Yukawa corrections to Higgs production in top partner models

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Higgs production from gluon fusion is sensitive to the properties of heavy colored fermions and to the Yukawa couplings, $\frac{Y_F M_F}{v}$, of these particles to the Higgs boson. We compute the two-loop, $\mathcal{O}((\frac{Y_F M_F}{v})^3)$ contributions of new high mass fermions to Higgs production. In the Standard Model, these contributions are part of the well-known electroweak corrections and are negligible. However, in models with TeV scale fermions, such as top partner or composite models, Yukawa corrections are enhanced by effects of $\mathcal{O}((\frac{Y_F M_F}{v})^3)$ and are potentially significant due to the large mass of the new quarks. We examine the size of these top partner Yukawa corrections to Higgs production for parameter choices which are allowed by precision electroweak constraints.

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I. INTRODUCTION

The discovery of a 126 GeV Higgs boson leads to the question of whether this particle is the single scalar field predicted by the Standard Model (SM), or whether it is the remnant of some more complicated theory. Composite models [1–3] and models where the Higgs is a pseudo-Goldstone boson of a broken symmetry such as little Higgs models [4–7] typically contain new heavy fermions which are not present in the Standard Model. These fermions can mix with the observed quarks and contribute to Higgs production and decay. The properties of new charged $-\frac{1}{3}$ fermions which can mix with the Standard Model b quark are greatly restricted by measurements of $Z \to b\bar{b}$ decays [8,9] and so we will concentrate on fermionic top partners which can mix with the Standard Model top quark. Heavy charged $\frac{2}{3}$ fermions have been searched for at the LHC, and depending on their decay modes, are restricted to be heavier than 600–700 GeV [10]. The properties of these potential new heavy fermions are also strongly constrained both by precision electroweak measurements [11–19], and by the requirement that the Higgs production rate, $gg \to H$, be close to the measured value [20–25]. Precision measurements of the Higgs production and decay rates offer a window into this possible new high scale physics, and in this paper we focus on quantifying the predictions of top partner models.

New heavy fermions can contribute to both $gg \to H$ and $H \to \gamma\gamma$ and, because of the large top partner mass, M_F, the Higgs-fermion Yukawa couplings, $\frac{Y_F M_F}{v}$, may generate large $\mathcal{O}((\frac{Y_F M_F}{v})^3)$ contributions at two loops. We compute the effects of the two-loop Yukawa couplings of top partners to Higgs production from gluon fusion using the low energy theorems valid in the $M_H \ll 2M_F$ limit [26,27]. These corrections are part of the complete two-loop electroweak corrections to Higgs production from gluon fusion. For the Standard Model, the Yukawa corrections have been known for some time [28,29] along with the complete two-loop electroweak corrections [30–33]. The full electroweak corrections are also known for a degenerate fourth generation of heavy fermions which does not mix with the Standard Model fermions [34]. The physical top mass, $m_t = 173$ GeV, is not large enough for the Yukawa corrections to be the dominant contribution to the electroweak corrections, but for large top masses (say $m_t \sim 700$ GeV), the Yukawa corrections would become the most significant contribution to the two-loop electroweak effect. Therefore, in top partner models where the fermion mass is at the TeV scale, these $\mathcal{O}((\frac{Y_F M_F}{v})^3)$ Yukawa corrections may be numerically significant.

Technical details of our calculation are contained in Sec. II. We begin with a review of the low energy theorem as applied to the two-loop Yukawa corrections to Higgs production and include a discussion of renormalization and our technique for expanding two-loop integrals. We demonstrate the validity of our techniques by reproducing the SM result for the Yukawa corrections to $gg \to H$ in Sec. III. Our new results are in Sec. IV, where we consider the class of models that contains a top partner which is an $SU(2)_L$ singlet that mixes with the Standard Model top quark. In Sec. IV D, we discuss the relevance of our results for Higgs precision measurements and the search for new physics effects through the measurement of Higgs properties.

II. CALCULATION TECHNIQUES

We are interested in the two-loop $\mathcal{O}((\frac{Y_F M_F}{v})^3)$ contributions to the gluon fusion production of a Higgs boson, where F is a heavy-quark ($M_F \gg M_H/2$) coupling to the Higgs boson. Since direct searches for top partners require $M_F > 700$ GeV [10], these contributions can potentially give effects enhanced by powers of M_F. The interactions of the heavy quarks with the Higgs boson are parametrized as

$$-\mathcal{L}_Y = \sum_F M_F^0 \left(1 + Y_F^0 \frac{H^0}{v^0}\right) \bar{F}^0 F^0, \tag{1}$$

where the superscript 0 denotes the unrenormalized values of the parameters. In the Standard Model, $Y_t = 1$.

A. Low energy theorem

We use the low energy theorem to compute the leading contribution in $\frac{M_H^2}{M_F^2}$ to the $gg \to H$ process. For a soft Higgs boson, $p_H \to 0$, the amplitude $\mathcal{A}_{gg\to H}$ is related to the gluon vacuum polarization amplitude, $\mathcal{A}_{gg} = -i\Pi_{\mu\nu}^{AB}$ [26,35–37],

$$\lim_{p_H\to 0} \mathcal{A}_{gg\to H} = \frac{1}{v^0} \sum_F Y_F^0 M_F^0 \frac{i\partial}{\partial M_F^0} \mathcal{A}_{gg}. \tag{2}$$

This is equivalent to inserting an additional heavy-quark propagator with the emission of a zero-momentum Higgs boson. (We have assumed that we are working with the quark mass eigenstates.) The differentiation is performed on the bare masses coming from propagators, while mass-dependent couplings are to be treated as constants. The renormalization is performed after taking the derivatives. It is straightforward to extend this approach to loop corrections to Higgs production [37–43].

At one loop, the gluon polarization tensor $\Pi_{AB}^{\mu\nu}(p^2)$ is

$$\Pi_{AB}^{\mu\nu\ 1L}(p^2) = \frac{\alpha_s^0}{\pi} \delta_{AB}(g^{\mu\nu}p^2 - p^\mu p^\nu)[N] \times \sum_F \left\{ (M_F^0)^{-2\epsilon} \left[\frac{1}{6\epsilon} + \mathcal{O}\left(\frac{p^2}{(M_F^0)^2}, \epsilon \right) \right] \right\}. \tag{3}$$

The amplitude for $gg \to H$ from Eq. (2) is

$$\mathcal{A}_{gg\to H}^{1L} = -\frac{\alpha_s^0}{3\pi v^0} \delta_{AB}(g^{\mu\nu}p^2 - p^\mu p^\nu)[N] \sum_F Y_F^0 (M_F^0)^{-2\epsilon} \equiv \Sigma_F \mathcal{A}_{gg\to H}^{1L,F}, \tag{4}$$

where

$$[N] = \Gamma(1+\epsilon)(4\pi\mu^2)^\epsilon \overset{\epsilon\to 0}{\to} 1, \tag{5}$$

and the sum is over all heavy fermions.

B. Techniques for two-loop integrals

We are interested in the two-loop contributions to the gluon two-point function which are enhanced by powers of the Yukawa couplings and so we neglect the $\mathcal{O}(g^2)$ contributions from W and Z exchange. In Landau gauge, the Goldstone bosons are massless and couple with Yukawa strength to the massive fermions, and so are included in the calculation. The $\mathcal{O}((\frac{Y_F M_F}{v})^3)$ contributions form a gauge invariant subset of the complete two-loop electroweak corrections.

Each of the diagrams has a contribution of the form (for external momentum p),

$$\Pi_i^{\mu\nu}(p^2) = a_i g^{\mu\nu} + b_i p^\mu p^\nu. \tag{6}$$

Gauge invariance requires that $\sum_i b_i = 0$. The coefficients are found by taking contractions with $g^{\mu\nu}$ and $p^\mu p^\nu$:

$$a_i = \frac{1}{d-1} \Pi_i^{\mu\nu}(p^2) \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right), \tag{7}$$

$$b_i = -\frac{1}{p^2(d-1)} \Pi_i^{\mu\nu}(p^2) \left(g_{\mu\nu} - d\frac{p_\mu p_\nu}{p^2} \right), \tag{8}$$

with $d = 4 - 2\epsilon$.

