Lepton dipole moments in supersymmetric low-scale seesaw models

Amon Ilakovac,[1] Apostolos Pilaftsis,[2,3] and Luka Popov[1]
[1]*Department of Physics, University of Zagreb, Bijenička cesta 32, P.O. Box 331, Zagreb, Croatia*
[2]*Department of Physics, CERN, Theory Division, CH-1211 Geneva 23, Switzerland*
[3]*Consortium for Fundamental Physics, School of Physics and Astronomy, University of Manchester, Manchester M13 9PL, United Kingdom*
(Received 22 August 2013; published 8 January 2014)

We study the anomalous magnetic and electric dipole moments of charged leptons in supersymmetric low-scale seesaw models with right-handed neutrino superfields. We consider a minimally extended framework of minimal supergravity, by assuming that *CP* violation originates from complex soft SUSY-breaking bilinear and trilinear couplings associated with the right-handed sneutrino sector. We present numerical estimates of the muon anomalous magnetic moment and the electron electric dipole moment, as functions of key model parameters, such as the Majorana mass scale m_N and tan β. In particular, we find that the contributions of the singlet heavy neutrinos and sneutrinos to the electron electric dipole moment are *naturally* small in this model, of order $10^{-27} - 10^{-28}$ $e$cm, and can be probed in present and future experiments.

DOI: 10.1103/PhysRevD.89.015001 PACS numbers: 12.60.Jv, 14.60.Pq

I. INTRODUCTION

The anomalous magnetic dipole moment (MDM) of the muon, a_μ, constitutes a high-precision observable extremely sensitive to physics beyond the Standard Model (SM). Its current experimental value $a_\mu^{\rm exp} = (116592089 \pm 63) \times 10^{-11}$ differs from the SM theoretical prediction $a_\mu^{\rm SM} = (116591802 \pm 49) \times 10^{-11}$ by [1]

$$\Delta a_\mu \equiv a_\mu^{exp} - a_\mu^{SM} = (287 \pm 80) \times 10^{-11}. \qquad (1.1)$$

Evidently, the deviation Δa_μ is at the 3.6 σ confidence level and has therefore been called the *muon anomaly.* Consequently, an important constraint on model building is derived by requiring that new-physics contributions to a_μ are smaller than Δa_μ.

Likewise, the electric dipole moment (EDM) of the electron, d_e, is a very sensitive probe for *CP* violation induced by new *CP* phases beyond the SM. The present upper limit on d_e is quoted to be [1–3]

$$d_e < 10.5 \times 10^{-28} e \text{ cm}. \qquad (1.2)$$

Future projected experiments utilizing paramagnetic systems, such as cesium, rubidium, and francium, may extend the current sensitivity to the 10^{-29}–$10^{-31} e$ cm level (see, e.g., [3] and references therein). In the SM, the predictions for d_e range from $10^{-38} e$ cm to $10^{-33} e$ cm, depending on whether the Dirac *CP* phase in the light neutrino mixing is zero or not [4]. Clearly, an observation of a nonzero value for d_e, much larger than $10^{-33} e$ cm, would signify *CP*-violating physics beyond the SM.

As an archetypal model of new physics, the so-called minimal supersymmetric Standard Model (MSSM) is of great interest. In general, models of softly broken supersymmetry (SUSY) at the 1–10 TeV scale, such as the MSSM, can account for the gauge hierarchy problem, predict rather accurate unification of gauge couplings near the Grand Unification Theory (GUT) scale, naturally explain the origin of spontaneous symmetry breaking of the SM gauge group, and predict viable candidates for solving the dark matter (DM) problem in the Universe. For a recent review, see [5].

To account for the observed light neutrino masses and mixings, we will consider SUSY extensions [6] to models with low-scale heavy neutrinos [7–10]. Specifically, the MSSM extended with low-scale right-handed neutrino superfields, which we denote hereafter as ν_RMSSM, predicts additional contributions to charged lepton flavor violation that do not exist in models with high-scale heavy neutrinos and are independent of the soft SUSY-breaking mechanism [11]. It is interesting to note that in the ν_RMSSM, Z-boson penguins [11,12] and box diagrams [13] dominate the amplitudes of processes, such as lepton → 3 leptons and $\mu \to e$ conversion, whereas photon-penguin LFV diagrams are subdominant and become only relevant to models with ultraheavy neutrinos close to the GUT scale [14]. In particular, our recent analysis has shown [13] that a significant region of the ν_RMSSM parameter space exists for which the branching ratios of charged lepton flavor violation processes are predicted to be close to the current experimental sensitivities, despite the fact that the soft SUSY-breaking scale has been pushed to values higher than 1 TeV, as a consequence of the discovery of a SM-like Higgs boson at the CERN Large Hadron Collider (LHC) [15] and the existing nonobservation limits on the gluino and squark masses that were also deduced from LHC data [16].

It is therefore of particular interest to investigate here whether the effects of low-scale heavy neutrinos and their

SUSY partners, the sneutrinos, contribute in a relevant manner to other high-precision observables, such as the muon anomalous MDM a_μ and the electron EDM d_e. We believe that the announced higher-precision measurement of a_μ by a factor of 4 in the future Fermilab experiment E989 [17,18] and the expected future sensitivities of the electron EDM down to the level of $\sim 10^{-31} e$ cm [3] render such an investigation both very interesting and timely.

Most studies on lepton dipole moments have been devoted to SUSY models realizing a high-scale seesaw mechanism [19–22]. Here instead, we consider the ν_RMSSM which provides potentially significant contributions to lepton dipole moments due to low-scale neutrinos and sneutrinos, as well as new sources of CP violation. In particular, an interesting possibility emerges if there exists CP violation beyond the SM which is sourced from the singlet sector of the ν_RMSSM. This new CP violation may originate from a complex soft trilinear sneutrino parameter A_ν or from a complex soft bilinear parameter B_ν. In addition, one may have new CP-odd phases residing in the 3×3 neutrino Yukawa-coupling matrix $\mathbf{h}_\nu$. Assuming that these are the only additional nonzero CP-odd phases in the ν_RMSSM, we find that the electron EDM is testable, but naturally small, typically of order $10^{-27} e$ cm, thereby avoiding to some extent the well-known problem of too large CP violation, from which SUSY extensions of the SM, such as the MSSM (see, e.g., [23]), usually suffer.

The outline of the paper is as follows. In Sec. II, we introduce our conventions and notation for the lepton dipole moments, as well as describe the new sources of CP violation that we are considering in the ν_RMSSM. Section III presents our numerical estimates for the lepton dipole moments a_μ and d_e. To this end, we specify our input parameters, including the neutrino Yukawa matrices adopted in our numerical analysis. Section IV summarizes our conclusions. Technical details pertinent to the lepton-dipole moment form factors are given in the Appendix.

