt{2}}[|M^0(t_a)\rangle|\bar{M}^0(t_b)\rangle - |\bar{M}^0(t_a)\rangle|M^0(t_b)\rangle], \\ &= \frac{1}{2\sqrt{2}}\{-\eta_2(e^{-i(\lambda_S t_a+\lambda_L t_b)} - e^{-i(\lambda_L t_a+\lambda_S t_b)})|M^0M^0\rangle + [(1-\xi)e^{-i(\lambda_S t_a+\lambda_L t_b)} + (1+\xi)e^{-i(\lambda_L t_a+\lambda_S t_b)}]|M^0\bar{M}^0\rangle \\ &\quad - [(1+\xi)e^{-i(\lambda_S t_a+\lambda_L t_b)} + (1-\xi)e^{-i(\lambda_L t_a+\lambda_S t_b)}]|\bar{M}^0M^0\rangle + \eta_1(e^{-i(\lambda_S t_a+\lambda_L t_b)} - e^{-i(\lambda_L t_a+\lambda_S t_b)})|\bar{M}^0\bar{M}^0\rangle\}, \end{aligned} \tag{22}$$

which can be rewritten as

$$|\Psi_-(t_a,t_b)\rangle = \frac{1}{\sqrt{2}}(|M_-(t_a)\rangle|M_+(t_b)\rangle - |M_+(t_a)\rangle|M_-(t_b)\rangle). \tag{23}$$

III. INTEGRATED RATES OF JOINT DECAYS

We denote the decay amplitudes

$$
\begin{aligned}
r^a &\equiv \langle\psi^a|\mathcal{H}|M^0\rangle, \qquad r^b \equiv \langle\psi^b|\mathcal{H}|M^0\rangle,\\
\bar{r}^a &\equiv \langle\psi^a|\mathcal{H}|\bar{M}^0\rangle, \qquad \bar{r}^b \equiv \langle\psi^b|\mathcal{H}|\bar{M}^0\rangle.
\end{aligned} \tag{24}
$$

Suppose that particle a decays to $|\psi^a\rangle$ at t_a, while particle b decays to $|\psi^b\rangle$ at t_b. The joint decay rate is obtained as

$$
\begin{aligned}
I(\psi^a,t_a;\psi^b,t_b) &= |\langle\psi^a\psi^b|\mathcal{H}_a\mathcal{H}_b|\Psi_+(t_a,t_b)\rangle|^2\\
&= \frac{1}{8}\{e^{-\Gamma_S(t_a+t_b)}|\Theta|^2 + 2e^{-(\Gamma_S t_a+\Gamma t_b)}\Re[\Theta^*\Xi e^{-i\Delta m t_b}] + 2e^{-(\Gamma t_a+\Gamma_S t_b)}\Re[\Theta^*\Phi e^{-i\Delta m t_a}]\\
&\quad + 2e^{-\Gamma(t_a+t_b)}\Re[\Theta^*\Lambda e^{-i\Delta m(t_a+t_b)}] + e^{-(\Gamma_S t_a+\Gamma_L t_b)}|\Xi|^2 + 2e^{-\Gamma(t_a+t_b)}\Re[\Xi^*\Phi e^{i\Delta m(t_b-t_a)}]\\
&\quad + 2e^{-(\Gamma t_a+\Gamma_L t_b)}\Re[\Xi^*\Lambda e^{i\Delta m t_a}] + e^{-(\Gamma_L t_a+\Gamma_S t_b)}|\Phi|^2 + 2e^{-(\Gamma_L t_a+\Gamma t_b)}\Re[\Phi^*\Lambda e^{i\Delta m t_b}] + e^{-\Gamma_L(t_a+t_b)}|\Lambda|^2\},
\end{aligned} \tag{25}
$$

where

$$
\begin{aligned}
&\Gamma \equiv \frac{1}{2}(\Gamma_S+\Gamma_L), \qquad \Delta m \equiv m_L - m_S, \qquad \Delta\Gamma \equiv \Gamma_L - \Gamma_S,\\
&\Theta = \eta_2(1-\xi)r^a r^b + (1-\xi^2)(r^a\bar{r}^b + r^b\bar{r}^a) + \eta_1(1+\xi)\bar{r}^a\bar{r}^b,\\
&\Xi = \eta_2\xi r^a r^b - \xi(1-\xi)r^a\bar{r}^b + \xi(1+\xi)r^b\bar{r}^a - \eta_1\xi\bar{r}^a\bar{r}^b,\\
&\Phi = \eta_2\xi r^a r^b + \xi(1+\xi)r^a\bar{r}^b - \xi(1-\xi)r^b\bar{r}^a - \eta_1\xi\bar{r}^a\bar{r}^b,\\
&\Lambda = -\eta_2(1+\xi)r^a r^b + (1-\xi^2)(r^a\bar{r}^b + r^b\bar{r}^a) - \eta_1(1-\xi)\bar{r}^a\bar{r}^b.
\end{aligned} \tag{26}
$$

Consider a fixed time interval $\Delta t = t_b - t_a$ between t_a and t_b. We obtain the time-integrated decay rate

$$
\begin{aligned}
I'(\psi^a,\psi^b,\Delta t) &= \int_0^\infty dt_a I(\psi^a,t_a;\psi^b,t_a+\Delta t)\\
&\equiv \frac{1}{8}\left\{\frac{e^{-\Gamma_S\Delta t}}{2\Gamma_S}|\Theta|^2 + 2e^{-\Gamma\Delta t}\Re\left[\frac{\Theta^*\Xi e^{-i\Delta m\Delta t}}{\Gamma_S+\Gamma+i\Delta m}\right] + 2e^{-\Gamma_S\Delta t}\Re\left[\frac{\Theta^*\Phi}{\Gamma_S+\Gamma+i\Delta m}\right]\right.\\
&\quad + e^{-\Gamma\Delta t}\Re\left[\frac{\Theta^*\Lambda e^{-i\Delta m\Delta t}}{\Gamma+i\Delta m}\right] + \frac{e^{-\Gamma_L\Delta t}}{\Gamma_S+\Gamma_L}|\Xi|^2 + \frac{e^{-\Gamma\Delta t}}{\Gamma}\Re[\Xi^*\Phi e^{i\Delta m\Delta t}] + 2e^{-\Gamma_L\Delta t}\Re\left[\frac{\Xi^*\Lambda}{\Gamma_L+\Gamma-i\Delta m}\right]\\
&\quad \left. + \frac{e^{-\Gamma_S\Delta t}}{\Gamma_S+\Gamma_L}|\Phi|^2 + 2e^{-\Gamma\Delta t}\Re\left[\frac{\Phi^*\Lambda e^{i\Delta m\Delta t}}{\Gamma_L+\Gamma-i\Delta m}\right] + \frac{e^{-\Gamma_L\Delta t}}{2\Gamma_L}|\Lambda|^2\right\}.
\end{aligned} \tag{27}
$$

Ignoring the higher orders of Δ_M and ϵ_M, we have

$$
\xi \approx 2\Delta_M, \qquad \eta_1 \approx 1-2\epsilon_M, \qquad \eta_2 \approx 1+2\epsilon_M, \tag{28}
$$

therefore

$$
I(\psi^a,t_a;\psi^b,t_b) \approx \frac{1}{2}\left[\frac{1}{4}\tilde{f}_0^{r^a r^b}(t_a,t_b) + \tilde{f}_1^{r^a r^b}(t_a,t_b)\Re\Delta_M + \tilde{f}_2^{r^a r^b}(t_a,t_b)\Im\Delta_M + \tilde{f}_3^{r^a r^b}(t_a,t_b)\Re\epsilon_M + \tilde{f}_4^{r^a r^b}(t_a,t_b)\Im\epsilon_M\right], \tag{29}
$$

where the functions $\tilde{f}_i^{r^a r^b}$, ($i = 0, 1, 2, 3, 4$), are as given in the Appendix. One also obtains the integrated rates

$$
I'(\psi^a,\psi^b,\Delta t) \approx \frac{1}{2}\left[\frac{1}{4}f_0^{r^a r^b}(\Delta t) + f_1^{r^a r^b}(\Delta t)\Re\Delta_M + f_2^{r^a r^b}(\Delta t)\Im\Delta_M + f_3^{r^a r^b}(\Delta t)\Re\epsilon_M + f_4^{r^a r^b}(\Delta t)\Im\epsilon_M\right], \tag{30}
$$

with

$$f_i^{r^a r^b}(\Delta t) = \int_0^\infty \tilde{f}_i^{r^a r^b}(t_a, t_a + \Delta t) dt_a, \tag{31}$$

the details of which are also given in the Appendix.

