Gluon bremsstrahlung by heavy quarks: Its effects on transport coefficients and equilibrium distribution

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The effects of gluon radiation by charm quarks on the transport coefficients, e.g., drag, longitudinal and transverse diffusions, and shear viscosity, have been studied within the ambit of perturbative quantum chromodynamics and kinetic theory. We found that soft gluon radiation by the charm quark has substantial effects on the transport coefficients. However, the radiative effects on the shape of its equilibrium distribution function is insignificant. We also observe that the shear viscosity to entropy ratio of the quark-gluon plasma is closer to the experimentally extracted value when the gluon radiation by the charm quark is included.

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I. INTRODUCTION

Recently the study of quark-gluon plasma (QGP), expected to be created in heavy-ion collisions at the Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) energies, has intrigued the scientific community with its multifarious interesting aspects. Therefore, in order to understand the different properties of QGP, we need to probe it. Among many, one efficient probe is the charm quark (CQ) produced in the early hard collisions of the partons from the colliding nuclei. Generally, the transport coefficients are sensitive to the interaction of the probes with the medium. Hence, the estimation of various transport coefficients of QGP by using CQs is a field of high contemporary interest. Moreover, strong coupling of the probes—the CQs here—with the medium may bring them into equilibrium with the bulk matter, so that the probes might follow a momentum distribution similar to that of the constituents of the medium. In the present work, we will consider the QGP as the thermal medium of light quarks, their antiparticles, and gluons, and we will consider the CQs as a probe. This will enable us to estimate the drag, diffusion, and shear viscous coefficients of QGP and to understand the nature of the equilibrium distribution of the CQs. The reasons behind choosing the CQ as a probe are twofold: (i) Being created from the early hard collisions, it can experience the hot/dense medium from its birth. The CQ distribution function is different from that of the medium particle, and being heavier than the constituent particles of QGP, it does not get equilibrated quickly, and hence it qualifies to act as a Brownian particle. (ii) The probability of the production of CQs (with mass M) inside a thermal medium of temperature T ($T \ll M$, where M is the mass of the CQ) is small; hence, the probability of the annihilation of CQs in the QGP is also small. Therefore, the CQs witness the entire evolution of the bath. The probability of the creation and annihilation of bottom quarks is even smaller; therefore, the present work can be extended to the bottom quarks as well. While propagating through the QGP, a CQ interacts with the medium particles via two dominant processes: (i) collisional or elastic interaction and (ii) inelastic interaction, like gluon bremsstrahlung or gluon radiation. In earlier works, while calculating the momentum diffusion coefficients, the gluon radiation by the CQ has either been ignored [1–4], or calculated for nonrelativistic CQ [5,6], or estimated by first determining the drag from radiative energy loss [7–9] and then using the Einstein relation between drag and diffusion coefficients [10].

In this work, we calculate, using perturbative quantum chromodynamics (pQCD), the transverse and longitudinal diffusion coefficients of the CQs undergoing radiative loss by emitting gluons while traveling through the QGP. We consider that the emitted gluons, being soft, get absorbed in the medium, resulting in energy transporting from the fast-moving CQs to the constituents of the bath. This transportation of momentum is reflected in the momentum diffusion coefficients of the CQs in QGP.

The values of the drag and diffusion coefficients can be used to characterize the distribution function of the probe. Therefore, these transport coefficients can be utilized to understand the departure of the CQ distribution from the thermal distribution of the bath particles. The shape of the equilibrium distribution function of the CQs has been studied using a generalized Einstein relation derived in Ref. [11]. We revisit this relation by including both the collisional and the radiative transport coefficients of the CQs.

The present work is organized as follows: In the next section, we discuss the formalism of the Fokker-Planck equation (FPE) and the procedure to evaluate transport coefficients for collisional and radiative processes. In Sec. III, the equilibrium distribution ($f_{\rm eq}^{\rm CQ}$) of CQs is elaborated in the context of the bremsstrahlung process. The impact of the radiative transport coefficients on the $f_{\rm eq}^{\rm CQ}$ is particularly highlighted. In Sec. IV, the shear viscosity (η)

to entropy density (s) ratio η/s of QGP is estimated using CQ transverse diffusion coefficients with particular emphasis on the radiative processes. Section V is dedicated to summary and discussion.

II. FORMALISM AND TRANSPORT COEFFICIENTS

Heavy quarks propagate as Brownian particles in the QGP medium. The ensemble of Brownian particles immersed in the thermal medium can be characterized by the single-particle distribution function, $f(\vec{x},\vec{p},t)$. The time evolution of f is governed by the master equation, a simplified version of which is the FPE.

