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Electroweak effective operators and Higgs physics

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We derive bounds from oblique parameters on the dimension-6 operators of an effective field theory of electroweak gauge bosons and the Higgs doublet. The loop-induced contributions to the ΔS, ΔT, and ΔU oblique parameters are sensitive to these contributions, and we pay particular attention to the role of renormalization when computing loop corrections in the effective theory. Limits on the coefficients of the effective theory from loop contributions to oblique parameters yield complementary information to direct Higgs production measurements.

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I. INTRODUCTION

With the discovery of a particle with the general appearance of a Higgs boson, the focus has turned to the detailed measurements of the properties of the Higgs boson candidate. This involves the measurement of as many Higgs production and decay channels as possible, along with the limits inferred from precision electroweak measurements [1]. Currently, all measurements are within $\sim 2-3\sigma$ of the predictions of the Standard Model (SM). A deviation from these predictions could be a signal of physics beyond the Standard Model.

A search for new physics in the Higgs sector can either involve examination of specific models or the use of effective field theories which respect the symmetries of the low energy physics. In this paper, we follow the later approach and assume that the Standard Model is a good approximation to physics at the weak scale and that all new physics is at a high scale. If the new physics is at a scale much higher than that probed experimentally, then an effective theory can be written in terms of an expansion in higher-dimension operators,

$$\mathcal{L} \sim \mathcal{L}_{\mathrm{SM}} + \Sigma_i \frac{f_i}{\Lambda^2}\mathcal{O}_i + \ldots . \tag{1}$$

The lowest-dimension operators, $\mathcal{O}_i$, which contribute to processes involving gauge bosons are dimension 6. The complete set of $SU(3)\times SU(2)_L \times U(1)_Y$ operators is quite large [2,3], but the assumption of flavor conservation, CP conservation, and SM physics in the lightest two fermion generations reduces the set of allowed operators considerably [4]. In this work, we further restrict the set of operators to those involving only the Higgs doublet and electroweak boson sectors [5–9]. The Lagrangian of Eq. (1) is valid at low energy ($\sim m_Z$) and reflects our ignorance of high scale physics.

At each order in the expansion in $1/\Lambda^n$, divergences appear in loop diagrams, which can be absorbed by renormalizing the coefficients of the operators appearing at one lower level in the loop expansion but at the same order in $1/\Lambda^n$, yielding a theory that is finite order by order in the expansion in $1/\Lambda$. The low energy effective field theory has the advantage that it can be matched to many possibilities for high scale physics. The Lagrangian of Eq. (1) can be used at energy scales much below Λ, where the observed physics approximates the SM up to small corrections. A specific model of new physics will predict the coefficients at the high scale, $f_i(\Lambda)$. The predictions must then be matched with experimental limits from low energy measurements, $f_i(\sim m_Z)$ and the renormalization group used to run the coefficients from the scale of the high energy predictions to that of the lower energy measurements [10–14].

New physics in the electroweak sector can be restricted by measurements of the oblique parameters, ΔS, ΔT, and ΔU, by restrictions on the deviations of the three gauge boson vertices and Higgs branching ratios from the Standard Model predictions, along with many SM processes. These effects and the resulting limits on the coefficients, f_i, of the dimension-6 operators have been studied by many authors [9,15–23]. The new feature of our work is a careful study of the dependence of the predictions on the renormalization scheme required by the effective theory formalism [5,24,25]. A complete analysis would include fermion operators and a careful choice of basis. Such a study is not necessary to illustrate our main point, however, which concerns the numerical effects of the renormalization scheme in the effective framework.

In Sec. II, we review the low energy effective Higgs electroweak theory and make the connection between our notation and the commonly used strongly interacting light Higgs (SILH) basis [26]. Section III contains analytic results for the oblique parameters, along with a discussion of the pinch technique needed to obtain gauge invariant results. Detailed appendices contain analytic results for the contributions to ΔS, ΔT and ΔU, as well as the required pinch contributions, in an R_ξ gauge. Numerical fits to the coefficients of the effective operators resulting from the oblique parameters and a discussion of the dependence on the renormalization scheme used to render the effective field theory finite are given in Sec. IV. As pointed out in Refs. [24,25], our limits are considerably weaker than those obtained using only the contributions to the oblique parameters proportional to $\log(\Lambda)$ given in Ref. [8]. We also compare the restrictions on the f_i couplings from the oblique parameters with those obtained from the the deviations of the experimental results from Standard Model predictions of Higgs branching ratios. Finally, Sec. V contains some conclusions.

II. EFFECTIVE THEORY

The effective Lagrangian we consider contains the $SU(2)_L \times U(1)_Y$ gauge fields and the Higgs $SU(2)_L$ doublet Φ. We assume that fermion interactions with the gauge bosons are those given by the Standard Model and that all new physics respects the $SU(2)_L \times U(1)_Y$ gauge invariance and conserves C and P. Possible new physics effects in the fermion sector are not considered here.

There are four operators which affect the gauge boson 2-point functions at tree level [5,27],

$$\begin{aligned}
\mathcal{O}_{DW} &= -\frac{g^2}{4}\,\mathrm{Tr}([D_\mu, \sigma^a \cdot W^a_{\nu\rho}][D^\mu, \sigma^b \cdot W^{b,\nu\rho}]) \\
\mathcal{O}_{DB} &= -\frac{g'^2}{2}(\partial_\mu B_{\nu\rho})(\partial^\mu B^{\nu\rho}) \\
\mathcal{O}_{BW} &= -\frac{gg'}{4}\Phi^\dagger B_{\mu\nu}\sigma^a \cdot W^{a,\mu\nu}\Phi \\
\mathcal{O}_{\Phi,1} &= (D_\mu\Phi)^\dagger(\Phi\Phi^\dagger)(D^\mu\Phi),
\end{aligned} \tag{2}$$

where[1]

$$\begin{aligned}
D_\mu &= \partial_\mu - i\frac{g}{2}B_\mu - i\frac{g'\sigma^a}{2}W^a_\mu \qquad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \\
W^\pm_{\mu\nu} &= \partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu \mp ig(W^3_\mu W^\pm_\nu - W^3_\nu W^\pm_\mu) \\
W^3_{\mu\nu} &= \partial_\mu W^3_\nu - \partial_\nu W^3_\mu - ig(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu).
\end{aligned} \tag{3}$$

$\mathcal{O}_{BW}$ gives $B - W^3$ mixing at tree level and contributes to ΔS, while $\mathcal{O}_{\Phi,1}$ affects m_Z, but not m_W, at tree level and thus contributes to ΔT.

There are six bosonic operators which contribute to the oblique parameters at one loop,

$$\begin{aligned}
\mathcal{O}_{WWW} &= -i\frac{g^3}{8}\,\mathrm{Tr}(\sigma^a \cdot W^{a,\mu}_\nu \sigma^b \cdot W^{b,\nu}_\rho \sigma^c \cdot W^{c,\rho}_\mu) \\
\mathcal{O}_W &= i\frac{g}{2}(D_\mu\Phi)^\dagger\sigma^a \cdot W^{a,\mu\nu}(D_\nu\Phi) \\
\mathcal{O}_B &= i\frac{g'}{2}(D_\mu\Phi)^\dagger B^{\mu\nu}(D_\nu\Phi) \\
\mathcal{O}_{WW} &= -\frac{g^2}{4}\Phi^\dagger\sigma^a \cdot W^{a,\mu\nu}\sigma^b \cdot W^b_{\mu\nu}\Phi \\
\mathcal{O}_{BB} &= -\frac{g'^2}{4}\Phi^\dagger B^{\mu\nu}B_{\mu\nu}\Phi \\
\mathcal{O}_{\Phi,2} &= \frac{1}{2}\partial^\mu(\Phi^\dagger\Phi)\partial_\mu(\Phi^\dagger\Phi).
\end{aligned} \tag{4}$$

The effects of $\mathcal{O}_{BB}$ and $\mathcal{O}_{WW}$ on three-gauge boson vertices can be eliminated by wave function renormalization and coupling redefinition, leaving only contributions to effective VVH vertices.

In addition, there are two operators that are neglected in our analysis,

$$\mathcal{O}_{\Phi,3} = \frac{1}{3}(\Phi^\dagger\Phi)^3 \qquad \mathcal{O}_{\Phi,4} = (D_\mu\Phi)^\dagger(D^\mu\Phi)(\Phi^\dagger\Phi). \tag{5}$$

$\mathcal{O}_{\Phi,3}$ only affects the Higgs self-interactions and is irrelevant for the oblique parameters. By using the equations of motion, $\mathcal{O}_{\Phi,4}$ can be written as a linear combination of $\mathcal{O}_{\Phi,2}$ and dimension-6 Yukawa operators. The latter have no effect on the self-energies of gauge bosons, and thus $\mathcal{O}_{\Phi,4}$ can also be excluded from our operator basis.

Finally, the Lagrangian for Higgs physics we consider is

$$\begin{aligned}
\mathcal{L} = \mathcal{L}_{\mathrm{SM}} &+ \frac{f_{DW}}{\Lambda^2}\mathcal{O}_{DW} + \frac{f_{DB}}{\Lambda^2}\mathcal{O}_{DB} + \frac{f_{BW}}{\Lambda^2}\mathcal{O}_{BW} + \frac{f_{\Phi,1}}{\Lambda^2}\mathcal{O}_{\Phi,1} \\
&+ \frac{f_{\Phi,2}}{\Lambda^2}\mathcal{O}_{\Phi,2} + \frac{f_{WWW}}{\Lambda^2}\mathcal{O}_{WWW} + \frac{f_W}{\Lambda^2}\mathcal{O}_W + \frac{f_B}{\Lambda^2}\mathcal{O}_B \\
&+ \frac{f_{WW}}{\Lambda^2}\mathcal{O}_{WW} + \frac{f_{BB}}{\Lambda^2}\mathcal{O}_{BB}.
\end{aligned} \tag{6}$$

The relationship between the coefficients of Eq. (6) and those of the SILH Lagrangian are given in Appendix A for convenience.

III. RESULTS

We are interested in the contributions to the oblique parameters from the operators of Eq. (6). The computation requires both the gauge boson 2-point functions and the pinch contributions in order to get gauge invariant results 28]]. The 2-point functions are defined as

$$\Pi^{\mu\nu}_{XY}(q^2) = g^{\mu\nu}\Pi_{XY}(q^2) - p^\mu p^\nu B_{XY}(q^2), \tag{7}$$

for $XY = WW$, ZZ, $\gamma\gamma$ and $Z\gamma$. The contributions (including tadpole diagrams) to $\Pi_{XY}(q^2)$ are given in Appendix B

[1]The convention for D_μ differs from that of Refs. [5,8,9], leading to some minus signs in the literature, relative to ours, which are described in Appendix A.

in the R_ξ gauge.[2] The Π_{XY} functions are gauge dependent and ultraviolet divergent.

The pinch contributions are defined in terms of the corrections to the $\bar{f}\gamma^\mu P_L f' V^\mu$ vertices, where $P_L = \frac{1}{2}(1-\gamma_5)$. Only left-handed contributions arise because there is always a coupling to a W boson. The vertex corrections are of the form

$$\Delta\Gamma_\mu^{Vff'}(q^2) = \gamma_\mu P_L \Delta\Gamma_L^{Vff'}(q^2). \tag{8}$$

We normalize

$$\Delta\Gamma_L^{Vff}(q^2) = gT_3^f \Delta\Gamma_L^V(q^2) \quad V = Z,\gamma$$
$$\Delta\Gamma_L^{Wff'}(p^2) = \frac{g}{\sqrt{2}}\Delta\Gamma_L^W(q^2). \tag{9}$$

Expressions for the pinch contributions, $\Delta\Gamma_L^V(q^2)$, in the R_ξ gauge are given in Appendix C.