IV. ASYMMETRIES BETWEEN DIFFERENT JOINT DECAY RATES

A. General formalism

In general, one can consider the asymmetry $\mathcal{A}(\psi^1\psi^2, \psi^3\psi^4; \Delta t)$ between the rate $I'(\psi^1, \psi^2, \Delta t)$ of the joint decays with $\psi^a = \psi^1$ and $\psi^b = \psi^2$ and the rate $I'(\psi^3, \psi^4, \Delta t)$ with $\psi^a = \psi^3$ and $\psi^b = \psi^4$,

$$\mathcal{A}(\psi^1\psi^2, \psi^3\psi^4; \Delta t) \equiv \frac{I'(\psi^1, \psi^2, \Delta t) - I'(\psi^3, \psi^4, \Delta t)}{I'(\psi^1, \psi^2, \Delta t) + I'(\psi^3, \psi^4, \Delta t)}, \tag{32}$$

$$\approx \frac{\frac{1}{4}(f_0^{r^1r^2} - f_0^{r^3r^4}) + \sum_{i=1}^4 (f_i^{r^1r^2} - f_i^{r^3r^4})\sigma_i}{\frac{1}{4}(f_0^{r^1r^2} + f_0^{r^3r^4}) + \sum_{i=1}^4 (f_i^{r^1r^2} + f_i^{r^3r^4})\sigma_i}, \tag{33}$$

where we have introduced shorthand notations $\sigma_1 \equiv \Re\Delta_M$, $\sigma_2 \equiv \Im\Delta_M$, $\sigma_3 \equiv \Re\epsilon_M$, and $\sigma_4 \equiv \Im\epsilon_M$.

In particular, we shall study the equal-state asymmetry

$$A(\psi^1\psi^1, \psi^2\psi^2; \Delta t) \equiv \frac{I'(\psi^1, \psi^1, \Delta t) - I'(\psi^2, \psi^2, \Delta t)}{I'(\psi^1, \psi^1, \Delta t) + I'(\psi^2, \psi^2, \Delta t)} \approx \frac{\frac{1}{4}(f_0^{r^1r^1} - f_0^{r^2r^2}) + \sum_{i=1}^4 (f_i^{r^1r^1} - f_i^{r^2r^2})\sigma_i}{\frac{1}{4}(f_0^{r^1r^1} + f_0^{r^2r^2}) + \sum_{i=1}^4 (f_i^{r^1r^1} + f_i^{r^2r^2})\sigma_i}, \tag{34}$$

and the unequal-state asymmetry

$$A(\psi^1\psi^2, \psi^2\psi^1; \Delta t) \equiv \frac{I'(\psi^1, \psi^2, \Delta t) - I'(\psi^2, \psi^1, \Delta t)}{I'(\psi^1, \psi^2, \Delta t) + I'(\psi^2, \psi^1, \Delta t)}$$

$$\approx \frac{\sum_{i=1}^4 (f_i^{r^1r^2} - f_i^{r^2r^1})\sigma_i}{\frac{1}{2} f_0^{r^1r^2} + \sum_{i=1}^4 (f_i^{r^1r^2} + f_i^{r^2r^1})\sigma_i}, \tag{35}$$

where we have used the property $f_0^{r^1r^2} = f_0^{r^2r^1}$, which can be seen from the expression of $f_0^{r^ar^b}$ in the Appendix. In the following, we will discuss three different kinds of processes.

B. Semileptonic-semileptonic processes

Consider the semileptonic-semileptonic decay processes with the final states $|\psi^1\rangle = |l^+\rangle$ and $|\psi^2\rangle = |l^-\rangle$, which are flavor eigenstates with eigenvalues 1 and -1, respectively. The decay amplitudes are $\langle l^+|\mathcal{H}|M^0\rangle \equiv R^+$, $\langle l^-|\mathcal{H}|M^0\rangle \equiv S^-$, $\langle l^+|\mathcal{H}|\bar{M}^0\rangle \equiv \bar{R}^+$, and $\langle l^-|\mathcal{H}|\bar{M}^0\rangle \equiv \bar{S}^-$.

From Eq. (25), $I(l^+, t_a; l^+, t_b)$ is obtained by substitution $(r^a, r^b, \bar{r}^a, \bar{r}^b) = (R^+, R^+, \bar{R}^+, \bar{R}^+)$, $I(l^+, t_a; l^-, t_b)$ by substitution $(r^a, r^b, \bar{r}^a, \bar{r}^b) = (R^+, S^-, \bar{R}^+, \bar{S}^-)$, $I(l_a^-, t_a; l_b^+, t_b)$ by substitution $(r^a, r^b, \bar{r}^a, \bar{r}^b) = (S^-, R^+, \bar{S}^-, \bar{R}^+)$, and $I(l_a^-, t_a; l_b^-, t_b)$ by substitution $(r^a, r^b, \bar{r}^a, \bar{r}^b) = (S^-, S^-, \bar{S}^-, \bar{S}^-)$.

In this case, with $|\psi^1\rangle = |l^+\rangle$ and $|\psi^2\rangle = |l^-\rangle$, the equal-flavor asymmetry is

$$A(l^+l^+, l^-l^-; \Delta t) \approx \frac{\frac{1}{4}(f_0^{R^+R^+} - f_0^{S^-S^-}) + \sum_{i=1}^4 (f_i^{R^+R^+} - f_i^{S^-S^-})\sigma_i}{\frac{1}{4}(f_0^{R^+R^+} + f_0^{S^-S^-}) + \sum_{i=1}^4 (f_i^{R^+R^+} + f_i^{S^-S^-})\sigma_i}, \tag{36}$$

while the unequal-flavor asymmetry is

$$A(l^+l^-, l^-l^+; \Delta t) \approx \frac{\sum_{i=1}^4 (f_i^{R^+S^-} - f_i^{S^-R^+})\sigma_i}{\frac{1}{2} f_0^{R^+S^-} + \sum_{i=1}^4 (f_i^{R^+S^-} + f_i^{S^-R^+})\sigma_i}. \tag{37}$$

They are obtained from Eqs. (34) and (35), respectively, using the substitution $r^1 = R^+$, $r^2 = S^-$, $\bar{r}^1 = \bar{R}^+$, and $\bar{r}^2 = \bar{S}^-$.

C. Hadronic-hadronic processes

For the hadronic-hadronic processes, we denote the two final states as $|\psi^1\rangle = |h_1\rangle$ and $|\psi^2\rangle = |h_2\rangle$. The decay amplitudes are $\langle h_1|\mathcal{H}|M^0\rangle \equiv Q_1$, $\langle h_2|\mathcal{H}|M^0\rangle \equiv Q_2$, $\langle h_1|\mathcal{H}|\bar{M}^0\rangle \equiv \bar{Q}_1$, $\langle h_2|\mathcal{H}|\bar{M}^0\rangle \equiv \bar{Q}_2$.

From Eq. (25), $I(h_1, t_a; h_1, t_b)$ is obtained by substitution $(r^a, r^b, \bar{r}^a, \bar{r}^b) = (Q_1, Q_1, \bar{Q}_1, \bar{Q}_1)$, $I(h_1, t_a; h_2, t_b)$ by substitution $(r^a, r^b, \bar{r}^a, \bar{r}^b) = (Q_1, Q_2, \bar{Q}_1, \bar{Q}_2)$, $I(h_2, t_a; h_1, t_b)$ by substitution $(r^a, r^b, \bar{r}^a, \bar{r}^b) = (Q_2, Q_1, \bar{Q}_2, \bar{Q}_1)$, and $I(h_2, t_a; h_2, t_b)$ by substitution $(r^a, r^b, \bar{r}^a, \bar{r}^b) = (Q_2, Q_2, \bar{Q}_2, \bar{Q}_2)$.