The form of the master equation, or the Boltzmann transport equation (BTE), governing the CQ distribution f is given by

$$\left[\frac{\partial}{\partial t}+\frac{\vec{p}}{E}.\frac{\partial}{\partial\vec{x}}+\vec{F}.\frac{\partial}{\partial\vec{p}}\right]f(\vec{x},\vec{p},t)=\left[\frac{\partial f}{\partial t}\right]_{\text{collisions}}. \quad (1)$$

In the absence of external force $\vec{F}$, and for a homogeneous plasma, we can write the BTE as follows:

$$\left[\frac{\partial f}{\partial t}\right]_{\text{collisions}}=\int d^3\vec{k}[w(\vec{p}+\vec{k},\vec{k})f(\vec{p}+\vec{k})-w(\vec{p},\vec{k})f(\vec{p})], \quad (2)$$

where $w(\vec{p},\vec{k})$ is the rate of collision of the CQ, changing its momentum from $\vec{p}$ to $\vec{p}-\vec{k}$. Considering only the soft scattering (small $|\vec{k}|$), we reduce the integro-differential Eq. (2) to the FPE:

$$\frac{\partial f}{\partial t}=\frac{\partial}{\partial p_i}\left[A_i(\vec{p})f+\frac{\partial}{\partial p_j}[B_{ij}(\vec{p})f]\right], \quad (3)$$

where the kernels are defined as

$$A_i=\int d^3\vec{k}w(\vec{p},\vec{k})k_i \quad (4)$$

and

$$B_{ij}=\frac{1}{2}\int d^3\vec{k}w(\vec{p},\vec{k})k_ik_j, \quad (5)$$

where A_i and B_{ij} are the drag and the diffusion coefficients of the CQ. Equation (3) is obtained by expanding the collision term [Eq. (2)] for small values of $\vec{k}$ and keeping terms up to the quadratic order. The two terms in the right-hand side of Eq. (3) have the same order of magnitude [12] because the averaging of the first power of k_i [through Eq. (4)] with its sign fluctuation involves greater degree of cancellation than the averaging of the quadratic term, k_ik_j [through Eq. (5)]. The higher-order terms, i.e., cubic, quartic, etc., are smaller compared to the terms kept in the above expression. With the assumption of small momentum transfers of the CQs with their thermal collision partners, the nonlinear BTE reduces to a linear FPE, which is much simpler to solve. (For the comparison of solutions of the BTE and FPE for the CQ see Ref. [13].)

Our motivation is to find out the drag and diffusion coefficients due to elastic and inelastic interactions of the CQs with the bath particles within the ambit of pQCD.

A. Transport coefficients for collisional processes

First, we concentrate on the two-body elastic processes. While propagating inside the plasma, the CQ (Q) encounters the following interactions with the bath particle: $Q(p)+g(q)\to Q(p')+g(q')$, $Q(p)+q(q)\to Q(p')+q(q')$, and $Q(p)+\bar{q}(q)\to Q(p')+\bar{q}(q')$, where the quantities within the brackets denote the momenta of the CQ, quark (q), antiquark ($\bar{q}$) and gluon (g). Therefore, A_i and B_{ij} are written in terms of invariant amplitude squared [1] as

$$A_i=\frac{1}{2E_p}\int\frac{d^3q}{(2\pi)^3 2E_q}\int\frac{d^3q'}{(2\pi)^3 2E_{q'}}\times\int\frac{d^3p'}{(2\pi)^3 2E_{p'}}\frac{1}{\gamma}\sum|M|^2_{2\to2}(2\pi)^4\delta^4(p+q-p'-q')\times\hat{f}(\mathbf{q})(1\pm\hat{f}(\mathbf{q}'))(p-p')_i, \quad (6)$$

$$A_i=\langle\langle(p-p')_i\rangle\rangle. \quad (7)$$

Similarly,

$$B_{ij}=\frac{1}{2}\langle\langle(p'-p)_i(p'-p)_j\rangle\rangle. \quad (8)$$

Since A_i and B_{ij} only depend on the three-momentum $\vec{p}$ and background temperature T, we can write them as

$$A_i=p_iA \quad (9)$$

and

$$B_{ij}=\left(\delta_{ij}-\frac{p_ip_j}{p^2}\right)B_\perp(p,T)+\frac{p_ip_j}{p^2}B_\parallel(p,T), \quad (10)$$

where $B_\perp$ and $B_\parallel$ are the transverse and longitudinal diffusion coefficients, respectively. Using Eqs. (7),(8),(9), and (10), we get

$$A(p)=\langle\langle 1\rangle\rangle-\frac{\langle\langle\vec{p}.p'\rangle\rangle}{p^2}, \quad (11)$$

$$B_\perp(p)=\frac{1}{4}\left[\langle\langle p'^2\rangle\rangle-\frac{\langle\langle(\vec{p}.\vec{p}')^2\rangle\rangle}{p^2}\right], \quad (12)$$

$$B_{\|}(p) = \frac{1}{2}\left[\frac{\langle\langle(\vec{p}.\vec{p}')^2\rangle\rangle}{p^2} - 2\langle\langle\vec{p}.\vec{p}'\rangle\rangle + p^2\langle\langle 1\rangle\rangle\right]. \quad (13)$$

Using Eqs. (11),(12), and (13) with the matrix elements of those elastic processes mentioned above, the drag and the transverse and longitudinal diffusion coefficients can be estimated.