II. MAGNETIC AND ELECTRIC DIPOLE MOMENTS

The anomalous MDM and EDM of a charged lepton l can be read off from the Lagrangian [24]:

$$\mathcal{L} = \bar{l}\left[\gamma_\mu(i\partial^\mu + eA^\mu) - m_l - \frac{e}{2m_l}\sigma^{\mu\nu}(F_l + iG_l\gamma_5)\partial_\nu A_\mu\right] l. \tag{2.1}$$

In the on-shell limit of the photon field A^μ, the form factor F_l defines the anomalous MDM of the lepton l, i.e., $a_l \equiv F_l$, while the form factor G_l defines its EDM, i.e., $d_l \equiv eG_l/m_l$. Given that the general form-factor decomposition of the photonic transition amplitude is given by [13]

$$i\mathcal{T}^{\gamma ll} = i\frac{e\alpha_w}{8\pi M_W^2}[(G_\gamma^L)_{ll} i\sigma_{\mu\nu}q^\nu P_L + (G_\gamma^R)_{ll} i\sigma_{\mu\nu}q^\nu P_R], \tag{2.2}$$

the anomalous MDM a_l and the EDM d_l of a lepton l are then, respectively, determined by

$$a_l = \frac{\alpha_w m_l}{8\pi M_W^2}[(G_\gamma^L)_{ll} + (G_\gamma^R)_{ll}], \tag{2.3}$$

$$d_l = \frac{e\alpha_w}{8\pi M_W^2} i[(G_\gamma^L)_{ll} - (G_\gamma^R)_{ll}]. \tag{2.4}$$

Here and in the following, we adopt the notation for the couplings and the form factors established in [13].

At the one-loop level, the EDM d_l of the lepton vanishes in the MSSM with universal soft SUSY-breaking boundary conditions and no soft CP phases, adopting the convention of a real superpotential Higgs-mixing parameter μ [21]. This result also holds true, even in extensions of the MSSM with heavy neutrinos, as long as the sneutrino sector is universal and CP-conserving as well.

As a minimal departure of the above universal scenario, we assume here that *only* the sneutrino sector is CP violating, due to soft CP phases in the bilinear and trilinear soft SUSY-breaking parameters:

$$\mathbf{b}_\nu \equiv \mathbf{B}_\nu \mathbf{m}_M = B_0 e^{i\theta} m_N \mathbf{1}_3, \tag{2.5}$$

$$\mathbf{A}_\nu = \mathbf{h}_\nu A_0 e^{i\varphi}, \tag{2.6}$$

where B_0 and A_0 are real parameters determined at the GUT scale, m_N is a real parameter input at the scale m_N, and θ and φ are physical, flavor blind CP-odd phases. In addition, $\mathbf{h}_\nu$ is the 3×3 neutrino Yukawa matrix to be specified in the next section. The soft SUSY-breaking terms corresponding to the $\mathbf{b}_\nu$ and $\mathbf{A}_\nu$ are obtained from the Lagrangian terms

$$-(\mathbf{A}_\nu)^{ij}\tilde{\nu}_{iR}^c(h_{uL}^+ \tilde{e}_{jL} - h_{uL}^0 \tilde{\nu}_{jL}) \tag{2.7}$$

and

$$(\mathbf{b}_\nu \mathbf{m}_M)_{ii}\tilde{\nu}_{Ri}\tilde{\nu}_{Ri}, \tag{2.8}$$

respectively. Correspondingly, $\tilde{\nu}_{iR}^c$, $\tilde{e}_{jL}$, h_{uL}^+, and h_{uL}^0 denote the heavy sneutrino, selectron, charged Higgs and neutral Higgs fields. The $O(3)$ flavor symmetry of the model for the heavy neutrinos assures that the heavy neutrino mass matrix $\mathbf{m}_N$ is proportional to the unit matrix $\mathbf{1}_3$ with eigenvalues m_N, up to small renormalization-group effects. To keep things simple, we also assume that the 3×3 soft bilinear mass matrix $\mathbf{b}_\nu$ is proportional to $\mathbf{1}_3$. In the standard SUSY seesaw scenarios with ultraheavy neutrinos of mass m_N, the CP-violating sneutrino contributions to electron

EDM d_e scale as B_0/m_N and A_0/m_N at the one-loop level, and practically decouple for heavy-neutrino masses m_N close to the GUT scale. Hence, sizeable effects on d_e should only be expected in low-scale seesaw scenarios, in which m_N can become comparable to B_0 and A_0.

Following the conventions of [13], the 12×12 sneutrino mass matrix may be cast into the 4×4 block form:

$$\mathbf{M}_{\tilde{\nu}}^2 = \begin{pmatrix} \mathbf{H}_1 & \mathbf{N} & \mathbf{0} & \mathbf{M} \\ \mathbf{N}^\dagger & \mathbf{H}_2^T & \mathbf{M}^T & \mathbf{b}_\nu^\dagger \\ \mathbf{0} & \mathbf{M}^* & \mathbf{H}_1^T & \mathbf{N}^* \\ \mathbf{M}^\dagger & \mathbf{b}_\nu & \mathbf{N}^T & \mathbf{H}_2 \end{pmatrix}. \tag{2.9}$$

The entries of $\mathbf{M}_{\tilde{\nu}}^2$ are expressed in terms of the 3×3 matrices:

$$\begin{aligned} \mathbf{H}_1 &= \mathbf{m}_{\tilde{L}}^2 + \mathbf{m}_D \mathbf{m}_D^\dagger + \frac{1}{2} M_Z^2 \cos 2\beta, \\ \mathbf{H}_2 &= \mathbf{m}_{\tilde{\nu}}^2 + \mathbf{m}_D^\dagger \mathbf{m}_D + \mathbf{m}_M \mathbf{m}_M^\dagger, \\ \mathbf{M} &= -\frac{v_2}{\sqrt{2}} \mathbf{A}_\nu^\dagger - \mu \mathbf{m}_D \cot \beta, \\ \mathbf{N} &= \mathbf{m}_D \mathbf{m}_M. \end{aligned} \tag{2.10}$$

Here $\mathbf{m}_{\tilde{L}}^2$, $\mathbf{m}_{\tilde{\nu}}^2$, and $\mathbf{A}_\nu$ are 3×3 soft SUSY-breaking matrices associated with the left-handed slepton doublets, the right-handed sneutrinos, and their trilinear couplings, respectively. We note that the bilinear soft 3×3 matrix $\mathbf{b}_\nu$ was neglected in Ref. [13], where the authors tacitly assumed that it was small compared to the other soft SUSY-breaking parameters in (2.9). Here, we take this term into account, but restrict the size of the universal bilinear mass parameter B_0, such that the sneutrino masses remain always positive and hence physical.