In this case, with $|\psi^1\rangle = |h_1\rangle$ and $|\psi^2\rangle = |h_2\rangle$, the equal-state asymmetry is

$$A(h_1h_1, h_2h_2; \Delta t) \approx \frac{\frac{1}{4}(f_0^{Q_1Q_1} - f_0^{Q_2Q_2}) + \sum_{i=1}^4 (f_i^{Q_1Q_1} - f_i^{Q_2Q_2})\sigma_i}{\frac{1}{4}(f_0^{Q_1Q_1} + f_0^{Q_2Q_2}) + \sum_{i=1}^4 (f_i^{Q_1Q_1} + f_i^{Q_2Q_2})\sigma_i}, \tag{38}$$

while the unequal-state asymmetry is

$$A(h_1h_2, h_2h_1; \Delta t) \approx \frac{\sum_{i=1}^4 (f_i^{Q_1Q_2} - f_i^{Q_2Q_1})\sigma_i}{\frac{1}{2} f_0^{Q_1Q_2} + \sum_{i=1}^4 (f_i^{Q_1Q_2} + f_i^{Q_2Q_1})\sigma_i}. \tag{39}$$

They are obtained from Eqs. (34) and (35), respectively, using the substitution $r^1 = Q_1$, $r^2 = Q_2$, $\bar{r}^1 = \bar{Q}_1$, and $\bar{r}^2 = \bar{Q}_2$.

D. Semileptonic-hadronic processes

For semileptonic-hadronic processes, consider $|\psi^a\rangle = |l^+\rangle$ or $|l^-\rangle$ while $|\psi^b\rangle = |h_1\rangle$ or $|h_2\rangle$, or vice versa. So there are eight cases of (ψ^a, ψ^b). From Eq. (25), $I(l^+, t_a; h_1, t_b)$ is obtained by substitution $(r^a, r^b, \bar{r}^a, \bar{r}^b) = (R^+, Q_1, \bar{R}^+, \bar{Q}_1)$, $I(h_1, t_a; l^+, t_b)$ by substitution $(r^a, r^b, \bar{r}^a, \bar{r}^b) = (Q_1, R^+, \bar{Q}_1, \bar{R}^+)$, $I(l^+, t_a; h_2, t_b)$ by substitution $(r^a, r^b, \bar{r}^a, \bar{r}^b) = (R^+, Q_2, \bar{R}^+, \bar{Q}_2)$, $I(h_2, t_a; l^+, t_b)$ by substitution $(r^a, r^b, \bar{r}^a, \bar{r}^b) = (Q_2, R^+, \bar{Q}_2, \bar{R}^+)$, $I(l^-, t_a; h_1, t_b)$ by substitution $(r^a, r^b, \bar{r}^a, \bar{r}^b) = (S^-, Q_1, \bar{S}^-, \bar{Q}_1)$, $I(h_1, t_a; l^-, t_b)$ by substitution $(r^a, r^b, \bar{r}^a, \bar{r}^b) = (Q_1, S^-, \bar{Q}_1, \bar{S}^-)$, $I(l^-, t_a; h_2, t_b)$ by substitution $(r^a, r^b, \bar{r}^a, \bar{r}^b) = (S^-, Q_2, \bar{S}^-, \bar{Q}_2)$, and $I(h_2, t_a; l^-, t_b)$ by substitution $(r^a, r^b, \bar{r}^a, \bar{r}^b) = (Q_2, S^-, \bar{Q}_2, \bar{S}^-)$, respectively.

For these eight different outcomes, one can define 28 different asymmetries according to (32). They are $A(l^+h_1, h_1l^+, \Delta t)$, $A(l^+h_1, l^+h_2, \Delta t)$, $A(l^+h_1, h_2l^+, \Delta t)$, $A(l^+h_1, l^-h_1, \Delta t)$, $A(l^+h_1, h_1l^-, \Delta t)$, $A(l^+h_1, l^-h_2, \Delta t)$, $A(l^+h_1, h_2l^-, \Delta t)$, $A(h_1l^+, l^+h_2, \Delta t)$, $A(h_1l^+, h_2l^+, \Delta t)$, $A(h_1l^+, l^-h_1, \Delta t)$, $A(h_1l^+, h_1l^-, \Delta t)$, $A(h_1l^+, l^-h_2, \Delta t)$, $A(h_1l^+, h_2l^-, \Delta t)$, $A(l^+h_2, h_2l^+, \Delta t)$, $A(l^+h_2, l^-h_1, \Delta t)$, $A(l^+h_2, h_1l^-, \Delta t)$, $A(l^+h_2, l^-h_2, \Delta t)$, $A(l^+h_2, h_2l^-, \Delta t)$, $A(h_2l^+, l^-h_1, \Delta t)$, $A(h_2l^+, h_1l^-, \Delta t)$, $A(h_2l^+, l^-h_2, \Delta t)$, $A(h_2l^+, h_2l^-, \Delta t)$, $A(l^-h_1, h_1l^-, \Delta t)$, $A(l^-h_1, l^-h_2, \Delta t)$, $A(l^-h_1, h_2l^-, \Delta t)$, $A(h_1l^-, l^-h_2, \Delta t)$, $A(h_1l^-, h_2l^-, \Delta t)$, and $A(l^-h_2, h_2l^-, \Delta t)$.

Among them there are four unequal-state asymmetries of the form of (35),

$$
\begin{aligned}
A(l^+h_1, h_1l^+, \Delta t) &\approx \frac{\sum_{i=1}^4 (f_i^{R^+Q_1} - f_i^{Q_1R^+})\sigma_i}{\frac{1}{2}f_0^{R^+Q_1} + \sum_{i=1}^4 (f_i^{R^+Q_1} + f_i^{Q_1R^+})\sigma_i}, \\
A(l^+h_2, h_2l^+, \Delta t) &\approx \frac{\sum_{i=1}^4 (f_i^{R^+Q_2} - f_i^{Q_2R^+})\sigma_i}{\frac{1}{2}f_0^{R^+Q_2} + \sum_{i=1}^4 (f_i^{R^+Q_2} + f_i^{Q_2R^+})\sigma_i}, \\
A(l^-h_1, h_1l^-, \Delta t) &\approx \frac{\sum_{i=1}^4 (f_i^{S^-Q_1} - f_i^{Q_1S^-})\sigma_i}{\frac{1}{2}f_0^{S^-Q_1} + \sum_{i=1}^4 (f_i^{S^-Q_1} + f_i^{Q_1S^-})\sigma_i}, \\
A(l^-h_2, h_2l^-, \Delta t) &\approx \frac{\sum_{i=1}^4 (f_i^{S^-Q_2} - f_i^{Q_2S^-})\sigma_i}{\frac{1}{2}f_0^{S^-Q_2} + \sum_{i=1}^4 (f_i^{S^-Q_2} + f_i^{Q_2S^-})\sigma_i}.
\end{aligned}
\tag{40}
$$

V. DETERMINING SYMMETRY VIOLATING PARAMETERS FROM DECAY ASYMMETRIES

We have discussed asymmetries of different decay modes, from which one can determine the CP and CPT violating parameters. There are four real numbers in the CP and CPT violating parameters. To derive the expressions of the four violating parameters, we need an equal number of decay asymmetries.

Suppose we consider four asymmetries $A_k \equiv A(\psi_k^1\psi_k^2, \psi_k^3\psi_k^4; \Delta t)$, with $k = 1, 2, 3, 4$ representing four different joint decay channels. According to (33),

$$
A_k = \frac{\frac{1}{4}\left(f_0^{r_k^1 r_k^2 +} - f_0^{r_k^3 r_k^4}\right) + \sum_{i=1}^4 \left(f_i^{r_k^1 r_k^2} - f_i^{r_k^3 r_k^4}\right)\sigma_i}{\frac{1}{4}\left(f_0^{r_k^1 r_k^2} + f_0^{r_k^3 r_k^4}\right) + \sum_{i=1}^4 \left(f_i^{r_k^1 r_k^2} + f_i^{r_k^3 r_k^4}\right)\sigma_i}. \tag{41}
$$

Defining

$$
\begin{aligned}
a_k &\equiv \frac{1}{4}\left[(1 - A_k)f_0^{r_k^1 r_k^2} - (1 + A_k)f_0^{r_k^3 r_k^4}\right], \\
K_{ki} &\equiv (A_k - 1)f_i^{r_k^1 r_k^2} + (A_k + 1)f_i^{r_k^3 r_k^4},
\end{aligned}
\tag{42}
$$

we can rewrite the four equations given by (41) as the following relation between these four asymmetries and the symmetry violating parameters,

$$
\begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} = K \begin{pmatrix} \Re\Delta_M \\ \Im\Delta_M \\ \Re\epsilon_M \\ \Im\epsilon_M \end{pmatrix}. \tag{43}
$$

Hence the CP and CPT symmetry violating parameters can be determined as

$$
\begin{pmatrix} \Re\Delta_M \\ \Im\Delta_M \\ \Re\epsilon_M \\ \Im\epsilon_M \end{pmatrix} = K^{-1} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix}, \tag{44}
$$

where K^{-1} is the inverse matrix of K. This provides a general relation between symmetry violating parameters and four arbitrarily chosen decay asymmetries.