The expression for the transport coefficient $[X(\vec{p},T)]$ can be schematically written as

$$X(p) = \int \text{phase space} \times \text{interaction} \times \text{transport part}. \quad (14)$$

In the case of drag (diffusion), the transport part involves momentum (square of the momentum) transfer of the CQ with the bath particle. The evaluation of the drag and diffusion coefficients with collisional processes is elaborated in Refs. [7,8]. Therefore, we refer to these references for details and do not repeat the discussions here.

B. Transport coefficients for radiative processes

Equation (14) can be used to evaluate the transport coefficients due to radiative processes by replacing the two-body phase space and invariant amplitude with their three-body counterparts and keeping the transport part the same. The $X(\vec{p},T)$ for the radiative process $Q(p)+\text{parton}(q)\to Q(p')+\text{parton}(q')+\text{gluon}(k_5)$ [where "parton" stands for light quarks, antiquarks, and gluons, and $k_5=(E_5,k_\perp,k_z)$] can be written as

$$X = \frac{1}{2E_p}\int\frac{d^3q}{(2\pi)^3 2E_q}\int\frac{d^3q'}{(2\pi)^3 2E_{q'}}\int\frac{d^3p'}{(2\pi)^3 2E_{p'}} \times\int\frac{d^3k_5}{(2\pi)^3 2E_5}\frac{1}{\gamma}\sum|M|^2_{2\to3}(2\pi)^4\delta^4(p+q-p'-q'-k_5) \times\hat{f}(E_q)(1\pm\hat{f}(E_{q'}))(1+\hat{f}(E_5)) \times\theta_1(\tau-\tau_F)\theta_2(E_p-E_5). \quad (15)$$

Equation (15) contains two theta functions: (i) $\theta(E_p-E_5)$ prohibits the emission of gluons with energy greater than E_p, the energy of the incoming heavy quark. (ii) $\theta(\tau-\tau_F)$ keeps the kinematics of the process strictly within the additive kinematic domain [14], wherein scattering centers are well separated enough that the gluon radiation is additive in nature.

In order to calculate $\sum|M|^2_{2\to3}$ for the process $Q(p)+\text{parton}(q)\to Q(p')+\text{parton}(q')+\text{gluon}(k_5)$, the necessary Mandelstam variables are defined as follows:

$$s=(p+q)^2, \qquad s'=(p'+q')^2, \quad (16)$$

$$t=(p-p')^2, \qquad t'=(q-q')^2, \quad (17)$$

$$u=(p-q')^2, \qquad u'=(q-p')^2, \quad (18)$$

with

$$s+t+u+s'+t'+u'=4M^2. \quad (19)$$

In the present work, we will consider the case of soft gluon emission (see Ref. [15] for details), i.e., when $k_5\to 0$, which implies $s'\to s$, $t'\to t$, $u'\to u$. For the kinematic region

$$\sqrt{s}\gg\sqrt{|t|}\sim q_\perp\gg k_\perp\gg m_D, \quad (20)$$

the invariant amplitude squared for a $2\to3$ process can be expressed in terms of a $2\to2$ process multiplied by the emitted gluon spectrum [16]:

$$|M|^2_{2\to3} = |M|^2_{2\to2}\times 12g_s^2\frac{1}{k_\perp^2}\left(1+\frac{M^2}{s}e^{2y}\right)^{-2}, \quad (21)$$

where M is the mass of the CQ. The last term in Eq. (21) within brackets is the dead cone factor, and y denotes the rapidity of the emitted gluon. Equation (21) provides the square of the invariant amplitude for light quarks for $M=0$. Following Eqs. (15) and (21), we have the radiative X:

$$X_{\text{rad}} = X_{\text{coll}}\times\int\frac{d^3k_5}{(2\pi)^3 2E_5}12g_s^2\frac{1}{k_\perp^2} \times\left(1+\frac{M^2}{s}e^{2y}\right)^{-2}[1+\hat{f}(E_5)] \times\theta(\tau-\tau_F)\theta(E_p-E_5). \quad (22)$$

After having calculated the radiative transport coefficients, we find our total or effective transport coefficient as the sum of the collisional and radiative contributions, i.e.,

$$X_{\text{eff}} = X_{\text{coll}} + X_{\text{rad}}, \quad (23)$$

where X_{coll} and X_{rad} are the transport coefficients for the collisional and radiative processes, respectively. In order to obtain the effective transport coefficient, we have added the collisional and radiative parts with the view in mind that though the invariant amplitude of three-body scattering can be expressed in terms of two-body scattering, the processes of collision and radiation take place inside the thermal medium independently.