The generation of a nonzero EDM d_e results from the soft sneutrino CP-odd phases θ and φ, as well as from complex neutrino Yukawa couplings $\mathbf{h}_\nu$. All these CP-odd phases are present in the photon dipole form factors $G_{ll\gamma}^{L,\tilde{N}}$ and $G_{ll\gamma}^{R,\tilde{N}}$, whose analytical forms may be found in [13]. In fact, we noticed that d_e may be generated by products of vertices that are not relatively complex conjugate to each other, such as [25]

$$\begin{aligned} \Delta_{CP}^{LR} &= \tilde{B}_{lkA}^{L,1} \tilde{B}_{lkA}^{R,1*} + \tilde{B}_{lkA}^{L,2} \tilde{B}_{lkA}^{R,2*}, \\ \Delta_{CP}^{RL} &= \tilde{B}_{lkA}^{R,1} B_{lkA}^{L,1*} + \tilde{B}_{lkA}^{R,2} \tilde{B}_{lkA}^{L,2*}. \end{aligned} \tag{2.11}$$

In the exact supersymmetric limit of softly broken SUSY theories, the anomalous MDM (as well as EDM) operator is forbidden, as a consequence of the Ferrara and Remiddi no-go theorem [26]. The theorem can be verified for every particle and its SUSY-counterpart contribution to the anomalous MDM a_μ. Besides the SM contribution, there are three additional contributions in the ν_RMSSM, which originate from: (i) heavy neutrinos, (ii) sneutrinos, and (iii) soft SUSY-breaking parameters. In the supersymmetric limit, the latter contribution (iii) vanishes. In the same limit, the heavy neutrino and sneutrino contributions read

$$\begin{aligned} (G_\gamma^{ll})^N &\to \frac{7}{6} B_{lN_a} B_{lN_a}^*, \\ (G_\gamma^{ll})^{\tilde{N}} &\to -\frac{7}{6} B_{lN_a} B_{lN_a}^*, \end{aligned} \tag{2.12}$$

where B_{lN_a} are the lepton-to-heavy neutrino mixings defined in the first article of Ref. [10] and in Ref. [27]. Obviously, the sum $(G_\gamma^{ll})^N + (G_\gamma^{ll})^{\tilde{N}}$ vanishes, thereby confirming the Ferrara-Remiddi theorem.

In the MSSM, the leading contribution to a_l behaves as [28,29]

$$a_l^{MSSM} \propto \frac{m_l^2}{M_{SUSY}^2} \tan \beta \, \mathrm{sign}(\mu M_{1,2}), \tag{2.13}$$

where M_{SUSY} is a typical soft SUSY-breaking mass scale, $\tan \beta = v_2/v_1$ is the ratio of the neutral Higgs vacuum expectation values, and $M_{1,2}$ are the soft gaugino masses associated with the $\mathrm{U}(1)_\mathrm{Y}$ and $SU(2)$ gauge groups, respectively. As we will see in the next section, the MSSM contribution (2.13) to a_μ remains dominant in the ν_RMSSM as well.

From (2.13) and (2.4), one naively expects d_l to behave at the one-loop level as

$$d_l^{MSSM} \propto \sin(\phi_{CP}) \frac{m_l}{M_{SUSY}^2} \tan \beta, \tag{2.14}$$

where φ_{CP} is a generic soft SUSY-breaking CP-odd phase. Nevertheless, beyond the one-loop approximation [30,21], other dependencies of d_l on $\tan \beta$ are possible in the MSSM. However, we show that in the ν_R MSSM at the one-loop level the $\tan \beta$ dependence is linear.

III. NUMERICAL RESULTS

In our numerical analysis, we adopt the procedure established in [13]. As a benchmark model, we choose a minimally extended scenario of minimal supergravity (mSUGRA), in which we allow for the bilinear and trilinear soft SUSY-breaking terms, $\mathbf{B}_\nu$ and $\mathbf{A}_\nu$, to acquire at the GUT scale overall CP-violating phases denoted as θ and φ, respectively. In addition, we choose the sign of the μ parameter to be positive. As for the neutrino Yukawa coupling matrix $\mathbf{h}_\nu$, we consider the approximate U(1)- and A_4-symmetric models introduced in [31] and [32], respectively. In these two scenarios, $\mathbf{h}_\nu$ can be expressed in terms of the real parameters a, b, and c and CP-odd phases that might be relevant for leptogenesis. Explicitly, the neutrino Yukawa-coupling matrix $\mathbf{h}_\nu$ in the U(1)-symmetric model is given by [31]

$$\mathbf{h}_\nu = \begin{pmatrix} 0 & 0 & 0 \\ ae^{-\frac{i\pi}{4}} & be^{-\frac{i\pi}{4}} & ce^{-\frac{i\pi}{4}} \\ ae^{\frac{i\pi}{4}} & be^{\frac{i\pi}{4}} & ce^{\frac{i\pi}{4}} \end{pmatrix}, \tag{3.1}$$

and in the model based on the A_4 discrete symmetry by [32]

$$\mathbf{h}_\nu = \begin{pmatrix} a & b & c \\ ae^{-\frac{2\pi i}{3}} & be^{-\frac{2\pi i}{3}} & ce^{-\frac{2\pi i}{3}} \\ ae^{\frac{2\pi i}{3}} & be^{\frac{2\pi i}{3}} & ce^{\frac{2\pi i}{3}} \end{pmatrix}. \tag{3.2}$$

It should be noted that the choices of the neutrino Yukawa matrices (3.1) and (3.2) both lead to massless light neutrinos for any value of the heavy neutrino mass scale m_N, as these are protected by the $U(1)$ and A_4 symmetries. The observed light neutrino masses and mixings can be obtained by introducing small symmetry-breaking parameters δ_{ij} (with $i, j = 1, 2, 3$), such that $\delta_{ij} \ll a, b, c$. Since lepton dipole moments remain practically unaffected by these small symmetry-breaking parameters, we do not consider them here in detail.

For definiteness, our numerical analysis in this section is based on the following baseline scenario:

$$m_0 = 1\,TeV, \quad M_{1/2} = 1\,TeV, \quad A_0 = -4\,TeV, \quad \tan\beta = 20,$$
$$m_N = 1\,TeV, \quad B_0 = 0.1\,TeV, \quad a = b = c = 0.05, \tag{3.3}$$

where m_0, $M_{1/2}$, and A_0 are the standard universal soft SUSY-breaking parameters. All mass parameters except m_N are defined at the GUT scale and m_N is taken in at m_N scale. It is understood that those parameters not explicitly quoted in the text assume their default values stated in (3.3). Likewise, unless it is explicitly stated otherwise, our default scenario for $\mathbf{h}_\nu$ is the one given in (3.2), with the specific choice $a = b = c = 0.05$ as given in (3.3).

In the following the ν_RMSSM contributions to the muon MDM not present in the SM are denoted by a_μ. We investigate the dependence of a_μ and d_e on several key theoretical parameters, by varying them around their baseline value given in (3.3), while keeping the remaining parameters fixed. In doing so, we also make sure that the displayed parameters can accommodate the LHC data for a SM-like Higgs boson with mass $m_H = 125.5 \pm 2$ GeV [15] and satisfy the current lower limits on gluino and squark masses [16], i.e., $m_{\tilde{g}} > 1500\,\mathrm{GeV}$ and $m_{\tilde{t}} > 500\,\mathrm{GeV}$. In the following, we present numerical results first for a_μ and then for d_e.