A good choice is to use the equal-state and unequal-state asymmetries defined for semileptonic-semileptonic processes and hadronic-hadronic processes. That is, we make the substitutions

$$
\begin{aligned}
A_1 &= A(l^+l^+, l^-l^-; \Delta t) \quad A_2 = A(l^+l^-, l^-l^+; \Delta t) \\
A_3 &= A(h_1h_1, h_2h_2; \Delta t) \quad A_4 = A(h_1h_2, h_2h_1; \Delta t).
\end{aligned}
\tag{45}
$$

Then

$$
\begin{aligned}
a_1 &\equiv \frac{1}{4}[(1 - A_1)f_0^{R^+R^+} - (1 + A_1)f_0^{S^-S^-}], \\
a_2 &\equiv -\frac{1}{2}A_2 f_0^{R^+S^-}, \\
a_3 &\equiv \frac{1}{4}[(1 - A_3)f_0^{Q_1Q_1} - (1 + A_3)f_0^{Q_2Q_2}], \\
a_4 &\equiv -\frac{1}{2}A_4 f_0^{Q_1Q_2},
\end{aligned}
\tag{46}
$$

while the matrix elements of K are given by

$$K_{1i} \equiv (A_1-1)f_i^{R^+R^+} + (A_1+1)f_i^{S^-S^-},$$
$$K_{2i} \equiv (A_2-1)f_i^{R^+S^-} + (A_2+1)f_i^{S^-R^+},$$
$$K_{3i} \equiv (A_3-1)f_i^{Q_1Q_1} + (A_3+1)f_i^{Q_2Q_2},$$
$$K_{4i} \equiv (A_4-1)f_i^{Q_1Q_2} + (A_4+1)f_i^{Q_2Q_1}, \quad (47)$$

with $i = 1, 2, 3, 4$ and $f_i^{r^ar^b} \equiv f_i^{r^ar^b}(\Delta t)$.

One can also use some of the asymmetries defined for the semileptonic-hadronic decay processes. For example, a convenient choice is to use the four unequal-state asymmetries in the semileptonic-hadronic decay processes. Hence one makes the substitutions

$$A_1 = A(l^+h_1, h_1l^+, \Delta t), \quad A_2 = A(l^+h_2, h_2l^+, \Delta t),$$
$$A_3 = A(l^-h_1, h_1l^-, \Delta t), \quad A_4 = A(l^-h_2, h_2l^-, \Delta t). \quad (48)$$

Then

$$a_1 \equiv -\frac{1}{2}A_5f_0^{R^+Q_1}, \qquad a_2 \equiv -\frac{1}{2}A_6f_0^{R^+\bar{Q}_2},$$
$$a_3 \equiv -\frac{1}{2}A_7f_0^{S^-Q_1}, \qquad a_4 \equiv -\frac{1}{2}A_8f_0^{S^-\bar{Q}_2}, \quad (49)$$

while the matrix elements of K are given by

$$K_{1i} = (A_1-1)f_i^{R^+Q_1} + (A_1+1)f_i^{Q_1R^+},$$
$$K_{2i} = (A_2-1)f_i^{R^+Q_2} + (A_2+1)f_i^{Q_2R^+},$$
$$K_{3i} = (A_3-1)f_i^{S^-Q_1} + (A_3+1)f_i^{Q_1S^-},$$
$$K_{4i} = (A_4-1)f_i^{S^-\bar{Q}_2} + (A_4+1)f_i^{\bar{Q}_2S^-}, \quad (50)$$

with $i = 1, 2, 3, 4$.

VI. SOME THEOREMS CONCERNING DECAY ASYMMETRIES AND CP AND CPT VIOLATIONS

First consider the following situation of equal-time joint decays. If we exchange ψ^a and ψ^b, then r^a and r^b are exchanged, thus Θ and Λ remain unchanged while Ξ and Φ are exchanged; consequently, (30) indicates that $I'(\psi^a, \psi^b, \Delta t = 0) = I'(\psi^b, \psi^a, \Delta t = 0)$, and thus $A(\psi^a\psi^b, \psi^b\psi^a; \Delta t = 0) = 0$.

Theorem 1: Consider joint decays of $|\Psi_+\rangle$. For $\Delta t = 0$, any unequal-state asymmetry $A(\psi^a\psi^b, \psi^b\psi^a; \Delta t = 0)$ always vanishes regardless of whether there is CP or CPT violation.

The same conclusion is also valid for $|\Psi_-\rangle$, and has been shown previously for the special cases of joint decays to flavor eigenstates and joint decays to CP eigenstates [32]. In the following we show that it is valid for any equal-time unequal-state asymmetry $A(\psi^a\psi^b, \psi^b\psi^a; \Delta t = 0)$. From (22), we obtain for $C = -1$ state,

$$\begin{aligned} I(\psi^a, t_a; \psi^b, t_b) &= |\langle\psi^a\psi^b|\mathcal{H}|\psi(t_a, t_b)\rangle|^2 \\ &= \frac{1}{8}[e^{-(\Gamma_St_a+\Gamma_Lt_b)}|\theta|^2 - 2e^{-\Gamma(t_a+t_b)}\Re(\theta^*\lambda e^{i\Delta m\Delta t}) \\ &\quad + e^{-(\Gamma_Lt_a+\Gamma_St_b)}|\lambda|^2], \end{aligned} \quad (51)$$

where

$$\theta \equiv \eta_2r^ar^b - (1-\xi)r^a\bar{r}^b + (1+\xi)\bar{r}^ar^b - \eta_1\bar{r}^a\bar{r}^b,$$
$$\lambda \equiv \eta_2r^ar^b + (1+\xi)r^a\bar{r}^b - (1-\xi)\bar{r}^ar^b - \eta_1\bar{r}^a\bar{r}^b. \quad (52)$$

Obviously when ψ^a and ψ^b are exchanged, so are θ and λ. Consequently, when $t_a = t_b = t$, $I(\psi^a, t; \psi^b, t) = I(\psi^b, t; \psi^a, t)$, which implies that any equal-time unequal-state asymmetry $A(\psi^a\psi^b, \psi^b\psi^a; \Delta t = 0)$ is zero.

Theorem 2: Consider joint decays of $|\Psi_-\rangle$. For $\Delta t = 0$, any unequal-state asymmetry $A(\psi^a\psi^b, \psi^b\psi^a; \Delta t = 0)$ always vanishes regardless of whether there is CP or CPT violation.

A. Semileptonic-semileptonic processes

(i) If CP is conserved indirectly, then $\epsilon_M = \Delta_M = 0$, thus $\Theta = (r^a + \bar{r}^a)(r^b + \bar{r}^b)$, $\Lambda = (r^a - \bar{r}^a)(r^b - \bar{r}^b)$, $\Xi = \Phi = 0$. Consequently, without making any approximation, we obtain exactly

$$A(l^+l^+, l^-l^-; \Delta t) = \frac{f_0^{R^+R^+}(\Delta t) - f_0^{S^-S^-}(\Delta t)}{f_0^{R^+R^+}(\Delta t) + f_0^{S^-S^-}(\Delta t)},$$
$$A(l^+l^-, l^-l^+; \Delta t) = 0. \quad (53)$$

Any deviation from these two equalities means indirect CP violation. In particular, a nonvanishing value of unequal-flavor asymmetry $A(l^+l^-, l^-l^+; \Delta t)$ is a signature of indirect CP violation.

Theorem 3: Consider joint decays of $|\Psi_+\rangle$. If the unequal-flavor asymmetry $A(l^+l^-, l^-l^+; \Delta t)$ is nonzero, then CP must be violated indirectly.