Gauge invariant 2-point functions can be constructed by forming the combinations[3]

$$\bar{\Pi}_{WW}(q^2) = \Pi_{WW}(q^2) + 2(q^2 - m_W^2)\Delta\Gamma_L^W(q^2)$$
$$\bar{\Pi}_{ZZ}(q^2) = \Pi_{ZZ}(q^2) + 2c(q^2 - m_Z^2)\Delta\Gamma_L^Z(q^2)$$
$$\bar{\Pi}_{\gamma Z}(q^2) = \Pi_{\gamma Z}(q^2) + sq^2\Delta\Gamma_L^Z(q^2) + c(q^2 - m_Z^2)\Delta\Gamma_L^\gamma(q^2)$$
$$\bar{\Pi}_{\gamma\gamma}(p^2) = \Pi_{\gamma\gamma}(p^2) + 2sq^2\Delta\Gamma_L^\gamma(q^2), \tag{10}$$

where $c \equiv \cos\theta_W$ and $s \equiv \sin\theta_W$. We have explicitly verified the cancellation of the ξ gauge parameters.

This allows the construction of gauge invariant oblique parameters [28],

$$\alpha\Delta S = \left(\frac{4s^2c^2}{m_Z^2}\right)\left\{\bar{\Pi}_{ZZ}(m_Z^2) - \bar{\Pi}_{ZZ}(0) - \bar{\Pi}_{\gamma\gamma}(m_Z^2) - \frac{c^2-s^2}{cs}(\bar{\Pi}_{\gamma Z}(m_Z^2))\right\}$$
$$\alpha\Delta T = \left(\frac{\bar{\Pi}_{WW}(0)}{m_W^2} - \frac{\bar{\Pi}_{ZZ}(0)}{m_Z^2}\right)$$
$$\alpha\Delta U = 4s^2\left\{\frac{\bar{\Pi}_{WW}(m_W^2) - \bar{\Pi}_{WW}(0)}{m_W^2} - c^2\left(\frac{\bar{\Pi}_{ZZ}(m_Z^2) - \bar{\Pi}_{ZZ}(0)}{m_Z^2}\right) - 2sc\left(\frac{\bar{\Pi}_{\gamma Z}(m_Z^2)}{m_Z^2}\right) - s^2\left(\frac{\bar{\Pi}_{\gamma\gamma}(m_Z^2)}{m_Z^2}\right)\right\}. \tag{11}$$

The oblique parameters effectively capture the new physics consequences on low energy observables with the assumption that all new physics is at a scale much larger than M_Z and that the new physics contributes only to the 2-point functions. The nonpinch terms which contribute to the vertex function are hence not included here but generate additional contributions from the effective Lagrangian, which is not captured in the STU formalism. We parametrize the oblique parameters as

$$\Delta S = C_S\frac{1}{\epsilon}\left(\frac{4\pi\mu^2}{m_Z^2}\right)^\epsilon\Gamma(1+\epsilon) + R_S$$
$$\Delta T = C_T\frac{1}{\epsilon}\left(\frac{4\pi\mu^2}{m_Z^2}\right)^\epsilon\Gamma(1+\epsilon) + R_T$$
$$\Delta U = C_U\frac{1}{\epsilon}\left(\frac{4\pi\mu^2}{m_Z^2}\right)^\epsilon\Gamma(1+\epsilon) + R_U, \tag{12}$$

where

$$C_S = \frac{m_H^2}{8\pi}\left\{\frac{f_B+f_W}{\Lambda^2}\right\} + \frac{m_Z^2}{24\pi\Lambda^2}\left\{f_B(20c^2+7) - 3f_W + 24(s^2f_{BB} + c^2f_{WW}) + 36c^2g^2f_{WWW} + \frac{8c^2}{g^2}f_{\Phi,2}\right\}$$
$$C_T = \frac{1}{16\pi c^2}\left\{9m_W^2\left(\frac{f_B+f_W}{\Lambda^2}\right) + 3m_H^2\frac{f_B}{\Lambda^2} - 12\frac{m_W^2}{g^2}\frac{f_{\Phi,2}}{\Lambda^2}\right\}$$
$$C_U = \frac{m_Z^2}{6\pi\Lambda^2}s^2f_W. \tag{13}$$

The logarithmic contributions to the oblique parameters coefficients were obtained in Ref. [8] and are obtained by the replacement in Eq. (12),

$$\frac{1}{\epsilon}\left(\frac{4\pi\mu^2}{m_Z^2}\right)^\epsilon\Gamma(1+\epsilon) \to \ln\left(\frac{\Lambda^2}{m_Z^2}\right). \tag{14}$$

All other terms are dropped, which gives the leading logarithmic result,

$$\Delta S^{LL} \to C_S\log\left(\frac{\Lambda^2}{m_Z^2}\right)$$
$$\Delta T^{LL} \to C_T\log\left(\frac{\Lambda^2}{m_Z^2}\right)$$
$$\Delta U^{LL} \to C_U\log\left(\frac{\Lambda^2}{m_Z^2}\right). \tag{15}$$

We take as inputs

$$m_H = 126\,\text{GeV},\quad m_Z = 91.1875\,\text{GeV},\quad m_W = 80.399\,\text{GeV}$$
$$G_F = 1.16637\times10^{-5}\,\text{GeV}^{-2},\quad m_t = 173\,\text{GeV}. \tag{16}$$

All other inputs are obtained using the tree level relationships of the SM. Equation (15) becomes

[2] Results in the unitary gauge can be found in Ref. [29].
[3] Gauge invariant expressions for the 2-point functions are found in Refs. [24,25]. Their construction differs from ours by finite, gauge invariant terms, as explained in Appendix C.

$$\Delta S^{LL} = [0.015 f_B + .0014 f_W + .0028 f_{BB} + .01 f_{WW} + .006 f_{WWW} + .0016 f_{\Phi,2}] \cdot \left(\frac{1\ \mathrm{TeV}}{\Lambda}\right)^2 \left[\frac{\log(\Lambda^2/m_Z^2)}{\log(1\ \mathrm{TeV}^2/m_Z^2)}\right]$$
$$\Delta T^{LL} = [0.013 f_B + .007 f_W - .0047 f_{\Phi,2}] \times \left(\frac{1\ \mathrm{TeV}}{\Lambda}\right)^2 \left[\frac{\log(\Lambda^2/m_Z^2)}{\log(1\ \mathrm{TeV}^2/m_Z^2)}\right]$$
$$\Delta U^{LL} = [0.0005 f_W]\left(\frac{1\ \mathrm{TeV}}{\Lambda}\right)^2 \left[\frac{\log(\Lambda^2/m_Z^2)}{\log(1\ \mathrm{TeV}^2/m_Z^2)}\right]. \quad (17)$$

Numerical fits to the oblique parameters using the logarithmic contributions of Eq. (17) have been given in Refs. [15,30]. The terms not associated with the divergences are[4]

$$R_S = -\frac{4\pi v^2}{\Lambda^2} f_{BW} + R_{S1} + R_{S2}\log(c) + R_{S3}\log\left(\frac{m_H}{m_Z}\right)$$
$$R_T = -\frac{v^2}{2\alpha\Lambda^2} f_{\Phi,1} + R_{T1} + R_{T2}\log(c) + R_{T3}\log\left(\frac{m_H}{m_Z}\right)$$
$$R_U = \frac{g^2 s^2}{c^2}\frac{8\pi v^2}{\Lambda^2} f_{DW} + R_{U1} + R_{U2}\log(c) + R_{U3}\log\left(\frac{m_H}{m_Z}\right). \quad (18)$$

Analytic results for R_S, R_S, and R_U are given in Appendix D. Numerically, we find

$$R_S = \{-0.76 f_{BW} + 10^{-3}(1.48 f_B - 1.4 f_W - 0.2 f_{BB} - 0.71 f_{WW} + 0.66 f_{WWW} + 1.96 f_{\Phi,2})\}\left(\frac{1\ \mathrm{TeV}}{\Lambda}\right)^2$$
$$R_T = \{-4.0 f_{\Phi,1} - 10^{-3}(0.13 f_B + 0.12 f_W - 3.97 f_{\Phi,2})\} \times \left(\frac{1\ \mathrm{TeV}}{\Lambda}\right)^2$$
$$R_U = \{0.20 f_{DW} + 10^{-3}(-0.02 f_B + 2.06 f_W + 0.14 f_{WW} + 2.1 f_{WWW} - 0.25 f_{\Phi,2})\}\left(\frac{1\ \mathrm{TeV}}{\Lambda}\right)^2. \quad (19)$$

The effective field theory is defined at the weak scale, $\mu \sim m_Z$ and encapsulates the effects of potential new physics which may occur at high scales. The divergences which arise at one loop, Eq. (12), can be eliminated by renormalizing the coefficients which enter the tree level results. The theory is thus rendered finite order by order in the expansion in powers of $1/\Lambda^2$ [5,7,24,25,31]. Using $\overline{\mathrm{MS}}$ renormalization, the renormalized coefficients relevant for a study of the oblique parameters are

$$f_{BW}(\mu) = f_{BW} - \frac{1}{\epsilon}(4\pi)^\epsilon \Gamma(1+\epsilon) C_S$$
$$f_{DW}(\mu) = f_{DW} - \frac{1}{\epsilon}(4\pi)^\epsilon \Gamma(1+\epsilon) C_U$$
$$f_{\Phi,1}(\mu) = f_{\Phi,1} - \frac{1}{\epsilon}(4\pi)^\epsilon \Gamma(1+\epsilon) C_T. \quad (20)$$

This renormalization prescription is equivalent to that of Ref. [5]. The large logarithms of Ref. [8] have been eliminated by the renormalization of the tree level couplings, and the only remaining contributions to the oblique corrections are the finite contributions. Our final result, taking $\mu = m_Z$, as appropriate for the low energy effective Lagrangian, is

$$\Delta S = R_S \qquad \Delta T = R_T \qquad \Delta U = R_U. \quad (21)$$

Note that this result is quite different from that of Ref. [8], since the $\log(\Lambda)$ terms have all been cancelled by the renormalization of the tree level coefficients. This is in agreement with the leading result of Ref. [10] for ΔS, which was obtained by scaling the coefficients in a new physics model from Λ to m_Z.

IV. PHENOMENOLOGY

A. Results from fits to oblique parameters

We do a χ^2 fit to the oblique parameters, using the results of the GFITTER group [1],

$$\Delta S = 0.03 \pm 0.10$$
$$\Delta T = 0.05 \pm 0.12$$
$$\Delta U = 0.03 \pm 0.10, \quad (22)$$

with the correlation matrix

$$\rho = \begin{pmatrix} 1.0 & 0.891 & -0.540 \\ 0.891 & 1.0 & -0.803 \\ -0.540 & -0.803 & 1.0 \end{pmatrix}. \quad (23)$$

The parameters f_{BW} and $f_{\Phi,1}$ contribute to ΔS and ΔT at tree level. We show the limits in Fig. 1.[5] These coefficients are highly restricted by the electroweak data, and we ignore them in the remaining fits, where we obtain limits pairwise

[4]Since f_{BW}, $f_{\Phi,1}$, and f_{DW} contribute at tree level, we do not consider the much smaller contributions of these operators at one loop to the oblique parameters.

[5]Reference [32] has done a similar analysis, and our fit to f_{BW} and $f_{\Phi,1}$ agrees with theirs.

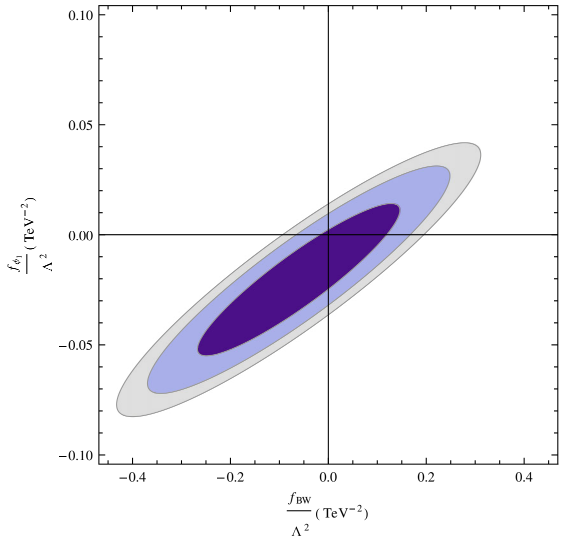

FIG. 1 (color online). Limits from the oblique parameters on $f_{\Phi,1}$ and f_{BW} for $\Lambda = 1$ TeV. These operators contribute at tree level and are significantly restricted. The curves (from outer to inner) are 99%, 95%, and 68% confidence level.

on various coefficients. We do not perform a global fit to the coefficients, since our point is simply to illustrate the numerical effects of the renormalization. Taking the 95% confidence level limits, and assuming $f_{BW}(m_Z)$ and $f_{\Phi,1}(m_Z)$ are $\mathcal{O}(1)$, the fit implies $\Lambda > 1.8$ TeV. Even though they are numerically constrained, f_{BW} and $f_{\Phi,1}$ cannot be set to 0 at the beginning, since they play a critical role in the renormalization, as seen in Eq. (20) [24,25].