A. Results for a_μ

Our numerical estimates for a_μ exhibit a direct quadratic dependence on the muon mass m_μ. In fact, we find that for the same set of soft SUSY-breaking parameters m_0, $M_{1/2}$, and A_0, the ratio a_μ/a_e remains constant to a good approximation, i.e., $a_\mu/a_e \approx m_\mu^2/m_e^2 \approx 42752.0$. In order to understand this parameter dependence, we have to carefully analyze the soft SUSY-breaking contributions to the form factors:

$$\begin{aligned} G_{ll\gamma}^{L,SB} = {}& \tilde{V}_{lma}^{0\ell R}\tilde{V}_{lma}^{0\ell R*}[m_l\lambda_{\tilde{e}_a}J_{41}^1(\lambda_{\tilde{e}_a},\lambda_{\tilde{\chi}_m^0})] \\ &+ \tilde{V}_{lma}^{0\ell L}\tilde{V}_{lma}^{0\ell L*}[m_l\lambda_{\tilde{e}_a}J_{41}^1(\lambda_{\tilde{e}_a},\lambda_{\tilde{\chi}_m^0})] \\ &+ \tilde{V}_{lma}^{0\ell L}\tilde{V}_{lma}^{0\ell R*}[2m_{\tilde{\chi}_m^0}\lambda_{\tilde{e}_a}J_{31}^0(\lambda_{\tilde{e}_a},\lambda_{\tilde{\chi}_m^0})], \end{aligned} \tag{3.4}$$

$$\begin{aligned} G_{ll\gamma}^{R,SB} = {}& \tilde{V}_{lma}^{0\ell L}\tilde{V}_{lma}^{0\ell L*}[m_l\lambda_{\tilde{e}_a}J_{41}^1(\lambda_{\tilde{e}_a},\lambda_{\tilde{\chi}_m^0})] \\ &+ \tilde{V}_{lma}^{0\ell R}\tilde{V}_{lma}^{0\ell R*}[m_l\lambda_{\tilde{e}_a}J_{41}^1(\lambda_{\tilde{e}_a},\lambda_{\tilde{\chi}_m^0})] \\ &+ \tilde{V}_{lma}^{0\ell R}\tilde{V}_{lma}^{0\ell L*}[2m_{\tilde{\chi}_m^0}\lambda_{\tilde{e}_a}J_{31}^0(\lambda_{\tilde{e}_a},\lambda_{\tilde{\chi}_m^0})], \end{aligned} \tag{3.5}$$

where the different terms that occur in (3.4) and (3.5) are defined in [13] and are also explicitly given in the Appendix. Observe that the neutralino vertices induce a term which is not manifestly proportional to the charged lepton mass, but to the neutralino mass. However, a closer inspection of the products of the mixing matrices $\tilde{V}_{lma}^{0\ell R}\tilde{V}_{lma}^{0\ell R*}$ and $\tilde{V}_{lma}^{0\ell R}\tilde{V}_{lma}^{0\ell L*}$ reveals [29] that these last expressions are by themselves proportional to the charged lepton mass m_l. The latter provides a nontrivial powerful check for the correctness of the results presented here.

In addition, our numerical analysis shows that the muon anomalous MDM a_μ is almost independent of the neutrino-Yukawa parameters a, b, and c, the heavy neutrino mass m_N and the soft trilinear parameter A_0. Hence, our results are almost insensitive to a particular choice for a neutrino Yukawa texture, e.g., as given in (3.1) and (3.2), and also independent of the CP-odd phases θ and φ.

In Fig. 1, we give numerical estimates for a_μ, as functions of the key theoretical parameters: $\tan\beta$, $M_{1/2}$, m_0, and m_N. In Fig. 1(a), we see that a_μ depends linearly on $\tan\beta$, as expected from (2.13). Likewise, we have investigated in Fig. 1 the dependence of a_μ on the soft SUSY-breaking parameters m_0 and $M_{1/2}$, for different kinematic situations, and obtained results consistent with the scaling behavior of $1/M_{SUSY}^2$ in (2.13).

In panel (e) of Fig. 1, we observe that the effect of the heavy right-handed neutrinos (N) and sneutrinos ($\tilde{N}$) on a_μ is negative, but small, in agreement with our discussion above. The size of their contributions alone to a_μ ranges from -10^{-12} to -4.8×10^{-15}, for $m_N = 0.5 - 10\,\mathrm{TeV}$. On the other hand, the left-handed sneutrino contributions to a_μ are approximately independent of the heavy Majorana mass m_N, reaching values $\approx 8.5 \times 10^{-11}$. The soft SUSY-breaking contributions are also approximately independent of the heavy Majorana mass m_N and have values $\approx 1.1 \times 10^{-12}$. Note that the light sneutrino contribution to the anomalous magnetic moment is the largest in magnitude, and it is already present in the MSSM contributions to a_μ. Finally, we have checked the dominance of the MSSM contributions by looking at the dependence of the parameter:

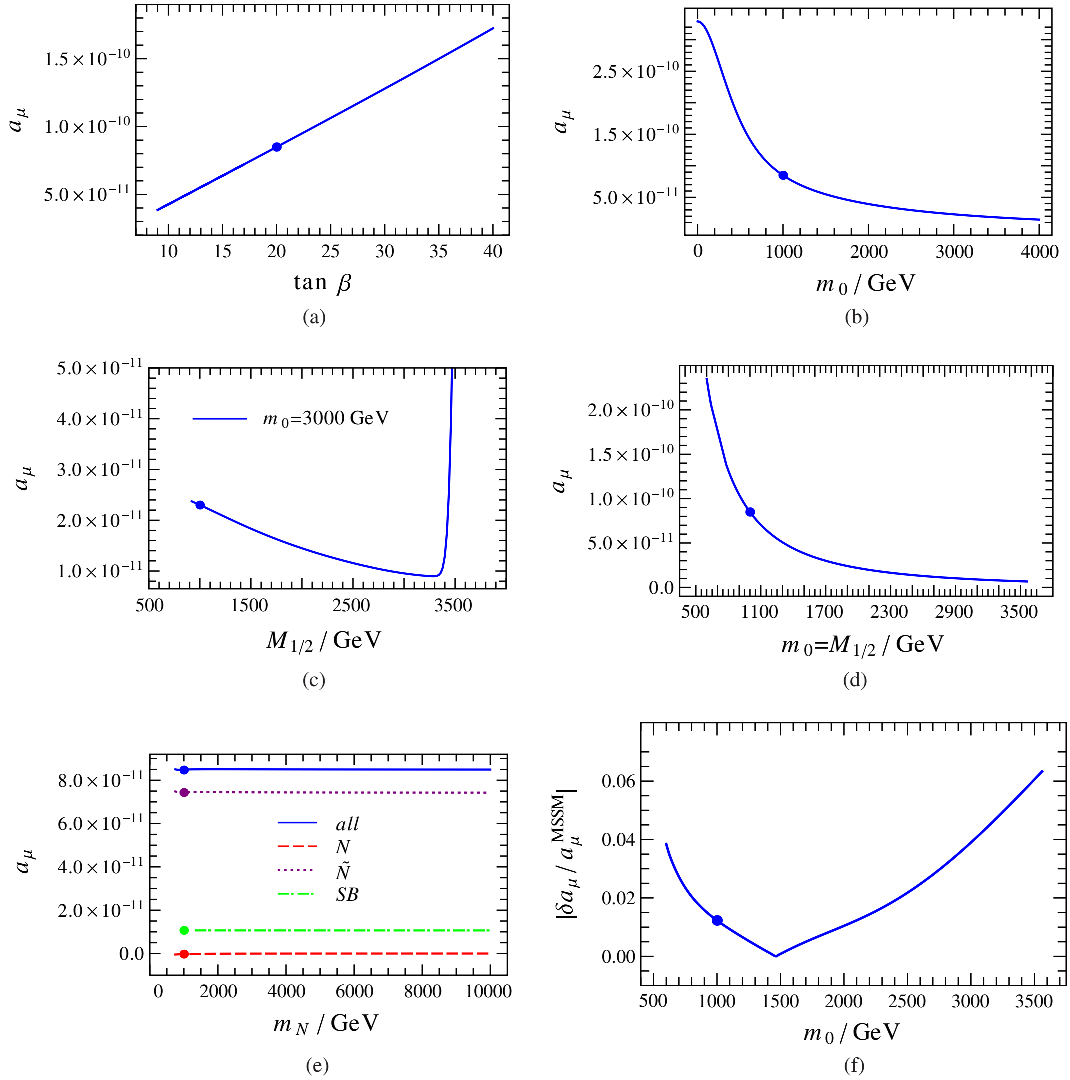

FIG. 1 (color online). Numerical estimates for the muon anomalous MDM a_μ, as functions of $\tan\beta$, $M_{1/2}$, m_N, m_0 and $m_0 = M_{1/2}$, in the ν_RMSSM. The default parameter set of the baseline model is given in (3.3). The panels (a), (b), (c), and (d) display a_μ dependencies on $\tan\beta$, m_0, $M_{1/2}$, and $m_0 = M_{1/2}$, respectively. Panel (e) shows the heavy neutrino (N), sneutrino ($\tilde{N}$), soft SUSY-breaking (SB), and *all* contributions to a_μ, as a function of m_N. Panel (f) displays an absolute value of the relative deviation $\delta a_\mu / a_\mu$ of the ν_RMSSM and MSSM predictions for a_μ [cf. (3.6)], as a function of m_0. The range of input parameters in all plots satisfies the current LHC constraints on Higgs, gluino, and squark masses. The heavy dots on the curves give the predicted values for a_μ evaluated for the default parameters (3.3).

$$\delta a_\mu = a_\mu^{\nu_R MSSM} - a_\mu^{MSSM}. \tag{3.6}$$

The difference δa_μ of the predictions for a_μ within the ν_RMSSM and the MSSM divided by a_μ is evaluated, and the absolute values of the results are displayed in panel (f) of Fig. 1, as a function of $m_0 = M_{1/2}$. The largest deviation from the MSSM is found for the largest allowed parameter value, $m_0 = 3600$ GeV, in which case $\delta a_\mu / a_\mu^{MSSM}$ is as large as 6.2×10^{-2}.

B. Results for d_e

We now study the dependence of the electron EDM d_e on several key model parameters, such as m_0, $M_{1/2}$, B_0, A_0, $\tan\beta$, θ, and φ. The predictions for d_μ may be obtained by using the naive scaling relation: $d_\mu \approx (m_\mu/m_e) d_e \approx 205 d_e$. We have found this scaling behavior is numerically satisfied very well. The maximal numerical values for d_e we obtained are of the order $\sim 10^{-27}$ e cm. Therefore predicted values for d_μ are always found to be less than $\sim 10^{-25} e\ cm$, which is several orders of magnitude below the present experimental upper bound: $d_\mu = 0.1 \pm 0.9 \times 10^{-19} e\ cm$ [1].

We note that heavy singlet neutrinos N do not contribute to d_e, even if the soft SUSY-breaking CP-odd phases φ and θ are nonzero. On the other hand, soft SUSY-breaking and right-handed neutrino effects induce nonvanishing d_e, if either θ or φ are nonzero. If both $\varphi = 0$ and $\theta = 0$, lepton

EDMs d_l numerically vanish. Therefore, the complex products of vertices (2.11) emerging in the ν_RMSSM do not induce the CP violation at one-loop level, in accord with the result of Ref. [21] obtained in the MSSM with a high-scale seesaw mechanism.

In Fig. 2, we present numerical estimates of d_e on the ν_RMSSM parameters $\tan\beta$, m_0, $M_{1/2}$, and m_N, for the maximal A_0 phase, $\varphi = \pi/2$. We also set $\theta = 0$, since the dependence of d_e on B_0 is weaker than the dependence on A_0. As shown in Fig. 2(a), d_e exhibits a linear dependence on $\tan\beta$ confirming the $\tan\beta$ naive scaling behavior in Eq. (2.14). Further, d_e is a decreasing function of m_0. As a function of $m_0 = M_{1/2}$, d_e assumes both positive and negative values, and is roughly proportional to $-1 - 2.4\,\mathrm{TeV}/m_0 + 6.3\,\mathrm{TeV}^2/m_0^2$. There is also a small region of parameter space for $m_0 = M_{1/2} \lesssim 800$ GeV, for which the prediction for d_e is of the order of the experimental upper limit on d_e (1.2). In addition, d_e decreases with increasing m_N: for the m_N values from panel (d) of Fig. 2, this behavior can be roughly approximated by a function $-0.13 + \mathrm{TeV}^{\frac{2}{3}} m_N^{-\frac{2}{3}}$; in the m_N range $10 < m_N < 100$TeV, d_e roughly scales as $1/m_N$; and above $m_N = 100\,\mathrm{TeV}$ it becomes a very slowly decreasing function in m_N.

In Fig. 3, we show the predicted numerical values for d_e, as functions of the soft SUSY-breaking parameters A_0 and B_0, and their corresponding CP phases φ and θ. In all panels except the panel (c), where $\varphi = 0$ and θ is a variable, φ assumes value $\pi/2$ or it is a variable and θ is taken to be equal zero. In the panel (a) of Fig. 3, the soft trilinear parameter A_0 is constrained by the LHC data pertinent to Higgs, gluino, and squark masses. The electron EDM d_e is a complicated function of $|A_0|$ that slowly rises for $|A_0|$ between 1.8 and 4.5 TeV, slowly decreases for $|A_0|$ between 4.5 and 6 TeV, and steeply rises for $|A_0| > 6$ TeV. This function cannot be precisely described by a simple Laurent series in $|A_0|$, but in the largest part of the allowed $|A_0|$ interval it can be roughly approximated by a constant. The φ dependence of d_e is almost sinusoidal with an amplitude a few times smaller than the experimental upper bound (1.2). Moreover, d_e is an approximately constant function of B_0, up to $B_0 \approx 600\,\mathrm{GeV}$. For larger values, i.e., $B_0 \gtrsim 600$ GeV, d_e steeply rises, somehow hinting at a numerical instability in the diagonalization of the sneutrino mass matrix, so our results in this regime are not valid. For $\varphi = \pi/2$, the electron EDM d_e attains values of order the experimental upper limit (1.2), but for $\varphi = \theta = 0$, the predictions are numerically consistent with zero. The dependence of d_e on θ is sinusoidal with an amplitude of order few $\times 10^{-30}$, while its average value strongly depends on the chosen value φ. From Figs. 2 and 3, the following dependence of d_l on m_l, $m_0 = M_{1/2}$, m_N, and $\tan\beta$ may be deduced:

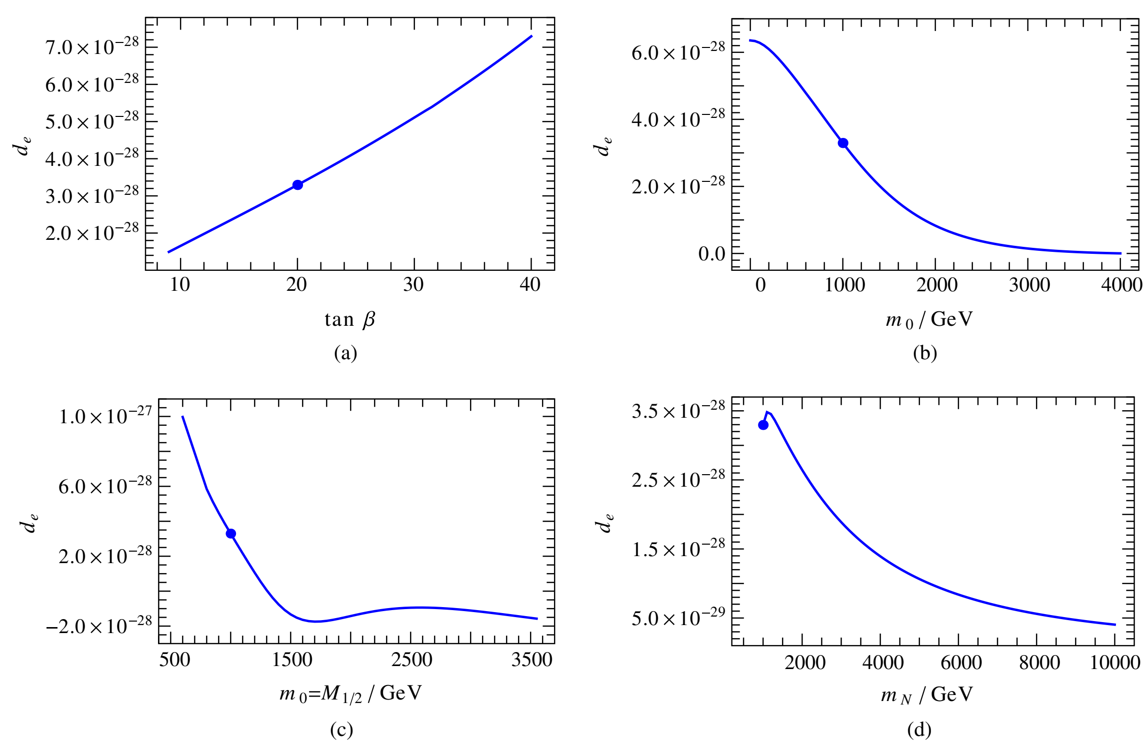

FIG. 2 (color online). Numerical estimates of the electron EDM d_e in the ν_RMSSM, as functions of $\tan\beta$, m_0, $m_0 = M_{1/2}$, and m_N, for $\varphi = \pi/2$ are shown in panels (a), (b), (c), and (d), respectively. The remaining parameters not shown assume the baseline values in (3.3). All input parameters are chosen so as to satisfy the LHC constraints on Higgs, gluino, and squark masses. The heavy dots on the curves indicate the predicted values for d_e evaluated for the default parameters (3.3).