In Fig. 1, we display the temperature dependence of the drag of CQs with momentum $p=5$ GeV. At low T, although the drag for radiative loss is comparable to that for collisional loss, at high temperatures the radiative drag tends to dominate. The difference between total and collisional transport coefficients broadens with increasing temperature. Even at temperatures attainable at RHIC, this

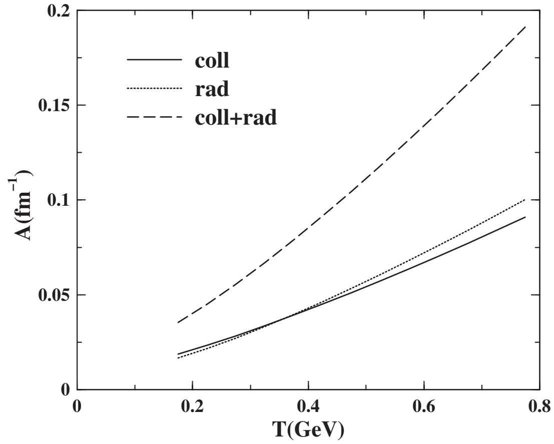

FIG. 1. Temperature dependence of the drag coefficient of CQs with momentum $p = 5$ GeV.

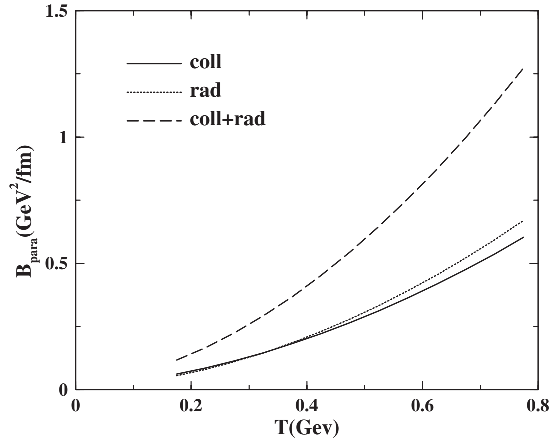

FIG. 3. Temperature dependence of the longitudinal diffusion coefficient of CQs with momentum $p = 5$ GeV.

distinction is significant enough to have a pronounced effect on certain experimental observables like nuclear modification factor, elliptic flow of CQs, etc. In the temperature range that may be achieved at LHC collision conditions, the radiative contributions to the drag may surpass the elastic contributions. Therefore, radiative processes will play a more dominant role at LHC than at RHIC. For a CQ (mass $M = 1.3$ GeV) with $p = 5$ GeV and $T = 300$ MeV, the drag coefficient attains a value almost double the value for the collisional case when radiation is included. At a temperature of 600 MeV, total drag becomes 2.12 times the collisional drag. The variation of drag with p at $T = 525$ MeV is depicted in Fig. 2. The dominance of radiative processes, in spite of dead cone suppression, is evident from the results for p beyond 5 GeV.

In Figs. 3 and 4, the variations of longitudinal diffusion coefficients with temperature and momentum, respectively, are displayed. Similar to drag, the contributions from radiative processes dominate over the collisional processes for higher T and p.

For $T = 300$ MeV, the radiative and collisional losses have similar contributions to $B_\perp$, but for T beyond 500 MeV, the radiative part exceeds the collisional part (Fig. 5). It is interesting to note the qualitative change in the momentum dependence of $B_\perp$ from $B_\parallel$ at fixed T (Fig. 6). The variation of $B_\perp$ with p is slower than that of $B_\parallel$. In this case, again the domination of the radiative transport coefficient over its collisional counterpart is evident. Though the nature of the momentum dependence of the diffusion coefficients is different from that of drag, it is always true that, save at very low momentum of the CQ, the radiative contribution is more than the elastic contribution at $T = 525$ MeV. Accordingly, for a relativistic CQ, inclusion of the radiative effects becomes imperative for the analysis of experimental data from nuclear collisions at RHIC and LHC. This statement can be put on a firmer ground if we quote some quantitative results comparing radiative and collisional contributions to the transport coefficients. The drag coefficient of a CQ having a momentum of 10 GeV is 0.038 fm^{-1} in case of elastic

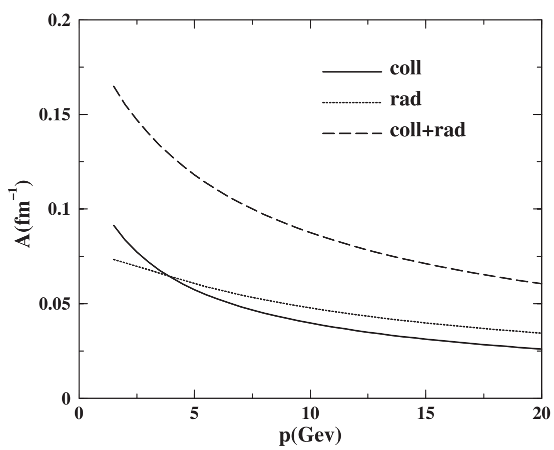

FIG. 2. Momentum dependence of the drag coefficient of CQs for a bath temperature of $T = 525$ MeV.

