Azimuthal asymmetries in semi-inclusive deep inelastic scattering with polarized beam and/or target and their nuclear dependences

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Using the formalism obtained from collinear expansion, we calculate the differential cross section and azimuthal asymmetries in the semi-inclusive deeply inelastic lepton-nucleon (nucleus) scattering process $e^- + N(A) \to e^- + q + X$ with both polarized beam and polarized target up to twist-3. We derive the azimuthal asymmetries in terms of twist-3 parton correlation functions. We simplify the results by using the QCD equation of motion that leads to a set of relationships between different twist-3 functions. We further study the nuclear dependence of azimuthal asymmetries and show that they have similar suppression factors as those in the unpolarized reactions.

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I. INTRODUCTION

Azimuthal asymmetries in semi-inclusive deeply inelastic lepton-nucleon scattering (SIDIS) play an important role in the study of the partonic structure of the nucleon, attracting much effort in both theory [1–15] and experiment [16–28]. In such studies, spin and nuclear dependences are often important and provide a useful tool to investigate these effects. Also because of this, higher twist contributions are often significant and need to be taken into account precisely. Besides, such higher twist effects usually depend on new higher twist parton correlation functions; hence the studies of them provide a new window to learn about the structure of the nucleon.

One of the most important issues in these studies is to establish the relationships between the experimentally measurable quantities and different parton distribution and/or correlation functions that describe the partonic structure of the nucleon and the properties of the hadronic interaction in a consistent theoretical framework. Collinear expansion seems to be the most promising technique that leads to such a framework. It was proposed in the 1980s and has been successfully applied to the inclusive processes [29–33]. It has been shown that, after collinear expansion, the differential cross section can be expressed as a convolution of the collinear expanded hard parts with the parton distribution and/or correlation functions in the nucleon. While the hard parts are calculable, the parton distribution and/or correlation functions can be defined in terms of gauge invariant matrix elements of the nucleon state. These matrix elements contain the information about parton distributions inside the nucleon. The gauge link inside the gauge invariant matrix elements is a result of multiple gluon scattering within the collinear expansion scheme. Within this scheme, one performs Taylor expansion of the hard parts around the collinear momenta. The leading twist contributions come from the zeroth order in the collinear expansion allowing all momenta taking their collinear values. Higher twist contributions from the higher orders of the Taylor expansion can be calculated consistently.

Higher twist effects in semi-inclusive deeply inelastic lepton-nucleon scattering have also been studied in literature, and calculations of the differential cross section up to the twist-3 level have been carried out [5,6,8,9]. However, most of these studies do not consider the application of collinear expansion. Instead, they usually start from the expressions obtained directly from the Feynman diagrams, extract the leading (twist-2) and the subleading (twist-3) twist contributions by making appropriate approximations, and insert the gauge link whenever needed to guarantee the gauge invariance of the parton distribution and/or correlation functions. It is thus unknown, if yes, how the collinear expansion is applicable here. It is not obvious where the gauge link comes from and which form it takes. It is also not known whether the calculations extend to an even higher twist. A systematic study leading to a consistent formalism is necessary but still lacking.

In Ref. [12], we made the first step toward this goal by applying the collinear expansion technique to the SIDIS process $e^- + N \to e^- + q + X$, where q denotes a quark that is equivalent to a jet in the experiment. We showed that the collinear expansion technique is applicable for this process and derived a formalism suitable for studying leading as well as higher twist contributions to $e^- + N \to e^- + q + X$ in a systematic way. This formalism is similar to what we have for the inclusive process, and similar expressions can be obtained for the differential cross section or the hadronic tensor as a convolution of the hard parts and the unintegrated or transverse momentum dependent (TMD) parton correlation functions. We carried out the calculations of the azimuthal asymmetries in the unpolarized cases up to twist-4 [14] and those in the case with transversely polarized targets up to twist-3 [12]. Furthermore, we also showed that the multiple gluon scattering contained in the gauge link leads to a significant nuclear dependence of the azimuthal asymmetries that can be studied experimentally [13,34].

In this paper, we present calculations of azimuthal asymmetries in the semi-inclusive process $e^- + N(A) \to e^- + q + X$ with the beam and target in different polarizations up to twist-3 using the formalism derived in [12]. For completeness, we summarize the formalism in Sec. II and present the results of the hadronic tensor. In Sec. III, we present the results of the differential cross sections and the azimuthal asymmetries. We study the nuclear dependence in Sec. IV and conclude in Sec. V.

II. THE HADRONIC TENSOR

The formalism that we use in our calculations are derived in [12] for the semi-inclusive process $e^- + N \to e^- + q + X$ and are summarized in [14]. It is obtained by applying collinear expansion and contains the contributions from the multiple gluon scattering. For completeness and also for comparison with other approaches such as those in [5,9], we summarize the most related results of this formalism in Sec. II.A and present the results for the hadronic tensors in different polarized cases up to twist-3 in other parts of this section.

A. The formalism

We consider the SIDIS process $e^- + N \to e^- + q + X$ and use l, l', p, k, and k' to denote the four-momenta of the electron, nucleon, and parton, respectively; those with primes are for the final state. The polarization vectors are denoted by s_l and s and are taken as $s_l^\mu = \lambda_l l^\mu/m_e + s_{l\perp}^\mu$, and $s^\mu = \lambda p^\mu/M + s_\perp^\mu$, where λ_l and λ are the helicities. We use light-cone coordinate $k^\mu = (k^+, k^-, \vec{k}_\perp)$ and take unit vectors as $\bar{n} = (1, 0, \vec{0}_\perp)$, $n = (0, 1, \vec{0}_\perp)$, and $n_\perp = (0, 0, \vec{n}_\perp)$. We choose the coordinate system such that $p = p^+\bar{n}$, $q = -x_B p + nQ^2/(2x_B p^+)$, $l_\perp = |\vec{l}_\perp| n_{\perp 1}$, where $x_B = Q^2/2p\cdot q$, and define $y = p\cdot q/p\cdot l$.

The differential cross section is given by

$$d\sigma = \frac{2\alpha_{\rm em}^2 e_q^2}{sQ^4} L^{\mu\nu}(l, l', s_l) \frac{d^2 W_{\mu\nu}}{d^2 k'_\perp} \frac{d^3 l' d^2 k'_\perp}{2E_{l'}}, \tag{1}$$

where $L^{\mu\nu}(l, l', s_l)$ is the leptonic tensor, and the hadronic tensor is given by

$$\frac{d^2 W_{\mu\nu}}{d^2 k'_\perp} = \int \frac{dk'_z}{(2\pi)^3 2E_{k'}} W_{\mu\nu}^{(\rm si)}(q, p, s, k'), \tag{2}$$

$$W_{\mu\nu}^{(\rm si)}(q, p, s, k') = \frac{1}{2\pi} \sum_X \langle p, s | j_\mu(0) | k', X\rangle \langle k', X | j_\nu(0) | p, s\rangle \times (2\pi)^4 \delta^4(p + q - p_X). \tag{3}$$

As discussed in [29,30] for inclusive DIS and in [12] for semi-inclusive DIS, to obtain the gauge invariant form of the hadronic tensor including leading and higher twist contributions in a systematic way, at the tree level, we need to consider the Feynman diagram series as illustrated in Fig. 1. The hadronic tensor is given by a sum of the contribution from each diagram $W_{\mu\nu}^{(\rm si)} = \sum_j W_{\mu\nu}^{(j,\rm si)}$. For example, for $j = 0$, 1, and 2,

$$W_{\mu\nu}^{(0,\rm si)} = \frac{1}{2\pi} \int \frac{d^4 k}{(2\pi)^4} {\rm Tr}[\hat{H}_{\mu\nu}^{(0)}(k, q) \hat{\phi}^{(0)}(k, p, S)] 2E_{k'} \times (2\pi)^3 \delta^3(\vec{k}' - \vec{k} - \vec{q}), \tag{4}$$

$$W_{\mu\nu}^{(1,\rm si)} = \frac{1}{2\pi} \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \sum_{c=L,R} 2E_{k'} \times (2\pi)^3 \delta^3(\vec{k}' - \vec{k}_c - \vec{q}) \times {\rm Tr}[\hat{H}_{\mu\nu}^{(1,c)\rho}(k_1, k_2, q) \hat{\phi}_\rho^{(1)}(k_1, k_2, p)], \tag{5}$$

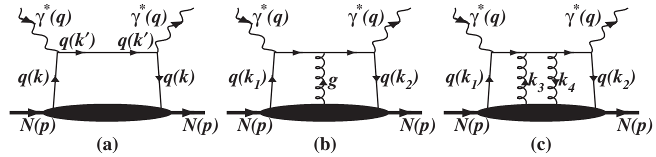

FIG. 1. Examples of the Feynman diagram series considered for $\gamma^* + N \to q + X$ with (a) $j = 0$, (b) $j = 1$, and (c) $j = 2$ gluons exchanged.