In Fig. 2, we show the allowed region for $\Lambda = 1$ TeV in the $f_{BB}(m_Z)$ and $f_{WW}(m_Z)$ plane, setting all other coefficients to zero. The left-hand side shows the result using the leading logarithmic result of Eq. (19). This figure is in agreement with Fig. 6 of Ref. [16]. After renormalizing the coefficients of the effective theory, as in Eq. (21), the result is shown on the right-hand side of Fig. 2. In Fig. 3 we show the allowed region for $\Lambda =$ 1 TeV in the $f_W(m_Z)$ and $f_{WW}(m_Z)$ plane, setting all other coefficients to zero. Again, the left-hand side shows the result using the leading logarithmic result of Eq. (19), while the right-hand side shows the result after renormalization of the couplings. As emphasized in Refs. [24,25], the limits are considerably weakened once the renormalization procedure, Eq. (20), is applied. In fact, taking $f_i \sim 1$, we see that no useful limits can be inferred.

B. Implications for Higgs decays

In this section, we demonstrate the complementarity of limits from oblique parameters to those obtained from measurements of Higgs branching ratios.

In the effective theory, the decay $H \to W^+W^-$ is modified [6,9],[6]

$$\begin{aligned}\mu_{WW} &\equiv \frac{\Gamma(H \to W^+W^-)}{\Gamma(H \to W^+W^-)|_{\mathrm{SM}}}\\ &= \frac{1}{4 - 4x_W + 3x_W^2}\left[2\left(x_W + 2f_W(m_Z)\frac{m_W^2}{\Lambda^2} + 2f_{WW}(m_Z)\frac{m_W^2}{\Lambda^2}(2 - x_W)\right)^2\right.\\ &\quad\left. + \left(2 - x_W + 2f_W(m_Z)\frac{m_W^2}{\Lambda^2} + 2f_{WW}(m_Z)\frac{m_W^2}{\Lambda^2}x_W\right)^2\right] - \frac{2}{g^2}\frac{m_W^2}{\Lambda^2}(f_{\Phi,1} + 2f_{\Phi,2})\\ &\sim 1 + [.0086f_{WW}(m_Z) + .017f_W(m_Z) - .03f_{\Phi,1}(m_Z) - .06f_{\Phi,2}(m_Z)]\left(\frac{1\ \mathrm{TeV}}{\Lambda}\right)^2 + \mathcal{O}\left(\frac{1}{\Lambda^4}\right), \end{aligned} \tag{24}$$

where $x_W = 4m_W^2/m_H^2$ and we have made explicit the dependence of the coefficients on the scale. The $f_{\Phi,1}$ and $f_{\Phi,2}$ contributions come from the Higgs wave function renormalization. Since we have assumed that there are no non-SM corrections to the fermion-Higgs couplings, we can use the measurements of $H \to W^+W^-$ from gluon fusion to limit f_{WW} and f_W in Eq. (24) [33,34]:

$$\mu_{WW} = .68 \pm .20 \quad (\mathrm{CMS}) \qquad \mu_{WW} = .99 \pm .30 \quad (\mathrm{ATLAS}). \tag{25}$$

In Fig. 4, we show the allowed region in $f_{WW}(m_Z)$ vs $f_W(m_Z)$ from $H \to W^+W^-$. In this case, the limits from Higgs decay and from the oblique parameters are similar. Note that the scale of Fig. 4, $f_i/\Lambda^2 \sim 200$, makes these limits meaningless.

[6]Note our differing convention for the sign of f_W from these references.

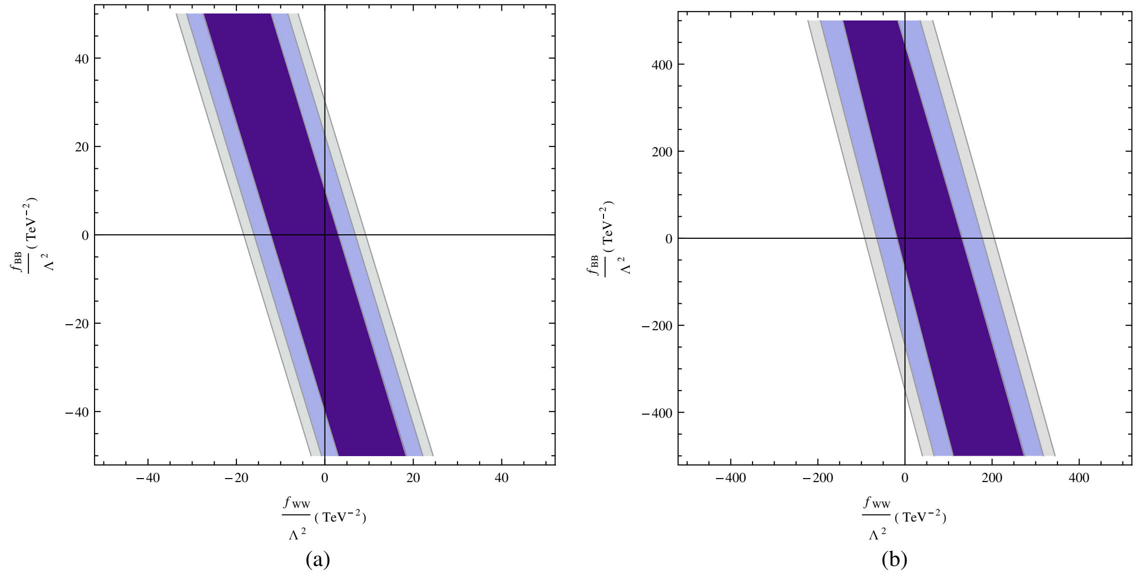

FIG. 2 (color online). (a) Limits from the oblique parameters on f_{BB} and f_{WW} for $\Lambda = 1$ TeV, using the leading logarithmic results of Eq. (17). The curves (from outer to inner) are 99%, 95%, and 68% confidence level. (b) Same as (a) except using the renormalized values of the coefficients, $f_{BB}(m_Z)$ and $f_{WW}(m_Z)$, Eqs. (19) and (21).

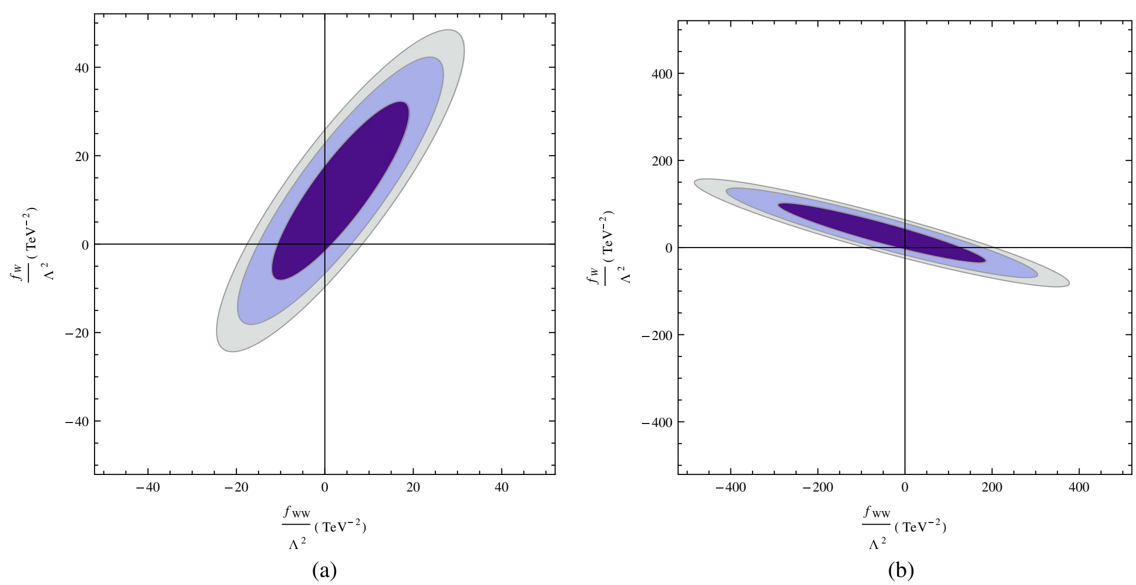

FIG. 3 (color online). (a) Limits from the oblique parameters on f_W and f_{WW} for $\Lambda = 1$ TeV, using the leading logarithmic results of Eq. (17). The curves (from outer to inner) are 99%, 95%, and 68% confidence level. (b) Same as (a) except using the renormalized values of the coefficients, $f_W(m_Z)$ and $f_{WW}(m_Z)$, Eqs. (19) and (21).

Only f_{BB}, f_{WW}, and $f_{BW}(m_Z)$ contribute to $H \to \gamma\gamma$. Using the well-known SM results [35], we find

$$\begin{aligned}\mu_{\gamma\gamma} &\equiv \frac{\Gamma(H\to\gamma\gamma)}{\Gamma(H\to\gamma\gamma)|_{\mathrm{SM}}} \\ &= \left\{1+\left(\frac{I_{\mathrm{real}}}{I_{\mathrm{real}}^2+I_{\mathrm{imag}}^2}\right)\frac{8\pi^2 v^2}{\Lambda^2}[f_{BB}(m_Z)+f_{WW}(m_Z)-f_{BW}(m_Z)]\right\}^2 \\ &\sim 1+1.47\left(\frac{1\ \mathrm{TeV}}{\Lambda}\right)^2[f_{BB}(m_Z)+f_{WW}(m_Z)-f_{BW}(m_Z)]+\mathcal{O}\left(\frac{1}{\Lambda^4}\right),\end{aligned} \tag{26}$$

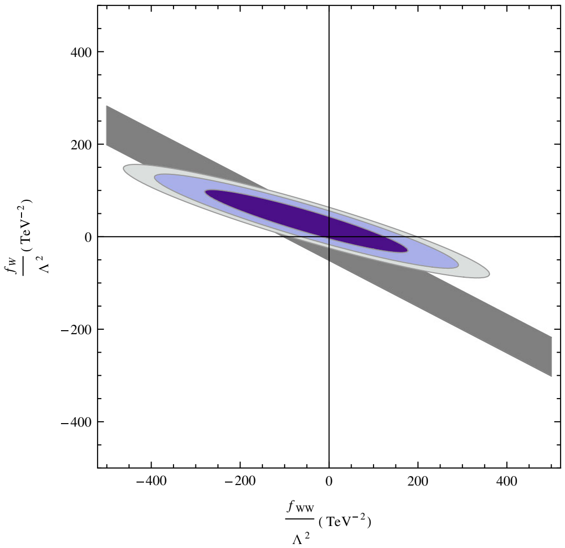

FIG. 4 (color online). 95% confidence level limits from the measurement of gluon fusion production with $H \to W^+W^-$ (black band), compared with the inferred limits from the oblique parameters.

