AdS/CFT for accelerator physics

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The crucial property of particle colliders is their ability to convert (e.g. electrical) energy into the mass of heavy particles. We have become used to the extremely low efficiency of this conversion and the severe limitations on the mass scale of heavy particles that can be reached. In view of this situation, it appears reasonable to ask whether a perfect conversion machine of this type (a perfect "collider") exists even in principle and whether there is a highest mass scale that can be reached by such a machine. It turns out that, with a number of assumptions, such a machine is conceivable in a world with a strongly coupled, approximately scale-invariant four-dimensional (4D) field theory with five-dimensional (5D) gravity dual. This machine can be realized as a 5D tower built on the IR brane (in Randall-Sundrum model language). Transporting mass to the tip of this tower is, under certain conditions, equivalent to producing heavy pointlike 4D particles. Hence, this can be thought of as a perfect "collider." In the simple, "pure Randall-Sundrum setting" that we analyze, this machine can only reach a certain maximal energy scale, which falls as the gravity-dual of the 4D quantum field theory (QFT) approaches the strong coupling domain. On these grounds, one might expect that a no-go theorem (in the spirit of that of Carnot for the conversion of heat into work) exists for generic weakly coupled QFTs.

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I. INTRODUCTION

The production of very heavy particles is one of the main goals of modern experimental particle physics. The method of choice is the acceleration of beams of charged particles (i.e. the conversion of electrical into kinetic energy) and their subsequent collision (i.e. the conversion of at least a small fraction of that kinetic energy into the mass of heavy particles). While in practice the longevity of these particles has always been very limited, the production of stable very heavy states (such as the famous WIMP possibly making up dark matter) is most certainly conceivable.

In this context, one naturally encounters the following apparently very basic and general question: Does there exist, at least in principle, a perfect machine for the conversion of work into mass of heavy particles? To give an extreme example, is it conceivable to take just 543 kWh $= 1.22 \times 10^{19}$ Gev from the electrical grid and covert them into one Planck mass particle?

It is, of course, well known that conventional colliders with this energy reach are very hard to imagine. Furthermore, even the production of e.g. 100 Higgs bosons is energetically much more expensive than the equivalent amount of electrical energy would suggest. But the question remains whether this is just due to our insufficient ingenuity or the limited technological progress made so far by mankind, or whether there exists some fundamental limitation.

Unfortunately, the present paper fails by a large margin to answer this extremely interesting question. However, it will at least outline a somewhat unusual (AdS/CFT-based [1]) way to think about problems of this type. In this context, a suggestion for a perfect energy conversion machine can be made. It will turn out that the reach of this type of machine is limited (at least in the simplest, pure-gravity models to be specified below). This range becomes small as the underlying four-dimensional (4D) quantum field theory (QFT) becomes weakly coupled. One may interpret this as a hint at the existence of a fundamental no-go theorem for a prefect machine in 4D weakly coupled QFT (which is what we are apparently stuck with in this part of the multiverse).

The paper is organized as follows: Sec. II shows that if our world were described by a Randall-Sundrum (RS) model [2], a five-dimensional (5D) tower built on the IR brane (and ideally reaching the UV brane) can be thought of as a perfect (Planck scale) collider. This is almost obvious since a simple 5D elevator, using electrical energy with an energy conversion efficiency near unity, could now be employed to "UV-shift" massive particles. Thus, as a first step, a very simple "toy-tower" (a horizontal mirror supported by radiation pressure) is considered. It turns out that, at least if one ignores all 5D field VEVs except the metric, only a limited height can be reached. Making use of specific, nonmetric 5D VEVs (as suggested e.g. by stringy settings), possibilities for avoiding this maximal height restriction exist. It remains open whether this loophole can actually be turned into a toy-model perfect collider of arbitrary energy reach.

Section III discusses an actual tower (made from some imagined 5D analogue of solid matter consisting e.g. of 5D

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"atoms") and its maximal height. The result turns out to be similar to that of the previous section (assuming again only metric VEVs in 5D). Thus, both models suggest that only a certain maximal energy can be reached by our ideal collider. The maximal height of the tower (and hence the 'collider' energy) falls with growing 5D curvature. As mentioned above, this conclusion may be avoided in certain supersymmetric models with flat directions. Nevertheless, we believe that our findings suggests that at least in large classes of weakly coupled 4d QFTs (where the AdS dual is strongly curved), no 'perfect collider' can be built even in principle.

The final section is devoted to a brief summary and discussion of open questions.

Previous ideas concerning 'Planck scale colliders' have appeared e.g. in [3–5]. In particular, Ref. [3] deals with possible fundamental limitations due to 4-dimensional gravity.