$$W_{\mu\nu}^{(2,\mathrm{si})} = \frac{1}{2\pi}\int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\sum_{c=L,R,M} 2E_{k'} \times (2\pi)^3\delta^3(\vec{k}' - \vec{k}_c - \vec{q}) \times \mathrm{Tr}[\hat{H}_{\mu\nu}^{(2,c)\rho\sigma}(k_1,k_2,k,q)\hat{\phi}_{\rho\sigma}^{(2)}(k_1,k_2,k,p,S), \quad (6)$$

where c denotes different cuts, $k_L = k_1$, $k_R = k_2$, $k_M = k$, and the hard parts are given by

$$\hat{H}_{\mu\nu}^{(0)}(q,k) = \gamma_\mu(\not{k} + \not{q})\gamma_\nu(2\pi)\delta_+((k-q)^2), \quad (7)$$

$$\hat{H}_{\mu\nu}^{(1,L)\rho}(k_1,k_2,q) = \gamma_\mu(\not{k}_2 + \not{q})\gamma^\rho\frac{\not{k}_1 + \not{q}}{(k_1+q)^2 - i\epsilon}\gamma_\nu(2\pi)\delta_+((k_2+q)^2), \quad (8)$$

$$\hat{H}_{\mu\nu}^{(2,L)\rho\sigma}(k_1,k_2,k,q) = \gamma_\mu(\not{k}_2 + \not{q})\gamma^\rho\frac{\not{k} + \not{q}}{(k+q)^2 - i\epsilon}\gamma^\sigma\frac{\not{k}_1 + \not{q}}{(k_1+q)^2 - i\epsilon}\gamma_\nu \times (2\pi)\delta_+((k_2+q)^2), \quad (9)$$

and the matrix elements or the correlators are defined as

$$\hat{\phi}^{(0)}(k,p,S) = \int d^4z e^{ikz}\langle p,S|\bar{\psi}(0)\psi(z)|p,S\rangle, \quad (10)$$

$$\hat{\phi}_\rho^{(1)}(k_1,k_2,p,S) = \int d^4y d^4z e^{ik_1z + i(k_2-k_1)y}\langle p,S|\bar{\psi}(0)gA_\rho(z)\psi(y)|p,S\rangle, \quad (11)$$

$$\hat{\phi}_{\rho\sigma}^{(2)}(k_1,k_2,k,p,S) = \int d^4y d^4y' d^4z e^{ik_1\cdot y + i(k-k_1)\cdot z' + i(k_2-k)\cdot z} \times \langle p,S|\bar{\psi}(0)gA_\rho(z)gA_\sigma(z')\psi(y)|p,S\rangle. \quad (12)$$

It is well known that, since the field operators in the correlators given by Eqs. (10)–(12) are at different space-time points, these correlators and the parton distribution and/or correlation functions defined from them are not gauge invariant. To reach the gauge invariant form, we need to do the collinear expansion. The expansion has been carried out and summarized in [12,14]. We present the main results in the following. For brevity, we show only $j = 0$ and 1 terms that are needed in our calculations in this paper up to twist-3.

It has been shown that [12], after collinear expansion, the hadronic tensor takes the form

$$\frac{d^2W_{\mu\nu}}{d^2k'_\perp} = \frac{d^2\tilde{W}_{\mu\nu}^{(0)}}{d^2k'_\perp} + \frac{d^2\tilde{W}_{\mu\nu}^{(1)}}{d^2k'_\perp} + \cdots, \quad (13)$$

$$\frac{d\tilde{W}_{\mu\nu}^{(0)}}{d^2k'_\perp} = \frac{1}{2\pi}\int dx d^2k_\perp \mathrm{Tr}[\hat{H}_{\mu\nu}^{(0)}(x)\hat{\Phi}^{(0)N}(x,k_\perp)] \times \delta^{(2)}(\vec{k}_\perp - \vec{k'}_\perp), \quad (14)$$

$$\frac{d\tilde{W}_{\mu\nu}^{(1)}}{d^2k'_\perp} = \frac{1}{2\pi}\int dx_1 d^2k_{1\perp} dx_2 d^2k_{2\perp} \times \sum_{c=L,R}\mathrm{Tr}[\hat{H}_{\mu\nu}^{(1,c)\rho}(x_1,x_2)\omega_\rho^{\rho'} \times \hat{\Phi}_{\rho'}^{(1)N}(x_1,k_{1\perp},x_2,k_{2\perp})]\delta^{(2)}(\vec{k}_{c\perp} - \vec{k'}_\perp), \quad (15)$$

where the symbols $\tilde{W}^{(j)}$ with tildes represent results after collinear expansion and $\omega_\rho{}^{\rho'} = g_\rho{}^{\rho'} - \bar{n}_\rho n^{\rho'}$ is a projection operator. The matrix elements take the gauge invariant form and are given by

$$\hat{\Phi}^{(0)N}(x,k_\perp) = \int \frac{p^+dy^-d^2y_\perp}{(2\pi)^3}e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp} \times \langle N|\bar{\psi}(0)\mathcal{L}(0;y)\psi(y)|N\rangle, \quad (16)$$

$$\hat{\Phi}_\rho^{(1)N}(x_1,k_{1\perp},x_2,k_{2\perp}) = \int \frac{p^+dy^-d^2y_\perp}{(2\pi)^3}\frac{p^+dz^-d^2z_\perp}{(2\pi)^3} \times e^{ix_2p^+z^- - i\vec{k}_{2\perp}\cdot\vec{z}_\perp + ix_1p^+(y^- - z^-) - i\vec{k}_{1\perp}\cdot(\vec{y}_\perp - \vec{z}_\perp)} \times \langle N|\bar{\psi}(0)\mathcal{L}(0;z)D_\rho(z)\mathcal{L}(z;y)\psi(y)|N\rangle, \quad (17)$$

where $\mathcal{L}(0;y)$ is the gauge link obtained in the collinear expansion and is given by

$$\mathcal{L}(0;y) = \mathcal{L}_\parallel^\dagger(\infty,\vec{0}_\perp;0,\vec{0}_\perp)\mathcal{L}_\perp(\infty;\vec{0}_\perp,\vec{y}_\perp)\mathcal{L}_\parallel(\infty,\vec{y}_\perp;y^-,\vec{y}_\perp), \quad (18)$$

$$\mathcal{L}_\parallel(\infty,\vec{y}_\perp;y^-,\vec{y}_\perp) = Pe^{-ig\int_{y^-}^{\infty}d\xi^- A^+(\xi^-,\vec{y}_\perp)}, \quad (19)$$

$$\mathcal{L}_\perp(\infty;\vec{0}_\perp,\vec{y}_\perp) = Pe^{-ig\int_{\vec{0}_\perp}^{\vec{y}_\perp} d\vec{\xi}_\perp\cdot\vec{A}_\perp(\infty,\vec{\xi}_\perp)}, \tag{20}$$

where P stands for path integral. The hard parts in these $\tilde{W}^{(j)}$'s with tildes are the first terms in the Taylor expansions at $k_i = x_i p$ of the corresponding hard parts obtained directly from the Feynman diagrams. They are given by [12]

$$\hat{H}^{(0)}_{\mu\nu}(x) = \frac{2\pi}{2q\cdot p}\gamma_\mu(\not{q}+x\not{p})\gamma_\nu\delta(x-x_B), \tag{21}$$

$$\hat{H}^{(1,L)\rho}_{\mu\nu}(x_1,x_2) = \frac{2\pi}{(2q\cdot p)^2}\frac{\gamma_\mu(\not{q}+x_2\not{p})\gamma^\rho(\not{q}+x_1\not{p})\gamma_\nu}{x_2-x_B-i\varepsilon}\delta(x_1-x_B). \tag{22}$$

Equations (13)–(22) form the basis for calculating the hadronic tensor in $e^- + N \to e^- + q + X$ at the tree level including leading and higher twist contributions. We emphasize once more that these equations are derived from the Feynman diagram series in Fig. 1 using collinear expansion. They are nothing else but a reorganization of $W^{(j,\mathrm{si})}$ given by Eqs. (4)–(6) obtained directly from this diagram series. We also note that $\tilde{W}^{(j)}$ differs distinctly from the corresponding $W^{(j,\mathrm{si})}$ and shows in particular the following features.

(1) None of the $\tilde{W}^{(j)}$'s with tildes corresponds to one single Feynman diagram in the diagram series given in Fig. 1. It contains contributions from all of the infinite number of diagrams in this diagram series with the exchange of $j = 0, 1, 2, \ldots$ gluon(s).

(2) The correlators acquire automatically the gauge links and are gauge invariant. The gauge link comes from the multiple gluon scattering shown in Fig. 1. Furthermore, in the quark-gluon-quark correlator, the covariant derivative is obtained to replace the gluon field operator in the original correlator before collinear expansion.

(3) All the parton momenta in the hard parts take only the $\bar{n}$ components, while the corresponding n and $n_\perp$ components are taken as zero. Also there are projection operators $\omega_\rho{}^{\rho'}$'s in the expressions for $\tilde{W}^{(j)}$ for $j > 0$ due to the collinear expansion.