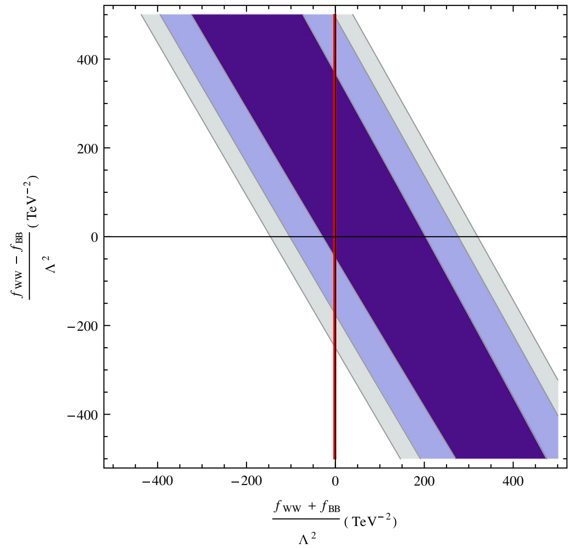

FIG. 5 (color online). Comparison of limits from oblique parameters (blue bands) and 95% confidence level limits from the measurement of gluon fusion production and the subsequent $H \to \gamma\gamma$ decay (red vertical line).

where

$$I_{\rm real} = \Sigma_f N_C Q_f^2 F_{1/2}^{\rm real}(x_f) + F_1^{\rm real}(x_W)$$
$$I_{\rm imag} = \Sigma_f N_C Q_f^2 F_{1/2}^{\rm imag}(x_f) + F_1^{\rm imag}(x_W), \quad (27)$$

$x_f = 4m_f^2/m_H^2$, and expressions for $F_{1/2}$ and F_1 are found in Ref. [35]. We can do a simple fit to the ATLAS and CMS results for $H \to \gamma\gamma$ [33,34]:

$$\mu_{\gamma\gamma} = .77 \pm .27 \quad ({\rm CMS})$$
$$\mu_{\gamma\gamma} = 1.55 \pm .31 \quad ({\rm ATLAS}). \quad (28)$$

Figure 5 shows the 95% confidence level limits from the gluon fusion of the Higgs boson, with the subsequent decay to $\gamma\gamma$, and contrasts the limit from the oblique parameters (setting $f_{BW}(m_Z) = 0$). The error band on the $H \to \gamma\gamma$ limits is not apparent on this scale, and again it is clear that the constraints from the oblique parameters cannot compete with those from Higgs decay even though they have a different shape in the f_{WW} and f_{BB} planes.

In a similar fashion, we can find the contribution to $H \to Z\gamma$,

$$\mu_{Z\gamma} \equiv \frac{\Gamma(H \to Z\gamma)}{\Gamma(H \to Z\gamma)|_{\rm SM}}$$
$$= 1 + \frac{2A_{\rm real}}{A_{\rm real}^2 + A_{\rm imag}^2} \frac{2\pi sc}{\alpha} \frac{m_Z^2}{\Lambda^2} g_1 + \mathcal{O}\left(\frac{1}{\Lambda^4}\right), \quad (29)$$

where [6]

$$g_1 = f_B(m_Z) - f_W(m_Z) + 4s^2 f_{BB}(m_Z) - 4c^2 f_{WW}(m_Z)$$
$$+ 2(c^2 - s^2) f_{BW}(m_Z), \quad (30)$$

and $A_{\rm real}$ and $A_{\rm imag}$ are the real and imaginary contributions of the sum of the fermion and W loops to the SM $H \to Z\gamma$ decay and can be found in Ref. [35]. In Fig. 6, we show the limits on combinations of $f_i(m_Z)$ which influence the decay $H \to Z\gamma$. Neglecting all parameters except $f_W(m_Z)$ and $f_B(m_Z)$, Fig. 6 corresponds to

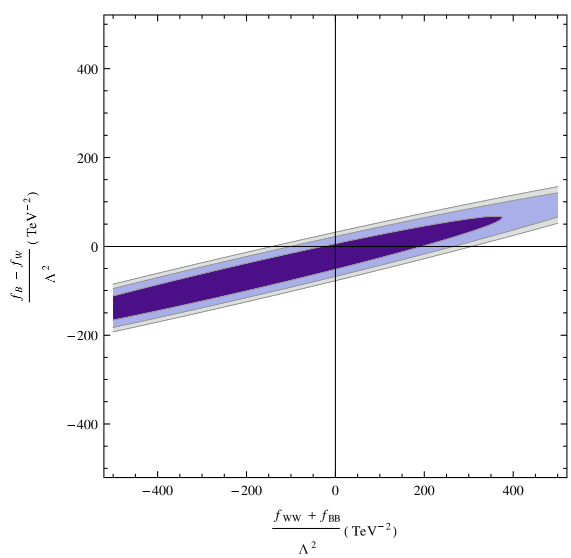

FIG. 6 (color online). Limits from oblique parameters which influence the decay $H \to Z\gamma$. The curves (from outer to inner) are 99%, 95%, and 68% confidence level.

$$-80 < [f_B(m_Z) - f_W(m_Z)]\left(\frac{1\ \text{TeV}}{\Lambda}\right)^2 < 35, \qquad (31)$$

which is not strong enough to give a limit on $\mu_{Z\gamma}$.

V. CONCLUSION

We have reexamined the prescription of Ref. [8] for obtaining limits on the couplings of an effective low energy theory of Higgs-gauge boson interactions by approximating ΔS, ΔT and ΔU by the leading logarithmic contributions. After renormalizing the coefficients of the operators which affect tree level results, however, the remaining contributions to the oblique parameters have no logarithmic enhancement, and the leading logarithmic approximation is inaccurate. We give both analytic and numerical results for the oblique parameters. Fits to the couplings of the effective theory using our prescription for ΔS, ΔT, and ΔU show that no meaningful limit on the couplings which contribute at one loop can be obtained from the oblique parameters. This is in contrast to the couplings which contribute at tree level, which are tightly constrained. This is in agreement with the results of Refs. [24,25]. Limits on the couplings, f_i, can, however, be extracted from Higgs decays and measurements of three-gauge boson vertices, and a complete fit was given in Ref. [30].

ACKNOWLEDGEMENTS

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APPENDIX A: RELATIONS BETWEEN HISZ AND SILH BASIS

In this appendix we present a prescription for converting between our effective Lagrangian and others which are frequently used in the literature. We begin by comparing our results with those of Ref. [5], which we label as Hagiwara-Ishihara-Szalapski-Zeppenfeld (HISZ). Reference [5] uses as its convention $D_\mu^{\text{HISZ}} \equiv \partial_\mu + ig'\frac{1}{2}B_\mu + ig\frac{\sigma^a}{2}W_\mu^a$, which is opposite from ours. To convert our final results to those of HISZ, the substitutions $f_{WWW} \to -f_{WWW}^{\text{HISZ}}$, $f_W \to -f_W^{\text{HISZ}}$ and $f_B \to -f_B^{\text{HISZ}}$ must be made in Eq. (12).

The HISZ operator basis [5] has 11 operators involving Higgs and electroweak gauge fields:

$$\mathcal{O}_{DW}^{\text{HISZ}},\quad \mathcal{O}_{DB}^{\text{HISZ}},\quad \mathcal{O}_{BW}^{\text{HISZ}},\quad \mathcal{O}_{\Phi,1}^{\text{HISZ}},\quad \mathcal{O}_{\Phi,2}^{\text{HISZ}},\quad \mathcal{O}_{\Phi,3}^{\text{HISZ}},\quad \mathcal{O}_{WWW}^{\text{HISZ}},\quad \mathcal{O}_{WW}^{\text{HISZ}},\quad \mathcal{O}_{BB}^{\text{HISZ}},\quad \mathcal{O}_{W}^{\text{HISZ}},\quad \mathcal{O}_{B}^{\text{HISZ}}.$$

The SILH operator basis [26] has the same number of operators (involving Higgs and electroweak gauge fields),

$$\mathcal{O}_{H}^{\text{SILH}},\quad \mathcal{O}_{T}^{\text{SILH}},\quad \mathcal{O}_{6}^{\text{SILH}},\quad \mathcal{O}_{W}^{\text{SILH}},\quad \mathcal{O}_{B}^{\text{SILH}},\quad \mathcal{O}_{2W}^{\text{SILH}},\quad \mathcal{O}_{2B}^{\text{SILH}},\quad \mathcal{O}_{BB}^{\text{SILH}},\quad \mathcal{O}_{HW}^{\text{SILH}},\quad \mathcal{O}_{HB}^{\text{SILH}},\quad \mathcal{O}_{3W}^{\text{SILH}},$$

where we use the definition in Ref. [13]. The operators are the same, but the normalization factors are more convenient.

The connections between operators are as follows:

- From SILH to our convention:

$$\mathcal{O}_H^{\text{SILH}} = \mathcal{O}_{\Phi,2} \qquad \mathcal{O}_T^{\text{SILH}} = \mathcal{O}_{\Phi,2} - 2\mathcal{O}_{\Phi,1} \qquad \mathcal{O}_6^{\text{SILH}} = 3\lambda\mathcal{O}_{\Phi,3} \qquad \mathcal{O}_W^{\text{SILH}} = 2\mathcal{O}_W - \mathcal{O}_{WW} - \mathcal{O}_{BW}$$
$$\mathcal{O}_B^{\text{SILH}} = 2\mathcal{O}_B - \mathcal{O}_{BB} - \mathcal{O}_{BW} \qquad \mathcal{O}_{2W}^{\text{SILH}} = \frac{1}{2g^2}\mathcal{O}_{DW} - \frac{2}{g^2}\mathcal{O}_{WWW} \qquad \mathcal{O}_{2B}^{\text{SILH}} = \frac{1}{2g'^2}\mathcal{O}_{DB} \qquad \mathcal{O}_{BB}^{\text{SILH}} = -4\mathcal{O}_{BB}$$
$$\mathcal{O}_{HW}^{\text{SILH}} = 2\mathcal{O}_W \qquad \mathcal{O}_{HB}^{\text{SILH}} = 2\mathcal{O}_B \qquad \mathcal{O}_{3W}^{\text{SILH}} = \frac{2}{3g^2}\mathcal{O}_{WWW}. \qquad (A1)$$

- From our convention to SILH:

$$\mathcal{O}_{DW} = 2g^2\mathcal{O}_{2W}^{\text{SILH}} + 6g^2\mathcal{O}_{3W}^{\text{SILH}} \qquad \mathcal{O}_{DB} = 2g'^2\mathcal{O}_{2B}^{\text{SILH}} \qquad \mathcal{O}_{BW} = -\mathcal{O}_B^{\text{SILH}} + \frac{1}{4}\mathcal{O}_{BB}^{\text{SILH}} + \mathcal{O}_{HB}^{\text{SILH}}$$
$$\mathcal{O}_{\Phi,1} = \frac{1}{2}\mathcal{O}_H^{\text{SILH}} - \frac{1}{2}\mathcal{O}_T^{\text{SILH}} \qquad \mathcal{O}_{\Phi,2} = \mathcal{O}_H^{\text{SILH}} \qquad \mathcal{O}_{\Phi,3} = \frac{1}{3\lambda}\mathcal{O}_6^{\text{SILH}} \qquad \mathcal{O}_{WWW} = \frac{3g^2}{2}\mathcal{O}_{3W}^{\text{SILH}}$$
$$\mathcal{O}_{WW} = \mathcal{O}_B^{\text{SILH}} - \mathcal{O}_W^{\text{SILH}} - \mathcal{O}_{HB}^{\text{SILH}} + \mathcal{O}_{HW}^{\text{SILH}} - \frac{1}{4}\mathcal{O}_{BB}^{\text{SILH}} \qquad \mathcal{O}_{BB} = -\frac{1}{4}\mathcal{O}_{BB}^{\text{SILH}} \qquad \mathcal{O}_W = \frac{1}{2}\mathcal{O}_{HW}^{\text{SILH}} \qquad \mathcal{O}_B = \frac{1}{2}\mathcal{O}_{HB}^{\text{SILH}}. \qquad (A2)$$

The connections between coefficients are as follows:

- From SILH to our convention:

$$c_H^{\rm SILH} = \frac{1}{2} f_{\Phi,1} + f_{\Phi,2} \qquad c_T^{\rm SILH} = -\frac{1}{2} f_{\Phi,1} \qquad c_6^{\rm SILH} = \frac{1}{3\lambda} f_{\Phi,3} \qquad c_W^{\rm SILH} = -f_{WW} \qquad c_B^{\rm SILH} = -f_{BW} + f_{WW}$$
$$c_{2W}^{\rm SILH} = 2g^2 f_{DW} \qquad c_{2B}^{\rm SILH} = 2g'^2 f_{DB} \qquad c_{BB}^{\rm SILH} = -\frac{1}{4} f_{BB} + \frac{1}{4} f_{BW} - \frac{1}{4} f_{WW} \qquad c_{HW}^{\rm SILH} = \frac{1}{2} f_W + f_{WW}$$
$$c_{HB}^{\rm SILH} = \frac{1}{2} f_B + f_{BW} - f_{WW} \qquad c_{3W}^{\rm SILH} = \frac{3}{2} g^2 f_{WWW} + 6g^2 f_{DW}. \tag{A3}$$