The strategy is to expand the loop integrals in powers of external momentum over the heavy mass scale in the loop, M_F. The numerators of the integrals have the form,

$$(k_1 \cdot p)^j (k_2 \cdot p)^m \times (\text{powers of } k_1^2, k_2^2, k_1 \cdot k_2), \tag{9}$$

where k_1, k_2 are the loop momenta. These integrals can be symmetrized using the techniques in the Appendix of Ref. [44].

In the limit where all the fermions in the loop are much heavier than the external mass scale [which will generically be of $\mathcal{O}(p^2 \sim M_H^2)$], we can calculate the two-loop integrals by expanding in powers of $\frac{p^2}{M_F^2}$ and retaining the leading term. Due to the small-momentum expansion, the integrals that we need to compute are all two-loop vacuum bubbles. If the Higgs and Goldstone boson interactions do not mix quarks with the same quantum number, as in the Standard Model and its four-generation extension, the vacuum bubbles only depend on one heavy mass scale, M_F. Their general form is

$$B(M_F, M_F, 0; n_1, n_2, 1) = \int \frac{d^d k_1}{(2\pi)^d} \int \frac{d^d k_2}{(2\pi)^d} \frac{1}{(k_1^2 - M_F^2)^{n_1} (k_2^2 - M_F^2)^{n_2} (k_1 + k_2)^2}. \tag{10}$$

Explicit expressions for these integrals are given in Refs. [44–47]. Alternatively, one can use integration by part identities to reduce these integrals to the master integral [44–47]

$$B(M_F, M_F, 0; 1, 1, 1) = -\frac{M_F^{2-4\epsilon}}{(4\pi)^4} [N]^2 \left(\frac{1}{\epsilon^2} + \frac{3}{\epsilon} + 7 \right). \tag{11}$$

We obtain these relations with the program AIR [48].

If more quarks with the same quantum numbers are present, and the Higgs and Goldstone boson interactions mix them, we also have the "off-diagonal" contribution where both the heavy quarks plus the boson run in the loops. We need the additional two-loop, two-masses scalar master integral,

$$B(M_F, M_{F'}, 0; 1, 1, 1) = \int \frac{d^d k_1}{(2\pi)^d} \frac{d^d k_2}{(2\pi)^d} \frac{1}{k_1^2 - M_F^2} \frac{1}{k_2^2 - M_{F'}^2} \frac{1}{(k_1 + k_2)^2}, \tag{12}$$

where M_F, $M_{F'}$ are the masses of the two heavy quarks. In the literature the integral with three massive lines is known [47],

$$\begin{aligned} B(M_F, M_{F'}, m; 1, 1, 1) &= \int \frac{d^d k_1}{(2\pi)^d} \frac{d^d k_2}{(2\pi)^d} \frac{1}{k_1^2 - M_F^2} \frac{1}{k_2^2 - M_{F'}^2} \frac{1}{(k_1 + k_2)^2 - m^2} \\ &= \frac{1}{2} \frac{m^{2-4\epsilon}}{(4\pi)^4} \frac{[N]^2}{(1-\epsilon)(1-2\epsilon)} \left\{ -\frac{1}{\epsilon^2}(1 + x + y) + \frac{2}{\epsilon}(x \ln x + y \ln y) - x \ln^2 x - y \ln^2 y \right. \\ &\left. \quad + (1 - x - y) \ln x \ln y - \lambda^2(x, y) \Phi^{(1)}(x, y) \right\}. \end{aligned} \tag{13}$$

The functions $\lambda^2(x, y)$ and $\Phi^{(1)}(x, y)$ are

$$\begin{aligned} \lambda(x, y) &= \sqrt{(1-x-y)^2 - 4xy}, \\ \Phi^{(1)}(x, y) &= \frac{1}{2\lambda} \{ 4\mathrm{Li}_2(1 - z_1) + 4\mathrm{Li}_2(1 - z_2) + 4\mathrm{Li}_2(1 - z_3) + \ln^2 z_1 + \ln^2 z_2 + \ln^2 z_3 + 2 \ln x \ln z_1 + 2 \ln y \ln z_2 \}, \end{aligned} \tag{14}$$

with

$$z_1 = \frac{(\lambda + x - y - 1)^2}{4y}, \quad z_2 = \frac{(\lambda + y - 1 - x)^2}{4x}, \quad z_3 = \frac{(\lambda + 1 - x - y)^2}{4xy}, \tag{15}$$

and $x = M_F^2/m^2$, $y = M_{F'}^2/m^2$. Using this result, we compute the two-loop gluon self-energy retaining the dependence on all three masses. We then take the limit $m \to 0$.

The virtual two-loop results for $g \to g$ depend on the specific model and will be given later. The two-loop contributions to $gg \to H$ from the heavy fermion loops are then found by applying the low energy theorem of Eq. (2).

C. Renormalization

Renormalization of the $gg \to H$ amplitude requires the quark mass and wave function counterterms, the Higgs wave function counterterm, and the $F\bar{F}g$ and $F\bar{F}H$ vertex counterterms. The quark wave function renormalization, $Z_{2,F}$, cancels against other counterterms and we do not need to compute it explicitly. We briefly review the renormalization of the quark mass. We start from the bare Lagrangian,

$$\mathcal{L} = \bar{F}^0 (i\not{\partial} - M_F^0) F^0 - g_s^0 \bar{F}^0 \gamma^\mu t^a G_\mu^{0,a} F^0. \tag{16}$$

The superscript "0" denotes bare fields and couplings which are related to the renormalized ones by the renormalization constants,

$$\begin{aligned} G_\mu^{0,a} &= \sqrt{Z_3} G_\mu^a, \quad & F^0 &= \sqrt{Z_{2,F}} F = \left(1 + \tfrac{\delta Z_{2,F}}{2}\right) F, \\ g_s^0 &= \tfrac{Z_1}{Z_{2,F}\sqrt{Z_3}} g_s, \quad & M_F^0 &= Z_M M_F = \left(1 + \tfrac{\delta M_F}{M_F}\right) M_F. \end{aligned} \tag{17}$$

With these conventions the $F\bar{F}g$ vertex is renormalized by Z_1 and due to the Ward identities, $Z_1 = Z_{2,F}$. The quark propagator counterterm is

$$\delta_F^{ct} = i[(\not{p} - M_F)\delta Z_{2,F} - \delta M_F]. \tag{18}$$

We require the renormalized quark propagator $-i\Sigma(M_F, \not{p})$ (including the counterterms) to be canonically normalized and to have a pole at the renormalized mass,

$$\Sigma(M_F, \not{p} = M_F) = 0, \qquad \Sigma'(M_F, \not{p})|_{\not{p} = M_F} = 0, \tag{19}$$

yielding, at one loop,

$$\begin{aligned} \delta M_F &= -\Sigma_{1L}(M_F, \not{p} = M_F), \\ \delta Z_{2,F} &= \Sigma_{1L}{}'(M_F, \not{p})|_{\not{p} = M_F}, \end{aligned} \tag{20}$$

where the sum of all the one-loop one-particle irreducible (1PI) insertions into the quark propagator is denoted as $-i\Sigma_{1L}(M_F, \not{p})$.