Because of the features mentioned above, in particular point (3), the expressions for $\tilde{W}^{(j)}$ can be further simplified. In fact, because of (3), the hard parts reduce to the following simple form:

$$\hat{H}^{(0)}_{\mu\nu}(x) = \pi\hat{h}^{(0)}_{\mu\nu}\delta(x-x_B), \tag{23}$$

$$\hat{H}^{(1,L)\rho}_{\mu\nu}(x_1,x_2)\omega_\rho{}^{\rho'} = \frac{\pi}{2q\cdot p}\hat{h}^{(1)\rho}_{\mu\nu}\omega_\rho{}^{\rho'}\delta(x_1-x_B), \tag{24}$$

where $\hat{h}^{(0)}_{\mu\nu} = \gamma_\mu\not{n}\gamma_\nu/p^+$, and $\hat{h}^{(1)\rho}_{\mu\nu} = \gamma_\mu\bar{\not{n}}\gamma^\rho\not{n}\gamma_\nu$. We see not only that their x dependences are contained only in the δ functions but also that $\hat{H}^{(1,L)\rho}_{\mu\nu}(x_1,x_2)\omega_\rho{}^{\rho'}$ depends only on x_1 but not on x_2. This implies that we can simply carry out the integration over k_2 in the correlator $\hat{\Phi}^{(1)}$ in $\tilde{W}^{(1)}$ and obtain

$$\frac{d^2\tilde{W}^{(0)}_{\mu\nu}}{d^2k_\perp} = \frac{1}{2}\,\mathrm{Tr}[\hat{h}^{(0)}_{\mu\nu}\hat{\Phi}^{(0)N}(x_B,k_\perp)], \tag{25}$$

$$\frac{d^2\tilde{W}^{(1,L)}_{\mu\nu}}{d^2k_\perp} = \frac{1}{4q\cdot p}\,\mathrm{Tr}[\hat{h}^{(1)\rho}_{\mu\nu}\omega_\rho{}^{\rho'}\hat{\varphi}^{(1)N}_{\rho'}(x_B,k_\perp)]. \tag{26}$$

The new correlator $\hat{\varphi}^{(1)}_\rho$ is defined as $\hat{\varphi}^{(1)N}_\rho(x_1,k_{1\perp}) \equiv \int dx_2 d^2k_{2\perp}\hat{\Phi}^{(1)N}_\rho(x_1,k_{1\perp},x_2,k_{2\perp})$ and is given by

$$\hat{\varphi}^{(1)N}_\rho(x,k_\perp) = \int\frac{p^+dy^-d^2y_\perp}{(2\pi)^3}e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp} \times\langle N|\bar{\psi}(0)D_\rho(0)\mathcal{L}(0;y)\psi(y)|N\rangle. \tag{27}$$

It depends only on one parton momentum k; the quark field operator $\bar{\psi}$ and the covariant derivative D_ρ are at the same space-time point. Here, we may note that, unlike what we do in the current approach, the calculations presented in, e.g., [9] start from the hadronic tensors $W^{(j)}$'s given by Eqs. (4) and (5) obtained directly from the Feynman diagrams given by Figs. 1(a) and 1(b). To obtain the corresponding results, they need to make approximations for the hard parts by keeping only $\bar{n}$ terms, inserting the gauge links into the matrix elements to guarantee the gauge invariance. Such operations are avoided in the formalism obtained using collinear expansion where the Feynman diagram series is considered systematically and the gauge links are obtained automatically.

B. Twist-3 contributions to $d^2W_{\mu\nu}/d^2k_\perp$

Up to twist-3, we need to consider the contributions from $d^2\tilde{W}^{(0)}_{\mu\nu}/d^2k_\perp$ and those from $d^2\tilde{W}^{(1)}_{\mu\nu}/d^2k_\perp$. Since the hard part $\hat{h}^{(0)}_{\mu\nu}$ and $\hat{h}^{(1)\rho}_{\mu\nu}$ have an odd number of γ matrices, only γ^α and $\gamma^\alpha\gamma_5$ terms of correlation matrices contribute. We decompose the correlation matrices as $\hat{\Phi}^{(0)} = (\Phi^{(0)}_\alpha\gamma^\alpha - \tilde{\Phi}^{(0)}_\alpha\gamma_5\gamma^\alpha)/2 + \cdots$, $\hat{\varphi}^{(1)}_\rho = (\varphi^{(1)}_{\rho\alpha}\gamma^\alpha - \tilde{\varphi}^{(1)}_{\rho\alpha}\gamma_5\gamma^\alpha)/2 + \cdots$, and obtain the hadronic tensors as

$$\frac{d^2\tilde{W}^{(0)}_{\mu\nu}}{d^2k_\perp} = \frac{1}{4}\,\mathrm{Tr}[\hat{h}^{(0)}_{\mu\nu}\gamma^\alpha]\Phi^{(0)}_\alpha - \frac{1}{4}\,\mathrm{Tr}[\hat{h}^{(0)}_{\mu\nu}\gamma_5\gamma^\alpha]\tilde{\Phi}^{(0)}_\alpha, \tag{28}$$

$$\frac{d^2\tilde{W}^{(1,L)}_{\mu\nu}}{d^2k_\perp} = \frac{1}{8p\cdot q}\,\mathrm{Tr}[\hat{h}^{(1)\rho}_{\mu\nu}\gamma^\alpha]\varphi^{(1)}_{\rho\alpha} - \frac{1}{8p\cdot q}\,\mathrm{Tr}[\hat{h}^{(1)\rho}_{\mu\nu}\gamma_5\gamma^\alpha]\tilde{\varphi}^{(1)}_{\rho\alpha}. \tag{29}$$

To proceed, we need to decompose the matrix elements involved in terms of the Lorentz covariants constructed from p, n, $k_\perp$, and S multiplied by the scalar functions of x and $k_\perp^2$. These scalar functions are just different components of the parton distribution and/or correlation functions. Such decompositions are the same as those discussed in different publications [9,35]. By inserting them into the above mentioned Eqs. (28) and (29), we can obtain the hadronic tensors in terms of these parton distribution and correlation functions. In the following, we calculate different contributions term by term. We first consider $d^2\tilde{W}^{(0)}_{\mu\nu}/d^2k_\perp$. Up to the twist-3 level, $\Phi^{(0)}_\alpha$ and $\tilde{\Phi}^{(0)}_\alpha$ are decomposed as [35]

$$\Phi^{(0)}_\alpha = (f_1 - \varepsilon_\perp^{ks} f_{1T}^\perp)p_\alpha + f^\perp k_{\perp\alpha} - f_T M\varepsilon_{\perp\alpha i}s_\perp^i - \frac{f_T^\perp}{M}\left(k_{\perp\alpha}k_{\perp\beta} - \frac{1}{2}k_\perp^2 d_{\alpha\beta}\right)\varepsilon_\perp^{\beta i}s_{\perp i} - \lambda f_L^\perp \varepsilon_{\perp\alpha i}k_\perp^i + \cdots, \tag{30}$$

$$\tilde{\Phi}^{(0)}_\alpha = -\left(\lambda g_{1L} - \frac{k_\perp\cdot s_\perp}{M}g_{1T}^\perp\right)p_\alpha + g^\perp \varepsilon_{\perp\alpha i}k_\perp^i - g_T M s_{\perp\alpha} + \frac{g_T^\perp}{M}\left(k_{\perp\alpha}k_{\perp\beta} - \frac{1}{2}k_\perp^2 d_{\alpha\beta}\right)s_\perp^\beta - \lambda g_L^\perp k_{\perp\alpha} + \cdots, \tag{31}$$

where $\varepsilon_\perp^{\mu\nu} \equiv \varepsilon^{\rho\sigma\mu\nu}\bar{n}_\rho n_\sigma$, $d_{\mu\nu} \equiv g_{\mu\nu} - \bar{n}_\mu n_\nu - \bar{n}_\nu n_\mu$, and $\varepsilon_\perp^{ks} \equiv (1/M)\varepsilon_\perp^{ij}k_{\perp i}s_{\perp j} = (1/M)(\vec{k}_\perp \times \vec{s}_\perp)\cdot\hat{z}$.

$$\mathrm{Tr}[\hat{h}^{(0)}_{\mu\nu}\not{p}] = -4d_{\mu\nu}, \tag{32}$$

$$\mathrm{Tr}[\hat{h}^{(0)}_{\mu\nu}\gamma_5\not{p}] = 4i\varepsilon_{\perp\mu\nu}, \tag{33}$$

$$\mathrm{Tr}[\hat{h}^{(0)}_{\mu\nu}\gamma_\alpha] = \frac{4}{p^+}n_{\{\mu}d_{\nu\}\alpha}, \tag{34}$$

$$\mathrm{Tr}[\hat{h}^{(0)}_{\mu\nu}\gamma_5\gamma_\alpha] = \frac{4i}{p^+}n_{[\mu}\varepsilon_{\perp\nu]\alpha}, \tag{35}$$

where $A_{\{\mu}B_{\nu\}} \equiv A_\mu B_\nu + A_\nu B_\mu$, and $A_{[\mu}B_{\nu]} \equiv A_\mu B_\nu - A_\nu B_\mu$. Hence, we obtain, up to twist-3,

$$\frac{d^2\tilde{W}^{(0)}_{\mu\nu}}{d^2k_\perp} = -d_{\mu\nu}(f_1 - \varepsilon_\perp^{ks}f_{1T}^\perp) + \frac{1}{p\cdot q}k_{\perp\{\mu}(q+x_Bp)_{\nu\}}(f^\perp - \varepsilon_\perp^{ks}f_T^\perp) - \frac{M}{p\cdot q}(q+x_Bp)_{\{\mu}\varepsilon_{\perp\nu\}i}s_\perp^i\hat{f}_T - \frac{\lambda}{p\cdot q}(q+x_Bp)_{\{\mu}\varepsilon_{\perp\nu\}i}k_\perp^i f_L^\perp + i\varepsilon_{\perp\mu\nu}\left(\lambda g_{1L} - \frac{k_\perp\cdot s_\perp}{M}g_{1T}^\perp\right) - \frac{i}{p\cdot q}k_{\perp[\mu}(q+x_Bp)_{\nu]}(g^\perp + \varepsilon_\perp^{ks}g_T^\perp) + \frac{iM}{p\cdot q}(q+x_Bp)_{[\mu}\varepsilon_{\perp\nu]i}s_\perp^i\hat{g}_T + \frac{i\lambda}{p\cdot q}(q+x_Bp)_{[\mu}\varepsilon_{\perp\nu]i}k_\perp^i g_L^\perp, \tag{36}$$

where $\hat{f}_T = f_T - \frac{k_\perp^2}{2M^2}f_T^\perp$ and $\hat{g}_T = g_T - \frac{k_\perp^2}{2M^2}g_T^\perp$.