- From our convention to SILH:

$$f_{DW} = \frac{1}{2g^2} c_{2W}^{\rm SILH} \qquad f_{DB} = \frac{1}{2g'^2} c_{2B}^{\rm SILH} \qquad f_{BW} = -c_B^{\rm SILH} - c_W^{\rm SILH} \qquad f_{\Phi,1} = -2c_T^{\rm SILH} \qquad f_{\Phi,2} = c_H^{\rm SILH} + c_T^{\rm SILH}$$
$$f_{\Phi,3} = 3\lambda c_6^{\rm SILH} \qquad f_{WWW} = \frac{2}{3g^2} c_{3W}^{\rm SILH} - \frac{2}{g^2} c_{2W}^{\rm SILH} \qquad f_{WW} = -c_W^{\rm SILH} \qquad f_{BB} = -c_B^{\rm SILH} - 4c_{BB}^{\rm SILH}$$
$$f_W = 2c_W^{\rm SILH} + 2c_{HW}^{\rm SILH} \qquad f_B = 2c_B^{\rm SILH} + 2c_{HB}^{\rm SILH}. \tag{A4}$$

APPENDIX B: SELF-ENERGIES IN R_ξ GAUGE

In this appendix we present the detailed formulas of the self-energies that contribute to Π_{XY} defined in Eq. (17) in the R_ξ gauge. We use different gauge parameters ξ_W and ξ_Z for the W and Z bosons, respectively. Π_{XY} can be written as

$$\Pi_{XY}(q^2) = \Sigma_i f_i \left\{ \Pi^i_{XY,FG}(q^2) + \Pi^i_{XY,\xi}(q^2) \right\}, \tag{B1}$$

where XY represents $\gamma\gamma$, Z γ, ZZ, and WW and Π^i is the part of the 2-point function which is proportional to f_i. $\Pi_{XY,FG}$ is independent of ξ and contains results in the Feynman gauge, and the second term collects terms that vanish when $\xi = 1$. In the following, we express our results in terms of scalar integral functions A_0 and B_0 [36]. Only nonzero contributions are listed here. We separate the contributions in proportion to each f_i:

$\mathcal{O}_B$:

$$\Pi_{\gamma\gamma,FG} = -\frac{f_B}{\Lambda^2} \frac{1}{72\pi} q^2 \alpha \{3(20m_W^2 + q^2) B_0(q^2, m_W^2, m_W^2) - 6A_0(m_W^2) + 2(q^2 - 6m_W^2)\}, \tag{B2}$$

$$\begin{aligned} \Pi_{\gamma\gamma,\xi} = \frac{f_B}{\Lambda^2} \frac{\alpha}{48\pi m_W^2} \{ & -2q^2(-2q^2 m_W^2(\xi_W - 2) + m_W^4(\xi_W^2 + 4\xi_W - 5) + q^4) B_0(q^2, m_W^2, m_W^2 \xi_W) \\ & + (q^6 - 4q^4 m_W^2 \xi_W) B_0(q^2, m_W^2 \xi_W, m_W^2 \xi_W) + (8q^4 m_W^2 + q^6) B_0(q^2, m_W^2, m_W^2) - 2q^2 m_W^2 (\xi_W + 5) A_0(m_W^2) \\ & + 2q^2 m_W^2 (\xi_W + 5) A_0(m_W^2 \xi_W)\}, \end{aligned} \tag{B3}$$

$$\begin{aligned} \Pi_{\gamma Z,FG} = \frac{f_B}{\Lambda^2} \frac{\alpha}{288\pi cs} \{ & 3(4m_Z^2(m_H^2 + q^2) - 2q^2 m_H^2 + m_H^4 - 5m_Z^4 + q^4) B_0(q^2, m_H^2, m_Z^2) \\ & + 3q^2(16(5s^2 - 2)m_W^2 + q^2(4s^2 - 1)) B_0(q^2, m_W^2, m_W^2) + 3A_0(m_Z^2)(m_H^2 + 5m_Z^2 - q^2) \\ & - 3A_0(m_H^2)(m_H^2 + 5m_Z^2 + q^2) + 6q^2(1 - 4s^2) A_0(m_W^2) + 2q^2(-3m_H^2 + (6 - 24s^2)m_W^2 - 3m_Z^2 + 4q^2 s^2)\}, \end{aligned} \tag{B4}$$

$$\Pi_{\gamma Z,\xi} = \frac{f_B}{\Lambda^2}\frac{\alpha}{96\pi c s m_W^2}\{(m_W^2 + q^2(2s^2-1))(4q^2 m_W^2 \xi_W - q^4)B_0(q^2, m_W^2\xi_W, m_W^2\xi_W) + 2(m_W^2 + q^2(2s^2-1))(-2q^2 m_W^2(\xi_W - 2) + m_W^4(\xi_W^2 + 4\xi_W - 5) + q^4)B_0(q^2, m_W^2, m_W^2\xi_W) - (8q^2 m_W^2 + q^4)(m_W^2 + q^2(2s^2-1))B_0(q^2, m_W^2, m_W^2) + 2m_W^2(\xi_W + 5)A_0(m_W^2)(m_W^2 + q^2(2s^2-1)) - 2m_W^2(\xi_W + 5)A_0(m_W^2\xi_W)(m_W^2 + q^2(2s^2-1))\}, \quad \text{(B5)}$$

$$\Pi_{ZZ,FG} = \frac{f_B}{\Lambda^2}\frac{\alpha}{144\pi c^2 q^2}\{(-3m_H^4(m_Z^2 + q^2) + 6m_H^2(-4q^2 m_Z^2 + m_Z^4 + q^4) - 3(m_Z^4 - q^4)(m_Z^2 - q^2))B_0(q^2, m_H^2, m_Z^2) + 3q^4(q^2(1-2s^2) - 8(5s^2-4)m_W^2)B_0(q^2, m_W^2, m_W^2) + 3A_0(m_H^2)(m_Z^2 + q^2)(m_H^2 - m_Z^2 + q^2) + 3A_0(m_Z^2)(-m_H^2(m_Z^2 + q^2) - 10q^2 m_Z^2 + m_Z^4 + q^4) + 6q^4(2s^2-1)A_0(m_W^2) + 2q^2(3m_H^2(m_Z^2 + q^2) - 2q^2((3-6s^2)m_W^2 + q^2 s^2) + 2q^2 m_Z^2 + 3m_Z^4)\}, \quad \text{(B6)}$$

$$\Pi_{ZZ,\xi} = \frac{f_B}{\Lambda^2}\frac{\alpha}{48\pi m_W^2}\{(m_Z^2 - q^2)(q^4 - 4q^2 m_W^2 \xi_W)B_0(q^2, m_W^2\xi_W, m_W^2\xi_W) - 2(m_Z^2 - q^2)(-2q^2 m_W^2(\xi_W - 2) + m_W^4(\xi_W^2 + 4\xi_W - 5) + q^4)B_0(q^2, m_W^2, m_W^2\xi_W) + (8q^2 m_W^2 + q^4)(m_Z^2 - q^2)B_0(q^2, m_W^2, m_W^2) + 2m_W^2(\xi_W + 5)A_0(m_W^2)(q^2 - m_Z^2) + 2m_W^2\{\xi_W + 5)(m_Z^2 - q^2)A_0(m_W^2\xi_W)\}, \quad \text{(B7)}$$

$$\Pi_{WW,FG} = \frac{f_B}{\Lambda^2}\frac{\alpha}{144\pi q^2}\{12(m_W^3 - q^2 m_W)^2 B_0(q^2, 0, m_W^2) - 3(-5q^4 m_Z^2 - m_W^4(7m_Z^2 + 8q^2) + 4q^2 m_Z^4 + 2m_W^2(-14q^2 m_Z^2 + m_Z^4 + 2q^4) + 4m_W^6 + m_Z^6)B_0(q^2, m_W^2, m_Z^2) - 3m_Z^2 A_0(m_W^2)(3m_W^2 + m_Z^2 + 5q^2) - 3A_0(m_Z^2)(-m_W^2(3m_Z^2 + 4q^2) + 5q^2 m_Z^2 + 4m_W^4 - m_Z^4) + 2q^2 m_Z^2(-3m_W^2 + 3m_Z^2 - q^2)\}, \quad \text{(B8)}$$

$$\Pi_{WW,\xi} = \frac{f_B}{\Lambda^2}\frac{\alpha}{48\pi q^2}\{(m_W^2 - q^2)^2(m_W^2 + 5q^2)B_0(q^2, 0, m_W^2) + (q^2 - m_W^2)(4q^2 m_W^2\xi_W + m_W^4\xi_W^2 - 5q^4)B_0(q^2, 0, m_W^2\xi_W) + (m_W^2 - q^2)(4q^2 m_W^2 \xi_W + m_Z^2(4q^2 - 2m_W^2\xi_W) + m_W^4\xi_W^2 + m_Z^4 - 5q^4)B_0(q^2, m_Z^2, m_W^2\xi_W) + (q^2 - m_W^2)(-2m_W^2(m_Z^2 - 2q^2) + 4q^2 m_Z^2 + m_W^4 + m_Z^4 - 5q^4)B_0(q^2, m_W^2, m_Z^2) + m_W^2(\xi_W - 1)A_0(m_Z^2)(m_W^2 - q^2) + m_Z^2(m_W^2 - q^2)A_0(m_W^2\xi_W) + m_Z^2 A_0(m_W^2)(q^2 - m_W^2)\}. \quad \text{(B9)}$$

$\mathcal{O}_W$:

$$\Pi_{\gamma\gamma,FG} = -\frac{f_W}{\Lambda^2}\frac{1}{72\pi}q^2\alpha\{3(20m_W^2 + q^2)B_0(q^2, m_W^2, m_W^2) - 6A_0(m_W^2) + 2(q^2 - 6m_W^2)\}, \quad \text{(B10)}$$

$$\Pi_{\gamma\gamma,\xi} = \frac{f_W}{\Lambda^2}\frac{\alpha}{48\pi m_W^2}\{-2q^2(-2q^2 m_W^2(\xi_W - 2) + m_W^4(\xi_W^2 + 4\xi_W - 5) + q^4)B_0(q^2, m_W^2, m_W^2\xi_W) + (q^6 - 4q^4 m_W^2\xi_W)B_0(q^2, m_W^2\xi_W, m_W^2\xi_W) + (8q^4 m_W^2 + q^6)B_0(q^2, m_W^2, m_W^2) - 2q^2 m_W^2(\xi_W + 5)A_0(m_W^2) + 2q^2 m_W^2(\xi_W + 5)A_0(m_W^2\xi_W)\}, \quad \text{(B11)}$$

$$\begin{aligned}\Pi_{\gamma Z,FG} = \frac{f_W}{\Lambda^2}\frac{\alpha}{288\pi cs}\{&-3(4m_Z^2(m_H^2+q^2)-2q^2m_H^2+m_H^4-5m_Z^4+q^4)B_0(q^2,m_H^2,m_Z^2)\\ &-3(4q^2(21-20s^2)m_W^2+48m_W^4+q^4(3-4s^2))B_0(q^2,m_W^2,m_W^2)-3A_0(m_Z^2)(m_H^2+5m_Z^2-q^2)\\ &+3A_0(m_H^2)(m_H^2+5m_Z^2+q^2)+6A_0(m_W^2)(24m_W^2+q^2(3-4s^2))\\ &+2(q^2(3m_H^2+3m_Z^2+4q^2(s^2-1))+6q^2(3-4s^2)m_W^2-72m_W^4)\},\end{aligned} \tag{B12}$$