We now turn to the $F\bar{F}H$ vertex counterterm. The interaction of a fermion F with the Higgs boson is

$$\mathcal{L}_Y = -\frac{H^0}{v^0} Y_F M_F^0 \bar{F}^0 F^0 = -\frac{g^0}{2M_W^0} H^0 Y_F M_F^0 \bar{F}^0 F^0. \tag{21}$$

The Y_F coupling and g receive no $\mathcal{O}(Y_F^2)$ renormalization to the order in which we are working [29,49]. We introduce the renormalization constants,

$$H^0=\sqrt{Z_H}H=\left(1+\frac{\delta Z_H}{2}\right)H,\quad (M_W^0)^2=M_W^2\left(1+\frac{\delta M_W^2}{M_W^2}\right). \tag{22}$$

In terms of the renormalized quantities, Eq. (21) becomes

$$\mathcal{L}=-\frac{g}{2M_W}HY_FM_F\bar{F}F\left(1+\frac{\delta M_F}{M_F}+\delta Z_{2,F}+\delta_3\right), \tag{23}$$

with

$$\delta_3=\left(\frac{\delta Z_H}{2}-\frac{1}{2}\frac{\delta M_W^2}{M_W^2}\right). \tag{24}$$

The Higgs wave function renormalization is computed from the sum of all 1PI insertions into the Higgs propagator, $-i\Pi_H(p^2)$,

$$Z_H=\left[1-\Pi_H'(p^2)|_{p^2=(M_H^0)^2}\right]^{-1}, \tag{25}$$

which at one-loop order yields

$$\delta Z_H=\Pi_H'(p^2)|_{p^2=(M_H^0)^2}. \tag{26}$$

Similarly, the W-mass renormalization can be computed from the sum of all 1PI insertions into the W propagator.

We now combine these result to obtain the two-loop $\mathcal{O}((\frac{Y_FM_F}{v})^3)$ counterterms for the $gg\to H$ amplitude in the limit $M_H << 2M_F$. We have

(i) from the quark mass counterterms on the internal legs,

$$\mathcal{A}_{M_F}^{2L,ct}=\sum_F\left[\left(i\frac{\partial\mathcal{A}_{gg\to H}^{1L,F}}{\partial M_F}\right)(-i\delta M_F)\right]$$
$$=\sum_F\left[\left(M_F\frac{\partial\mathcal{A}_{gg\to H}^{1L,F}}{\partial M_F}\right)\left(\frac{\delta M_F}{M_F}\right)\right]. \tag{27}$$

The derivative of the one-loop amplitude with respect to the mass of the fermion gives an extra fermion propagator, upon which we insert the mass counterterm. We emphasize that the mass counterterm for the fermion F needs to be inserted only upon the one-loop amplitude containing that fermion. As in the low energy theorem of Eq. (2), the derivatives only act on mass terms coming from the internal propagators, and not on the masses from the Yukawa interactions. We should notice that in the result, Eq. (4), there is cancellation between a mass from the Yukawa vertex, $Y_F\frac{M_F^0}{v^0}$, and a mass from the propagator. With this in mind,

$$M_F\frac{\partial\mathcal{A}_{gg\to H}^{1L,F}}{\partial M_F}=-(2\epsilon+1)\mathcal{A}_{gg\to H}^{1L,F}, \tag{28}$$

(ii) from the quark wave function renormalization,

$$\mathcal{A}_{Z_{2,F}}^{2L,ct}=3\sum_F[(i\mathcal{A}_{gg\to H}^{1L,F})(i\delta Z_{2,F})]$$
$$=-3\sum_F[\delta Z_{2,F}\mathcal{A}_{gg\to H}^{1L,F}], \tag{29}$$

(iii) from the $F\bar{F}g$ vertex counterterm,

$$\mathcal{A}_{Z_1}^{2L,ct}=2\sum_F[\delta Z_{2,F}\mathcal{A}_{gg\to H}^{1L,F}], \tag{30}$$

(iv) from the $F\bar{F}H$ vertex of Eq. (23),

$$\mathcal{A}_{Z_Y}^{2L,ct}=\sum_F\left[\mathcal{A}_{gg\to H}^{1L,F}\left(\frac{\delta M_F}{M_F}+\delta Z_{2,F}+\delta_3\right)\right]. \tag{31}$$

Combining these results we obtain

$$\mathcal{A}_{gg\to H}^{2L,ct}=\sum_F\left\{\left[-(2\epsilon+1)\frac{\delta M_F}{M_F}+\left(\frac{\delta M_F}{M_F}+\delta_3\right)\right]\mathcal{A}_{gg\to H}^{1L,F}\right\} \tag{32}$$

$$=\sum_F\left[\left(-2\epsilon\frac{\delta M_F}{M_F}+\delta_3\right)\mathcal{A}_{gg\to H}^{1L,F}\right]. \tag{33}$$

Since the one-loop result is finite, the counterterm receives a finite contribution from the pole of the quark mass renormalization and divergencies in the counterterm can only come from δ_3.

III. RESULTS

A. Standard Model

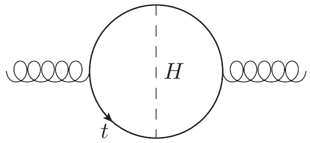
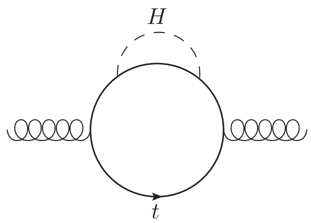
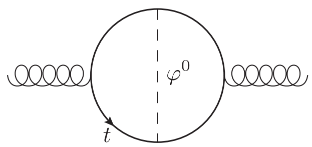
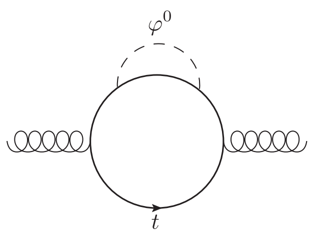
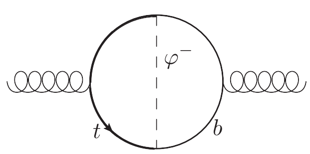
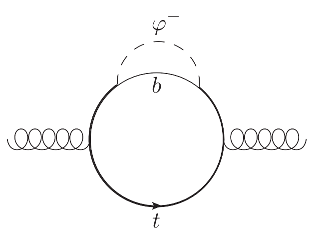
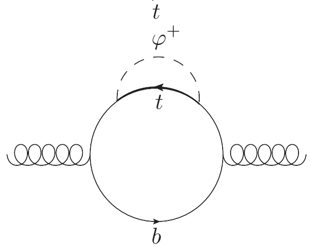
As a check of our technique, we reproduce the well-known $\mathcal{O}(\frac{m_t^3}{v^3})$ contributions to the gluon two-point function and to the $gg\to H$ amplitude in the limit $M_H\to 0$ [28,29]. We compute, for each diagram, the contractions with $g^{\mu\nu}$ and $p^\mu p^\nu$, which are shown in Table 1. The Standard Model with a massless b quark corresponds to $m_b=Y_b=0$, $Y_t=1$, and as a shorthand notation we define $\frac{m_t}{v}\equiv y_t$. A massless b quark first enters at two loops. The terms of $\mathcal{O}(\frac{p^4}{m_t^4})$ do not enter into our final results, but are included as a check of our method and demonstration of gauge invariance.

The diagrams where the bosons (H, $\varphi^\pm$, φ^0) propagate on a leg (rows 2, 4, 6, and 7 of Table I) have a symmetry factor of 2. The sum of the entries in Table I is gauge invariant and gives the Standard Model result

$$\Pi_{AB}^{\mu\nu,2L}|_{SM}=\frac{\alpha_s}{16\pi^3}\delta_{AB}(g^{\mu\nu}p^2-p^\mu p^\nu)[N]^2\frac{y_t^2}{3}m_t^{-4\epsilon}, \tag{34}$$

with $[N]$ defined according to Eq. (5). This result is finite and therefore, using the low energy theorem of the

TABLE I. Individual results for the Standard Model contractions. There is a prefactor $\mathcal{F}_t = -\delta^{ab}(g^{\mu\nu} - p^\mu p^\nu/p^2)\frac{\alpha_s}{16\pi^3}[N]^2 m_t^{2-4\epsilon} y_t^2$, with $y_t = m_t/v$.