(ii) If CP is conserved directly, then $R^+ = \bar{S}^-$ and $S^- = \bar{R}^+$; consequently,

$$f_0^{R^+R^+} = f_0^{S^-S^-}, \qquad f_i^{R^+R^+} = -f_i^{S^-S^-},$$
$$f_j^{R^+S^-} = f_j^{S^-R^+} = 0, \qquad f_0^{R^+S^-} = f_0^{S^-R^+}, \quad (54)$$

where $i = 1, 2, 3, 4$ and $j = 3, 4$. Then

$$A(l^+l^+, l^-l^-; \Delta t) \approx \frac{4\sum_{i=1}^4 f_i^{R^+R^+}\sigma_i}{f_0^{R^+R^+}},$$
$$A(l^+l^-, l^-l^+; \Delta t) \approx \frac{\sum_{i=1}^2(f_i^{R^+S^-} - f_i^{S^-R^+})\sigma_i}{\frac{1}{2}f_0^{R^+S^-} + \sum_{i=1}^2(f_i^{R^+S^-} + f_i^{S^-R^+})\sigma_i}, \quad (55)$$

which says that $A(l^+l^-, l^-l^+; \Delta t)$ does not depend on ϵ_M up to its first order. Furthermore, if CPT is also assumed to be conserved indirectly, i.e., $\Delta_M = 0$, then $A(l^+l^-, l^-l^+; \Delta t) \sim O(\epsilon_M^2)$.

Theorem 4: Consider joint decays of $|\Psi_+\rangle$. If the unequal-flavor asymmetry $A(l^+l^-, l^-l^+; \Delta t)$ depends on the first order of ϵ_M, then CP must be violated directly.

(iii) If the semileptonic decays respect the $\Delta\mathcal{F} = \Delta Q$ rule, where $\mathcal{F}$ is the flavor quantum number and Q is the charge number, then $\bar{R}^+ = S^- = 0$. Consequently,

$$f_0^{R^+R^+} = \left|\frac{R^+}{\bar{S}^-}\right|^4 f_0^{S^-S^-}, \qquad f_i^{R^+R^+} = -\left|\frac{R^+}{\bar{S}^-}\right|^4 f_i^{S^-S^-},$$
$$f_0^{R^+S^-} = f_0^{S^-R^+}, \qquad f_j^{R^+S^-} = f_j^{S^-R^+} = 0, \tag{56}$$

with $i = 1, 2, 3, 4$, $j = 3, 4$. Then

$$A(l^+l^+, l^-l^-; \Delta t) \approx \frac{\frac{1}{4}(1 - |\frac{R^+}{S^-}|^4) f_0^{S^-S^-} + \sum_{i=1}^4 (1 + |\frac{R^+}{S^-}|^4) f_i^{S^-S^-}\sigma_i}{\frac{1}{4}(1 + |\frac{R^+}{S^-}|^4) f_0^{S^-S^-} + \sum_{i=1}^4 (1 - |\frac{R^+}{S^-}|^4) f_i^{S^-S^-}\sigma_i},$$
$$A(l^+l^-, l^-l^+; \Delta t) \approx \frac{\sum_{i=1}^2 (f_i^{R^+S^-} - f_i^{S^-R^+})\sigma_i}{\frac{1}{2} f_0^{R^+S^-} + \sum_{i=1}^2 (f_i^{R^+S^-} + f_i^{S^-R^+})\sigma_i}; \tag{57}$$

that is, $A(l^+l^-, l^-l^+; \Delta t)$ does not depend on ϵ_M up to its first order in this situation.

Theorem 5: Consider joint decays of $|\Psi_+\rangle$. If the unequal-flavor asymmetry $A(l^+l^-, l^-l^+; \Delta t)$ depends on the first order of ϵ_M, then the $\Delta\mathcal{F} = \Delta Q$ rule must be violated.

B. Hadronic-hadronic processes

(i) For the hadronic-hadronic processes, first we consider the situation of $|h_1\rangle = CP|h_2\rangle$; that is, $|h_1\rangle$ and $|h_2\rangle$ are mutual CP conjugates. For example, $B^0\bar{B}^0 \to D^+K^-D^-K^+$, $\pi^+D_S^-\pi^-D_S^+$. If CP is conserved directly, then $Q_1 = \bar{Q}_2$ and $Q_2 = \bar{Q}_1$, and one can obtain

$$f_0^{Q_1Q_1} = f_0^{Q_2Q_2}, \qquad f_i^{Q_1Q_1} = -f_i^{Q_2Q_2}, \qquad f_0^{Q_1Q_2} = f_0^{Q_2Q_1},$$
$$f_j^{Q_1Q_2} = f_j^{Q_2Q_1} = 0, \tag{58}$$

where $i = 1, 2, 3, 4$, $j = 3, 4$. It is obtained that

$$A(h_1h_1, h_2h_2; \Delta t) \approx \frac{4\sum_{i=1}^4 f_i^{Q_1Q_1}\sigma_i}{f_0^{Q_1Q_1}},$$
$$A(h_1h_2, h_2h_1; \Delta t) \approx \frac{\sum_{i=1}^2 (f_i^{Q_1Q_2} - f_i^{Q_2Q_1})\sigma_i}{\frac{1}{2} f_0^{Q_1Q_2} + \sum_{i=1}^2 (f_i^{Q_1Q_2} + f_i^{Q_2Q_1})\sigma_i}. \tag{59}$$

So $A(h_1h_2, h_2h_1; \Delta t)$ does not depend on ϵ_M up to its first order. Furthermore, if CPT is also assumed to be conserved indirectly, then $\Delta_M = 0$, and consequently $A(h_1h_2, h_2h_1; \Delta t) \sim O(\epsilon_M^2)$. One can see the similarity between $A(h_1h_1, h_2h_2; \Delta t)$ and $A(l^+l^-, l^-l^+; \Delta t)$, as well as that between $A(h_1h_2, h_2h_1; \Delta t)$ and $A(l^+l^-, l^-l^+; \Delta t)$.

Theorem 6: Consider joint decays of $|\Psi_+\rangle$. Suppose $|h_1\rangle$ and $|h_2\rangle$ are CP conjugates. If $A(h_1h_1, h_2h_2; \Delta t)$ depends on the first order of ϵ_M, then CP must be violated directly.

(ii) Consider the situation of $CP|h_1\rangle = |h_1\rangle$ and $CP|h_2\rangle = -|h_2\rangle$; that is, $|h_1\rangle$ and $|h_2\rangle$ are CP eigenstates with eigenvalues 1 and -1, respectively. From the expression (17) of $|\Psi_+\rangle$ in terms of CP eigenstates, it is immediately seen that with $\Delta t = 0$, if CP is conserved both directly and indirectly, then the decay products of the two particles should always be CP eigenstates with an equal eigenvalue; hence, $I(h_1, t_a; h_2, t_b) = I(h_2, t_a; h_1, t_b) = 0$.

Theorem 7: Consider joint decays of $|\Psi_+\rangle$. Suppose $|h_1\rangle$ and $|h_2\rangle$ are CP eigenstates with eigenvalues 1 and -1, respectively. The deviation of $I(h_1, t_a; h_2, t_a)$ or $I(h_2, t_a; h_1, t_a)$ from zero implies CP violation, direct or indirect or both.

In more quantitative details, let us first assume that CP is conserved directly, then the decay amplitudes satisfy $Q_1 = \bar{Q}_1$ and $Q_2 = -\bar{Q}_2$; therefore, for $|\psi^a\rangle = |h_1\rangle$ while $|\psi^b\rangle = |h_2\rangle$, $I(h_1, t_a; h_2, t_b)$ is given by (25) with $\Theta = [\eta_2 - \eta_1 - (\eta_2 + \eta_1)\xi]Q_1Q_2$, $\Xi = (\eta_1 + \eta_2 + 2)\xi Q_1Q_2$, $\Phi = (\eta_1 + \eta_2 - 2)\xi Q_1Q_2$, $\Lambda = [\eta_1 - \eta_2 - (\eta_2 + \eta_1)\xi]Q_1Q_2$. Similarly, if $|\psi^a\rangle = |h_2\rangle$ while $|\psi^b\rangle = |h_1\rangle$, then $I(h_2, t_a; h_1, t_b)$ is given by (25) with $\Theta = [\eta_2 - \eta_1 - (\eta_2 + \eta_1)\xi]Q_1Q_2$, $\Xi = (\eta_1 + \eta_2 - 2)\xi Q_1Q_2$, $\Phi = (\eta_1 + \eta_2 + 2)\xi Q_1Q_2$, $\Lambda = [\eta_1 - \eta_2 - (\eta_2 + \eta_1)\xi]Q_1Q_2$. Consequently, $I(h_1, t_a; h_2, t_b)$ and $I(h_2, t_a; h_1, t_b)$ are of the order of $O(\Delta_M^2)$ and $O(\epsilon_M^2)$. Moreover, if CP is also indirectly conserved, then $\xi = 0$, thus $\Theta = \Lambda$ while $\Xi = \Phi$; consequently, $I(h_1, t_a; h_2, t_a) = I(h_2, t_a; h_1, t_b) = 0$, confirming Theorem 7.