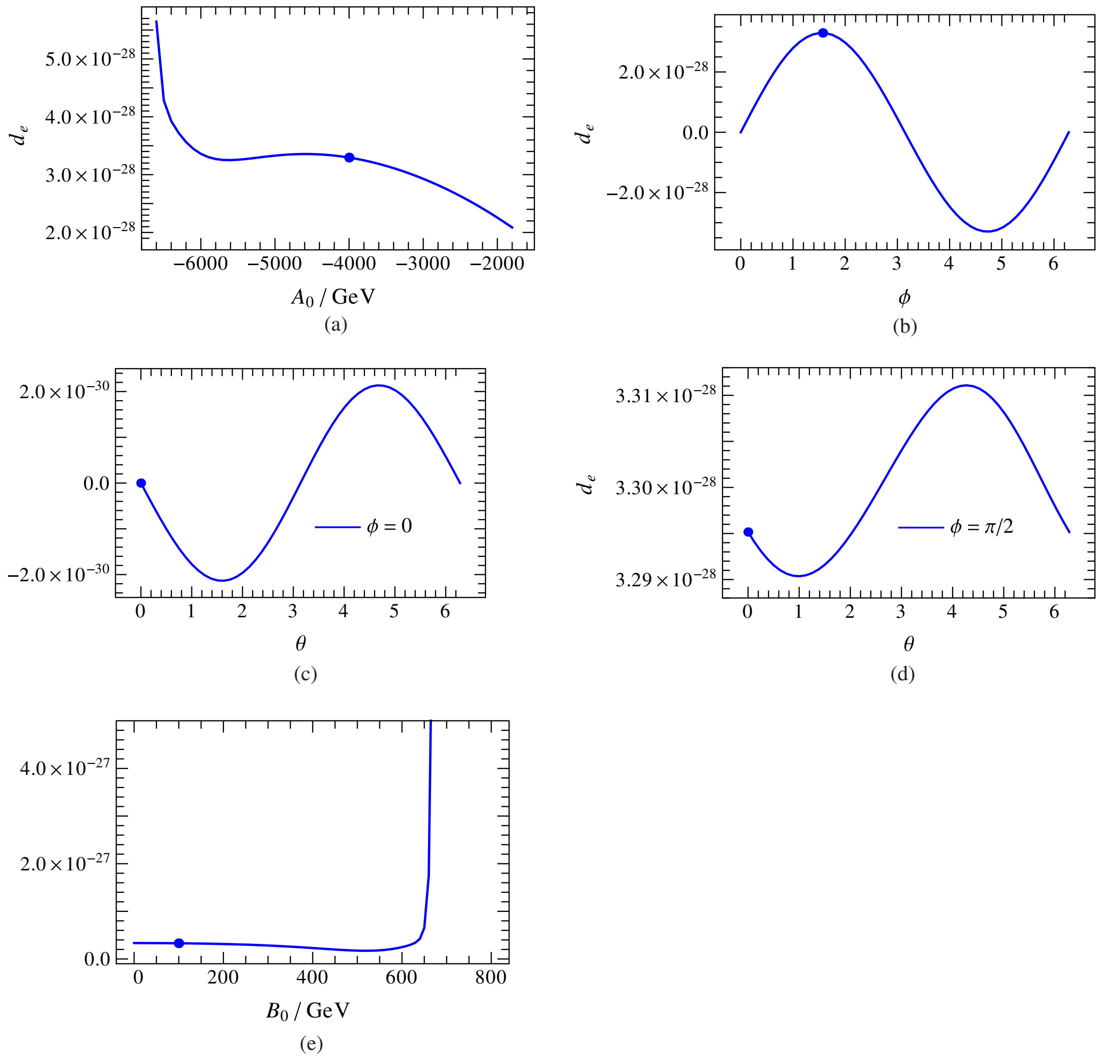

FIG. 3 (color online). Predicted numerical values for the electron EDM d_e versus the soft SUSY-breaking parameters A_0 [panel (a)] and B_0 [panel (e)] and their corresponding soft CP-odd phases φ [panel (b)] and θ [panels (c) and (d)] in the ν_RMSSM, for the baseline scenario in (3.3). If not shown φ assumes value $\pi/2$. The range of input parameters shown in the plots is compatible with the LHC constraints on Higgs, gluino, and squark masses. The heavy dots show the predicted values for d_e, using the default parameters (3.3).

$$d_l \propto \tan\beta \times m_l \times \frac{f(m_0)}{m_N^x}, \qquad m_N < 10\,\text{TeV}, \qquad (3.7)$$

where x assumes values between $2/3$ and 1, and $f(m_0)$ is roughly proportional to the function $-1 - 2.4\,\text{TeV}/m_0 + 6.3\,\text{TeV}^2/m_0^2$. The last factor in Eq. (3.7) corresponds to the scaling factor $1/M_{SUSY}^2$ in the naive approximation (2.14), and in the approximate expressions for lepton EDM derived in [21].

IV. CONCLUSIONS

We have systematically studied the one-loop contributions to the muon anomalous MDM a_μ and the electron EDM d_e in the ν_RMSSM. In particular, we have paid special attention to the effect of the sneutrino soft SUSY-breaking parameters, B_ν and A_ν, and their universal CP phases, θ and φ, on a_μ and d_e. To the best of our knowledge, lepton dipole moments have not been analyzed in detail before, within SUSY models with low-scale singlet (s)neutrinos.

For the anomalous MDM a_μ of the muon, we have found that the heavy singlet neutrino and sneutrino contributions to a_μ are small, typically 1 to 2 orders of magnitude below the muon anomaly Δa_μ. Instead, left-handed sneutrinos and sleptons give the largest effect on Δa_μ, exactly as is the case in the MSSM. The dependence of a_μ on the muon mass m_μ, $\tan\beta$, and the soft SUSY-breaking mass scale M_{SUSY} have been carefully analyzed and their scaling behavior according to (2.13) has been confirmed. Finally, the dependence of a_μ on the universal soft trilinear parameter A_0, the neutrino Yukawa couplings $\mathbf{h}_\nu$ and the heavy neutrino mass m_N are negligible.

Furthermore, we have analyzed the electron EDM d_e in the ν_RMSSM. The heavy singlet neutrinos do not contribute to d_e, and soft SUSY-breaking and sneutrino terms contribute only if the phases φ and/or θ have a nonzero value. The contribution from the possible CP violating terms arising from the relatively complex products of the vertices exposed in (2.11) is numerically shown to be equal to zero. On the other hand, the contribution due to a nonzero value of φ is the largest and may give rise to values for the electron EDM d_e comparable to its present experimental upper limit. The effect of the CP-odd phase θ on d_e is approximately 1 to 2 orders of magnitude smaller than that of φ. The size of d_e increases with $\tan\beta$ and the mass of the lepton m_ℓ; it is approximatively independent of A_0 and B_0, but it generically decreases, as functions of the soft SUSY-breaking parameters m_0, $M_{1/2}$.

Based on our numerical results, we have also derived approximate semianalytical expressions, which differ from those presented in the existing literature for SUSY models realizing a GUT-scale seesaw mechanism. Specifically, the flavor blind CP-odd phases lead to a scaling of the lepton EDM $d_l \propto m_l \tan\beta/m_N^y$, where $2/3 < y < 1$. Further d_l generally decreases with M_{SUSY}, but that cannot be described with a simple scaling law. The dependences on SUSY-breaking parameters A_0 and B_0 are weak in the largest part of the parameter space. The linear dependence on $\tan\beta$ and the dependence on heavy neutrino mass are new results of this paper. In comparison the $\tan\beta$ dependence in Ref. [21] is, depending on its magnitude, either cubic or constant. Given the current experimental limits on d_e, we identified a significant portion of the ν_RMSSM parameter space with maximal CP phase $\varphi = \pi/2$, where the electron EDM d_e can have values comparable to the present and future experimental sensitivities. The effect of sneutrino-sector CP violation on the neutron and Mercury EDMs is expected to be suppressed, which is a distinctive feature for the class of the ν_RMSSM scenarios studied in this paper.

ACKNOWLEDGMENTS

The work of A. P. is supported in part by the Lancaster-Manchester-Sheffield Consortium for Fundamental Physics under STFC Grant No. ST/J000418/1. A. P. also acknowledges partial support by an IPPP associateship from Durham University. The work of A. I. and L. P. was supported by the Ministry of Science, Sports and Technology under Contract No. 119-0982930-1016.

APPENDIX

Here we present detailed analytical expressions for all the quantities that appear in the form factors $G^{L,SB}_{ll\gamma}$ and $G^{R,SB}_{ll\gamma}$, given in (3.4) and (3.5), respectively. To start with, the variables λ_X are defined as $\lambda_X = m_X^2/M_W^2$, for instance, $\lambda_{\tilde{e}} = m_{\tilde{e}}^2/M_W^2$. The integrals J^a_{bc} derived from loop integrations [13] are UV finite. These are given by

$$J^a_{bc} = (-1)^{a-n_b-n_c} \int_0^\infty \frac{dx x^{1+a}}{(x+\lambda_b)^{n_b}(x+\lambda_c)^{n_c}}. \tag{A1}$$

The couplings $\tilde{V}^{0\ell L}_{lma}$ and $\tilde{V}^{0\ell R}_{lma}$ read

$$\tilde{V}^{0\ell L}_{lma} = -\sqrt{2}t_w Z^*_{m1}(R^{\tilde{e}}_R)^*_{al} - \frac{(m_e)_l}{\sqrt{2}c_\beta M_W} Z^*_{m3}(R^{\tilde{e}}_L)^*_{al} \tag{A2}$$

$$\tilde{V}^{0\ell R}_{lma} = \frac{1}{\sqrt{2}c_w}(c_w Z_{m2} + s_w Z_{m1})(R^{\tilde{e}}_L)^*_{al} - \frac{(m_e)_l}{\sqrt{2}c_\beta M_W} Z_{m3}(R^{\tilde{e}}_R)^*_{al}, \tag{A3}$$

where $t_w = \tan\theta_w$, $c_w = \cos\theta_w$, $s_w = \sin\theta_w$, and $c_\beta = \cos\beta$. The unitary matrices $\mathcal{U}$ and $\mathcal{V}$, which diagonalize the chargino mass matrix, and the unitary matrix Z diagonalizing the neutralino mass matrix are taken from [33]. Finally, the following lepton-slepton disalignment matrices may be defined:

$$R^{\tilde{e}L}_{ak} = U^{\tilde{e}}_{ia} U^{e_L *}_{ik}, \quad R^{\tilde{e}R}_{ak} = U^{\tilde{e}}_{i+3a} U^{e_R *}_{ik}, \tag{A4}$$

where U^{e_L}, U^{e_R}, and $U^{\tilde{e}}$ are unitary matrices diagonalizing the lepton and slepton mass matrices, with $a = 1, \ldots, 6$ and $i, k = 1, 2, 3$.

