Possible Anderson localization in a holographic superconductor

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We study the effect of disorder in a holographic superconductor by introducing a quasiperiodic chemical potential. When the condensation of the superconductor is sufficiently small compared with the strength of disorder, we find that there exists a discontinuous phase transition from superconducting state to normal state with increasing disorder strength. For relatively large condensation, we find that disorder suppresses but does not completely destroy superconductivity.

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I. INTRODUCTION

The effect of disorder in a superconductor has intrigued scientists for several decades. Soon after the BCS theory [1], Anderson found that weak disorder cannot destroy the superconductivity [2]. Until now, both theories and experiments have confirmed that a strong disorder will eventually destruct superconductivity, driving the system into an insulating state or a normal metal state [3–10]. However, the effect of interactions in a disordered superconductor is still not well understood. As a natural way to study a strongly coupled quantum field theory system, the AdS/CFT correspondence [17] has been used to study the interplay of disorder and interaction [11–16]. The holographic correspondence has also been proved to be successful to study various properties of superconductors [18,19]. In Ref. [20] the authors first studied a dirty holographic superconductor, then found that the disordered superconductor always has a larger critical temperature relative to the to the T_c for the uniform one. In this paper we focus on understanding another important issue, the possible Anderson localization in a holographic superconductor. Technically, the weak disorder effect is introduced by a quasiperiodic chemical potential on the boundary field theory, and the strength of the disorder is controlled by a parameter α. By tuning α we find that when the condensation is small, the weak disorder will destroy the superconductivity; clearly this is a holographic realization of Anderson localization in superconductors.

II. MODEL AND DEFINITION OF DISORDER

The starting action in the usual gravity dual of a holographic superconductor is [18] $S = \int d^4x\sqrt{-g}[R - 2\Lambda - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |\nabla\psi - iA\psi|^2 - m^2|\psi|^2]$', where $\Lambda = -d(d-1)/2L^2$ is the cosmological constant, d is the dimension of the boundary, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the strength of the gauge field. The metric is an AdS Schwarzschild black hole, $ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(dx^2 + dy^2)$ with $f(r) = r^2/L^2(1 - r_0^3/r^3)$, r being the bulk radial coordinate, r_0 the horizon position, and x, y the boundary coordinates. Without loss of generality, we set $L = 1$. The temperature of the black hole is $T = \frac{3r_0}{4\pi}$.

We use the ansatz of $\psi = \psi(r,x)$ and $A = (A_t(r,x), 0, 0, 0)$, where x is the spatial coordinate of the boundary field theory, and choose $m^2 = -2$. In the probe limit, with the scaling of $\psi \to \psi/r$ and working in the coordinates with $z = 1/r$, we have the following equations of motion (EOMs):

$$(1 - z^3)A_t^{(2,0)}(z,x) + A_t^{(0,2)}(z,x) - 2A_t(z,x)\psi(z,x)^2 = 0, \tag{1}$$

$$\psi(z,x)(A_t(z,x)^2 + z^4 - z) + (1 - z^3)\psi^{(0,2)}(z,x) + (z^3 - 1)^2\psi^{(2,0)}(z,x) + 3(z^3 - 1)z^2\psi^{(1,0)}(z,x) = 0. \tag{2}$$

The superscripts on the fields mean the derivative of z and x; for example, $A_t^{(2,0)}(z,x)$ means $\partial_z^2 A_t(z,x)$ and $A_t^{(0,2)}(z,x)$ means $\partial_x^2 A_t(z,x)$. The expansions of ψ and A_t near the infinite boundary are

$$\psi(r,x) \sim \psi^{(0)}(x) + \psi^{(1)}(x)z + \cdots, \tag{3}$$

$$A_t(r,x) \sim \mu(x) + \rho(x)z + \cdots. \tag{4}$$

We choose the quantization such that $\psi^{(0)}(x) = 0$ and $\psi^{(1)}(x) = \langle\mathcal{O}(x)\rangle$ is the order parameter. We introduce the disorder through a quasiperiodic chemical potential on the boundary as

$$\mu(x) = \mu_a + (1 - \alpha)(\mu' - \mu_a)\cos(2k_1\pi x/2) + \alpha(\mu' - \mu_a)\cos(2k_2\pi x/2), \tag{5}$$

where $2k_1$ and $2k_2$ are two coprime positive integers, μ_a is the average value of $\mu(x)$, $0 \le \alpha \le 1$ controls the pattern of $\mu(x)$, μ' controls the maximal value $\mu_{\max} = \mu'$ of $\mu(x)$, and