Theorem 8: Consider joint decays of $|\Psi_+\rangle$. Suppose $|h_1\rangle$ and $|h_2\rangle$ are CP eigenstates with eigenvalues 1 and -1, respectively. If $I(h_1, t_a; h_2, t_b)$ and $I(h_2, t_a; h_1, t_b)$ are the order of $O(\Delta_M)$ and $O(\epsilon_M)$, then CP is violated directly.

On the other hand, if we first assume that CP is conserved indirectly, then $\xi = 0$ and $\eta_1 = \eta_2 = 1$, thus $|M_+(t)\rangle = e^{-i\lambda_S t}|M_+\rangle$, $|M_-(t)\rangle = e^{-i\lambda_L t}|M_-\rangle$. Consequently,

$$|\psi(t_a, t_b)\rangle = \frac{1}{\sqrt{2}}[e^{-i\lambda_S(t_a+t_b)}|M_+\rangle|M_+\rangle - e^{-i\lambda_L(t_a+t_b)}|M_-\rangle|M_-\rangle], \tag{60}$$

which implies $I(h_1, t_a; h_2, t_b) = I(h_2, t_a; h_1, t_b) = |e^{-i\Lambda_S(t_a+t_b)}(Q_1 + \bar{Q}_1)(Q_2 + \bar{Q}_2) - e^{-i\Lambda_L(t_a+t_b)}(Q_1 - \bar{Q}_1) \times (Q_2 - \bar{Q}_2)|^2$. Moreover, if CP is also conserved directly, then $Q_1 = \bar{Q}_1$, $Q_2 = -\bar{Q}_2$; consequently. $I(h_1, t_a; h_2, t_b) = I(h_2, t_a; h_1, t_b) = 0$, again confirming Theorem 7.

C. Semileptonic-hadronic processes

For semileptonic-hadronic processes, here we consider some of the asymmetries, for which the assumption of CP conservation can lead to relatively simple results.

(i) Consider the case that the hadronic decay products satisfy $CP|h_1\rangle = |h_2\rangle$. If CP is conserved directly, then $R^+ = \bar{S}^-$, $\bar{R}^+ = S^-$, $Q_1 = \bar{Q}_2$, and $Q_2 = \bar{Q}_1$. Consequently,

$$f_0^{R^+Q_1} = f_0^{S^-Q_2}, \qquad f_i^{R^+Q_1} = -f_i^{S^-Q_2}, \qquad f_0^{R^+Q_2} = f_0^{S^-Q_1}, \qquad f_i^{R^+Q_2} = -f_i^{S^-Q_1}, \tag{61}$$

with $i = 1, 2, 3, 4$. So to the order of $O(\Delta_M)$ and $O(\epsilon_M)$, we have

$$\begin{aligned} A(l^+h_1, l^-h_2; \Delta t) &\approx 4\left(\frac{f_1^{R^+Q_1}}{f_0^{R^+Q_1}}\Re\epsilon_M + \frac{f_2^{R^+Q_1}}{f_0^{R^+Q_1}}\Im\epsilon_M + \frac{f_3^{R^+Q_1}}{f_0^{R^+Q_1}}\Re\Delta_M + \frac{f_4^{R^+Q_1}}{f_0^{R^+Q_1}}\Im\Delta_M\right), \\ A(l^+h_2, l^-h_1; \Delta t) &\approx 4\left(\frac{f_1^{R^+Q_2}}{f_0^{R^+Q_2}}\Re\epsilon_M + \frac{f_2^{R^+Q_2}}{f_0^{R^+Q_2}}\Im\epsilon_M + \frac{f_3^{R^+Q_2}}{f_0^{R^+Q_2}}\Re\Delta_M + \frac{f_4^{R^+Q_2}}{f_0^{R^+Q_2}}\Im\Delta_M\right). \end{aligned} \tag{62}$$

(ii) Consider the case that the hadronic decay products are CP eigenstates, which satisfy $CP|h_1\rangle = |h_1\rangle$ and $CP|h_2\rangle = -|h_2\rangle$. If CP is conserved directly, then $R^+ = \bar{S}^-$, $\bar{R}^+ = S^-$, $Q_1 = \bar{Q}_1$ and $Q_2 = -\bar{Q}_2$. Consequently,

$$f_0^{R^+Q_k} = f_0^{S^-Q_k} = f_0^{Q_kS^-}, \qquad f_i^{R^+Q_k} = -f_i^{S^-Q_k} = -f_i^{Q_kS^-}, \tag{63}$$

with $i = 1, 2, 3, 4$ and $k = 1, 2$. So up to the order of $O(\Delta_M)$ and $O(\epsilon_M)$, we have

$$\begin{aligned} A(l^+h_k, l^-h_k; \Delta t) &\approx A(l^+h_k, h_kl^-; \Delta t) \\ &\approx 4\left(\frac{f_1^{R^+Q_k}}{f_0^{R^+Q_k}}\Re\epsilon_M + \frac{f_2^{R^+Q_k}}{f_0^{R^+Q_k}}\Im\epsilon_M + \frac{f_3^{R^+Q_k}}{f_0^{R^+Q_k}}\Re\Delta_M + \frac{f_4^{R^+Q_k}}{f_0^{R^+Q_k}}\Im\Delta_M\right), \end{aligned} \tag{64}$$

with $k = 1, 2$.

Theorem 9: Consider the semileptonic-hadronic decay asymmetries $A(l^+h_k, h_kl^-; \Delta t)$ and $A(l^+h_k, l^-h_k; \Delta t)$ of $|\Psi_+\rangle$, $(k = 1, 2)$. Suppose $|h_1\rangle$ and $|h_2\rangle$ are CP eigenstates with eigenvalues 1 and -1, respectively. If $A(l^+h_k, h_kl^-; \Delta t) \neq A(l^+h_k, l^-h_k; \Delta t)$ even in the first order of CP and CPT violating parameters, then CP must be violated directly.

VII. SUMMARY

To summarize, we have studied the $C = +1$ entangled state $|\Psi_+\rangle$ of pseudoscalar neutral meson pairs. We have calculated various integrated joint decay rates of semileptonic-semileptonic processes, hadronic-hadronic processes, and semileptonic-hadronic processes, as well as experimentally observable asymmetries defined for them, including equal-state asymmetries, unequal-state asymmetries, and more general ones, which are functions of the CP and CPT violating parameters. Any four of these asymmetries can be used to determine the real and imaginary parts of the indirect symmetry violating parameters ϵ_M and Δ_M. For example, one can choose the equal-state and unequal-state asymmetries in semileptonic-semileptonic and hadronic-hadronic decays. Alternatively, one can choose four asymmetries in semileptonic-hadronic decays. The coefficients in these equations depend on whether CP is violated directly or whether the $\Delta\mathcal{F} = \Delta Q$ rule is violated indirectly or directly. Through these relations we can examine whether various symmetries or rules are violated, and determine the symmetry violating parameters. Also note that these relations are for a given Δt; hence, by using various different values, one can obtain the quantities in many times, and make averages.

We also make some simple statements concerning the joint decays of $|\Psi_+\rangle$ presented as theorems, as the following. If the unequal-flavor asymmetry $A(l^+l^-, l^-l^+; \Delta t)$ is nonzero, then CP must be violated indirectly. If $A(l^+l^-, l^-l^+; \Delta t)$ depends on the first order of ϵ_M, then CP must be violated directly, and the $\Delta\mathcal{F} = \Delta Q$ rule is violated. If $A(h_1h_1, h_2h_2; \Delta t)$ for CP conjugates $|h_1\rangle$ and $|h_2\rangle$ depends on the first order of ϵ_M, then CP is violated directly. For CP eigenstates $|h_1\rangle$ and $|h_2\rangle$ with eigenvalues 1 and -1, respectively, the deviation of $I(h_1, t_a; h_2, t_b)$ or $I(h_2, t_a; h_1, t_b)$ from zero implies CP violation. For CP eigenstates $|h_1\rangle$ and $|h_2\rangle$ with eigenvalues 1 and -1, respectively, consider the semileptonic-hadronic decay asymmetries $A(l^+h_k, h_kl^-; \Delta t)$ and $A(l^+h_k, l^-h_k; \Delta t)$, $(k = 1, 2)$. If $A(l^+h_k, h_kl^-; \Delta t) \neq A(l^+h_k, l^-h_k; \Delta t)$ even in the first order of CP and CPT violating parameters, then CP must be violated directly.