	$g_{\mu\nu}$ contraction	$p_\mu p_\nu/p^2$ contraction
	$\mathcal{F}_t[1 + \frac{3}{\epsilon} - \frac{p^2}{m_t^2}(\frac{157}{72} - \frac{1}{12\epsilon} + \frac{1}{4\epsilon^2}) - \frac{p^4}{m_t^4}(\frac{509}{1350} + \frac{1}{10\epsilon})]$	$\mathcal{F}_t[\frac{5}{8} + \frac{3}{4\epsilon} + \frac{p^2}{m_t^2}(\frac{1}{48} - \frac{1}{24\epsilon}) - \frac{p^4}{m_t^4}\frac{1}{240}]$
	$\mathcal{F}_t[-\frac{1}{2} - \frac{3}{2\epsilon} + \frac{p^2}{m_t^2}(\frac{161}{288} - \frac{29}{48\epsilon} + \frac{1}{8\epsilon^2}) - \frac{p^4}{m_t^4}(\frac{11}{75} + \frac{1}{10\epsilon})]$	$\mathcal{F}_t[-\frac{5}{16} - \frac{3}{8\epsilon} - \frac{p^2}{m_t^2}(\frac{1}{96} - \frac{1}{48\epsilon}) + \frac{p^4}{m_t^4}\frac{1}{480}]$
	$\mathcal{F}_t[3 + \frac{1}{\epsilon} + \frac{p^2}{m_t^2}(\frac{5}{24} + \frac{1}{12\epsilon} - \frac{1}{4\epsilon^2}) + \frac{p^4}{m_t^4}(\frac{19}{180} - \frac{1}{10\epsilon})]$	$\mathcal{F}_t[\frac{7}{8} + \frac{1}{4\epsilon} - \frac{p^2}{m_t^2}(\frac{5}{144} + \frac{1}{24\epsilon}) - \frac{p^4}{m_t^4}\frac{1}{144}]$
	$\mathcal{F}_t[-\frac{3}{2} - \frac{1}{2\epsilon} + \frac{p^2}{m_t^2}(-\frac{5}{96} + \frac{19}{48\epsilon} + \frac{1}{8\epsilon^2}) + \frac{p^4}{m_t^4}(\frac{1}{18} + \frac{1}{10\epsilon})]$	$\mathcal{F}_t[-\frac{7}{16} - \frac{1}{8\epsilon} + \frac{p^2}{m_t^2}(\frac{5}{288} + \frac{1}{48\epsilon}) + \frac{p^4}{m_t^4}\frac{1}{288}]$
	$\mathcal{F}_t[2 + \frac{1}{\epsilon} - \frac{p^2}{m_t^2}(\frac{3}{16} - \frac{7}{24\epsilon}) + \frac{p^4}{m_t^4}(\frac{167}{2160} + \frac{17}{360\epsilon})]$	$\mathcal{F}_t[\frac{5}{8} + \frac{1}{4\epsilon} - \frac{p^2}{m_t^2}(\frac{11}{144} + \frac{1}{24\epsilon}) - \frac{1}{72}\frac{p^4}{m_t^4}]$
	$\mathcal{F}_t[-1 - \frac{1}{2\epsilon} + \frac{p^2}{m_t^2}(\frac{7+4\pi^2}{96} - \frac{5}{48\epsilon} + \frac{1}{8\epsilon^2}) - \frac{7}{180}\frac{p^4}{m_t^4}]$	$\mathcal{F}_t[-\frac{5}{16} - \frac{1}{8\epsilon} + \frac{p^2}{m_t^2}(\frac{11}{288} + \frac{1}{48\epsilon}) + \frac{1}{144}\frac{p^4}{m_t^4}]$
	$\mathcal{F}_t[-1 - \frac{1}{2\epsilon} - \frac{p^2}{m_t^2}(\frac{4\pi^2-9}{96} + \frac{1}{16\epsilon} + \frac{1}{8\epsilon^2}) - \frac{1}{18}\frac{p^4}{m_t^4}]$	$\mathcal{F}_t[-\frac{5}{16} - \frac{1}{8\epsilon} + \frac{p^2}{m_t^2}(\frac{11}{288} + \frac{1}{48\epsilon}) + \frac{1}{144}\frac{p^4}{m_t^4}]$

previous section, the two-loop $\mathcal{O}(y_t^3)$ contribution to the ggH amplitude is of order $\mathcal{O}(\epsilon)$.

The terms needed for the renormalization of the one-loop amplitude are [29,49,50]

$$\frac{\delta m_t}{m_t}|_{SM} = \frac{y_t^2}{32\pi^2}[N]m_t^{-2\epsilon}\left(\frac{3}{\epsilon}+8\right),$$
$$\frac{\delta Z_H}{2}|_{SM} = -\frac{N_C}{16\pi^2}y_t^2[N]m_t^{-2\epsilon}\left[\frac{1}{\epsilon}-\frac{2}{3}+\mathcal{O}(\epsilon^2)\right],$$
$$\frac{\delta M_W^2}{M_W^2}|_{SM} = -\frac{N_C}{8\pi^2}\frac{m_t^2}{v^2}[N]m_t^{-2\epsilon}\left(\frac{1}{\epsilon}+\frac{1}{2}\right), \quad (35)$$

where $N_C = 3$. Therefore, from Eq. (24),

$$\delta_3|_{SM} = \frac{7}{6}\frac{N_C}{16\pi^2}\frac{m_t^2}{v^2}, \quad (36)$$

and the final two-loop $\mathcal{O}(y_t^3)$ contribution is

$$\mathcal{A}^{2L}_{gg\to H}|_{SM} = \mathcal{A}^{2L,ct}_{gg\to H}|_{SM} = \frac{m_t^2}{16\pi^2 v^2}\left[\frac{7}{6}N_C - 3\right]\mathcal{A}^{1L}_{gg\to H}|_{SM}$$
$$= 0.0016\left(\frac{m_t}{173\text{ GeV}}\right)^2\mathcal{A}^{1L}_{gg\to H}|_{SM}, \quad (37)$$

with

$$\mathcal{A}^{1L}_{gg\to H}|_{SM} = -\frac{\alpha_s}{3\pi v}\delta_{AB}(g^{\mu\nu}p^2 - p^\mu p^\nu). \quad (38)$$

Equation (37) agrees with the results of Refs. [28,29].

The results of Eq. (37) are to be compared with the total electroweak contribution to $gg\to H$ [28,30–32,51]. Assuming that the QCD and electroweak interactions factorize [33], the electroweak effects increase the total cross section by ~5% at the LHC [52]. The dominant role is played by light-fermion loops. The contribution from the top quark, also beyond the infinite-mass approximation, is just a few % of the light-quark contribution [31]. In order for the $\mathcal{O}(y_t^3)$ contributions to the cross section to be $\mathcal{O}(5\%)$, we would have required $m_t \sim 700$ GeV, suggesting that in models with heavy fermions the two-loop Yukawa corrections might be the dominant electroweak contribution [34]. We will examine this possibility in the following section.

IV. TOP PARTNER SINGLET MODEL

A. The model

We consider a model with an additional vectorlike charge $\frac{2}{3}$ quark, $\mathcal{T}^2$, which mixes with the Standard Model top quark [12–14,20,53–55].

For simplicity we make the following assumptions:

(i) The electroweak gauge group is the standard $SU(2)_L \times U(1)_Y$ group.

(ii) There is only a single Standard Model Higgs $SU(2)_L$ doublet, Φ.

(iii) We neglect generalized Cabibbo-Kobayashi-Maskawa mixing and only allow mixing between the Standard-Model-like top quark and the new charge $\frac{2}{3}$ singlet quark.

The Standard-Model-like fermions are

$$\psi_L^1 = \begin{pmatrix}\mathcal{T}_L^1\\ b_L\end{pmatrix}, \quad \mathcal{T}_R^1, b_R, \quad (39)$$

with the Lagrangian describing fermion masses,

$$-\mathcal{L}_M^{SM} = \lambda_1\bar{\psi}_L^1\Phi b_R + \lambda_2\bar{\psi}_L^1\tilde{\Phi}\mathcal{T}_R^1 + \text{h.c.}, \quad (40)$$

where $\tilde{\Phi} = i\sigma_2\Phi^*$. After electroweak symmetry breaking, the Higgs field is given by

$$\Phi = \begin{pmatrix}\phi^+\\ \frac{1}{\sqrt{2}}(H+v-i\phi^0)\end{pmatrix}. \quad (41)$$

Note that regardless of the Yukawa couplings, the Higgs boson and the neutral Goldston boson always enter in the combination $H - i\phi^0$.