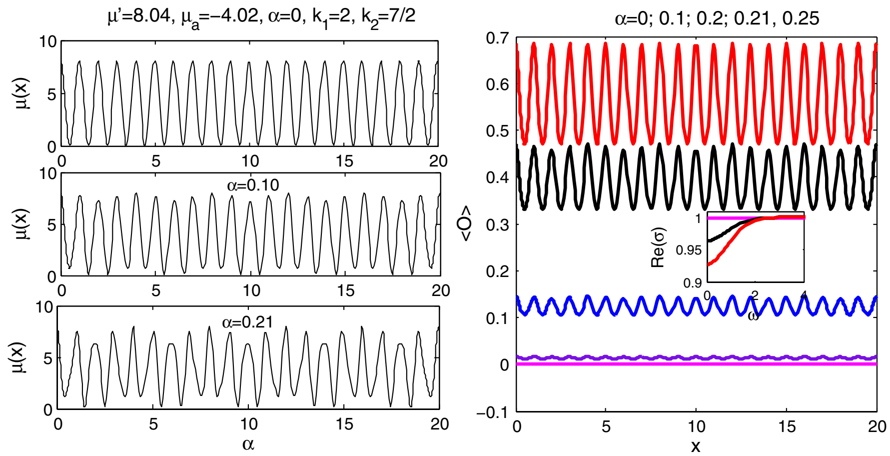

FIG. 1 (color online). Left: the plot of $\mu(x)$ for the cases of $\mu_a = 4.02$, $\mu' = 8.04$ and $\alpha = 0; 0.1; 0.21$ with $k_1 = 2$, $k_2 = 7/2$. Right: the order parameter $\langle \mathcal{O}(x) \rangle$ for five cases $\alpha = 0; 0.1; 0.2; 0.21; 0.25$ (from top to bottom) with $\mu_a = 4.02$, $\mu' = 8.04$, $k_1 = 2$, $k_2 = 7/2$. We see that a phase transition happens when increasing α. Inset of the right plot: the average values of real part of conductivity along the y direction for three cases $\alpha = 0; 0.1; 0.25$ in the right plot.

the minimal value $\mu_{\min} = -\mu' + 2\mu_a$ of $\mu(x)$. Thus, the amplitude of the oscillating $\mu(x)$ is $2(\mu' - \mu_a)$. Then α is the parameter of the disorder strength after fixing μ_a and μ', μ' is the parameter of the amplitude of the oscillation after fixing μ_a and α. Similar kind of quasiperiodic lattice has been already used to study the Anderson localization in Refs. [21–23]. Strictly, k_1/k_2 should an irrational number for the quasiperiodic case; however, by using two coprime positive integers $2k_1$ and $2k_2$ we can still induce some weak disorder effect as shown in Fig. 1.

The EOMs are solved by using the Chebyshev spectral method [24]. We discretize the EOMs on a two-dimensional Chebyshev grid with 20 points along the z direction and 400 points in the x direction. In all the calculations we choose the length l of the sample to be $l = 20$.

III. INTERPLAY OF DISORDER EFFECT AND PERIODIC EFFECT

A holographic superconductor with periodic chemical potential has been studied in [25–27]. In [26,27] the authors found that the superconductivity is enhanced by the presence of the periodic chemical potential. In Fig. 2 we plot the average value of the order parameter $\langle \mathcal{O}_a \rangle$ as a function of α for various combinations of k_1 and k_2 with fixed μ' and μ_a. The lowest pink lines in Fig. 2 are the homogeneous solutions with $\mu(x) = \mu_a = 4.05$ and 5. $\mu_c = 4.06$ is the critical value for the homogeneous configuration; after μ_c we will see no superconductivity [18]. From the left plot it can be seen that $\langle \mathcal{O}_a \rangle = 0$ when $\mu(x) = 4.05 < 4.06$, while for the periodic or quasiperiodic cases we have nonzero condensation for some regions of α. Similar phenomena also happen for the case of $\mu_a = 5$. In all cases, both periodic and quasiperiodic chemical potential induce a larger value of order parameter compared to the homogeneous case.