The uses of $|\Psi_+\rangle$ and $|\Psi_-\rangle$ well complement each other. Two outstanding examples are as follows. In the flavor basis, their relative phases between $|M^0\rangle|\bar{M}^0\rangle$ and $|\bar{M}^0\rangle|M^0\rangle$ are opposite. Consequently, CP must be violated indirectly if the unequal-flavor asymmetry in $|\Psi_+\rangle$ is nonzero, while the same conclusion can be drawn if the equal-flavor asymmetry in $|\Psi_-\rangle$ is nonzero [32]. On the other hand, in the CP basis, $|\Psi_+\rangle$ is a superposition of equal-CP terms $|M_+\rangle|M_+\rangle$ and $|M_-\rangle|M_-\rangle$, while $|\Psi_-\rangle$ is a superposition of unequal-CP terms $|M_-\rangle|M_+\rangle$ and $|M_+\rangle|M_-\rangle$. Consequently, CP must be violated if any unequal-CP joint decay rate of $|\Psi_+\rangle$ is nonzero, while the same conclusion can be drawn if any equal-CP joint decay rate of $|\Psi_-\rangle$ is nonzero [32]. Besides, $|\Psi_+\rangle$ and $|\Psi_-\rangle$ also share some common phenomena. For example, for both $|\Psi_+\rangle$ and $|\Psi_-\rangle$, any equal-time unequal-state asymmetry $A(\psi^a\psi^b, \psi^b\psi^a; \Delta t = 0)$ must always vanish regardless of whether CP and CPT are violated.

We hope the present study of $|\Psi_+\rangle$ motivates its use in studying CP and CPT violations. Among various reasons, note the availability of $|\Psi_+\rangle$ in an energy range just above the $\Upsilon(4\mathbf{S})$ resonance with 100% branch ratio [33].

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APPENDIX: DETAILED EXPRESSIONS OF $\tilde{f}_i^{r^a r^b \bar{r}^a \bar{r}^b}$ and $f_i^{r^a r^b \bar{r}^a \bar{r}^b}$

$$\begin{aligned}\tilde{f}_0^{r^a r^b}(t_a, t_b) = {} & |(r^a + \bar{r}^a)(r^b + \bar{r}^b)|^2 e^{-\Gamma_S(t_a+t_b)} + |(r^a - \bar{r}^a)(r^b - \bar{r}^b)|^2 e^{-\Gamma_L(t_a+t_b)} \\ & - 2e^{-\Gamma(t_a+t_b)} \Re[(r^a + \bar{r}^a)^*(r^b + \bar{r}^b)^*(r^a - \bar{r}^a)(r^b - \bar{r}^b)e^{-i\Delta m(t_a+t_b)}],\end{aligned}$$

$$\begin{aligned}f_0^{r^a r^b}(\Delta t) = e^{-\Gamma\Delta t}\Bigg\{ & \frac{|(r^a + \bar{r}^a)(r^b + \bar{r}^b)|^2}{2\Gamma_S} e^{\frac{\Delta\Gamma\Delta t}{2}} + \frac{|(r^a - \bar{r}^a)(r^b - \bar{r}^b)|^2}{2\Gamma_L} e^{-\frac{\Delta\Gamma\Delta t}{2}} \\ & - \Re\left[\frac{(r^a + \bar{r}^a)^*(r^b + \bar{r}^b)^*(r^a - \bar{r}^a)(r^b - \bar{r}^b)}{(\Gamma + i\Delta m)} e^{-i\Delta m\Delta t}\right]\Bigg\},\end{aligned}$$

$$\begin{aligned}\tilde{f}_1^{r^a r^b}(t_a, t_b) = {} & -e^{-\Gamma_S(t_a+t_b)} \Re[(r^a r^b - \bar{r}^a \bar{r}^b)^*(r^a + \bar{r}^a)(r^b + \bar{r}^b)] + e^{-\Gamma_L(t_a+t_b)} \Re[(r^a r^b - \bar{r}^a \bar{r}^b)^*(r^a - \bar{r}^a)(r^b - \bar{r}^b)] \\ & - e^{-\Gamma(t_a+t_b)} \Re[(r^a r^b - \bar{r}^a \bar{r}^b)^*(r^a + \bar{r}^a)(r^b + \bar{r}^b)e^{i\Delta m(t_a+t_b)}] \\ & + e^{-\Gamma(t_a+t_b)} \Re[(r^a r^b - \bar{r}^a \bar{r}^b)^*(r^a - \bar{r}^a)(r^b - \bar{r}^b)e^{-i\Delta m(t_a+t_b)}] + e^{-(\Gamma t_a + \Gamma_S t_b)}|r^b \\ & + \bar{r}^b|^2 \Re[(r^a + \bar{r}^a)^*(r^a - \bar{r}^a)e^{-i\Delta m t_a}] + e^{-(\Gamma_S t_a + \Gamma t_b)}|r^a + \bar{r}^a|^2 \Re[(r^b + \bar{r}^b)^*(r^b - \bar{r}^b)e^{-i\Delta m t_b}] - e^{-(\Gamma t_a + \Gamma_L t_b)}|r^b \\ & - \bar{r}^b|^2 \Re[(r^a - \bar{r}^a)^*(r^a + \bar{r}^a)e^{i\Delta m t_a}] - e^{-(\Gamma_L t_a + \Gamma t_b)}|r^a - \bar{r}^a|^2 \Re[(r^b - \bar{r}^b)^*(r^b + \bar{r}^b)e^{i\Delta m t_b}],\end{aligned}$$

$$\begin{aligned}f_1^{r^a r^b}(\Delta t) = e^{-\Gamma\Delta t}\Bigg\{ & -\frac{\Re[(r^a r^b - \bar{r}^a \bar{r}^b)^*(r^a + \bar{r}^a)(r^b + \bar{r}^b)]}{2\Gamma_S} e^{\frac{\Delta\Gamma\Delta t}{2}} + \frac{\Re[(r^a r^b - \bar{r}^a \bar{r}^b)^*(r^a - \bar{r}^a)(r^b - \bar{r}^b)]}{2\Gamma_L} e^{-\frac{\Delta\Gamma\Delta t}{2}} \\ & - \Re\left[\frac{(r^a r^b - \bar{r}^a \bar{r}^b)^*(r^a + \bar{r}^a)(r^b + \bar{r}^b)}{2(\Gamma - i\Delta m)} e^{i\Delta m\Delta t}\right] + \Re\left[\frac{(r^a r^b - \bar{r}^a \bar{r}^b)^*(r^a - \bar{r}^a)(r^b - \bar{r}^b)}{2(\Gamma + i\Delta m)} e^{-i\Delta m\Delta t}\right] \\ & + |r^b + \bar{r}^b|^2 \Re\left[\frac{(r^a + \bar{r}^a)^*(r^a - \bar{r}^a)}{\Gamma + \Gamma_S + i\Delta m}\right] e^{-\frac{\Delta\Gamma\Delta t}{2}} + |r^a + \bar{r}^a|^2 \Re\left[\frac{(r^b + \bar{r}^b)^*(r^b - \bar{r}^b)}{\Gamma + \Gamma_S + i\Delta m} e^{-i\Delta m\Delta t}\right] \\ & - |r^b - \bar{r}^b|^2 \Re\left[\frac{(r^a - \bar{r}^a)^*(r^a + \bar{r}^a)}{\Gamma + \Gamma_L - i\Delta m}\right] e^{\frac{\Delta\Gamma\Delta t}{2}} - |r^a - \bar{r}^a|^2 \Re\left[\frac{(r^b - \bar{r}^b)^*(r^b + \bar{r}^b)}{\Gamma + \Gamma_L - i\Delta m} e^{i\Delta m\Delta t}\right]\Bigg\},\end{aligned}$$