$$\begin{aligned}\Pi_{\gamma Z,\xi} = \frac{f_W}{\Lambda^2}\frac{\alpha}{96\pi s m_W m_Z}\{&q^2(2q^2-m_Z^2)(q^2-4m_W^2\xi_W)B_0(q^2,m_W^2\xi_W,m_W^2\xi_W)\\ &+(4m_W^4(\xi_W-1)(m_Z^2(\xi_W+2)-q^2(\xi_W+5))+4q^2m_W^2(m_Z^2(\xi_W+4)+2q^2(\xi_W-2))\\ &-4(2q^4m_Z^2+q^6))B_0(q^2,m_W^2,m_W^2\xi_W)+(9q^4m_Z^2-8m_W^2(3q^2m_Z^2-2q^4)+2q^6)B_0(q^2,m_W^2,m_W^2)\\ &+A_0(m_W^2)(m_W^2(4m_Z^2(\xi_W+2)-4q^2(\xi_W+5))-10q^2m_Z^2)\\ &+A_0(m_W^2\xi_W)(m_W^2(4q^2(\xi_W+5)-4m_Z^2(\xi_W+2))+10q^2m_Z^2)-4q^2m_W^2m_Z^2(\xi_W-1)\},\end{aligned} \tag{B13}$$

$$\begin{aligned}\Pi_{ZZ,FG} = -\frac{f_W}{\Lambda^2}\frac{\alpha}{144\pi q^2 s^2}\{&3(m_H^4(m_Z^2+q^2)-2m_H^2(-4q^2m_Z^2+m_Z^4+q^4)+(m_Z^2-q^2)^2(m_Z^2+q^2))B_0(q^2,m_H^2,m_Z^2)\\ &+3q^2B_0(q^2,m_W^2,m_W^2)(4m_W^2(4m_Z^2+q^2(11-10s^2))-10q^2m_Z^2+48m_W^4+q^4(1-2s^2))\\ &-3A_0(m_H^2)(m_Z^2+q^2)(m_H^2-m_Z^2+q^2)+A_0(m_Z^2)(3m_H^2(m_Z^2+q^2)-3(-10q^2m_Z^2+m_Z^4+q^4))\\ &-6q^2A_0(m_W^2)(24m_W^2-10m_Z^2+q^2(1-2s^2))-2q^2(3m_H^2(m_Z^2+q^2)+6m_W^2(2m_Z^2+q^2(1-2s^2))\\ &-72m_W^4+3m_Z^4+2q^4s^2-2q^4)\},\end{aligned} \tag{B14}$$

$$\begin{aligned}\Pi_{ZZ,\xi} = -\frac{f_W}{\Lambda^2}\frac{\alpha}{48\pi q^2 s^2 m_Z^2}\{&(m_Z^2-q^2)(q^6-4q^4m_W^2\xi_W)B_0(q^2,m_W^2\xi_W,m_W^2\xi_W)\\ &-2(m_Z^2-q^2)(5q^4m_Z^2-2q^2m_W^2(2m_Z^2(\xi_W+1)+q^2(\xi_W-2))\\ &+m_W^4(\xi_W-1)(m_Z^2(1-\xi_W)+q^2(\xi_W+5))+q^6)B_0(q^2,m_W^2,m_W^2\xi_W)\\ &+(m_Z^2-q^2)(10q^4m_Z^2+8q^2m_W^2(q^2-2m_Z^2)+q^6)B_0(q^2,m_W^2,m_W^2)\\ &-2A_0(m_W^2)(m_Z^2-q^2)(m_W^2(m_Z^2(1-\xi_W)+q^2(\xi_W+5))+5q^2m_Z^2)\\ &+2(m_Z^2-q^2)A_0(m_W^2\xi_W)(m_W^2(m_Z^2(1-\xi_W)+q^2(\xi_W+5))+5q^2m_Z^2)+4q^2m_W^2m_Z^2(\xi_W-1)(q^2-m_Z^2)\},\end{aligned} \tag{B15}$$

$$\begin{aligned}\Pi_{WW,FG} = -\frac{f_W}{\Lambda^2}\frac{\alpha}{144\pi q^2 s^2}\{&3(m_H^4(m_W^2+q^2)-2m_H^2(-4q^2m_W^2+m_W^4+q^4)+(m_W^2-q^2)^2(m_W^2+q^2))B_0(q^2,m_H^2,m_W^2)\\ &-12s^2m_W^2(m_W^2-q^2)^2B_0(q^2,0,m_W^2)+3(4q^4m_Z^2+m_W^4(8m_Z^2+q^2(59-8s^2))-5q^2m_Z^4\\ &+m_W^2(10q^2m_Z^2-6m_Z^4+2q^4(2s^2+15))+(4s^2-2)m_W^6+q^6)B_0(q^2,m_W^2,m_Z^2)\\ &-3A_0(m_H^2)(m_W^2+q^2)(m_H^2-m_W^2+q^2)\\ &+3A_0(m_W^2)(m_H^2(m_W^2+q^2)+m_W^2(q^2-6m_Z^2)-5q^2m_Z^2+5m_W^4-2q^4)\\ &+3A_0(m_Z^2)(m_W^2(6m_Z^2-q^2(4s^2+19))+5q^2m_Z^2+(4s^2-6)m_W^4-q^4)\\ &+2q^2(-3m_H^2(m_W^2+q^2)+m_W^2(36m_Z^2-3q^2)-3q^2m_Z^2+21m_W^4+2q^4)\},\end{aligned} \tag{B16}$$

$$\Pi_{WW,\xi} = -\frac{f_W}{\Lambda^2}\frac{\alpha}{48\pi q^2 s^2 m_Z^2}\{(m_W^2-q^2)^2(2m_W^2(5q^2-m_Z^2\xi_Z)+(q^2-m_Z^2\xi_Z)^2+m_W^4)B_0(q^2,m_W^2,m_Z^2\xi_Z)$$
$$-q^2(q^2-m_W^2)(-2q^2(m_W^2\xi_W+m_Z^2\xi_Z)+(m_W^2\xi_W-m_Z^2\xi_Z)^2+q^4)B_0(q^2,m_W^2\xi_W,m_Z^2\xi_Z)$$
$$-(m_W^2-q^2)(m_W^2-m_Z^2)(4q^2m_W^2\xi_W+m_W^4\xi_W^2-5q^4)B_0(q^2,0,m_W^2\xi_W)$$
$$+(m_W^2-q^2)(4q^2m_W^2+m_W^4-5q^4)(m_W^2-m_Z^2)B_0(q^2,0,m_W^2)$$
$$+(m_W^2-q^2)(-4q^4m_Z^2-m_W^4\xi_W(2m_Z^2+q^2(\xi_W-4))+5q^2m_Z^4$$
$$+m_W^2(-4q^2m_Z^2(\xi_W-1)+m_Z^4+q^4(2\xi_W-5))+m_W^6\xi_W^2-q^6)B_0(q^2,m_Z^2,m_W^2\xi_W)$$
$$-(q^2-m_W^2)(4q^4m_Z^2+m_W^4(4m_Z^2-13q^2)-5q^2m_Z^4-2m_W^2(-q^2m_Z^2+m_Z^4-7q^4)-2m_W^6+q^6)B_0(q^2,m_W^2,m_Z^2)$$
$$+m_W^2A_0(m_Z^2)(m_W^2-q^2)(m_W^2(\xi_W-2)+m_Z^2+q^2(-(\xi_W+10)))$$
$$+m_W^2(m_W^2-q^2)A_0(m_Z^2\xi_Z)(m_W^2-m_Z^2\xi_Z+q^2(\xi_W+10))-m_Z^2(q^2-m_W^2)A_0(m_W^2\xi_W)(m_W^2(1-\xi_W)+q^2(\xi_Z+10))$$
$$+m_Z^2A_0(m_W^2)(q^2-m_W^2)(m_W^2(1-\xi_Z)+q^2(\xi_Z+10))+2q^2m_W^2m_Z^2(q^2-m_W^2)(\xi_W+\xi_Z-2)\}. \tag{B17}$$

$\mathcal{O}_{BB}$:

$$\Pi_{\gamma\gamma,FG} = -\frac{f_{BB}}{\Lambda^2}\frac{\alpha}{4\pi m_H^2}q^2\{m_H^2A_0(m_H^2)+6m_W^2A_0(m_W^2)+3m_Z^2A_0(m_Z^2)-2(2m_W^4+m_Z^4)\}, \tag{B18}$$

$$\Pi_{\gamma Z,FG} = \frac{f_{BB}}{\Lambda^2}\frac{\alpha}{4\pi csm_H^2}(c^2-1)\{m_H^2m_Z^2(-m_H^2+m_Z^2+q^2)B_0(q^2,m_H^2,m_Z^2)-m_Z^2A_0(m_Z^2)(m_H^2+3q^2)$$
$$+m_H^2A_0(m_H^2)(m_Z^2-q^2)-6q^2m_W^2A_0(m_W^2)+2q^2(2m_W^4+m_Z^4)\}, \tag{B19}$$

$$\Pi_{ZZ,FG} = \frac{f_{BB}}{\Lambda^2}\frac{\alpha}{4\pi c^2m_H^2}(1-c^2)\{2m_H^2m_Z^2(-m_H^2+m_Z^2+q^2)B_0(q^2,m_H^2,m_Z^2)+m_H^2A_0(m_H^2)(2m_Z^2-q^2)$$
$$-m_Z^2A_0(m_Z^2)(2m_H^2+3(m_Z^2+q^2))-6q^2m_W^2A_0(m_W^2)+2(2q^2m_W^4+q^2m_Z^4+m_Z^6)\}, \tag{B20}$$

$$\Pi_{WW,FG} = -\frac{f_{BB}}{\Lambda^2}\frac{\alpha}{4\pi m_H^2}(c^2-1)m_Z^4\{2m_Z^2-3A_0(m_Z^2)\}. \tag{B21}$$

$\mathcal{O}_{WW}$:

$$\Pi_{\gamma\gamma,FG} = -\frac{f_{WW}}{\Lambda^2}\frac{\alpha}{4\pi m_H^2}q^2\{m_H^2A_0(m_H^2)+6m_W^2A_0(m_W^2)+3m_Z^2A_0(m_Z^2)-2(2m_W^4+m_Z^4)\}, \tag{B22}$$

$$\Pi_{\gamma Z,FG} = \frac{f_{WW}}{\Lambda^2}\frac{\alpha}{4\pi sm_H^2}c\{m_H^2m_Z^2(-m_H^2+m_Z^2+q^2)B_0(q^2,m_H^2,m_Z^2)$$
$$-m_Z^2A_0(m_Z^2)(m_H^2+3q^2)+m_H^2A_0(m_H^2)(m_Z^2-q^2)-6q^2m_W^2A_0(m_W^2)+2q^2(2m_W^4+m_Z^4)\}, \tag{B23}$$

$$\Pi_{ZZ,FG} = \frac{f_{WW}}{\Lambda^2}\frac{\alpha}{4\pi s^2m_H^2}c^2\{2m_H^2m_Z^2(-m_H^2+m_Z^2+q^2)B_0(q^2,m_H^2,m_Z^2)+m_H^2A_0(m_H^2)(2m_Z^2-q^2)$$
$$-m_Z^2A_0(m_Z^2)(2m_H^2+3(m_Z^2+q^2))-6A_0(m_W^2)(q^2m_W^2+m_Z^4)+2(2q^2m_W^4+q^2m_Z^4+2m_W^2m_Z^4+m_Z^6)\}, \tag{B24}$$

$$\Pi_{WW,FG} = -\frac{f_{WW}}{\Lambda^2}\frac{\alpha}{4\pi s^2 m_H^2}\{-2m_H^2 m_W^2(-m_H^2+m_W^2+q^2)B_0(q^2,m_H^2,m_W^2)+m_H^2A_0(m_H^2)(q^2-2m_W^2)$$
$$+2m_W^2A_0(m_W^2)(m_H^2+3(m_W^2+q^2))+3A_0(m_Z^2)(q^2m_Z^2+m_W^4)-2(m_W^4(m_Z^2+2q^2)+q^2m_Z^4+2m_W^6)\}. \quad \text{(B25)}$$