When $\alpha = 0$ or $\alpha = 1$, we recover the cases of periodic chemical potentials: $\mu(x) = \mu_a + (\mu' - \mu_a)\cos(k_1 \pi x)$ and $\mu(x) = \mu_a + (\mu' - \mu_a)\cos(k_2 \pi x)$. From Fig. 2, we can see that $\langle \mathcal{O}_a \rangle$ decreases with increasing k in the periodic cases. As a check we see that when $k = k_2 = 7/2; 9/2$, which is greater than $k_1 = 2$, $\langle \mathcal{O}_a \rangle$ for a periodic $\mu(x)$ with $k = k_2$ is small than that of $k = k_1$. If we keep increasing k (the results are not include here), the condensation $\langle \mathcal{O}_a \rangle$ asymptotes some constant value. These results have also been found in [25,27].

Looking at the two red lines with dots in the top of Fig. 2, we see the condensation does not monotonically increase with increasing α. The condensation decreases first then

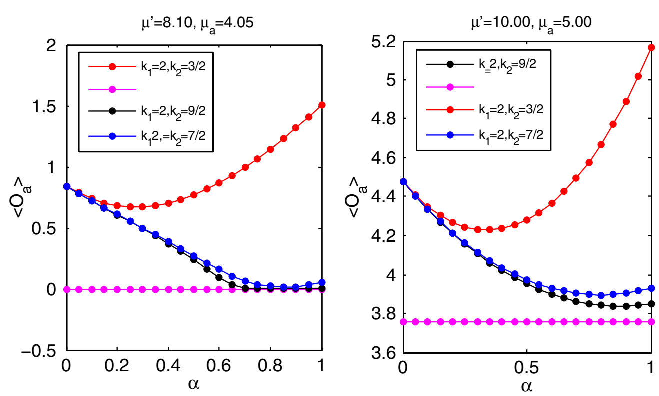

FIG. 2 (color online). The average value of order parameter $\langle \mathcal{O}_a \rangle$ as a function of α for different k_2 with a fixed $k_1 = 2$. In the left plot $\mu' = 8.1$, $\mu_a = 4.05$; in the right plot $\mu' = 10$, $\mu_a = 5$. The lowest two pink dotted lines are the homogeneous case with $\mu(x) = 4.05$ and $\mu(x) = 5$, respectively. In all the plots we increase α from 0 to 1 with a step $\delta\alpha = 0.05$.