The mass eigenstates are t, T and b, where t and b are the observed top and bottom quarks. The mass eigenstates in the top sector can be found by the rotations:

$$\chi_L^t \equiv \begin{pmatrix}t_L\\ T_L\end{pmatrix} \equiv U_L^t\begin{pmatrix}\mathcal{T}_L^1\\ \mathcal{T}_L^2\end{pmatrix}, \quad (42)$$

with $\Psi_{L,R} \equiv \frac{1\pm\gamma_5}{2}\Psi$. Similar rotations are introduced for the right-handed fermions. The matrices U_L^t and U_R^t are unitary matrices and are parametrized as

$$U_L^t = \begin{pmatrix}\cos\theta_L & -\sin\theta_L\\ \sin\theta_L & \cos\theta_L\end{pmatrix},$$
$$U_R^t = \begin{pmatrix}\cos\theta_R & -\sin\theta_R\\ \sin\theta_R & \cos\theta_R\end{pmatrix}. \quad (43)$$

The most general CP conserving fermion mass terms allowed by the $SU(2)_L\times U(1)_Y$ gauge symmetry are

$$-\mathcal{L}_M = -\mathcal{L}_M^{SM} + \lambda_3\bar{\psi}_L^1\tilde{\Phi}\mathcal{T}_R^2 + \lambda_4\bar{\mathcal{T}}_L^2\mathcal{T}_R^1 + \lambda_5\bar{\mathcal{T}}_L^2\mathcal{T}_R^2 + h.c.$$
$$= \bar{\chi}_L^t\left[U_L^t M^t U_R^{t\dagger}\right]\chi_R^t + \lambda_1\frac{v}{\sqrt{2}}\bar{b}_L b_R + h.c., \quad (44)$$

where

$$M^t = \begin{pmatrix}\lambda_2\frac{v}{\sqrt{2}} & \lambda_3\frac{v}{\sqrt{2}}\\ \lambda_4 & \lambda_5\end{pmatrix}. \quad (45)$$

We can always rotate $\mathcal{T}^2$ such that $\lambda_4 = 0$ and so there are 3 independent parameters in the top sector, which we take to be the physical masses, m_t and M_T, along with the left mixing angle, θ_L. In the following we will abbreviate $s_L \equiv \sin\theta_L$, $c_L \equiv \cos\theta_L$.

The couplings of the heavy charge $\frac{2}{3}$ quarks to the Higgs boson are [20]

$$-\mathcal{L}_H = \frac{m_t}{v} c_L^2 \bar{t}_L t_R H + \frac{M_T}{v} s_L^2 \bar{T}_L T_R H + s_L c_L \frac{M_T}{v} \bar{t}_L T_R H + s_L c_L \frac{m_t}{v} \bar{T}_L t_R H + h.c.$$
$$= \frac{m_t}{v} c_L^2 \bar{t} t H + \frac{M_T}{v} s_L^2 \bar{T} T H + s_L c_L \frac{M_T + m_t}{2v} (\bar{t} T + \bar{T} t) H + s_L c_L \left(\frac{M_T - m_t}{2v}\right) (\bar{t}\gamma_5 T - \bar{T}\gamma_5 t) H. \quad (46)$$

The charged current interactions are

$$\mathcal{L}^{CC} = -\frac{g}{\sqrt{2}} (c_L \bar{t}_L \gamma_\mu b_L + s_L \bar{T}_L \gamma_\mu b_L) W^\mu + \text{h.c.} \quad (47)$$

Finally, the neutral current interactions are

$$\mathcal{L}^{NC} = \frac{g}{\cos\theta_W} \Sigma_{i=t,T} \left\{ \bar{f}_i \gamma^\mu \left[(g_L^i + \delta g_L^i) \left(\frac{1-\gamma_5}{2}\right) + (g_R^i + \delta g_R^i) \left(\frac{1+\gamma_5}{2}\right) \right] f_i \right\} Z_\mu$$
$$+ \frac{g}{\cos\theta_W} \Sigma_{i\neq j} \left\{ \bar{f}_i \gamma^\mu \left[\delta g_L^{ij} \left(\frac{1-\gamma_5}{2}\right) + \delta g_R^{ij} \left(\frac{1+\gamma_5}{2}\right) \right] f_j \right\} Z_\mu, \quad (48)$$

where $g_L^i = T_3^i - Q_i s_W^2$, $g_R^i = -Q_i s_W^2$, s_W is the sine of the Weinberg angle, Q_i the electric charge of the quark and $T_3^i = \pm\frac{1}{2}$. The anomalous couplings are

$$\delta g_L^t = \delta g_L^T = -\frac{s_L^2}{2}, \qquad \delta g_R^t = \delta g_L^T = \delta g_R^{tT} = 0, \qquad \delta g_L^{tT} = \frac{s_L c_L}{2}. \quad (49)$$

It is straightforward to use the above expressions to calculate the contributions of the top partners to the oblique parameters, ΔS, ΔT and ΔU, and to parameters measured in $Z \to b\bar{b}$ [13,14,20]. The most stringent restrictions are found from the oblique parameters and are shown in Fig. 1. In the limit $M_T \sim m_t \gg M_W$, the top partner contributions to the T parameter are [20,56]

$$\Delta T \sim \frac{3}{16\pi \sin^2\theta_W} \left(\frac{M_T^2 - m_t^2}{M_W^2}\right) s_L^2. \quad (50)$$

A scan over parameter space in the top singlet model [20] using the exact results for ΔS, ΔT and ΔU confirms the accuracy of the approximate relationship of Eq. (50) in the experimentally allowed region. It is clear that the heavy T contributions decouple in the limit $s_L \to 0$. Comparison with Eq. (46) shows that the mixed $\bar{t}\gamma_5 T H$ pseudoscalar couplings of the Higgs to top partners are proportional to ΔT and hence must be highly suppressed. We therefore neglect these pseudoscalar couplings in the next section. We also note that the T particle can be very heavy without being restricted by the

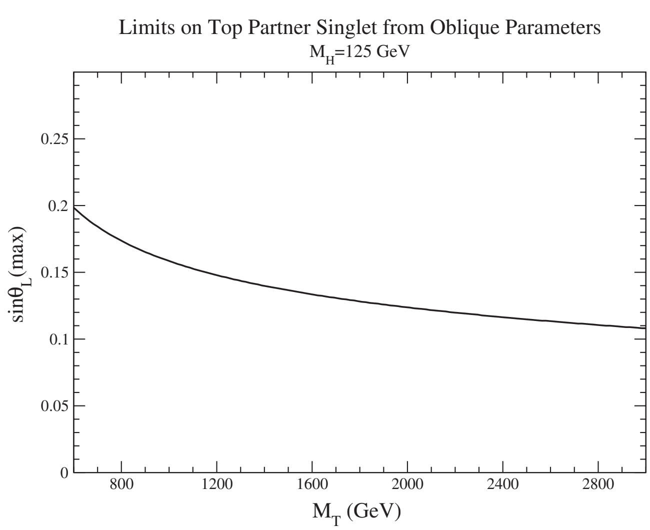


FIG. 1. Maximum allowed mixing angle, $\sin\theta_L$, in the singlet top partner model from oblique parameters [20].

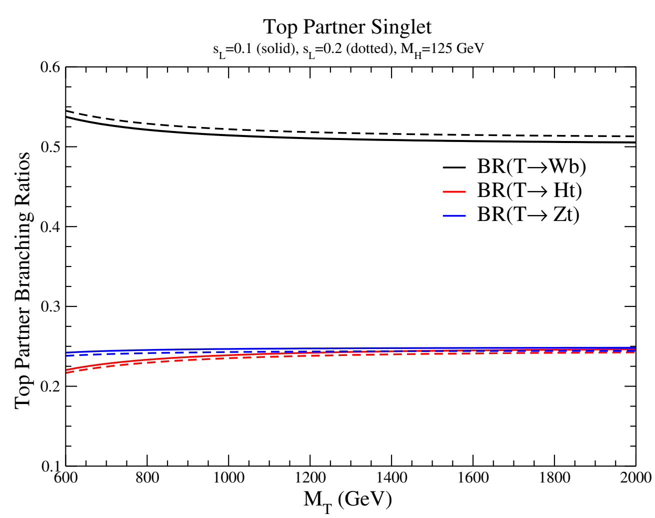


FIG. 2 (color online). Branching ratios of the top partner, T, in the singlet top partner model.

requirement of perturbative unitarity in $F\bar{F} \to F\bar{F}$ scattering [57,58],

$$s_L^2 M_T < 550 \text{ GeV (unitarity bound)}. \tag{51}$$

For example $M_T = 2$ TeV requires only $s_L < 0.5$ to preserve unitarity.