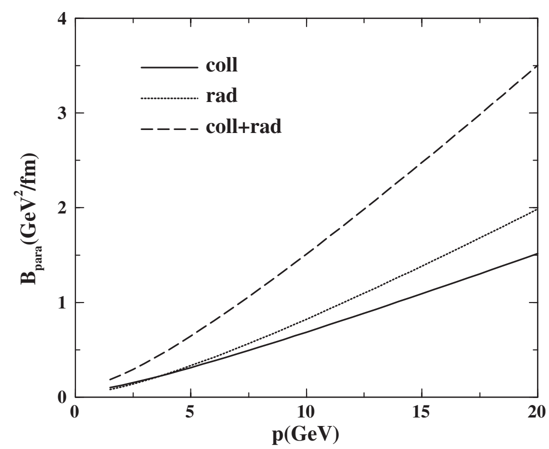

FIG. 4. Momentum dependence of the longitudinal diffusion coefficient of CQs for a bath temperature of $T = 525$ MeV.

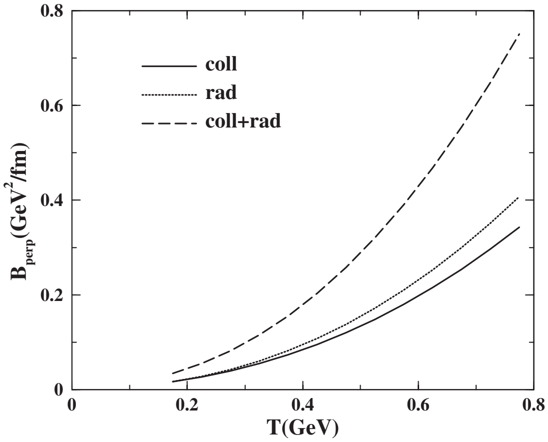

FIG. 5. Temperature dependence of the transverse diffusion coefficient of CQs with momentum $p = 5$ GeV.

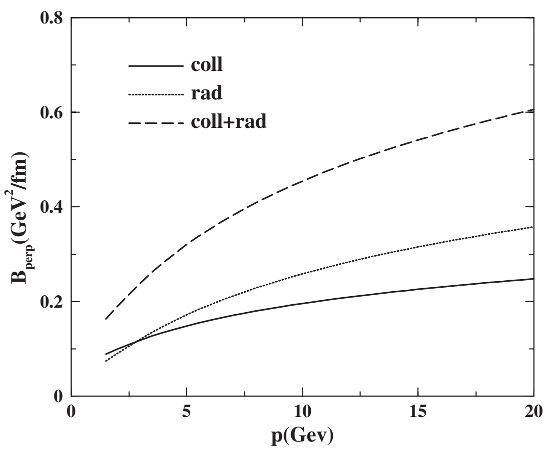

FIG. 6. Momentum dependence of the transverse diffusion coefficient of CQs for a bath temperature of $T = 525$ MeV.

loss, whereas the radiative contribution is 0.047 fm^{-1}. Radiative $B_\perp$ is about 1.33 times its collisional counterpart. In the case of longitudinal diffusion coefficient, the radiative contribution is 1.2 times the elastic one.

III. EQUILIBRIUM DISTRIBUTION OF CHARM QUARKS

Having calculated the diffusion coefficients of CQs including both collisional as well as radiative effects, we would like to investigate the fate of the equilibrium distribution function of a CQ undergoing elastic as well as radiative processes. A generalized Einstein relation involving the three transport coefficients, i.e., drag, transverse, and longitudinal diffusion coefficients, is obtained in Ref. [11] to establish the shape of the distribution of CQs after it gets equilibrated due to its collisional interaction with the medium. In Ref. [11], the radiative process was not taken into account. We would like to explore the role of the radiative processes in the characterization of the equilibrium distribution and to check whether the CQs become a part of the thermal medium abiding by the same class of statistics, which is the Boltzmann-Jüttner distribution, followed by the bath particles.

We discuss the generalized Einstein relation by examining the Fokker-Planck equation in the absence of any external force in a homogeneous QGP:

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial p_i}\left(A_i f + \frac{\partial}{\partial p_j B_{ij} f}\right) = -\vec{\nabla}_p.\vec{\wp}. \tag{24}$$

A relationship among the transport coefficients can be derived by demanding that $\partial f/\partial t$ equal zero; i.e., the probability current $\vec{\wp}$ vanishes when Eq. (24) is satisfied by the equilibrium distribution function, $f_{\mathrm{eq}}^{\mathrm{CQ}}$.