$\mathcal{O}_{WWW}$:

$$\Pi_{\gamma\gamma,FG} = \frac{f_{WWW}}{\Lambda^2}\frac{q^2s^2g_w^4}{16\pi^2}\{3(q^2-4m_W^2)B_0(q^2,m_W^2,m_W^2)-6A_0(m_W^2)+6m_W^2-q^2\}, \quad \text{(B26)}$$

$$\Pi_{\gamma Z,FG} = -\frac{f_{WWW}}{\Lambda^2}\frac{cq^2sg_w^4}{16\pi^2}\{(12m_W^2-3q^2)B_0(q^2,m_W^2,m_W^2)+6A_0(m_W^2)-6m_W^2+q^2\}, \quad \text{(B27)}$$

$$\Pi_{ZZ,FG} = -\frac{f_{WWW}}{\Lambda^2}\frac{c^2q^2g_w^4}{16\pi^2}\{(12m_W^2-3q^2)B_0(q^2,m_W^2,m_W^2)+6A_0(m_W^2)-6m_W^2+q^2\}, \quad \text{(B28)}$$

$$\Pi_{WW,FG} = -\frac{f_{WWW}}{\Lambda^2}\frac{g_w^4}{16\pi^2}\{-3s^2(m_W^2-q^2)^2B_0(q^2,0,m_W^2)+3(-m_W^2(m_Z^2+2q^2(s^2-2))$$
$$+(s^2+1)m_W^4+q^4(s^2-1))B_0(q^2,m_W^2,m_Z^2)+3A_0(m_Z^2)(s^2m_W^2-q^2(s^2-1))$$
$$+3q^2A_0(m_W^2)-6q^2m_W^2+q^4\}. \quad \text{(B29)}$$

$\mathcal{O}_{\Phi,2}$:[7]

$$\Pi_{ZZ} = \frac{f_{\Phi,2}}{\Lambda^2}\frac{m_Z^2}{144\pi^2q^2}[3(m_H^4-2m_H^2(m_Z^2+q^2)+m_Z^4+10m_Z^2q^2+q^4)B_0(q^2,m_H^2,m_Z^2)$$
$$-3(m_H^2-m_Z^2-2q^2)A_0(m_H^2)+3(m_H^2-m_Z^2-q^2)A_0(m_Z^2)-2q^2(3m_H^2+3m_Z^2-q^2)], \quad \text{(B30)}$$

$$\Pi_{WW} = \frac{f_{\Phi,2}}{\Lambda^2}\frac{m_W^2}{144\pi^2q^2}[3(m_H^4-2m_H^2(m_W^2+q^2)+m_W^4+10m_W^2q^2+q^4)B_0(q^2,m_H^2,m_W^2)$$
$$-3(m_H^2-m_W^2-2q^2)A_0(m_H^2)+3(m_H^2-m_W^2-q^2)A_0(m_W^2)-2q^2(3m_H^2+3m_W^2-q^2)]. \quad \text{(B31)}$$

APPENDIX C: PINCH TERMS IN R_ξ GAUGE

Analytic results for Γ_L^V as defined in Eq. (9) are given here in the R_ξ gauge and contain both a pinch contribution and a nonpinch contribution,

$$\Gamma_L^V = \Gamma_L^V(\text{pinch}) + \Gamma_L^V(\text{nonpinch}). \quad \text{(C1)}$$

The "pinch" contribution is defined as in Refs. [37,38] to be the contribution of the 3-point interaction which exactly cancels the internal fermion propagator. We note that Γ_L^V(nonpinch) is gauge invariant. References [24,25] included this nonpinch contribution in their definition of $\bar{\Pi}_{XY}$. From here on, we use Γ_L^V to denote the pinch contribution only, as is used in the definition of the oblique parameters in Ref. [28]. We use different gauge parameters ξ_W and ξ_Z for the W and Z bosons, respectively, and they cancel separately in our final result. $\Delta\Gamma_L^V$ can be written as

$$\Delta\Gamma_L^V = \Delta\Gamma_{L,FG}^V + \Delta\Gamma_{L,\xi}^V, \quad \text{(C2)}$$

[7]The contributions to Π_{ZZ} and Π_{WW} from $\mathcal{O}_{\Phi,2}$ are gauge independent without the addition of tadpole diagrams. Since the tadpole diagrams cancel in the calculation of the oblique parameters, we do not include them for the $\mathcal{O}_{\Phi,2}$ 2-point functions. The tadpole contributions *are* included in the 2-point functions listed above for all other operators.

where V represents γ, Z, and W. The first term $\Pi_{XY,FG}$ contains results in the Feynman gauge and is independent of ξ. The second term collects terms that vanish when $\xi = 1$. In the following, we express our results in terms of scalar integral functions A_0 and B_0 [36]. Only nonzero pinch contributions are listed here, and we separate the contributions in terms proportional to each of the individual f_i:

$\mathcal{O}_B$:

$$\begin{aligned}\Delta\Gamma^{\gamma}_{L,\xi} = \frac{f_B}{\Lambda^2}\frac{\alpha}{96\pi s m_W^2}\{&q^2(8m_W^2+q^2)(-B_0(q^2,m_W^2,m_W^2)) + (4q^2m_W^2\xi_W - q^4)B_0(q^2,m_W^2\xi_W,m_W^2\xi_W)\\ &+2(-2q^2m_W^2(\xi_W-2)+m_W^4(\xi_W^2+4\xi_W-5)+q^4)B_0(q^2,m_W^2,m_W^2\xi_W)\\ &+2m_W^2(\xi_W+5)A_0(m_W^2)-2m_W^2(\xi_W+5)A_0(m_W^2\xi_W)\},\end{aligned} \tag{C3}$$

$$\begin{aligned}\Delta\Gamma^{Z}_{L,\xi} = \frac{f_B}{\Lambda^2}\frac{\alpha}{96\pi c m_W^2}\{&(q^4-4q^2m_W^2\xi_W)B_0(q^2,m_W^2\xi_W,m_W^2\xi_W)\\ &+(4q^2m_W^2(\xi_W-2)-2m_W^4(\xi_W^2+4\xi_W-5)-2q^4)B_0(q^2,m_W^2,m_W^2\xi_W)\\ &+(8q^2m_W^2+q^4)B_0(q^2,m_W^2,m_W^2)-2m_W^2(\xi_W+5)A_0(m_W^2)+2m_W^2(\xi_W+5)A_0(m_W^2\xi_W)\},\end{aligned} \tag{C4}$$

$$\Delta\Gamma^{W}_{L,FG} = -\frac{f_B}{\Lambda^2}\frac{\alpha}{8\pi}c^2m_Z^2(B_0(q^2,0,m_W^2)-B_0(q^2,m_W^2,m_Z^2)), \tag{C5}$$

$$\begin{aligned}\Delta\Gamma^{W}_{L,\xi} = \frac{f_B}{\Lambda^2}\frac{\alpha}{96\pi q^2}\{&(-4q^2m_W^2\xi_W-m_W^4\xi_W^2+5q^4)B_0(q^2,0,m_W^2\xi_W)\\ &+(4q^2m_W^2\xi_W+m_Z^2(4q^2-2m_W^2\xi_W)+m_W^4\xi_W^2+m_Z^4-5q^4)B_0(q^2,m_Z^2,m_W^2\xi_W)\\ &+(-4q^2m_W^2-4q^2m_Z^2+2m_W^2m_Z^2-m_W^4-m_Z^4+5q^4)B_0(q^2,m_W^2,m_Z^2)\\ &+(4q^2m_W^2+m_W^4-5q^4)B_0(q^2,0,m_W^2)+m_W^2(\xi_W-1)A_0(m_Z^2)+m_Z^2A_0(m_W^2\xi_W)-m_Z^2A_0(m_W^2)\}.\end{aligned} \tag{C6}$$

$\mathcal{O}_W$:

$$\begin{aligned}\Delta\Gamma^{\gamma}_{L,\xi} = \frac{f_W}{\Lambda^2}\frac{\alpha}{96\pi s m_W^2}\{&q^2(8m_W^2+q^2)(-B_0(q^2,m_W^2,m_W^2)\}+(4q^2m_W^2\xi_W-q^4)B_0(q^2,m_W^2\xi_W,m_W^2\xi_W)\\ &+2(-2q^2m_W^2(\xi_W-2)+m_W^4(\xi_W^2+4\xi_W-5)+q^4)B_0(q^2,m_W^2,m_W^2\xi_W)\\ &+2m_W^2(\xi_W+5)A_0(m_W^2)-2m_W^2(\xi_W+5)A_0(m_W^2\xi_W)),\end{aligned} \tag{C7}$$

$$\Delta\Gamma^{Z}_{L,FG} = -\frac{f_W}{\Lambda^2}\frac{\alpha}{4\pi s^2}cm_Z^2B_0(q^2,m_W^2,m_W^2), \tag{C8}$$

$$\begin{aligned}\Delta\Gamma^{Z}_{L,\xi} = \frac{f_W}{\Lambda^2}\frac{\alpha}{96\pi q^2 m_W(cm_W-m_Z)}\{&(q^6-4q^4m_W^2\xi_W)B_0(q^2,m_W^2\xi_W,m_W^2\xi_W)\\ &+(-2m_W^4(\xi_W-1)(m_Z^2(1-\xi_W)+q^2(\xi_W+5))\\ &+4q^2m_W^2(2m_Z^2(\xi_W+1)+q^2(\xi_W-2))-2(5q^4m_Z^2+q^6))B_0(q^2,m_W^2,m_W^2\xi_W)\\ &+(10q^4m_Z^2+8q^2m_W^2(q^2-2m_Z^2)+q^6)B_0(q^2,m_W^2,m_W^2)\\ &+A_0(m_W^2)(-2m_W^2(m_Z^2(1-\xi_W)+q^2(\xi_W+5))-10q^2m_Z^2)\\ &+A_0(m_W^2\xi_W)(2m_W^2(m_Z^2(1-\xi_W)+q^2(\xi_W+5))+10q^2m_Z^2)-4q^2m_W^2m_Z^2(\xi_W-1)\},\end{aligned} \tag{C9}$$

$$\Delta\Gamma^{W}_{L,FG} = \frac{f_W}{\Lambda^2}\frac{\alpha}{8\pi s^2} c^2 m_Z^2\{(s^2-2)B_0(q^2, m_W^2, m_Z^2) - s^2 B_0(q^2, 0, m_W^2)\}, \tag{C10}$$

$$\begin{aligned}\Delta\Gamma^{W}_{L,\xi} = &-\frac{f_W}{\Lambda^2}\frac{\alpha}{96\pi q^2 s^2 m_Z^2}\{(m_W^2-q^2)(2m_W^2(5q^2-m_Z^2\xi_Z)+(q^2-m_Z^2\xi_Z)^2+m_W^4)B_0(q^2,m_W^2,m_Z^2\xi_Z)\\
&+q^2(-2q^2(m_W^2\xi_W+m_Z^2\xi_Z)+(m_W^2\xi_W-m_Z^2\xi_Z)^2+q^4)B_0(q^2,m_W^2\xi_W,m_Z^2\xi_Z)\\
&-(m_W^2-m_Z^2)(4q^2m_W^2\xi_W+m_W^4\xi_W^2-5q^4)B_0(q^2,0,m_W^2\xi_W)+(4q^2m_W^2+m_W^4-5q^4)(m_W^2-m_Z^2)B_0(q^2,0,m_W^2)\\
&+(-4q^4m_Z^2-m_W^4\xi_W(2m_Z^2+q^2(\xi_W-4))+5q^2m_Z^4\\
&+m_W^2(-4q^2m_Z^2(\xi_W-1)+m_Z^4+q^4(2\xi_W-5))+m_W^6\xi_W^2-q^6)B_0(q^2,m_Z^2,m_W^2\xi_W)\\
&+(4q^4m_Z^2+m_W^4(4m_Z^2-13q^2)-5q^2m_Z^4-2m_W^2(-q^2m_Z^2+m_Z^4-7q^4)-2m_W^6+q^6)B_0(q^2,m_W^2,m_Z^2)\\
&+m_W^2A_0(m_Z^2)(m_W^2(\xi_W-2)+m_Z^2+q^2(-(\xi_W+10)))+m_W^2A_0(m_Z^2\xi_Z)(m_W^2-m_Z^2\xi_Z+q^2(\xi_W+10))\\
&+m_Z^2A_0(m_W^2)(m_W^2(\xi_Z-1)-q^2(\xi_Z+10))\\
&+m_Z^2A_0(m_W^2\xi_W)(m_W^2(1-\xi_W)+q^2(\xi_Z+10))-2q^2m_W^2m_Z^2(\xi_W+\xi_Z-2)\}.\end{aligned} \tag{C11}$$