Limits on the direct production of the top partner have been obtained by CMS [10] as a function of the branching ratios, $T \to W^+ b$, $T \to Zt$, and $T \to Ht$. These branching ratios are easily computed and are shown in Fig. 2,

$$\begin{aligned}\Gamma(T \to W^+ b) &= \frac{G_F}{8\pi\sqrt{2}} M_T \lambda^{1/2}(M_T, m_b, M_W) s_L^2 \\ &\quad \times (1 + x_W^2 - 2x_W^4), \\ \Gamma(T \to Zt) &= \frac{G_F}{16\pi\sqrt{2}} M_T \lambda^{1/2}(M_T, m_t, M_Z) s_L^2 c_L^2 \\ &\quad \times (1 + x_Z^2 - 2x_t^2 - 2x_Z^4 + x_t^4 + x_Z^2 x_t^2), \\ \Gamma(T \to Ht) &= \frac{G_F}{16\pi\sqrt{2}} M_T \lambda^{1/2}(M_T, m_t, M_H) s_L^2 c_L^2 \\ &\quad \times (1 + 6x_t^2 - x_H^2 + x_t^4 - x_t^2 x_H^2),\end{aligned} \tag{52}$$

where $\lambda(a,b,c) = a^4 + b^4 + c^4 - 2(a^2b^2 + a^2c^2 + b^2c^2)$, $x_i = \frac{M_i}{M_T}$, and we neglect the b mass. The results are rather insensitive to s_L, as is obvious from Fig. 2.

In the following sections, we compute the two-loop Yukawa enhanced contribution to $gg \to H$ in the top partner singlet model using the low energy theorem.

B. Contributions from off-diagonal terms

We first present results for the two-loop corrections to the gluon self-energy coming from diagrams involving two heavy quarks, T and t, and a neutral boson, either H or ϕ^0. Examples of such diagrams are shown in Fig. 3. We consider a general interaction Lagrangian

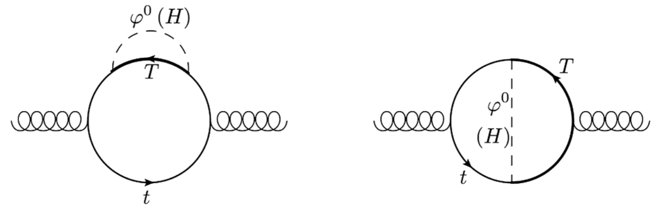


FIG. 3. Two-loop contributions to the gluon self-energy from "mixed" diagrams.

$$\begin{aligned}-\mathcal{L}_\Phi^{N.C.} &= (\bar{t}_L y_{tt} t_R + \bar{t}_L y_{tT} T_R + \bar{T}_L y_{TT} T_R + \bar{T}_L y_{Tt} t_R) \\ &\quad \times (H + i\phi^0) + h.c. \\ &= H[\bar{t} Y_{tt} t + \bar{t} Y_{tT} T + \bar{T} Y_{TT} T + \bar{T} Y_{Tt} t \\ &\quad + \gamma_5(\bar{t} A_{tT} T + \bar{T} A_{Tt} t)] + i\phi^0[\bar{t} A_{tT} T + \bar{T} A_{Tt} t \\ &\quad + \gamma_5(\bar{t} Y_{tT} T + \bar{T} Y_{Tt} t)],\end{aligned} \tag{53}$$

where the couplings are assumed real. We defined

$$Y_{qq'} = \frac{y_{qq'} + y_{q'q}}{2}, \qquad A_{qq'} = \frac{y_{qq'} - y_{q'q}}{2}. \tag{54}$$

The diagonal interactions are pure scalar, as in the Standard Model. The corresponding contributions to the two-loop gluon self-energy can be obtained by rescaling the results of Table I. We report here the contributions from the off–diagonal terms and the corresponding effects on the ggH interaction in terms of the general Lagrangian, Eq. (53). We will then adapt them to the top partner singlet model.

The two-loop mixed diagrams with two different quarks and a Higgs boson exchange contribute

$$\begin{aligned}\Pi_{AB}^{\mu\nu,2L}|_{\text{mixed},H} = {} & \frac{\alpha_s}{192\pi^3}\delta_{AB}(g^{\mu\nu}p^2 - p^\mu p^\nu)[N]^2 m_t^{-4\epsilon}\left[\frac{1}{\epsilon}\left(\Delta_- + 4\frac{a^2+1}{a}\Delta_+\right) + \frac{5}{2}\Delta_- + 4\frac{a^2+1}{a}\Delta_+ \right. \\ & \left. + 4\frac{\log a}{a^2-1}\left(\Delta_- - \frac{2a^4 - a^2 - 2}{a}\Delta_+\right)\right],\end{aligned} \tag{55}$$

where we introduced the shorthand notation $a = M_T/m_t$, $\Delta_+ = Y_{tT}Y_{Tt} + A_{tT}A_{Tt}$, and $\Delta_- = Y_{tT}Y_{Tt} - A_{tT}A_{Tt}$. This result correctly reproduces the limit for $M_T \to m_t$ of Table I for zero pseudoscalar couplings and $Y_{tT} = Y_{Tt} \to m_t Y_t / v$. From the Lagrangian of Eq. (53), the contribution from the mixed diagrams with the exchange of a neutral Goldstone boson is

$$\Pi_{AB}^{\mu\nu,2L}|_{\text{mixed},\phi^0} = -\Pi_{AB}^{\mu\nu,2L}|_{\text{mixed},H}(Y_{qq'} \leftrightarrow A_{qq'}), \tag{56}$$

so that in the sum the terms which are symmetric under the exchange of T and t cancel and the total result from the mixed diagrams of Fig. 3 is

$$\Pi^{\mu\nu,2L}_{AB}|_{\rm mixed} = \frac{\alpha_s}{96\pi^3}\delta_{AB}(g^{\mu\nu}p^2 - p^\mu p^\nu)[N]^2 m_t^{-4\epsilon}\Delta_-\left(\frac{1}{\epsilon}+\frac{5}{2}+4\frac{\log\, a}{a^2-1}\right). \tag{57}$$

Again, this correctly reproduces the limit $M_T \to m_t$ from the sum of the first four entries of Table I [accounting for a factor of 2 for the two heavy quarks and $(3-2\epsilon)^{-1}$ for the projector of Eq. (8)].

Applying the low energy theorem, Eq. (2), the scalar ggH vertex receives a finite correction

$$\mathcal{A}^{0,2L}_{gg\to H}|_{\rm mixed} = \frac{\alpha_s}{24\pi^3 v}\delta_{AB}(g^{\mu\nu}p^2 - p^\mu p^\nu)\Delta_- \frac{1}{a^2-1}\left[Y_T - a^2 Y_t - 2(Y_T - Y_t)\frac{a^2}{a^2-1}\ \log\, a\right]. \tag{58}$$

The off-diagonal couplings yield new contributions to the renormalization of the quark mass and of the Higgs wave function (Fig. 4),

$$\frac{\delta m_t}{m_t}|_{\rm mixed} = \frac{1}{16\pi^2}[N]m_t^{-2\epsilon}\Delta_-\left[\frac{1}{\epsilon}+2+a^2-2a^4\ \log\, a+(a^4-1)\log|a^2-1|\right],$$
$$\frac{\delta Z_H}{2}|_{\rm mixed} = -\frac{N_C}{8\pi^2}[N]m_t^{-2\epsilon}\left[\frac{\Delta_-}{\epsilon}+\frac{\Delta_-}{2}\frac{a^6-7a^4+7a^2-1-4a^4(a^2-3)\log\, a}{(a^2-1)^3} - \Delta_+ a\frac{a^4-1-4a^2\ \log\, a}{(a^2-1)^3}\right]. \tag{59}$$

These results correctly reproduce the Standard Model limit, Eq. (35), from the top quark contribution when $\Delta_+ = \Delta_- \to (\frac{m_t^2}{v^2})$ and $a \to 1$. (Note that for the quark mass renormalization one also needs to add the contribution from the b quark loop, as in the Standard Model). The W mass receives contributions from $t-b$ and $T-b$ loops. They have the same form as in Eq. (35) up to rescaling factors $V_{tb}V^*_{bt}$, $V_{Tb}V^*_{bT}$ from modifications of the heavy-quark couplings to the W bosons with respect to the Standard Model. These modifications are related to the deviations in the $F\bar{F}'H$ vertex. In the singlet top partner model we consider, both come from the mixing among quarks of the same quantum numbers. Poles will cancel once we consider an explicit model where these relations are clear. Since the two-loop amplitude, Eq. (58), is finite and the quark mass renormalization only contributes a finite term [Eq. (33)], we expect δ_3 to be finite.