Then, we calculate the contributions from $d^2\tilde{W}^{(1)}_{\mu\nu}/d^2k_\perp$. Up to twist-3, in the correlation matrices $\varphi^{(1)}_{\rho\alpha}$ and $\tilde{\varphi}^{(1)}_{\rho\alpha}$, we need to consider the p_α terms as given in the following:

$$\varphi^{(1)}_{\rho\alpha} = p_\alpha\left[\varphi^\perp k_{\perp\rho} - \varphi_T M\varepsilon_{\perp\rho i}s_\perp^i - \frac{\varphi_T^\perp}{M}\left(k_{\perp\alpha}k_{\perp\beta} - \frac{1}{2}k_\perp^2 d_{\alpha\beta}\right)\varepsilon_\perp^{\beta i}s_{\perp i} - \lambda\varphi_L^\perp\varepsilon_{\perp\rho i}k_\perp^i\right] + \cdots \tag{37}$$

$$\tilde{\varphi}^{(1)}_{\rho\alpha} = ip_\alpha\left[-\tilde{\varphi}^\perp\varepsilon_{\perp\rho i}k_\perp^i + \tilde{\varphi}_T M s_{\perp\rho} - \frac{\tilde{\varphi}_T^\perp}{M}\left(k_{\perp\alpha}k_{\perp\beta} - \frac{1}{2}k_\perp^2 d_{\alpha\beta}\right)s_\perp^\beta + \lambda\tilde{\varphi}_L^\perp k_{\perp\rho}\right] + \cdots. \tag{38}$$

The corresponding hard factors are given by

$$\mathrm{Tr}[\hat{h}^{(1)\rho}_{\mu\nu}\not{p}] = -8p_\mu d_\nu{}^\rho, \tag{39}$$

$$\mathrm{Tr}[\hat{h}^{(1)\rho}_{\mu\nu}\gamma_5 \not{p}] = -8ip_\mu \varepsilon_{\perp\nu}{}^{\rho}. \quad (40)$$

We insert them into Eq. (29) and obtain

$$\frac{d^2\tilde{W}^{(1,L)}_{\mu\nu}}{d^2k'_\perp} = -\frac{p_\mu}{p\cdot q}[(\varphi^\perp - \varepsilon^{ks}_\perp \varphi^\perp_T)k_{\perp\nu} - \hat{\varphi}_T M\varepsilon_{\perp\nu i}s^i_\perp - \lambda\varphi^\perp_L \varepsilon_{\perp\nu i}k^i_\perp] - \frac{p_\mu}{p\cdot q}[(\tilde{\varphi}^\perp + \varepsilon^{ks}_\perp \tilde{\varphi}^\perp_T)k_{\perp\nu} + \hat{\tilde{\varphi}}_T M\varepsilon_{\perp\nu i}s^i_\perp + \lambda\tilde{\varphi}^\perp_L \varepsilon_{\perp\nu i}k^i_\perp], \quad (41)$$

where $\hat{\varphi}_T = \varphi_T - \frac{k^2_\perp}{2M^2}\varphi^\perp_T$ and $\hat{\tilde{\varphi}}_T = \tilde{\varphi}_T - \frac{k^2_\perp}{2M^2}\tilde{\varphi}^\perp_T$.

C. Simplifying $d^2W_{\mu\nu}/d^2k_\perp$ with QCD EOM relations

The quark field operator $\psi(y)$ satisfies the QCD equation of motion (EOM) for massless quark $\gamma\cdot D(y)\psi(y) = 0$. Hence, the correlation functions defined in Eqs. (30), (31), (37), and (38) are not independent from each other. We have in particular, for $\rho = 1, 2$,

$$x\Phi^{(0)}_\rho = -\frac{n^\alpha}{p^+}(\mathrm{Re}\varphi^{(1)}_{\rho\alpha} - \varepsilon_{\perp\rho}{}^{\sigma}\mathrm{Im}\tilde{\varphi}^{(1)}_{\sigma\alpha}), \quad (42)$$

$$x\tilde{\Phi}^{(0)}_\rho = -\frac{n^\alpha}{p^+}(\mathrm{Re}\tilde{\varphi}^{(1)}_{\rho\alpha} + \varepsilon_{\perp\rho}{}^{\sigma}\mathrm{Im}\varphi^{(1)}_{\sigma\alpha}). \quad (43)$$

We make Lorentz contractions of both sides of Eq. (42) with $k^\rho_\perp$ and $\varepsilon^{\rho i}_\perp k_{\perp i}$, and obtain

$$xf^\perp = -\mathrm{Re}(\varphi^\perp - \tilde{\varphi}^\perp), \quad (44)$$

$$xf_T = -\mathrm{Re}(\varphi_T + \tilde{\varphi}_T), \quad (45)$$

$$xf^\perp_L = -\mathrm{Re}(\varphi^\perp_L + \tilde{\varphi}^\perp_L), \quad (46)$$

$$xf^\perp_T = -\mathrm{Re}(\varphi^\perp_T + \tilde{\varphi}^\perp_T). \quad (47)$$

Similarly, after Lorentz contractions of both sides of Eq. (43) with $k^\rho_\perp$ and $\varepsilon^{\rho i}_\perp k_{\perp i}$, we obtain

$$xg^\perp = \mathrm{Im}(\varphi^\perp - \tilde{\varphi}^\perp), \quad (48)$$

$$xg_T = -\mathrm{Im}(\varphi_T + \tilde{\varphi}_T), \quad (49)$$

$$xg^\perp_L = -\mathrm{Im}(\varphi^\perp_L + \tilde{\varphi}^\perp_L), \quad (50)$$

$$xg^\perp_T = -\mathrm{Im}(\varphi^\perp_T + \tilde{\varphi}^\perp_T). \quad (51)$$

We note that similar relations have also been obtained earlier in, e.g., [9]. However, the twist-3 parton correlation functions in the corresponding equations in [9] are defined using the quark-gluon-quark correlater where gluon field A_ρ is used instead of D_ρ used here. We see clearly the similarities and differences between those relations obtained there and those that are listed above.

Using these relations, we rewrite the contributions from $\tilde{W}^{(1)}_{\mu\nu}$ as

$$2\mathrm{Re}\frac{d^2\tilde{W}^{(1,L)}_{S,\mu\nu}}{d^2k_\perp} = \frac{x_B}{p\cdot q}\{p_{\{\mu}k_{\perp\nu\}}(f^\perp - \varepsilon^{ks}_\perp f^\perp_T) - Mp_{\{\mu}\varepsilon_{\perp\nu\}i}s^i_\perp \hat{f}_T - \lambda p_{\{\mu}\varepsilon_{\perp\nu\}i}k^i_\perp f^\perp_L\}, \quad (52)$$

$$2\,\mathrm{Im}\frac{d^2\tilde{W}^{(1,L)}_{A,\mu\nu}}{d^2k_\perp} = \frac{x_B}{p\cdot q}\{p_{[\mu}k_{\perp\nu]}(g^\perp + \varepsilon^{ks}_\perp g^\perp_T) + Mp_{[\mu}\varepsilon_{\perp\nu]i}s^i_\perp \hat{g}_T + \lambda p_{[\mu}\varepsilon_{\perp\nu]i}k^i_\perp g^\perp_L)\}. \quad (53)$$

It is very interesting to see that, up to twist-3, all the contributions can be expressed by the coefficient functions of $\hat{\Phi}^{(0)}$.

We add the contributions from $\tilde{W}^{(1)}_{\mu\nu}$ to those from $\tilde{W}^{(0)}_{\mu\nu}$ and obtain the final result for the hadronic tensor up to twist-3 as

$$\frac{d^2W_{\mu\nu}}{d^2k_\perp} = -d_{\mu\nu}(f_1 - \varepsilon^{ks}_\perp f^\perp_{1T}) + \frac{1}{p\cdot q}k_{\perp\{\mu}(q+2x_Bp)_{\nu\}}(f^\perp - \varepsilon^{ks}_\perp f^\perp_T) - \frac{M}{p\cdot q}(q+2x_Bp)_{\{\mu}\varepsilon_{\perp\nu\}i}s^i_\perp \hat{f}_T - \frac{\lambda}{p\cdot q}(q+2x_Bp)_{\{\mu}\varepsilon_{\perp\nu\}i}k^i_\perp f^\perp_L + i\varepsilon_{\perp\mu\nu}\left(\lambda g_{1L} - \frac{k_\perp\cdot s_\perp}{M}g^\perp_{1T}\right) - \frac{i}{p\cdot q}k_{\perp[\mu}(q+2x_Bp)_{\nu]}(g^\perp + \varepsilon^{ks}_\perp g^\perp_T) + \frac{iM}{p\cdot q}(q+2x_Bp)_{[\mu}\varepsilon_{\perp\nu]i}s^i_\perp \hat{g}_T + \frac{i\lambda}{p\cdot q}(q+2x_Bp)_{[\mu}\varepsilon_{\perp\nu]i}k^i_\perp g^\perp_L. \quad (54)$$

We see that the result satisfies the electromagnetic gauge invariance $q^\mu d^2W_{\mu\nu}/d^2k_\perp = q^\nu d^2W_{\mu\nu}/d^2k_\perp = 0$ explicitly. The result is expressed in terms of 12 TMD parton distribution and/or correlation functions. They contain the information from the hadron structure and that from the multiple gluon scattering. We discuss them briefly in the following section.