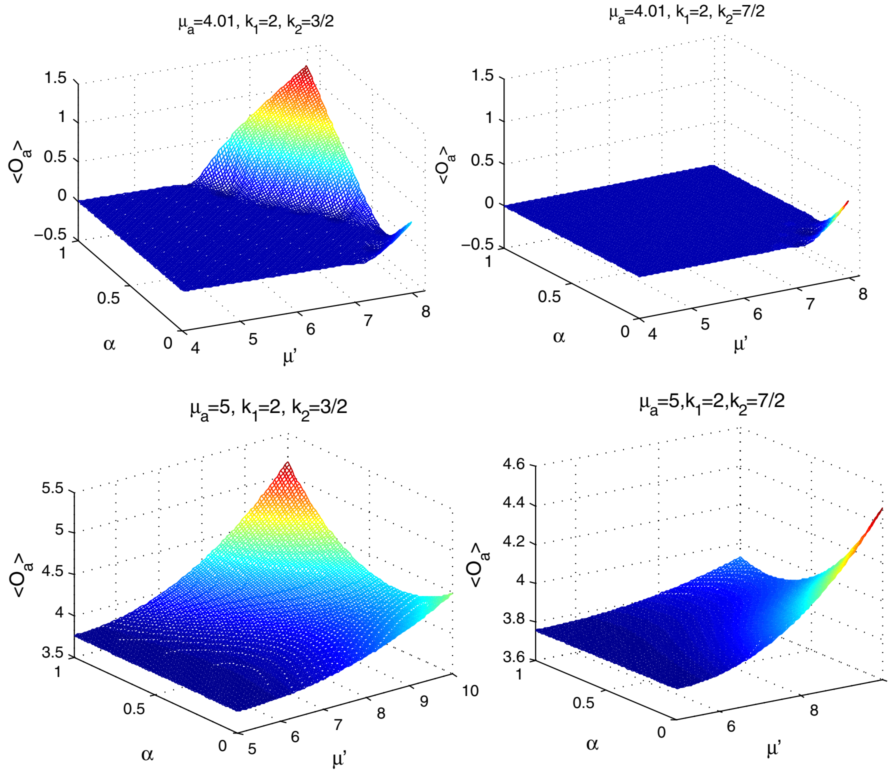

FIG. 3 (color online). $\langle \mathcal{O}_a \rangle$ as a function of both α and μ' for a fixed $\mu_a = 4.01$ and $\mu_a = 5$. There are regions in which the condensation is zero when $\mu_a = 4.01$, which means that there is phase transition when increasing α when $7.1 \preceq \mu' < 8.02$.

increases when we increase the portion of the case of $k_2 = 3/2$ by tuning α. This means that there is an interplay between the disorder effect and the periodic effect: periodic chemical potential favors an increasing condensation, while disorder favors a decreasing one. In the left plot of Fig. 2, similar nonmonotonic behaviors of the two cases with $k_2 = 7/2$ (blue lines) also confirm the existence of disorder effect. We can also see a phase transition from the superconducting phase to a normal phase at $\alpha_c \sim 0.8$ when $k_1 = 2$, $k_2 = 9/2$, but the main reason of the phase transition is the periodic effect since the transition happens at $\alpha_c > 0.5$ and the periodic case with $k = k_2 = 9/2$ is of a vanishing condensation.

We also studied how the condensation behaves when we tune both μ' and α with a fixed μ_a. Figure 3 shows $\langle \mathcal{O}_a \rangle$ as a function of both α and μ', where $0 < \mu' < 2\mu_a$ is chosen in order to have positive chemical potentials. The important information from Fig. 3 is that when we reduce μ' (the oscillating amplitude) with fixed α, μ_a, k_1 and k_2, the condensation will be decreased.

The two parameters α and μ' control the properties of the disorder effect, and the quasiperiodic $\mu(x)$ affect the superconductor in a complex way. With a fixed α, increasing the amplitude $2(\mu' - \mu_a)$ of $\mu(x)$ enhances the superconductivity, as shown in Fig. 3. When $\alpha = 0$ or 1 we reproduce the result that the superconductivity of a striped holographic superconductor will be enhanced [26–28].

However, with a fixed amplitude $2(\mu' - \mu_a)$, the disorder can always suppress the superconductivity when by turning α from zero to a finite value, as shown in Figs. 1–4.

The interplay between the disorder effect and the periodic effect with fixed μ' and μ_a will result in a

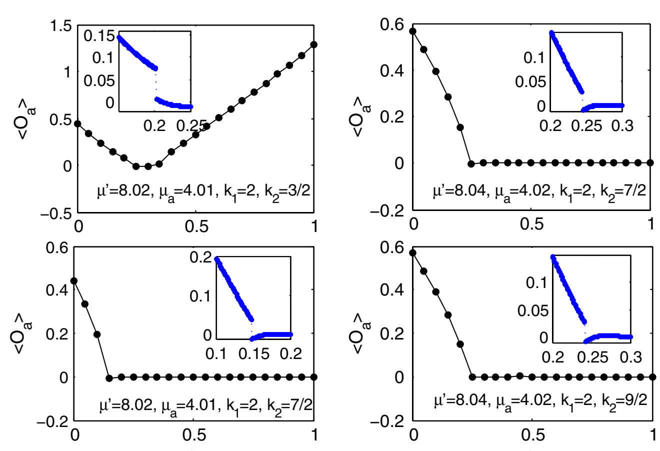

FIG. 4 (color online). The Anderson localization phase transition with $\mu_a = 4.01, \mu' = 8.02$ and $\mu_a = 4.02, \mu' = 8.04$. The four insets are the detail plot of the region when phase transition happens, in which we increase α step by step with the distance $\delta\alpha = 0.001$.