C. Results for top partner model

We now turn to the top partner singlet model described at the beginning of this section. The two-loop gluon self-energy containing only heavy quarks of one kind, either t or T, is finite, as shown in Eq. (34). Therefore, there is no contribution to the unrenormalized two-loop amplitude, $\mathcal{A}^{0,2L}_{gg\to H}$, coming from the diagonal fermion interactions of Eq. (46). Applying the couplings of Eq. (46) to the general result of Eq. (59), the diagrams containing off-diagonal mixings between different heavy quarks and the bosons inside the loop yield a contribution,[1]

$$\mathcal{A}^{0,2L}_{gg\to H}|_s = \mathcal{A}^{0,2L}_{gg\to H}|_{\rm mixed} = -\frac{m_t^2}{16\pi^2 v^2}s_L^2 c_L^2\frac{a^2+1}{a^2-1} \times\left[s_L^2 - a^2 c_L^2 + (c_L^2 - s_L^2)\frac{2a^2}{a^2-1}\ \log\, a\right] \times \mathcal{A}^{1L}_{gg\to H}|_s. \tag{60}$$

We normalized the result to the one-loop ggH amplitude in the top partner singlet model,

$$\mathcal{A}^{1L}_{gg\to H}|_s = -\frac{\alpha_s}{3\pi v}\delta_{AB}(g^{\mu\nu}p^2 - p^\mu p^\nu). \tag{61}$$

Note that in the infinite-mass approximation, this is the same as the Standard Model amplitude, Eq. (38). Only finite mass corrections yield deviations from the Standard Model result [20].

The low energy theorem as formulated in Eq. (2) does not reproduce the diagrams where the external Higgs boson couples to two different quarks. From Eq. (46), we see that these pseudoscalar couplings are proportional to $s_L(M_T - m_t) \sim \Delta T$ and are restricted by the measurements shown in Fig. 1 to be small. Neglecting them is thus a reasonable approximation.

From Eq. (47), the couplings of the heavy quarks to the W boson in the singlet model are rescaled by $V_{tb} = V^*_{bt} = c_L$, $V_{Tb} = V^*_{bT} = s_L$, and the W-mass renormalization is

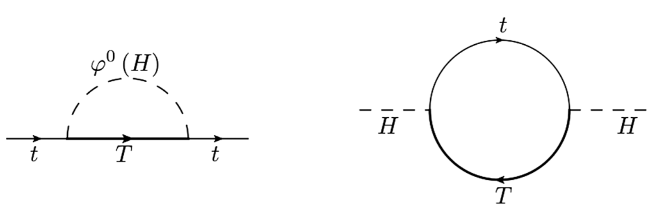


FIG. 4. Off-diagonal contributions to the quark (left) and Higgs (right) self-energy.

[1] We use the subscript "s" to denote quantities in the top singlet model.

$$\frac{\delta M_W^2}{M_W^2}|_s = -\frac{N_C}{8\pi^2 v^2}[N]\left(\frac{1}{\epsilon}+\frac{1}{2}\right)m_t^{2-2\epsilon}[c_L^2 + s_L^2 a^2(1-4\log a)]. \tag{62}$$

The wave function renormalization is found by rescaling the Standard Model results of Eq. (35) by c_L^4 for the t contribution, s_L^4 for the T contribution, and adding the mixed contribution of Eq. (59) using the couplings of Eq. (46),

$$\frac{\delta Z_H}{2}|_s = -\frac{N_C}{16\pi^2 v^2}[N]m_t^{2-2\epsilon}\left[\frac{a^2 s_L^2 + c_L^2}{\epsilon} - \frac{2}{3}(a^2 s_L^4 + c_L^4) - 2a^2 \log a s_L^4 + s_L^2 c_L^2 \frac{a^8 - 10a^6 + 10a^2 - 1 - 4a^4(a^4 - 2a^2 - 7)\log a}{2(a^2-1)^3}\right]. \tag{63}$$

From the general result of Eq. (24), we obtain

$$\delta_3|_s = \frac{N_C}{96\pi^2 v^2}m_t^2\left[4(c_L^4 + s_L^4 a^2) + s_L^2 a^2(3 - 12c_L^2 \log a) + 3c_L^2\left(1 - s_L^2\frac{a^6 - 9a^4 - 9a^2 + 1}{(a^2-1)^2}\right) + 12 s_L^2 c_L^2 a^4 \log a \frac{a^4 - 2a^2 - 7}{(a^2-1)^3}\right]. \tag{64}$$

As we anticipated, the poles in the W mass and the Higgs wave function renormalization cancel and δ_3 is finite. In the limits $c_L \to 1$ or $s_L \to 1$ only one heavy quark (t or T, respectively) couples to the Higgs, and Eq. (64) correctly reproduces the Standard Model infinite-mass result of Eq. (36). The final ingredient that we need for the two-loop renormalization are the poles of the heavy-quark mass renormalization constants. Combining Eqs. (35) and (59),

$$\frac{\delta m_t}{m_t}|_{s,\epsilon} = \frac{1}{\epsilon}\frac{1}{32\pi^2 v^2}m_t^2 c_L^2[3c_L^2 + s_L^2(a^2+1)], \qquad \frac{\delta M_T}{M_T}|_{s,\epsilon} = \frac{1}{\epsilon}\frac{1}{32\pi^2 v^2}m_t^2 s_L^2[3s_L^2 a^2 + c_L^2(a^2+1)]. \tag{65}$$

From Eq. (33) the two-loop counterterm is

$$\mathcal{A}_{gg\to H}^{2L,ct}|_s = \frac{m_t^2}{64\pi^2 v^2}\frac{1}{(a^2-1)^2}\left[a^6 + 5a^4 + 5a^2 + 1 - (a^2-1)^3\frac{11\cos(2\theta_L) - 3\cos(6\theta_L)}{8} - 6a^2(a^2+1)\cos(4\theta_L) + 6a^2 \log a \frac{a^4 - 10a^2 + 1}{a^2 - 1}\sin^2(2\theta_L)\right] \times \mathcal{A}_{gg\to H}^{1L}|_s, \tag{66}$$

where for simplicity we set $N_C = 3$. The renormalized two-loop amplitude then reads

$$\mathcal{A}_{gg\to H}^{2L}|_s = \frac{1}{256\pi^2 v^2}\frac{m_t^2}{(a^2-1)^3}\Big\{5a^8 + 14a^6 - 14a^2 - 5 + 8\sin^2(2\theta_L)a^2 \log a[3(a^4 - 10a^2 + 1) - (a^4-1)\cos(2\theta_L)] - \cos(2\theta_L)(a^2-1)^2(5a^4 - 12a^2 + 5) - \cos(4\theta_L)(a^8 + 22a^6 - 22a^2 - 1) + \cos(6\theta_L)(a^2-1)^2(a^4 - 4a^2 + 1)\Big\} \times \mathcal{A}_{gg\to H}^{1L}|_s. \tag{67}$$