Using the following form of $f_{\mathrm{eq}}^{\mathrm{CQ}}$,

$$f_{\mathrm{eq}}^{\mathrm{CQ}}(p;T,q) = N\ \exp[-\Phi(p;T,q)], \tag{25}$$

the desired relation can be found out, where N is the normalization factor and T, q are parameters needed to specify the shape of the distribution. Using Eqs. (9) and (10) and the fact that $f_{\mathrm{eq}}^{\mathrm{CQ}}$ depends only on the magnitude of momentum for the spatially homogeneous case, we arrive at the general Einstein relation:

$$A(p,T) = \frac{1}{p}\frac{d\Phi}{dp}B_\parallel(p,T) - \frac{1}{p}\frac{dB_\parallel}{dp} - \frac{2}{p^2}[B_\parallel(p,T) - B_\perp(p,T)]. \tag{26}$$

This relation is valid for any momentum of CQ and can be reduced to the well-known Einstein relation $D = \gamma MT$ in the nonrelativistic limit, where $A = \gamma$ and $B_\perp = B_\parallel = D$, i.e., $B_{ij} = D\delta_{ij}$ and $\Phi = p^2/(2\ \mathrm{MT})$.

From Eq. (26), it is clear that if the three transport coefficients are known, then one can infer the correct equilibrium distribution function obeyed by CQs and ascertain whether or not CQs will fall under the Boltzmann-Jütner class of statistics. It is clear from the variation of $d\Phi/dp$ [calculated from Eq. (26)] with p (Fig. 7) that $d\Phi/dp$ deviates significantly from $d/dp(\sqrt{p^2+m^2}/T)$, i.e., CQs seem to be away from the Boltzmann-like distribution. In principle, we should have ascertained the precise form of Φ from Eq. (26) had we been able to include nonperturbative effects in A, $B_\perp$, and $B_\parallel$. Therefore, to study the equilibrium distribution and its deviation from the Boltzmann-Jüttner distribution quantitatively, we consider the Tsallis distribution [17], for which Φ is given by

$$\Phi_{Ts} = \frac{1}{1-q}\ \ln[1-(1-q)E(p)/T_T], \tag{27}$$

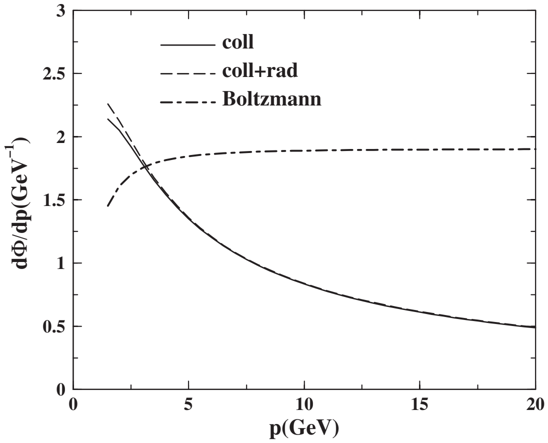

FIG. 7. The variation of $d\Phi/dp$ with p for CQs propagating in a QGP having temperature $T = 525$ MeV.

where T_T (temperature-like) and q are parameters. Φ_{Ts} reduces to the Boltzmann distribution in the limit $q \to 1$ and $T_T \to T$ (where T is the temperature of the heat bath). The values of T_T and q will dictate the form of $f_{\rm eq}^{\rm CQ}$. Putting Eq. (27) into Eq. (26), we get [11]

$$T_T + (q-1)E = \frac{dE}{dp}\frac{1}{p\frac{A}{B_\parallel} + \frac{1}{B_\parallel}\frac{dB_\parallel}{dp} + \frac{2}{p}(1 - \frac{B_\perp}{B_\parallel})}. \quad (28)$$