$$
\begin{aligned}
\tilde{f}_2^{r^a r^b}(t_a, t_b) = & -e^{-\Gamma_S(t_a+t_b)}\Im[(r^a r^b - \bar{r}^a\bar{r}^b)^*(r^a + \bar{r}^a)(r^b + \bar{r}^b)] + e^{-\Gamma_L(t_a+t_b)}\Im[(r^a r^b - \bar{r}^a\bar{r}^b)^*(r^a - \bar{r}^a)(r^b - \bar{r}^b)] \\
& - e^{-\Gamma(t_a+t_b)}\Im[(r^a r^b - \bar{r}^a\bar{r}^b)^*(r^a + \bar{r}^a)(r^b + \bar{r}^b)e^{i\Delta m(t_a+t_b)}] \\
& + e^{-\Gamma(t_a+t_b)}\Im[(r^a r^b - \bar{r}^a\bar{r}^b)^*(r^a - \bar{r}^a)(r^b - \bar{r}^b)e^{-i\Delta m(t_a+t_b)}] - e^{-(\Gamma t_a+\Gamma_S t_b)}|r^b \\
& + \bar{r}^b|^2\Im[(r^a + \bar{r}^a)^*(r^a - \bar{r}^a)e^{-i\Delta m t_a}] - e^{-(\Gamma_S t_a+\Gamma t_b)}|r^a + \bar{r}^a|^2\Im[(r^b + \bar{r}^b)^*(r^b - \bar{r}^b)e^{-i\Delta m t_b}] + e^{-(\Gamma t_a+\Gamma_L t_b)}|r^b \\
& - \bar{r}^b|^2\Im[(r^a - \bar{r}^a)^*(r^a + \bar{r}^a)e^{i\Delta m t_a}] + e^{-(\Gamma_L t_a+\Gamma t_b)}|r^a - \bar{r}^a|^2\Im[(r^b - \bar{r}^b)^*(r^b + \bar{r}^b)e^{i\Delta m t_b}],
\end{aligned}
$$

$$
\begin{aligned}
f_2^{r^a r^b}(\Delta t) = e^{-\Gamma\Delta t}\Bigg\{ & -\frac{\Im[(r^a r^b - \bar{r}^a\bar{r}^b)^*(r^a + \bar{r}^a)(r^b + \bar{r}^b)]}{2\Gamma_S}e^{\frac{\Delta\Gamma\Delta t}{2}} + \frac{\Im[(r^a r^b - \bar{r}^a\bar{r}^b)^*(r^a - \bar{r}^a)(r^b - \bar{r}^b)]}{2\Gamma_L}e^{-\frac{\Delta\Gamma\Delta t}{2}} \\
& - \Im\left[\frac{(r^a r^b - \bar{r}^a\bar{r}^b)^*(r^a + \bar{r}^a)(r^b + \bar{r}^b)}{2(\Gamma - i\Delta m)}e^{i\Delta m\Delta t}\right] + \Im\left[\frac{(r^a r^b - \bar{r}^a\bar{r}^b)^*(r^a - \bar{r}^a)(r^b - \bar{r}^b)}{2(\Gamma + i\Delta m)}e^{-i\Delta m\Delta t}\right] \\
& - |r^b + \bar{r}^b|^2\Im\left[\frac{(r^a + \bar{r}^a)^*(r^a - \bar{r}^a)}{\Gamma + \Gamma_S + i\Delta m}\right]e^{\frac{\Delta\Gamma\Delta t}{2}} - |r^a + \bar{r}^a|^2\Im\left[\frac{(r^b + \bar{r}^b)^*(r^b - \bar{r}^b)}{\Gamma + \Gamma_S + i\Delta m}e^{-i\Delta m\Delta t}\right] \\
& + |r^b - \bar{r}^b|^2\Im\left[\frac{(r^a - \bar{r}^a)^*(r^a + \bar{r}^a)}{\Gamma + \Gamma_L - i\Delta m}\right]e^{-\frac{\Delta\Gamma\Delta t}{2}} + |r^a - \bar{r}^a|^2\Im\left[\frac{(r^b - \bar{r}^b)^*(r^b + \bar{r}^b)}{\Gamma + \Gamma_L - i\Delta m}e^{i\Delta m\Delta t}\right]\Bigg\},
\end{aligned}
$$

$$
\begin{aligned}
\tilde{f}_3^{r^a r^b}(t_a, t_b) = & \, e^{-\Gamma_S(t_a+t_b)}\Re[(r^a r^b - \bar{r}^a\bar{r}^b)^*(r^a + \bar{r}^a)(r^b + \bar{r}^b)] + e^{-\Gamma_L(t_a+t_b)}\Re[(r^a r^b - \bar{r}^a\bar{r}^b)^*(r^a - \bar{r}^a)(r^b - \bar{r}^b)] \\
& - e^{-\Gamma(t_a+t_b)}\Re[(r^a r^b - \bar{r}^a\bar{r}^b)^*(r^a + \bar{r}^a)(r^b + \bar{r}^b)e^{i\Delta m(t_a+t_b)}] \\
& - e^{-\Gamma(t_a+t_b)}\Re[(r^a r^b - \bar{r}^a\bar{r}^b)^*(r^a - \bar{r}^a)(r^b - \bar{r}^b)e^{-i\Delta m(t_a+t_b)}],
\end{aligned}
$$

$$
\begin{aligned}
f_3^{r^a r^b}(\Delta t) = e^{-\Gamma\Delta t}\Bigg\{ & \frac{\Re[(r^a r^b - \bar{r}^a\bar{r}^b)^*(r^a + \bar{r}^a)(r^b + \bar{r}^b)]}{2\Gamma_S}e^{\frac{\Delta\Gamma\Delta t}{2}} + \frac{\Re[(r^a r^b - \bar{r}^a\bar{r}^b)^*(r^a - \bar{r}^a)(r^b - \bar{r}^b)]}{2\Gamma_L}e^{-\frac{\Delta\Gamma\Delta t}{2}} \\
& - \Re\left[\frac{(r^a r^b - \bar{r}^a\bar{r}^b)^*(r^a + \bar{r}^a)(r^b + \bar{r}^b)}{2(\Gamma - i\Delta m)}e^{i\Delta m\Delta t}\right] - \Re\left[\frac{(r^a r^b - \bar{r}^a\bar{r}^b)^*(r^a - \bar{r}^a)(r^b - \bar{r}^b)}{2(\Gamma + i\Delta m)}e^{-i\Delta m\Delta t}\right]\Bigg\},
\end{aligned}
$$

$$
\begin{aligned}
\tilde{f}_4^{r^a r^b}(t_a, t_b) = & \, e^{-\Gamma_S(t_a+t_b)}\Im[(r^a r^b - \bar{r}^a\bar{r}^b)^*(r^a + \bar{r}^a)(r^b + \bar{r}^b)] + e^{-\Gamma_L(t_a+t_b)}\Im[(r^a r^b - \bar{r}^a\bar{r}^b)^*(r^a - \bar{r}^a)(r^b - \bar{r}^b)] \\
& - e^{-\Gamma(t_a+t_b)}\Im[(r^a r^b - \bar{r}^a\bar{r}^b)^*(r^a + \bar{r}^a)(r^b + \bar{r}^b)e^{i\Delta m(t_a+t_b)}] \\
& - e^{-\Gamma(t_a+t_b)}\Im[(r^a r^b - \bar{r}^a\bar{r}^b)^*(r^a - \bar{r}^a)(r^b - \bar{r}^b)e^{-i\Delta m(t_a+t_b)}],
\end{aligned}
$$

$$
\begin{aligned}
f_4^{r^a r^b}(\Delta t) = e^{-\Gamma\Delta t}\Bigg\{ & \frac{\Im[(r^a r^b - \bar{r}^a\bar{r}^b)^*(r^a + \bar{r}^a)(r^b + \bar{r}^b)]}{2\Gamma_S}e^{\frac{\Delta\Gamma\Delta t}{2}} + \frac{\Im[(r^a r^b - \bar{r}^a\bar{r}^b)^*(r^a - \bar{r}^a)(r^b - \bar{r}^b)]}{2\Gamma_L}e^{-\frac{\Delta\Gamma\Delta t}{2}} \\
& - \Im\left[\frac{(r^a r^b - \bar{r}^a\bar{r}^b)^*(r^a + \bar{r}^a)(r^b + \bar{r}^b)}{2(\Gamma - i\Delta m)}e^{i\Delta m\Delta t}\right] - \Im\left[\frac{(r^a r^b - \bar{r}^a\bar{r}^b)^*(r^a - \bar{r}^a)(r^b - \bar{r}^b)}{2(\Gamma + i\Delta m)}e^{-i\Delta m\Delta t}\right]\Bigg\}.
\end{aligned}
$$

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