D. TMD quark distribution/correlation functions

As can be seen from Eq. (54), up to twist-3, 12 TMD parton distribution and/or correlation functions are involved for the semi-inclusive DIS scattering process $e^- + N \to e^- + q + X$. Six of them are from the expansion of $\Phi_\alpha^{(0)} = \mathrm{Tr}[\gamma_\alpha \hat{\Phi}_\alpha^{(0)}]/2$ and six from $\tilde{\Phi}_\alpha^{(0)} = \mathrm{Tr}[\gamma_5\gamma_\alpha \hat{\Phi}_\alpha^{(0)}]/2$. They are defined in Eqs. (30) and (31). By reversing these two equations, we can obtain the operator expressions for these quark distribution and correlation functions. Four of them are leading twist parton distribution functions that are quite familiar to us and can be found in different literature, e.g., in [36]. They all have clear physical interpretations, have attracted much attention, and have been given much effort both theoretically [9,37–52] and experimentally [23–25,27,53–61]. As can be seen in Sec. II.B, in the jet production process $e + N \to e + q + X$ where only one hadron state is involved, the hard parts contain an odd number of γ matrices. Hence, in the decomposition of correlation matrices, chiral-odd distribution and/or correlation functions, such as transversity and Boer-Mulders functions, involve an even number of γ matrices and will not contribute. Such functions can be studied in the hadron production process $e + N \to e + h + x$ or the Drell-Yan process $p + p \to l\bar{l} + X$, where two hadron states are involved.

The other eight are twist-3 and have the following operator expressions [35]:

$$k_\perp^2 f^\perp(x,k_\perp) = \int \frac{p^+dy^-d^2y_\perp}{2(2\pi)^3} e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp} \times \langle p|\bar{\psi}(0)\not{k}_\perp \mathcal{L}(0;y)\psi(y)|p\rangle, \tag{55}$$

$$k_\perp^2 g^\perp(x,k_\perp) = -\int \frac{p^+dy^-d^2y_\perp}{2(2\pi)^3} e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp} \times \langle p|\bar{\psi}(0)\varepsilon_\perp^{ij} k_{\perp j}\gamma_{\perp i}\gamma_5 \mathcal{L}(0;y)\psi(y)|p\rangle, \tag{56}$$

$$\varepsilon_\perp^{ks}(k_\perp\cdot s_\perp) f_T(x,k_\perp) = -\frac{1}{M^2}\int \frac{p^+dy^-d^2y_\perp}{2(2\pi)^3} e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp} \times \left\langle p, s_\perp^{\uparrow\downarrow}|\bar{\psi}(0)\left(k_\perp^i k_\perp^j - \frac{1}{2}k_\perp^2 d^{ij}\right) \times \gamma_{\perp i} s_{\perp j}\mathcal{L}(0;y)\psi(y)|p, s_\perp^{\uparrow\downarrow}\right\rangle, \tag{57}$$

$$\varepsilon_\perp^{ks}(k_\perp\cdot s_\perp) g_T(x,k_\perp) = \frac{1}{M^2}\int \frac{p^+dy^-d^2y_\perp}{2(2\pi)^3} e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp} \times \left\langle p, s_\perp^{\uparrow\downarrow}|\bar{\psi}(0)\left(k_\perp^i k_\perp^j - \frac{1}{2}k_\perp^2 d^{ij}\right) \times \gamma_{\perp i}\varepsilon_{\perp jl} s_\perp^l \gamma_5 \mathcal{L}(0;y)\psi(y)|p, s_\perp^{\uparrow\downarrow}\right\rangle, \tag{58}$$

$$\varepsilon_\perp^{ks}(k_\perp\cdot s_\perp) f_T^\perp(x,k_\perp) = -\int \frac{p^+dy^-d^2y_\perp}{2(2\pi)^3} e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp} \times \langle p, s_\perp^{\uparrow\downarrow}|\bar{\psi}(0)\not{s}_\perp \mathcal{L}(0;y)\psi(y)|p, s_\perp^{\uparrow\downarrow}\rangle, \tag{59}$$

$$\varepsilon_\perp^{ks}(k_\perp\cdot s_\perp) g_T^\perp(x,k_\perp) = -\int \frac{p^+dy^-d^2y_\perp}{2(2\pi)^3} e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp} \times \langle p, s_\perp^{\uparrow\downarrow}|\bar{\psi}(0)\varepsilon_\perp^{ij} s_{\perp j}\gamma_{\perp i}\gamma_5 \mathcal{L}(0;y)\psi(y)|p, s_\perp^{\uparrow\downarrow}\rangle, \tag{60}$$

$$k_\perp^2 f_L^\perp(x,k_\perp) = -\int \frac{p^+dy^-d^2y_\perp}{2(2\pi)^3} e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp} \times \langle p,+|\bar{\psi}(0)\varepsilon_\perp^{ij} k_{\perp j}\gamma_{\perp i}\mathcal{L}(0;y)\psi(y)|p,+\rangle, \tag{61}$$

$$k_\perp^2 g_L^\perp(x,k_\perp) = \int \frac{p^+dy^-d^2y_\perp}{2(2\pi)^3} e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp} \times \langle p,+|\bar{\psi}(0)\not{k}_\perp\gamma_5 \mathcal{L}(0;y)\psi(y)|p,+\rangle. \tag{62}$$

Among them, $f^\perp$ and $g^\perp$ are related to the unpolarized case; f_T, g_T, $f_T^\perp$, and $g_T^\perp$ are related to the transverse polarization, and $f_L^\perp$ and $g_L^\perp$ are related to the longitudinal polarization.

These twist-3 quark correlation functions have no simple probabilistic interpretation. In fact, as we can see from the derivations that lead to these results, these twist-3 correlation functions come from the interference terms between amplitudes for scattering without multiple gluon scattering and that with one gluon scattering.

If we integrate over $\int d^2k_\perp$, we obtain the hadronic tensor $W_{\mu\nu}$ from $d^2W_{\mu\nu}/d^2k_\perp$. Since all parton distribution and/or correlation functions f's and g's are scalar functions of $k_\perp$, all the terms that are linearly dependent on $k_\perp$ vanish after the integration and we obtain from Eq. (54) that

$$W_{\mu\nu} = -d_{\mu\nu} f_1(x) - \frac{M}{p\cdot q}(q+2x_B p)_{\{\mu}\varepsilon_{\perp\nu\}i} s_\perp^i f_T(x) + i\varepsilon_{\perp\mu\nu}\lambda g_{1L}(x) + \frac{iM}{2p\cdot q}(q+2x_B p)_{[\mu}\varepsilon_{\perp\nu]i} s_\perp^i g_T(x), \quad (63)$$

where $f_1(x) \equiv \int d^2k_\perp f_1(x,k_\perp)$, $g_{1L}(x) \equiv \int d^2k_\perp g_{1L}(x,k_\perp)$, and,

$$f_T(x) \equiv \int d^2k_\perp f_T(x,k_\perp), \quad (64)$$

$$g_T(x) \equiv \int d^2k_\perp g_T(x,k_\perp). \quad (65)$$

The $g_T(x)$ term is the only twist-3 contribution to the structure function in inclusive DIS with a longitudinally polarized lepton beam and a transversely polarized nucleon target, as discussed in [62]. The $f_T(x)$ term is a time-reversal-odd term corresponding to the T-odd term $p_{\{\mu}\varepsilon_{\nu\}\rho\sigma\tau}p^\rho q^\sigma s^\tau$ in $W_{\mu\nu}$. It can be shown that, under time reversal invariance, $f_T(x)=0$.

The situation as considered in [2] and [4] can be recovered by putting $g=0$. In this case, there is no multiple gluon scattering and $\mathcal{L}=1$. Consequently the T-odd TMD distribution and/or correlation functions must be zero. The twist-3 quark correlation functions reduce to

$$xf^\perp(x,k_\perp)|_{g=0} = f_1(x,k_\perp)|_{g=0}, \quad (66)$$

$$f_T(x,k_\perp)|_{g=0} = f_L^\perp(x,k_\perp)|_{g=0} = f_T^\perp(x,k_\perp)|_{g=0} = 0, \quad (67)$$

$$xg_L^\perp(x,k_\perp)|_{g=0} = g_{1L}(x,k_\perp)|_{g=0}, \quad (68)$$

$$xg_T(x,k_\perp)|_{g=0} = -\frac{k_\perp^2}{2M^2}xg_T^\perp(x,k_\perp)|_{g=0} = -\frac{k_\perp^2}{2M^2}g_{1T}^\perp(x,k_\perp)|_{g=0}, \quad (69)$$

$$g^\perp(x,k_\perp)|_{g=0} = 0. \quad (70)$$

The hadronic tensor reduces to

$$\left.\frac{d^2\tilde{W}_{\mu\nu}}{d^2k_\perp}\right|_{g=0} = -\left[d_{\mu\nu} - \frac{1}{x_B p\cdot q}k_{\perp\{\mu}(q+2x_B p)_{\nu\}}\right] f_1(x,k_\perp)|_{g=0} + i\lambda\left[\varepsilon_{\perp\mu\nu} + \frac{1}{x_B p\cdot q}(q+2x_B p)_{[\mu}\varepsilon_{\perp\nu]i}k_\perp^i\right] g_{1L}(x,k_\perp)|_{g=0} - i\frac{k_\perp\cdot s_\perp}{M}\left[\varepsilon_{\perp\mu\nu} + \frac{1}{x_B p\cdot q}(q+2x_B p)_{[\mu}\varepsilon_{\perp\nu]i}k_\perp^i\right] g_{1T}^\perp(x,k_\perp)|_{g=0}. \quad (71)$$

This just corresponds to the results obtained using the simple parton model with intrinsic transverse momentum as discussed in [2] and [4] for the unpolarized and the longitudinally polarized cases, respectively. The deviations from this result come from the multiple gluon scattering.