$\mathcal{O}_{WWW}$:

$$\Delta\Gamma^{\gamma}_{L,FG} = -\frac{f_{WWW}}{\Lambda^2}\frac{3\alpha}{8\pi s} q^2 g_w^2 B_0(q^2, m_W^2, m_W^2), \tag{C12}$$

$$\Delta\Gamma^{Z}_{L,FG} = -\frac{f_{WWW}}{\Lambda^2}\frac{3\alpha}{8\pi s^2} c q^2 g_w^2 B_0(q^2, m_W^2, m_W^2), \tag{C13}$$

$$\Delta\Gamma^{W}_{L,FG} = \frac{f_{WWW}}{\Lambda^2}\frac{-3\alpha}{8\pi s^2} q^2 g_w^2 (c^2 B_0(q^2, m_W^2, m_Z^2) + s^2 B_0(q^2, 0, m_W^2)). \tag{C14}$$

APPENDIX D: ANALYTIC RESULTS FOR OBLIQUE PARAMETERS

The finite contributions to the oblique parameters are

$$\begin{aligned}R_{S1} = &\frac{m_Z^2}{24\pi\Lambda^2}\left\{\frac{29}{3}(5c^2+1)f_B - c^2(64c^2+15)f_W + 72(s^2 f_{BB}+c^2 f_{WW}) + 72c^2 f_{WWW}g^2\right.\\
&+3\left(\frac{m_H}{m_Z}\right)^2(c^2 f_B + s^2 f_W - 8c^2 f_{WW} - 8s^2 f_{BB}) + \left(\frac{m_H}{m_Z}\right)^4[(2c^2-3)f_B + (2s^2-3)f_W]\\
&+-48(s^2 f_{BB}+c^2 f_{WW}) + 2\left(\frac{m_H}{m_Z}\right)^2[(2s^2+1)f_B + (2c^2+1)f_W + 12(s^2 f_{BB}+c^2 f_{WW})]\\
&+\left(\frac{m_H}{m_Z}\right)^4[(2s^2+1)f_B+(2c^2+1)f_W]\left(\frac{m_H}{m_Z}\right)\sqrt{4-\left(\frac{m_H}{m_Z}\right)^2}\cos^{-1}\left(\frac{m_H}{2m_Z}\right)\\
&\left.-2[(32c^2+1)f_B + 36c^2 f_{WWW}g^2 - (32c^4+8c^2-1)f_W]\sqrt{4c^2-1}\sin^{-1}\left(\frac{1}{2c}\right)\right\}\\
&+\frac{f_{\Phi,2}}{\Lambda^2}\frac{m_W^2 s^2}{72\pi^2\alpha}\left[79-27\left(\frac{m_H}{m_Z}\right)^2+6\left(\frac{m_H}{m_Z}\right)^4\right.\\
&\left.-6\left(12-4\left(\frac{m_H}{m_Z}\right)^2+\left(\frac{m_H}{m_Z}\right)^4\right)\left(\frac{m_H}{m_Z}\right)\sqrt{4-\left(\frac{m_H}{m_Z}\right)^2}\cos^{-1}\left(\frac{m_H}{2m_Z}\right)\right],\end{aligned} \tag{D1}$$

$$R_{S2} = \frac{m_Z^2}{12\pi\Lambda^2}\{-(1+30c^2)f_B - (1-10c^2)f_W - 36c^2g^2f_{WWW}\}, \tag{D2}$$

$$\begin{aligned} R_{S3} = {} & \frac{m_H^2}{24\pi\Lambda^2}\left\{(f_B+f_W)\left(\frac{m_H^4}{m_Z^4}-12\right) + 2(s^2f_B+c^2f_W)\left(\frac{m_H^4}{m_Z^4}+12-18\frac{m_H^2}{m_H^2-m_Z^2}\right)\right. \\ & \left. + 24(s^2f_{BB}+c^2f_{WW})\left(\frac{m_H^2-4m_Z^2}{m_Z^2}\right)\right\} \\ & - \frac{f_{\Phi,2}}{\Lambda^2}\frac{m_Z^4s^2c^2}{12\pi^2\alpha(m_H^2-m_Z^2)} \times \left(\frac{m_H}{m_Z}\right)^2\left[\left(\frac{m_H}{m_Z}\right)^6 - 7\left(\frac{m_H}{m_Z}\right)^4 + 24\left(\frac{m_H}{m_Z}\right)^2 - 36\right], \end{aligned} \tag{D3}$$

$$R_{T1} = \frac{m_Z^2}{32\pi\Lambda^2}\left\{15f_B + \left(\frac{3-23c^2}{s^2}\right)f_W\right\} + \frac{5m_H^2}{32\pi c^2\Lambda^2}f_B - \frac{5m_Z^2s^2}{32\pi^2\alpha\Lambda^2}f_{\Phi,2}, \tag{D4}$$

$$R_{T2} = \frac{m_W^2}{8\pi s^2\Lambda^2}\left\{5f_B - 3\left(\frac{m_W^2}{m_H^2-m_W^2} + \frac{2(c^2+4)}{3s^2}\right)f_W\right\} + \frac{3m_W^4}{8\pi^2\alpha\Lambda^2(m_H^2-m_W^2)}f_{\Phi,2}, \tag{D5}$$

$$R_{T3} = -\frac{3}{8\pi c^2}\frac{m_H^4}{\Lambda^2(m_H^2-m_Z^2)}\left\{f_B + \frac{m_W^2}{m_H^2-m_W^2}f_W\right\} + \frac{3m_H^4m_Z^2s^2}{8\pi^2\alpha\Lambda^2(m_H^2-m_Z^2)(m_H^2-m_W^2)}f_{\Phi,2}, \tag{D6}$$

$$\begin{aligned} R_{U1} = {} & \frac{m_Z^2}{24\pi\Lambda^2c^4}\left\{\left[48c^6f_{WW} - \left(\frac{m_H}{m_Z}\right)^2(2s^2c^4f_B + 2(c^4+c^6)f_W + 24c^6f_{WW})\right.\right. \\ & \left. - \left(\frac{m_H}{m_Z}\right)^4(s^2c^4f_B + (c^4+c^6)f_W)\right]\left(\frac{m_H}{m_Z}\right)\sqrt{4-\left(\frac{m_H}{m_Z}\right)^2}\cos^{-1}\left(\frac{m_H}{2m_Z}\right) \\ & + 4\left[-24c^4f_{WW} + 2\left(\frac{m_H}{m_Z}\right)^2c^2\left(f_W+6f_{WW}\right) + \left(\frac{m_H}{m_Z}\right)^4f_W\right]\left(\frac{m_H}{m_Z}\right)\sqrt{4c^2-\left(\frac{m_H}{m_Z}\right)^2}\cos^{-1}\left(\frac{m_H}{2m_W}\right) \\ & - 2[(80c^8+116c^6+90c^4+22c^2-11)f_W + 216f_{WWW}g^2]\sqrt{4c^2-1}\sin^{-1}\left(\frac{1}{2c}\right) \\ & + 2\left[(-40c^4+6c^2+1)\frac{s^2}{c^2}f_B + 36c^2(4c^2-1)f_{WWW}g^2\right]\sqrt{4c^2-1}\cos^{-1}\left(\frac{1}{2c}\right) \\ & + \left[\frac{2}{3}c^2(240c^6-121c^4-62c^2+33) + 11\pi\sqrt{4c^2-1}(8c^4+2c^2-1)\right]f_W \\ & - (84c^6-95c^4+9c^2+2)f_B + 96s^2c^4f_{WWW}g^2 + 3\left(\frac{m_H}{m_Z}\right)^2s^2c^4(f_B-f_W-16f_{WW}) \\ & \left. + 2\left(\frac{m_H}{m_Z}\right)^4s^2c^2[c^2f_B-(c^2+2)f_W]\right\} + \frac{f_{\Phi,2}}{\Lambda^2}\frac{m_Z^2s^2}{24\pi^2\alpha c^4}\left(\frac{m_H}{m_Z}\right)\left[-9c^4s^2\left(\frac{m_H}{m_Z}\right) + 2c^2s^2(1+c^2)\left(\frac{m_H}{m_Z}\right)^3\right. \\ & + 2c^6\left[\left(\frac{m_H}{m_Z}\right)^4 - 4\left(\frac{m_H}{m_Z}\right)^2 + 12\right]\sqrt{4-\left(\frac{m_H}{m_Z}\right)^2}\cos^{-1}\left(\frac{m_H}{2m_Z}\right) \\ & \left. - 2\left[12c^4 - 4c^2\left(\frac{m_H}{m_Z}\right)^2 + \left(\frac{m_H}{m_Z}\right)^4\right]\sqrt{4c^2-\left(\frac{m_H}{m_Z}\right)^2}\cos^{-1}\left(\frac{m_H}{2m_W}\right)\right], \end{aligned} \tag{D7}$$

$$\begin{aligned}R_{U2} &= \frac{1}{12\pi c^2\Lambda^2 m_W^4}\Big\{-24c^2 f_{WW} m_W^2(2m_W^4 - 4m_H^2 m_W^2 + m_H^4) + 36(1-6c^2) f_{WWW} g^2 m_Z^2 m_W^4 \\ &+ (34c^8 - 118c^6 + 58c^4 - 3c^2 - 1) f_B m_Z^6 \\ &+ \frac{c^2 f_W}{s^2(m_W^2 - m_H^2)}(2m_H^2 s^2(-3m_W^6 - 6m_H^2 m_W^4 - m_H^4 m_W^2 + m_H^6) \\ &+ (18c^8 - 146c^6 + 24c^4 + 55c^2 - 11) m_Z^6 (m_H^2 - m_W^2))\Big\} \\ &+ \frac{m_Z^4 s^2}{12\pi^2\alpha\Lambda^2 c^4 (m_H^2 - m_W^2)}\left[2c^8 - 38c^6\left(\frac{m_H}{m_Z}\right)^2 + 24c^4\left(\frac{m_H}{m_Z}\right)^4 - 7c^2\left(\frac{m_H}{m_Z}\right)^6 + \left(\frac{m_H}{m_Z}\right)^8\right] f_{\Phi,2}, \end{aligned} \tag{D8}$$

$$\begin{aligned}R_{U3} &= \frac{m_H^2 s^2}{12\pi\Lambda^2}\Big\{f_{WW}\frac{24((c^2+1)m_H^2 - 4m_W^2)}{m_W^2} + f_W\left[\frac{18m_Z^2 m_W^2}{(m_H^2 - m_Z^2)(m_H^2 - m_W^2)} + \frac{(c^4 + 2c^2 + 2)m_H^4}{m_W^4} - 6\right] \\ &+ f_B\frac{m_H^6 - m_H^4 m_Z^2 - 6m_H^2 m_Z^4 - 12m_Z^6}{m_Z^6 - m_H^2 m_Z^4}\Big\} + \frac{m_Z^6 s^4}{12\pi^2\alpha\Lambda^2 c^4 (m_H^2 - m_W^2)(m_H^2 - m_Z^2)}\left(\frac{m_H}{m_Z}\right)^2 \\ &\times\Bigg[-(c^4 + c^2 + 1)\left(\frac{m_H}{m_Z}\right)^8 + (c^6 + 8c^4 + 8c^2 + 1)\left(\frac{m_H}{m_Z}\right)^6 \\ &- c^2(7c^4 + 31c^2 + 7)\left(\frac{m_H}{m_Z}\right)^4 + 24c^4(c^2+1)\left(\frac{m_H}{m_Z}\right)^2 - 36c^6\Bigg] f_{\Phi,2}. \end{aligned} \tag{D9}$$

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