Our aim is to calculate the right-hand side of Eq. (28) and determine the values of T_T and q by studying the variation of $T_T + (q-1)E$ with E and parametrizing the variation by a straight line. It is important to note that the quantities on the right-hand side of Eq. (28) involve the ratios of the transport coefficients rather than their absolute values. First, we consider the elastic processes only. The dependence of $T_T + (q-1)E$ on E for CQs of mass $M = 1.3$ GeV propagating inside a heat bath of temperature $T = 525$ MeV is plotted in Fig. 8, considering A, $B_\perp$, and $B_\parallel$ for collisional loss only. We get $q = 1.101$ and $T_T = 184$ MeV. Φ_{Ts} with these values of T_T and q is far from being that of Boltzmann-Jüttner statistics (shown by the long-dashed line). Results displayed in Fig. 8 also indicate that the inclusion of radiative effects on the drag and diffusion coefficients does not make any noteworthy change on the shape of the equilibrium distribution of CQs. In Figs. 8 and 9, the long-dashed horizontal lines represent the Boltzmann-Jüttner distribution ($q = 1$ and $T_T = T$) which is obeyed by the constituent of QGP. The values of T_T and q do not get altered with the inclusion of the radiative effects. As a matter of fact, this effect is not quite unexpected. By looking at Eq. (28), we might conclude that it is not the magnitude of the transport coefficients, but rather their ratio which decides the shape of the equilibrium distribution. Therefore, it is not surprising that although the value of the relaxation time of CQs is dictated by the magnitude of the drag coefficient (in which the radiative contribution is substantial), the shape is largely independent of the magnitude of the transport coefficients. In turn, this means that the nature of the underlying interaction of a CQ with the bath particles, i.e., whether it suffers only elastic collisions or undergoes bremsstrahlung also, has very little to do with the ultimate shape of $f_{\rm eq}^{\rm CQ}$. This conclusion remains unaltered even when we increase the bath temperature T to 725 MeV. At $T = 725$ MeV, the slope of the straight line remains almost unchanged, i.e., the value of the parameter q comes out to be 1.095, which is close to the value obtained for $T = 525$ MeV. However, the value of the other parameter of the Tsallis distribution, T_T, is found out to be 335 MeV. At this temperature for the heat bath too, the incorporation of the radiative drag/diffusion coefficients hardly has any bearing as far as the shape of $f_{\rm eq}^{\rm CQ}$ is concerned. For the probe—the CQs—to become a part of the system, i.e., to follow the same statistics as that of the bath particles, both the parameter q and the ratio T_T/T should be 1. Instead, we

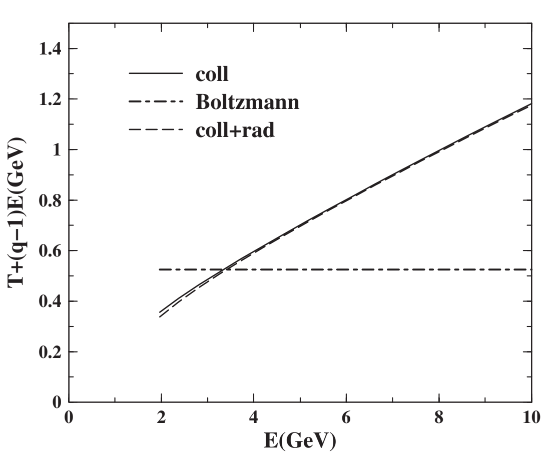

FIG. 8. Plot of rhs of Eq. (28) vs E for collisional as well as total transport coefficients at $T = 525$ MeV. The long-dashed line shows the plot expected for Boltzmann-Jüttner distribution.

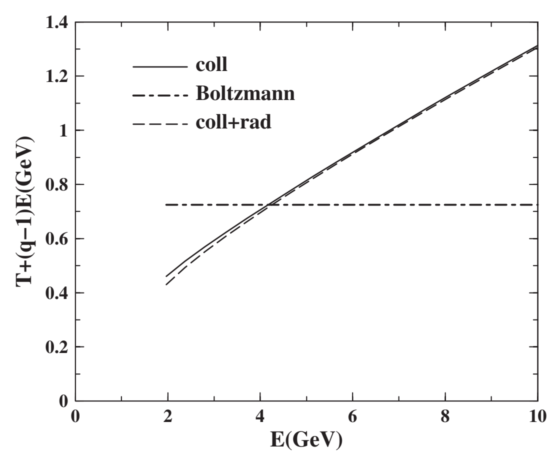

FIG. 9. Plot of rhs of Eq. (28) vs E for collisional as well as total transport coefficients at $T = 725$ MeV. The long-dashed line shows the plot expected for Boltzmann-Jüttner distribution.

notice that q and T/T_T (this ratio is 2.85 at $T = 525$ MeV and 2.164 at $T = 725$ MeV) are never equal to unity. Therefore, it may be concluded that although the CQ may equilibrate while propagating through QGP, it may not share the same distribution with the bath particles, i.e., with the light quarks and gluons, for a wide range of CQ energies and bath temperatures.

IV. SHEAR VISCOSITY (η) TO ENTROPY DENSITY (s) RATIO OF QGP PROBED BY THE CHARM QUARK

The value of the shear viscosity (η) to entropy density (s) ratio, η/s, plays a pivotal role in deciding the nature of QGP, i.e., whether the medium behaves like a weakly coupled gas or a strongly coupled liquid. In this work we evaluate η/s by calculating the transport parameter, $\hat{q}$, which is a measure of the squared average momentum exchange between the probe and the bath particles per unit length [18–20]. The $\hat{q}$, which has been found to be $\sim$1 GeV^2/fm in Ref. [20], can be related to the transverse diffusion coefficient of the CQ, which is calculated here. When a CQ with a certain momentum propagates in QGP, a transverse momentum exchange with the bath particles occurs. Hence, the momentum of the energetic CQ is shared by the low-momentum (on the average) bath particles, which is expressed through the transverse diffusion coefficients. The transverse diffusion coefficients cause the minimization of the momentum (or velocity) gradient in the system. Therefore, it must be related to the shear viscous coefficients of the system which drive the system toward a depleted velocity gradient. The transverse momentum diffusion coefficient $B_\perp$ can be written as

$$B_\perp = \frac{1}{2}\left(\delta_{ij} - \frac{p_i p_j}{p^2}\right) B_{ij}. \tag{29}$$