III. CROSS SECTIONS AND AZIMUTHAL ASYMMETRIES

Making the Lorentz contraction of the hadronic tensor $d^2W_{\mu\nu}/d^2k_\perp$ as given by Eq. (54) with the leptonic tensor $L^{\mu\nu}(l,l')$, we obtain the differential cross section of the process $e^-(l,s_l)+N(p,s)\to e^-(l')+q(k')+X$ as

$$\frac{d\sigma}{dx_B dy d^2k_\perp} = \frac{2\pi\alpha_{\rm em}^2 e_q^2}{Q^2 y}(W_{UU} + \lambda_l W_{LU} + s_\perp W_{UT} + \lambda W_{UL} + \lambda_l\lambda W_{LL} + \lambda_l s_\perp W_{LT}), \quad (72)$$

where $W_{s_l s}$ represents the contribution in the different polarization case, and we use the superscript $s_l = U$ or L to denote whether the lepton is unpolarized or longitudinally polarized, while $s=U$, L, or T denotes whether the nucleon is unpolarized, longitudinally polarized, or transversely polarized [63]. These different contributions are given by

$$W_{UU} = A(y)f_1 - \frac{2x_B|\vec{k}_\perp|}{Q}B(y)f^\perp \cos\phi, \tag{73}$$

$$W_{UT} = \frac{|\vec{k}_\perp|}{M}A(y)f_{1T}^\perp \sin(\phi-\phi_s) - \frac{2x_BM}{Q}B(y)\left[f_T \sin\phi_s - \frac{k_\perp^2}{2M^2}f_T^\perp \sin(2\phi-\phi_s)\right], \tag{74}$$

$$W_{UL} = -\frac{2x_B|\vec{k}_\perp|}{Q}B(y)f_L^\perp \sin\phi, \tag{75}$$

$$W_{LU} = -\frac{2x_B|\vec{k}_\perp|}{Q}D(y)g^\perp \sin\phi, \tag{76}$$

$$W_{LL} = C(y)g_{1L} - \frac{2x_B|\vec{k}_\perp|}{Q}D(y)g_L^\perp \cos\phi, \tag{77}$$

$$W_{LT} = \frac{|\vec{k}_\perp|}{M}C(y)g_{1T}^\perp \cos(\phi-\phi_s) - \frac{2x_BM}{Q}D(y)\left[g_T \cos\phi_s - \frac{k_\perp^2}{2M^2}g_T^\perp \cos(2\phi-\phi_s)\right], \tag{78}$$

where $A(y) = 1+(1-y)^2$, $B(y) = 2(2-y)\sqrt{1-y}$, $C(y) = y(2-y)$, $D(y) = 2y\sqrt{1-y}$, $\cos\phi = \vec{l}_\perp\cdot\vec{k}_\perp/|\vec{l}_\perp||\vec{k}_\perp|$, $\sin\phi = (\vec{l}_\perp\times\vec{k}_\perp)\cdot\vec{e}_z/|\vec{l}_\perp||\vec{k}_\perp|$, $\cos\phi_s = \vec{l}_\perp\cdot\vec{s}_\perp/|\vec{l}_\perp||\vec{s}_\perp|$, and $\sin\phi_s = (\vec{l}_\perp\times\vec{s}_\perp)\cdot\vec{e}_z/|\vec{l}_\perp||\vec{s}_\perp|$.

We note that, except for the slightly different notations [63], these results are almost the same as those obtained in [9] for jet production. They have the same structures, and the forms of the coefficients in each term are the same [64]. This is expected since the kinematics is the same and the approximations made in the hard parts in [9] should be equivalent to keep the leading and subleading terms in the collinear expansion. Also, because of the relationship given by Eqs. (48)–(51) obtained from the equation of motion, all the results are expressed by the different components of $\hat{\Phi}^{(0)}$ defined by Eq. (15), which is identical to the original $\hat{\phi}^{(0)}$ given by Eq. (10) except for the gauge link. Hence, the difference in defining higher twist correlators such as that between $\hat{\varphi}^{(1)}$ given by Eq. (27) and $\hat{\phi}^{(1)}$ given by Eq. (11) does not show up in the final results. However, it seems not to be the case for an even higher twist [14]. Other features of the results are summarized in the following.

We see that there are leading twist contributions in the UU, UT, LL, and LT cases, while there are twist-3 contributions in all cases. The different azimuthal asymmetries are defined by the average values of the corresponding sine or cosine of the angles. There are two leading twist azimuthal asymmetries as given by

$$\langle\sin(\phi-\phi_s)\rangle_{UT} = s_\perp\frac{|\vec{k}_\perp|}{2M}\frac{f_{1T}^\perp(x,k_\perp)}{f_1(x,k_\perp)}, \tag{79}$$

$$\langle\cos(\phi-\phi_s)\rangle_{LT} = \lambda_l s_\perp\frac{|\vec{k}_\perp|}{2M}\frac{C(y)}{A(y)}\frac{g_{1T}^\perp(x,k_\perp)}{f_1(x,k_\perp)}. \tag{80}$$

The azimuthal asymmetry $\langle\cos\phi\rangle$ exists at twist-3 for the unpolarized case. It receives also a twist-3 contribution in the LL case but also a leading twist contribution in the LT case, i.e.,

$$\langle\cos\phi\rangle_{UU} = -\frac{|\vec{k}_\perp|}{Q}\frac{B(y)}{A(y)}\frac{x_Bf^\perp(x_B,k_\perp)}{f_1(x_B,k_\perp)}, \tag{81}$$

$$\langle\cos\phi\rangle_{LL} = -\frac{|\vec{k}_\perp|}{Q}\frac{B(y)x_Bf^\perp(x_B,k_\perp)+\lambda_l\lambda D(y)x_Bg_L^\perp(x_B,k_\perp)}{A(y)f_1(x_B,k_\perp)+\lambda_l\lambda C(y)g_{1L}(x_B,k_\perp)}, \tag{82}$$

$$\langle\cos\phi\rangle_{LT} = \frac{|\vec{k}_\perp|}{2M}\frac{\lambda_l s_\perp C(y)g_{1T}^\perp(x_B,k_\perp)\cos\phi_s - \frac{2M}{Q}B(y)x_Bf^\perp(x_B,k_\perp)}{A(y)f_1(x_B,k_\perp) - \lambda_l s_\perp\frac{2M}{Q}x_Bg_T(x_B,k_\perp)\cos\phi_s}. \tag{83}$$

We note in particular that there exists a twist-3 asymmetry $\langle \sin\phi \rangle$ for the LU or UL case, i.e., when the lepton or nucleon is longitudinally polarized while the other is unpolarized. It is given by

$$\langle \sin\phi \rangle_{LU} = -\lambda_l \frac{|\vec{k}_\perp|}{Q} \frac{D(y)}{A(y)} \frac{x_B g^\perp(x_B, k_\perp)}{f_1(x_B, k_\perp)}, \qquad (84)$$

$$\langle \sin\phi \rangle_{UL} = -\lambda \frac{|\vec{k}_\perp|}{Q} \frac{B(y)}{A(y)} \frac{x_B f_L^\perp(x_B, k_\perp)}{f_1(x_B, k_\perp)}. \qquad (85)$$

They are determined by the TMD parton correlation $g^\perp$ and $f_T^\perp$, respectively.

It is also interesting to see that, if we integrate over ϕ, we obtain

$$\frac{d\sigma}{|\vec{k}_\perp| dx_B dy d|\vec{k}_\perp|} = \frac{4\pi^2 \alpha_{em}^2 e_q^2}{Q^2 y} \Big\{ A(y) f_1 - s_\perp \frac{2x_B M}{Q} B(y) f_T \sin\phi_s \qquad (86)$$

$$+ \lambda_l \lambda C(y) g_{1L} - \lambda_l s_\perp \frac{2x_B M}{Q} D(y) g_T \cos\phi_s \Big\}. \qquad (87)$$

We see that the transverse spin asymmetry exists for the semi-inclusive process $e^- + N \to e^- + q + X$ at the twist-3 level both in the target singly polarized case UT and in the case LT where the lepton is also longitudinally polarized. But the asymmetries in these two cases are different and are given by

$$\langle \sin\phi_s \rangle_{UT} = -s_\perp \frac{M}{Q} \frac{B(y)}{A(y)} \frac{x_B f_T(x_B, k_\perp)}{f_1(x_B, k_\perp)}, \qquad (88)$$

$$\langle \cos\phi_s \rangle_{LT} = -\lambda_l s_\perp \frac{M}{Q} \frac{D(y)}{A(y)} \frac{x_B g_T(x_B, k_\perp)}{f_1(x_B, k_\perp)}. \qquad (89)$$

We also note that, in experiments, we usually measure for a given $|\vec{k}_\perp|$ interval. In this case, we need to carry out the integration over $|\vec{k}_\perp|$. For example, if we integrate over the whole $|\vec{k}_\perp|$ region, we obtain

$$\langle\langle \sin\phi \rangle\rangle_{LU} = -\lambda_l \frac{B(y)}{A(y)} \frac{2\pi}{Q} \frac{\int \vec{k}_\perp^2 d|\vec{k}_\perp| x_B g^\perp(x_B, k_\perp)}{f_1(x_B)}. \qquad (90)$$

By carrying out the integration over $d^2k_\perp$, we obtain the differential cross section $d\sigma/dx_B dy$ for the inclusive DIS process $e^- + N \to e^- + X$ as

$$\frac{d\sigma}{dx_B dy} = \frac{2\pi\alpha_{em}^2 e_q^2}{Q^2 y} \Big\{ A(y) f_1(x_B) + \lambda\lambda_l C(y) g_{1L}(x_B) - \lambda_l s_\perp \frac{2x_B M}{Q} D(y) g_T'(x_B) \cos\phi_s \Big\}, \qquad (91)$$

where we see clearly that the only twist-3 contribution exists for the case that the lepton is longitudinally polarized and the nucleon is transversely polarized.