phase transition from the superconducting state to a nonsuperconducting state in some regions of parameters as shown in Fig. 1 and Fig. 3. The DC conductivity along the y direction of the nonsuperconducting state is finite, as shown by the inset in Fig. 1 ($\alpha = 0.25$), which means that the nonsuperconducting state is a normal metal state rather than an insulating state.

IV. DISCONTINUOUS PHASE TRANSITION FROM SUPERCONDUCTING TO NORMAL STATE

With the results in the above section, we already see that there is a phase transition when the superconductor is close to T_c ($\mu_a \approx \mu_c = 4.06$) by increasing α from zero to a finite value (< 0.5) for $\mu_a = 4.02$ and $\mu_a = 4.01$ as shown in Figs. 1 and 3. Figure 4 shows the critical value of $\alpha_c < 0.5$, at which a phase transition from the superconducting state to the normal state occurs when $\mu_a = 4.02$ and $\mu_a = 4.01$. We note that the value of α_c for the case of $\mu' = 8.02$, $\mu_a = 4.01$, $k_1 = 2$, $k_2 = 3/2$ ($\simeq$0.2) is larger than that for the case of $\mu' = 8.02$, $\mu_a = 4.01$, $k_1 = 2$, $k_2 = 7/2$, ($\simeq$0.15), which is a consequence of the interplay between the disorder effect and the periodic effect as studied above. From the four insets in Fig. 4 (blue lines), we see the order parameter goes discontinuous at α_c, which indicates that the superconducting to normal phase transition is a discontinuous one. By computing many cases with other values of μ_a systematically, we find that when $\mu_a > 4.03$ there is no disorder-driven phase transition anymore with $k_1 = 2$.

When the phase transition happens, the free energy of the superconductor is also obtained by computing the on-shell action according to the AdS/CFT dictionary. The results are shown in Fig. 5. It is clear to see that the free energy also goes discontinuously at α_c, which means that this is a zeroth-order phase transition. More details for the calculations of conductivity and free energy will be presented elsewhere [29].

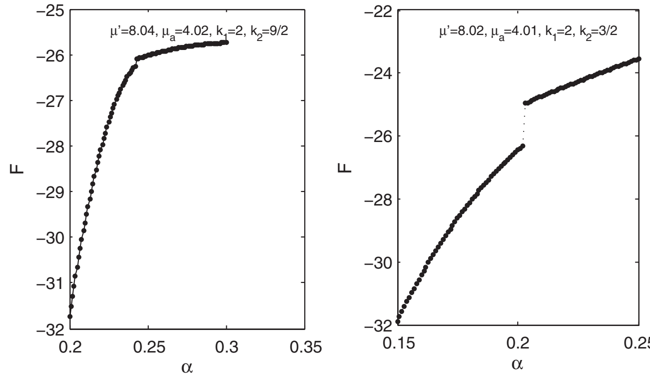

FIG. 5. The free energy of the superconductor when the phase transition happens. (left) $\mu' = 8.04$, $\mu_a = 4.02$, $k_1 = 2$, $k_2 = 9/2$. (right) $\mu' = 8.02$, $\mu_a = 4.01$, $k_1 = 2$, $k_2 = 3/2$.

V. CONCLUSION

In this paper, we systematically studied the interplay of disorder effect and periodic effect in two-dimensional s-wave holographic superconductors. We reproduced the results in condensed matter physics that the disorder will suppress superconductivity and finally result in a discontinuous superconducting-to-normal-state phase transition when the gap is sufficiently small relative to the strength of the disorder.

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