This reproduces the Standard Model result of Eq. (37) for $\theta_L = 0$ and in the $a \to 1$ limit for $\theta_L = \pi/2$, i.e., when only one heavy quark of mass m_t runs in the loops with Standard-Model-like couplings. For small mixing, as required by the precision electroweak results, Eq. (67) reduces to

$$\mathcal{A}_{gg\to H}^{2L}|_s \to \frac{m_t^2}{32\pi^2 v^2}\left[1 + \frac{2\theta_L^2}{(a^2-1)^2}\left(15a^4 + 8a^2 + 1 + 4a^2 \log a \frac{a^4 - 15a^2 + 2}{a^2 - 1}\right)\right]\mathcal{A}_{gg\to H}^{1L}|_s. \tag{68}$$

In the limit of small $\delta \equiv M_T - m_t$, but for arbitrary mixing,

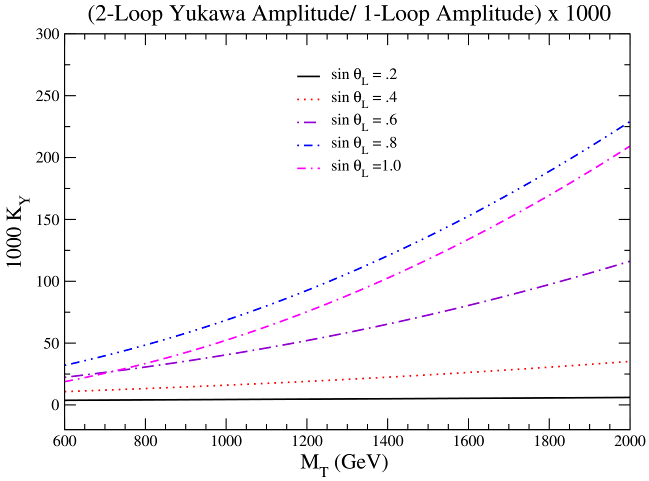


FIG. 5 (color online). 1000 times the contribution from the two-loop Yukawa amplitude of Eq. (67) divided by the one-loop Yukawa amplitude in the top partner singlet model as a function of the mixing with the Standard Model top quark.

$$\mathcal{A}^{2L}_{gg\to H}|_s \to \frac{m_t^2}{32\pi^2 v^2}\left[3-2\cos(4\theta_L)-\frac{\delta}{6m_t}\sin^2\theta_L(7\cos(4\theta_L) -34\cos(2\theta_L)-53)\right]\mathcal{A}^{1L}_{gg\to H}|_s. \quad (69)$$

Finally, for almost degenerate quarks with small mixing both these expansions reduce to

$$\mathcal{A}^{2L}_{gg\to H}|_s \to \frac{m_t^2}{96\pi^2 v^2}\left(3+48\theta_L^2+40\frac{\delta}{m_t}\theta_L^2\right)\mathcal{A}^{1L}_{gg\to H}|_s. \quad (70)$$

The first correction in the small $\frac{\delta}{m_t}$ parameter is further suppressed by the small mixing, and is therefore subleading with respect to the θ_L^2 correction.

In the $\delta \to 0$ limit there are no $t\gamma_5\bar{t}H$ contributions, and the low energy theorem reproduces all contributions.

D. Phenomenology

In this section, we consider the phenomenological implications of the Yukawa corrections to the top partner model given in the previous section. At one-loop, the amplitude for $gg \to H$ is identical to the Standard Model rate up to corrections of $\mathcal{O}(\frac{M_H^2}{M_T^2})$ [20], and large deviations are therefore first possible at the two-loop level. In Fig. 5, we show the effects of the two-loop contributions relative to the one-loop contribution (including only the Yukawa terms calculated here), without keeping into account the bounds from electroweak precision data. We quantify these effects through the K factor

$$K_Y = \frac{\mathcal{A}^{2L}_{gg\to H}|_s}{\mathcal{A}^{1L}_{gg\to H}|_s}. \quad (71)$$

Only for ridiculously large values of the mixing parameter, $s_L \sim 1$, do the effects of the Yukawa corrections reach the level of a few %. Effects are even smaller if we restrict ourselves to the allowed region of Fig. 1 (Figs. 6 and 7). In Fig. 6, we show how the Yukawa corrections increase with the mixing angle for a fixed $M_T = 800$ GeV, in the range allowed by precision electroweak measurements. The behavior is consistent with the small-angle expansion of Eq. (68). Finally, in Fig. 7, we show the dependence on the heavy mass M_T. For large values of M_T to be allowed, we need to restrict ourselves to a small mixing angle. It is clear that these two-loop Yukawa corrections are always at the subpercent level. In order to obtain large Yukawa corrections, we would need to construct a more complicated model where large mixing with the Standard Model

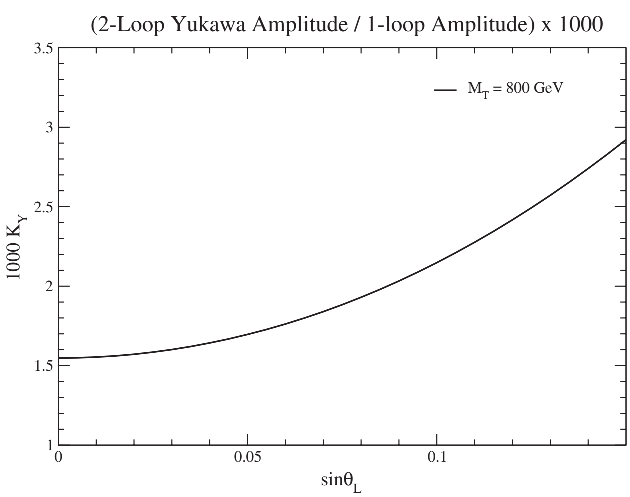


FIG. 6. 1000 times the contribution from the two-loop Yukawa amplitude of Eq. (67) divided by the one-loop Yukawa amplitude in the top partner singlet model with $M_T = 800$ GeV.

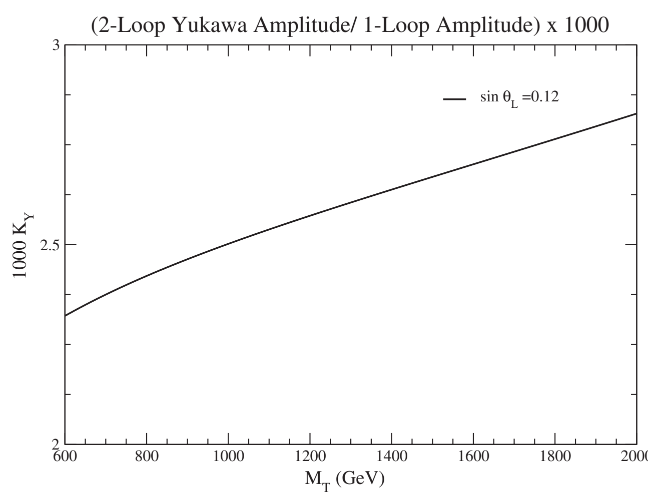


FIG. 7. 1000 times the contribution from the two-loop Yukawa amplitude of Eq. (67) divided by the one-loop Yukawa amplitude in the top partner singlet model with $\sin\theta_L = 0.12$.

fermions was not forbidden by electroweak precision measurements.

V. CONCLUSIONS

We have considered the two-loop $\mathcal{O}((\frac{Y_F M_F}{v})^3)$ contributions to $gg \to H$ using the low energy theorem and our analytic results will be of use to future model builders. These corrections are well known and small for the Standard Model. In the singlet top partner model there are contributions of $\mathcal{O}((\frac{Y_F M_F}{v})^3)$ to Higgs production via gluon fusion which are potentially important. These corrections are suppressed, however, by a mixing angle, s_L, which is restricted by precision electroweak measurements to be small, and we find that the Yukawa corrections in this model are at the subpercent level. This reinforces our conclusions from a previous work, that the singlet top partner model represents an example where the gluon fusion Higgs production rate will be almost identical to that of the Standard Model and hence precision measurements of the rate will be insensitive to the new physics. Exploring this class of models will require the direct observation of the top partners.

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