At $g = 0$, the cross section reduces to the result obtained in the simple parton model with intrinsic transverse momentum [2,4]. By inserting the results given by Eqs. (66)–(70) into Eqs. (73)–(78), we obtain

$$W_{UU}|_{g=0} = \Big[A(y) - \frac{2|\vec{k}_\perp|}{Q} B(y) \cos\phi \Big] f_1(x, k_\perp)|_{g=0}, \qquad (92)$$

$$W_{UT}|_{g=0} = F_{UL}|_{g=0} = F_{LU}|_{g=0} = 0, \qquad (93)$$

$$W_{LL}|_{g=0} = \Big[C(y) - \frac{2|\vec{k}_\perp|}{Q} D(y) \cos\phi \Big] g_{1L}(x, k_\perp)|_{g=0}, \qquad (94)$$

$$W_{LT}|_{g=0} = \frac{|\vec{k}_\perp|}{M} \Big[C(y) - \frac{2|\vec{k}_\perp|}{Q} D(y) \cos\phi \Big] g_{1T}^\perp(x, k_\perp)|_{g=0} \times \cos(\phi - \phi_s). \qquad (95)$$

Correspondingly, for the azimuthal asymmetries discussed above, we obtain

$$\langle \sin(\phi - \phi_s) \rangle_{UT}|_{g=0} = 0, \qquad (96)$$

$$\langle \cos(\phi - \phi_s) \rangle_{LT}|_{g=0} = \lambda_l s_\perp \frac{|\vec{k}_\perp|}{2M} \frac{C(y)}{A(y)} \frac{g_{1T}^\perp(x, k_\perp)}{f_1(x, k_\perp)}, \qquad (97)$$

$$\langle \cos\phi \rangle_{UU}|_{g=0} = -\frac{B(y)}{A(y)} \frac{|\vec{k}_\perp|}{Q}, \qquad (98)$$

$$\langle \cos\phi \rangle_{LL}|_{g=0} = -\frac{|\vec{k}_\perp|}{Q} \frac{B(y) f_1(x, k_\perp) + \lambda_l \lambda D(y) g_{1L}(x, k_\perp)}{A(y) f_1(x, k_\perp) + \lambda_l \lambda C(y) g_{1L}(x, k_\perp)}, \qquad (99)$$

$$\langle\cos\phi\rangle_{LT}|_{g=0} = \frac{|\vec{k}_\perp|}{2M}\frac{\lambda_l s_\perp C(y) g_{1T}^\perp(x_B,k_\perp)\cos\phi_s - \frac{2M}{Q}B(y)x_B f_1(x_B,k_\perp)}{A(y)f_1(x_B,k_\perp) + \lambda_l s_\perp \frac{k_\perp^2}{MQ}D(y)g_{1T}^\perp(x,k_\perp)\cos\phi_s}, \tag{100}$$

$$\langle\sin\phi\rangle_{LU}|_{g=0} = \langle\sin\phi\rangle_{UL}|_{g=0} = \langle\sin\phi_s\rangle_{UT}|_{g=0} = 0, \tag{101}$$

$$\langle\cos\phi_s\rangle_{LT}|_{g=0} = \lambda_l s_\perp \frac{D(y)}{A(y)}\frac{k_\perp^2}{MQ}\frac{g_{1T}^\perp(x_B,k_\perp)}{f_1(x_B,k_\perp)}. \tag{102}$$

Clearly, a systematic study of these asymmetries should provide very important information on the structure of the nucleon and the properties of strong interaction. In particular, the deviations from the results given by Eqs. (96)–(102) tell us the influences from the multiple gluon scattering.

IV. NUCLEAR DEPENDENCE

The above mentioned calculations apply to $e^- + N \to e^- + q + X$ as well as $e^- + A \to e^- + q + X$, i.e., for reactions using a nucleus target. Similar results are obtained with only a replacement of the state $|N\rangle$ by $|A\rangle$ in the definitions of the parton distribution and/or correlation functions. It has also been shown [34] that the multiple gluon scattering contained in the gauge link leads to a strong nuclear dependence for these TMD parton distribution and/or correlation functions. Such nuclear dependences can manifest themselves in the azimuthal asymmetries in SIDIS [13,14]. In this section, we present the results for the parton distributions and azimuthal asymmetries given in the Sec. III.

A. *A* dependence of the parton correlation functions

If we replace the state $|N\rangle$ by $|A\rangle$, the multiple gluon scattering in the gauge link can be connected to different nucleons in the nucleus A, thus giving rise to nuclear dependence. It has been shown that, under the "maximal two gluon approximation" [34], a TMD quark distribution $\Phi_\alpha^A(x,k_\perp)$ in nucleus defined in the form

$$\Phi_\alpha^A(x,k_\perp) \equiv \int \frac{p^+ dy^- d^2y_\perp}{(2\pi)^3} e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp}\langle A|\bar\psi(0)\Gamma_\alpha\mathcal{L}(0;y)\psi(y)|A\rangle \tag{103}$$

is given by a convolution of the corresponding distribution $\Phi_\alpha^N(x,k_\perp)$ in the nucleon and a Gaussian broadening [34], i.e.,

$$\Phi_\alpha^A(x,k_\perp) \approx \frac{A}{\pi\Delta_{2F}}\int d^2\ell_\perp e^{-(\vec{k}_\perp-\vec{\ell}_\perp)^2/\Delta_{2F}}\Phi_\alpha^N(x,\ell_\perp), \tag{104}$$

where Γ_α is any gamma matrix, Δ_{2F} is the broadening width, $\Delta_{2F} = \int d\xi_N^- \hat{q}_F(\xi_N)$, and $\hat{q}_F(\xi_N) = (2\pi^2\alpha_s/N_c)\rho_N^A(\xi_N)[xf_g^N(x)]_{x=0}$ is the quark transport parameter, where $\rho_N^A(\xi_N)$ is the spatial nucleon number density inside the nucleus, $f_g^N(x)$ is the gluon distribution function in the nucleon, and the superscript A or N denotes that it is for the nucleus or the nucleon.

The derivations in [34] apply to any nucleon and nucleus in the unpolarized case. Since both $\Phi_\alpha^{(0)}$ and $\tilde\Phi_\alpha^{(0)}$ defined in Eqs. (30) and (31) are of the form given by Eq. (104), Eq. (104) applies and derives the A dependences of different parton distribution and/or correlation functions in the unpolarized case. For those involved in the differential cross section up to twist-3, we obtain

$$f_1^A(x,k_\perp) \approx \frac{A}{\pi\Delta_{2F}}\int d^2\ell_\perp e^{-(\vec{k}_\perp-\vec{\ell}_\perp)^2/\Delta_{2F}} f_1^N(x,\ell_\perp), \tag{105}$$

$$|\vec{k}_\perp|^2 f^{\perp A}(x,k_\perp) \approx \frac{A}{\pi\Delta_{2F}}\int d^2\ell_\perp e^{-(\vec{k}_\perp-\vec{\ell}_\perp)^2/\Delta_{2F}}(\vec{k}_\perp\cdot\vec{\ell}_\perp) f^{\perp N}(x,\ell_\perp), \tag{106}$$

$$|\vec{k}_\perp|^2 g^{\perp A}(x,k_\perp) \approx \frac{A}{\pi\Delta_{2F}}\int d^2\ell_\perp e^{-(\vec{k}_\perp-\vec{\ell}_\perp)^2/\Delta_{2F}}(\vec{k}_\perp\cdot\vec{\ell}_\perp) g^{\perp N}(x,\ell_\perp). \tag{107}$$

To illustrate the dependence more clearly, we take the Gaussian ansatz for the transverse momentum dependence, i.e.,

$$f_1^N(x,\ell_\perp) = \frac{1}{\pi\alpha}f_1^N(x)e^{-\vec{\ell}_\perp^2/\alpha}, \tag{108}$$

$$f^{\perp N}(x,\ell_\perp) = \frac{1}{\pi\beta}f^{\perp N}(x)e^{-\vec{\ell}_\perp^2/\beta}, \tag{109}$$