By Eq. (8) and using the notation $(p' - p)_i = k_i$,

$$\begin{aligned} B_\perp &= \frac{1}{2}\left(\delta_{ij} - \frac{p_i p_j}{p^2}\right)\frac{1}{2}\langle\langle k_i k_j\rangle\rangle \\ &= \frac{1}{4}\left\langle\left\langle \vec{k}^2 - \frac{(\vec{p}.\vec{k})^2}{\vec{p}^2}\right\rangle\right\rangle. \end{aligned}$$

If we take $\vec{p}$ to be along the z axis,

$$\begin{aligned} B_\perp &= \frac{1}{4}\langle\langle \vec{k}^2 - k_z^2\rangle\rangle \\ &= \frac{1}{4}\langle\langle k_\perp^2\rangle\rangle \\ &= \frac{1}{4}\hat{q}. \end{aligned} \tag{30}$$

With this definition of $\hat{q}$, we calculate the η/s of QGP from the following expression [18]:

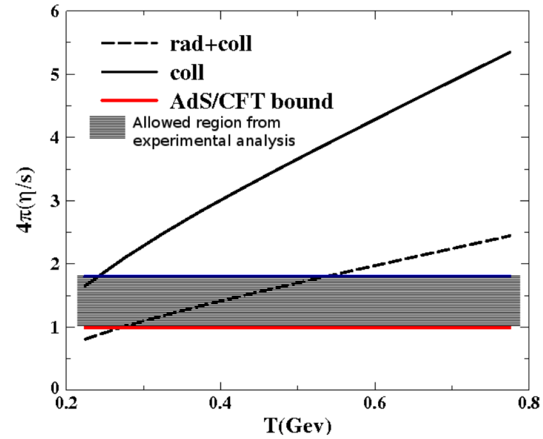

FIG. 10 (color online). The value of η/s for a CQ with momentum $\langle p_T\rangle = 5$ GeV propagating in a QGP of temperature T.

$$\frac{\eta}{s} \approx 1.25\frac{T^3}{\hat{q}}. \tag{31}$$

Therefore,

$$4\pi\frac{\eta}{s} \approx 1.25\pi\frac{T^3}{B_\perp}. \tag{32}$$

Equation (32) indicates that the η/s can be estimated from $B_\perp$. From the analysis of the experimental data [20], it was found that $4\pi\frac{\eta}{s} = 1.4 \pm 0.4$, which may be compared with the AdS/CFT bound $4\pi\frac{\eta}{s} \geq 1$ [21]. We display $4\pi\frac{\eta}{s}$ against T when the CQ undergoes both collisional and radiative processes.

From the results shown in Fig. 10, it should be noted that the value of η/s changes substantially with the inclusion of the radiative effects. The inclusion of the radiative loss in $B_\perp$ brings the theoretical values closer to the experimental findings [22]. This highlights the importance of the radiative loss of the CQ in QGP. It is interesting to note that the value of $\hat{q}$ for $T = 300$ MeV is about 2 GeV^2/fm. This value is close to the one obtained in the Gyulassy-Levai-Vitev approach to energy loss [23] but lower than the value extracted from Baier-Dokhshitzer-Mueller-Peigne-Schiff [24] or Armesto-Salgado-Wiedemann [25] approaches.

V. SUMMARY AND CONCLUSION

Transport coefficients, i.e., drag and the transverse and longitudinal diffusion coefficients, of CQs propagating in QGP have been evaluated using pQCD by including both the elastic collision of CQs with the constituent particles of the bath along with soft gluon radiation (which gets absorbed in the medium subsequently). Radiative drag/diffusion coefficients are seen to exceed the collisional ones for high bath temperatures and CQ momenta. A relation between the transverse diffusion coefficients ($B_{\perp}$) and η/s is established. We obtain a reasonable value of η/s for the QGP when the contributions from the gluon bremsstrahlung of the CQ are added with the collisional contributions. We also investigate the dependence of the shape of the equilibrium distribution function of CQ on the three transport coefficients. We find that the incorporation of radiation does not alter the shape of the equilibrium distribution significantly, owing to the fact that the shape counts on the ratios of the transport coefficients instead of their absolute values. The present work has been performed for the CQ. However, its extension for the bottom quark is straightforward, where the mass of the charm quark has to be replaced by that of the bottom quark.

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