[1] J. Beringer et al. (Paticle Data Group), Phys. Rev. D 86, 010001 (2012).
[2] J. J. Hudson, D. M. Kara, I. J. Smallman, B. E. Sauer, M. R. Tarbutt, and E. A. Hinds, Nature (London) 473, 493 (2011).
[3] M. Jung, J. High Energy Phys. 05 (2013) 168.
[4] M. Pospelov and A. Ritz, Ann. Phys. (Amsterdam) 318, 119 (2005).
[5] D. I. Kazakov, Nucl. Phys. B, Proc. Suppl. 203–204, 118 (2010).
[6] F. Deppisch and J. W. F. Valle, Phys. Rev. D 72, 036001 (2005).
[7] D. Wyler and L. Wolfenstein, Nucl. Phys. B218, 205 (1983).
[8] R. N. Mohapatra, Phys. Rev. Lett. 56, 561 (1986); R. N. Mohapatra and J. W. F. Valle, Phys. Rev. D 34, 1642

(1986); S. Nandi and U. Sarkar, Phys. Rev. Lett. **56**, 564 (1986).

[9] G. C. Branco, W. Grimus, and L. Lavoura, Nucl. Phys. **B312**, 492 (1989).

[10] A. Pilaftsis, Z. Phys. C **55**, 275 (1992); P. S. B. Dev and A. Pilaftsis, Phys. Rev. D **86**, 113001 (2012).

[11] A. Ilakovac and A. Pilaftsis, Phys. Rev. D **80**, 091902 (2009); , Nucl. Phys. B, Proc. Suppl. **218**, 26 (2011).

[12] M. Hirsch, F. Staub, and A. Vicente, Phys. Rev. D **85**, 113013 (2012); A. Abada, D. Das, A. Vicente, and C. Weiland, J. High Energy Phys. 09 (**2012**) 015.

[13] A. Ilakovac, A. Pilaftsis, and L. Popov, Phys. Rev. D **87**, 053014 (2013).

[14] F. Borzumati and A. Masiero, Phys. Rev. Lett. **57**, 961 (1986); J. Hisano, T. Moroi, K. Tobe, M. Yamaguchi, and T. Yanagida, Phys. Lett. B **357**, 579 (1995); J. Hisano, T. Moroi, K. Tobe, and M. Yamaguchi, Phys. Rev. D **53**, 2442 (1996); J. Hisano and D. Nomura, Phys. Rev. D **59**, 116005 (**1999**); D. F. Carvalho, J. R. Ellis, M. E. Gomez, and S. Lola, Phys. Lett. B **515**, 323 (2001); J. R. Ellis, J. Hisano, S. Lola, and M. Raidal, Nucl. Phys. **B621**, 208 (2002); E. Arganda and M. Herrero, Phys. Rev. D **73**, 055003 (2006); J. Hisano, M. Nagai, P. Paradisi, and Y. Shimizu, J. High Energy Phys. 12 (**2009**) 030.

[15] G. Aad *et al.* (ATLAS Collaboration), Phys. Lett. B **716**, 1 (2012); S. Chatrchyan *et al.* (CMS Collaboration), Phys. Lett. B **716**, 30 (2012); S. Chatrchyan *et al.* (CMS Collaboration), Phys. Rev. Lett. **110**, 081803 (2013).

[16] G. Aad *et al.* (ATLAS Collaboration), J. High Energy Phys. 07 (**2012**) 167; S. Chatrchyan *et al.* (CMS Collaboration) Phys. Rev. Lett. **109**, 171803 (2012).

[17] B. L. Roberts, Nucl. Phys. B, Proc. Suppl. **218**, 237 (2011); http://physics.bu.edu/people/show/61; Nucl. Phys. B, Proc. Suppl., **155**, 372 (2006)).

[18] G. Venanzoni, J. Phys. Conf. Ser. **349**, 012008 (2012); Frascati Phys. Ser. **54**, 52 (2012); E989 experiment at Fermilab: http://gm2.fnal.gov.

[19] J. Ellis, J. Hisano, M. Raidal, and Y. Shimizu, Phys. Lett. B **528**, 86 (2002).

[20] I. Masina, Nucl. Phys. **B671**, 432 (2003).

[21] Y. Farzan and M. E. Peskin, Phys. Rev. D **70**, 095001 (2004).

[22] A. Dedes, H. E. Haber, and J. Rosiek, J. High Energy Phys. 11 (**2007**) 059.

[23] J. R. Ellis, J. S. Lee, and A. Pilaftsis, J. High Energy Phys. 10 (**2008**) 049.

[24] G. C. Branco, L. Lavoura, and J. P. Silva, *CP Violation* (Clarendon Press, Oxford, 1999).

[25] See Eq. (C3) in Ref. [13].

[26] S. Ferrara and E. Remiddi, Phys. Lett. **53B**, 347 (1974).

[27] A. Ilakovac and A. Pilaftsis, Nucl. Phys. **B437**, 491 (1995).

[28] U. Chattopadhyay and P. Nath, Phys. Rev. D **53**, 1648 (1996); T. Moroi, Phys. Rev. D **53**, 6565 (1996); Phys. Rev. D **56**, 4424(E) (1997); M. S. Carena, G. F. Giudice, and C. E. M. Wagner, Phys. Lett. B **390**, 234 (1997).

[29] D. Stöckinger, J. Phys. G **34**, R45 (2007).

[30] A. Pilaftsis, Nucl. Phys. **B644**, 263 (2002).

[31] A. Pilaftsis, Phys. Rev. Lett. **95**, 081602 (2005); A. Pilaftsis and T. E. J. Underwood, Phys. Rev. D **72**, 113001 (2005); F. F. Deppisch and A. Pilaftsis, Phys. Rev. D **83**, 076007 (2011).

[32] J. Kersten and A. Y. Smirnov, Phys. Rev. D **76**, 073005 (2007).

[33] M. Drees, M. Goldbole, and P. Roy, *Theory and Phenomenology of Sparticles* (World Scientific, Singapore, 2004).