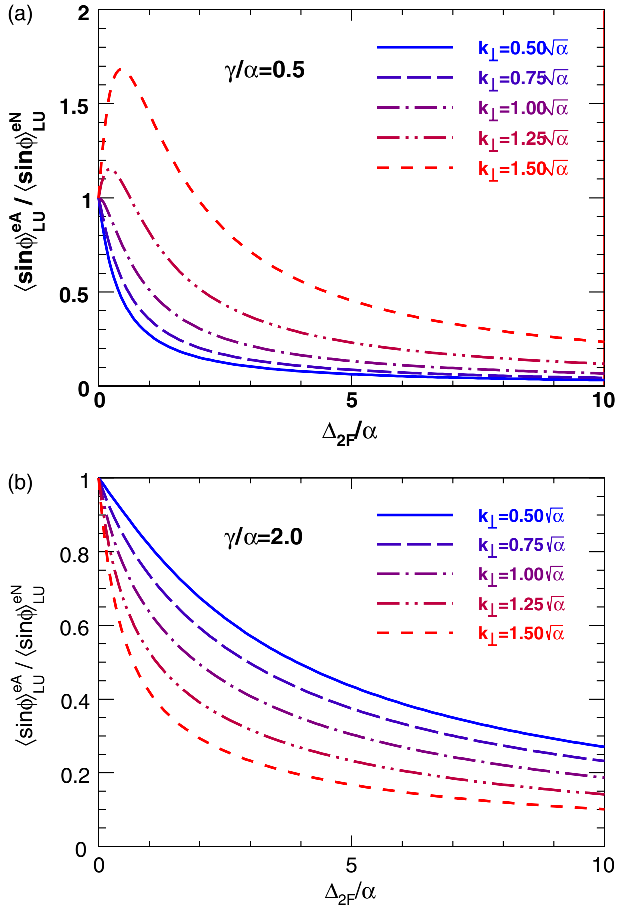

FIG. 2 (color online). Ratio of $\langle \sin\phi\rangle_{LU}^{eA}/\langle \sin\phi\rangle_{LU}^{eN}$ as a function of Δ_{2F} for different $k_\perp$ and γ.

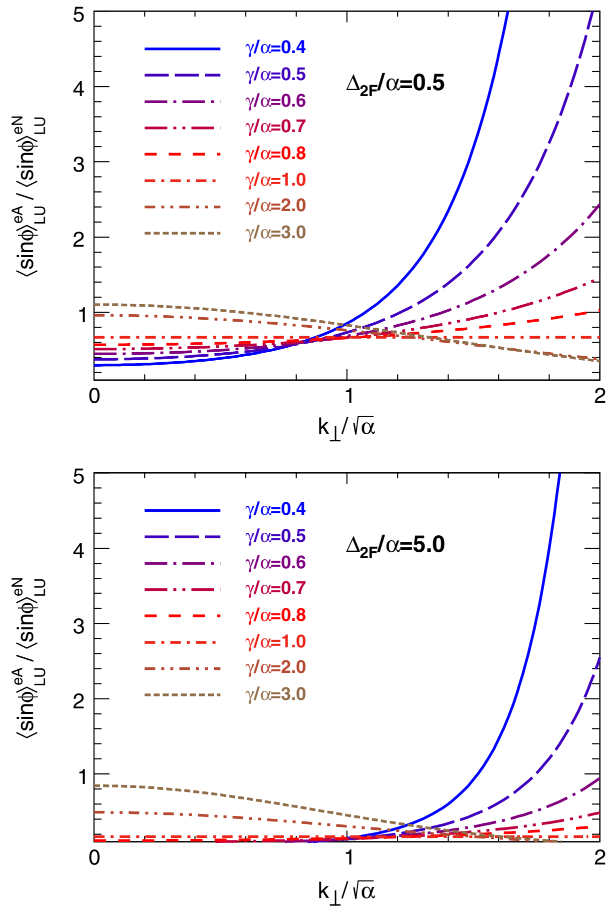

FIG. 3 (color online). Ratio of $\langle \sin\phi\rangle_{LU}^{eA}/\langle \sin\phi\rangle_{LU}^{eN}$ as a function of $k_\perp$ for different γ and Δ_{2F}.

$$g^{\perp N}(x,\ell_\perp) = \frac{1}{\pi\gamma} g^{\perp N}(x) e^{-\vec{\ell}_\perp^2/\gamma}, \tag{110}$$

and obtain immediately

$$f_1^A(x,k_\perp) \approx \frac{A}{\pi\alpha_A} f_1^N(x) e^{-\vec{k}_\perp^2/\alpha_A}, \tag{111}$$

$$f^{\perp A}(x,k_\perp) \approx \frac{A}{\pi\beta_A}\frac{\beta}{\beta_A} f^{\perp N}(x) e^{-\vec{k}_\perp^2/\beta_A}, \tag{112}$$

$$g^{\perp A}(x,k_\perp) \approx \frac{A}{\pi\gamma_A}\frac{\gamma}{\gamma_A} g^{\perp N}(x) e^{-\vec{k}_\perp^2/\gamma_A}, \tag{113}$$

where $\alpha_A = \alpha + \Delta_{2F}$, $\beta_A = \beta + \Delta_{2F}$, and $\gamma_A = \gamma + \Delta_{2F}$. We see that all the TMD distribution/correlation functions have p_T broadening with the magnitude Δ_{2F}, but the twist-3 parton correlation function $f^\perp(x,k_\perp)$ or $g^\perp(x,k_\perp)$ has an extra suppression factor β/β_A or γ/γ_A.

B. *A* dependence of the azimuthal asymmetry

Having the nuclear dependences of the TMD parton distribution and correlation functions and the expressions for the azimuthal asymmetries presented in the previous sections, we can calculate the nuclear dependence of the azimuthal asymmetries in a straightforward manner with the Gaussian ansatz for the TMD distributions and/or correlations.

For reactions with an unpolarized target, the results are just the same as those for the unpolarized reaction as discussed in [13]. This applies to the asymmetry $\langle \sin\phi\rangle_{LU}$ given by Eq. (84), for which we obtain

$$\frac{\langle \sin\phi\rangle_{LU}^{eA}}{\langle \sin\phi\rangle_{LU}^{eN}} \approx \frac{\alpha_A}{\alpha}\left(\frac{\gamma}{\gamma_A}\right)^2 \exp\left[\left(\frac{1}{\alpha_A} - \frac{1}{\alpha} - \frac{1}{\gamma_A} + \frac{1}{\gamma}\right)\vec{k}_\perp^2\right]. \tag{114}$$

For $\alpha = \gamma$, it simply reduces to

$$\frac{\langle \sin\phi\rangle_{LU}^{eA}}{\langle \sin\phi\rangle_{LU}^{eN}} \approx \frac{\alpha}{\alpha + \Delta_{2F}}. \tag{115}$$

Integrated over $|\vec{k}_\perp|$, we have

$$\frac{\langle\langle \sin\phi\rangle\rangle_{LU}^{eA}}{\langle\langle \sin\phi\rangle\rangle_{LU}^{eN}} \approx \sqrt{\frac{\gamma}{\gamma+\Delta_{2F}}}. \tag{116}$$

We see that, also in this case, the asymmetry is suppressed in reactions using the nucleus target in a similar manner as in the unpolarized case discussed in [13,14].

The width γ can in general be different from α. Hence, we present as an example in Figs. 2(a) and 2(b) the ratio as a function of the k_T-broadening parameter Δ_{2F}. We see that it is very similar to $\langle \cos\phi\rangle_{UU}$ discussed in [13]. We also plot the k_T dependence of the ratio in Figs. 3(a) and 3(b). It is easy to see that for $\gamma/\alpha < 1$, the ratio of $\langle \sin\phi\rangle_{LU}$ is quite sensitive to the value of γ/α in the large $k_\perp$ region.

V. SUMMARY

We present a systematic calculation of the hadronic tensor and azimuthal asymmetries in the semi-inclusive deep-inelastic scattering $e^- + N \to e^- + q + X$ with a polarized beam and/or a polarized target based on the collinear expansion formalism in leading order pQCD and up to twist-3 contributions. The results depend on a number of new TMD parton correlation functions. We showed that measurements of the corresponding azimuthal asymmetries and their $k_\perp$ dependence can provide much information on these TMD correlation functions that in turn can shed light on the properties of the multiple gluon interaction in hadronic processes. We presented the results also for reactions with the nucleus target $e^- + A \to e^- + q + X$ and discuss the nuclear dependence. We show that the relationship between these TMD correlation functions inside large nuclei and that of a nucleon under two-gluon correlation approximation. One can study the nuclear dependence of the different azimuthal asymmetries that are determined by the corresponding parton distribution and correlation functions. With the Gaussian ansatz for the TMD parton correlation functions inside the nucleon, we also illustrate numerically that the asymmetries are suppressed in the corresponding SIDIS with a nuclear target.

Experimental studies of the azimuthal asymmetries have been carried out in both unpolarized and polarized semi-inclusive deep inelastic scattering with a nucleon target [16–27]. More results are expected from CLAS at JLab and COMPASS at CERN. The available data seem to be consistent with the Gaussian ansatz for the transverse momentum dependence of the TMD matrix elements in the unpolarized case [65]. However, these data are still not adequate enough to provide any precise constraints on the form of the higher twist matrix elements. Our calculations of the azimuthal asymmetries are most valid in the small transverse momentum region where next-to-leading order pQCD corrections are not dominant. The high twist effects are also most accessible in the intermediate region of Q^2. One expects that future experiments such as those at the proposed Electron Ion Collider [52] will be better equipped to study these high twist effects in detail.

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