II. COLLIDERS VS ELEVATORS IN THE RANDALL-SUNDRUM MODEL

A. How towers in RS models can be used to produce heavy particles

Our use of the AdS/CFT proposal will be limited to its simple yet very concrete and intuitive implementation in RS type models. To be very specific, we take the AdS metric in the form

$$ds^2 = e^{2ky}dx^2 + dy^2, \tag{1}$$

where k sets the AdS curvature scale. Our discussion is based on the action [2]

$$S = \int_0^{y_{UV}} d^4x dy \sqrt{-g_5}\left(\frac{1}{2}M_5^2\mathcal{R} - \mathcal{L}_{5d}\right) + \int d^4x\sqrt{-g_{IR}}\mathcal{L}_{IR} + \int d^4x\sqrt{-g_{UV}}\mathcal{L}_{UV}. \tag{2}$$

Here the compact space is the interval $y \in [0, y_{UV}]$, with gravity and some 5D field theory in the bulk and two 4D theories at the boundaries (coupled to the induced metrics g_{IR} and g_{UV}). Appropriate 5D and 4D cosmological constants have been absorbed in the Lagrangians for brevity. As in the celebrated proposal for the solution of the hierarchy problem [2], we take "our" QFT to be IR brane localized. Furthermore, and this is the crucial and nontrivial step, we imagine that future technology will allow us to penetrate the bulk and construct "5D robots" capable of manipulating structures in 5D, at least near the IR brane (cf. Fig. 1).

To be very clear, the point here is not that an RS model will actually be discovered at the LHC. Neither do we really hope that we will learn to manipulate structures at length scales of TeV^{-1} (which is equivalent to manipulating structures in the bulk). We are here considering a "model universe," not too dissimilar from our own, where the question

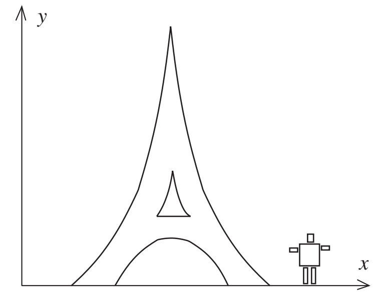

FIG. 1. A tower standing on the IR brane of the Randall-Sundrum model, built by a "5D robot," which is able to manipulate 5D (i.e. sub-TeV^{-1}-sized) structures.

of probing the Planck scale appears with an interesting twist (as we will presently explain).

Before doing so, we recall some familiar facts about the setting described above (so far without any robots) and its AdS/CFT interpretation (see e.g. [6,7]): First, we dimensionally reduce to 4D and Weyl-rescale the 4D metric g_4 to ensure that $g_4 = g_{IR}$. The resulting 4D effective theory of this compactification includes 4D gravity (with a Planck scale set by $M_4^2 \sim kM_5^3 \exp(2ky_{UV})$) and a strongly coupled sector (the KK modes of 5D gravity and $\mathcal{L}_{5d}$). This sector is approximately conformal in the energy range $k \ll E \ll k \exp(ky_{UV})$. Furthermore, the 4D effective theory also includes the two (by assumption weakly coupled) 4D field theories governed by $\mathcal{L}_{IR}$ and $\mathcal{L}_{UV}$. If these two Lagrangians, as they appear in (2), are governed by mass parameters M_1 and M_2, then the two corresponding sectors of the resulting 4D effective theory will be governed by mass parameters M_1 and $M_2 \exp(ky_{UV})$ respectively. This is the due to the different induced metrics at the two boundaries of our slice of AdS space. For simplicity, we set $M_1 = M_2 = M$ from now on.

From the 4D perspective, this setting looks rather conventional: one may think of it as the "Standard Model" ($\mathcal{L}_{IR}$ with mass scale $M \sim \mathrm{TeV}$), some form of technicolor, 4D gravity, and a weakly coupled sector with very heavy particles ($\mathcal{L}_{UV}$ with mass scale $M \exp(ky_{UV})$). The point is that, if we can build a 5D tower (in the AdS interpretation of this model, cf. Fig. 1), then this corresponds to a perfect collider (in the sense of a machine for producing very heavy pointlike particles) on the 4D side. We will shortly estimate the maximal height our 5D tower can reach, but before doing so let us argue in some detail that such a tower would be able to do the job of a conventional particle collider: Indeed, let us assume that $\mathcal{L}_{5d}$ contains some fundamental field of mass m ($m \sim M$ for simplicity). Corresponding particles can hence be produced by a conventional (i.e. IR-brane-bound) collider. This 5D field may also couple to a set of UV-brane fields, allowing e.g. its decay to two UV-brane particles of mass ϵm and $(1 - \epsilon)m$. Thus, if a 5D tower reaching the UV brane could be build, this would be equivalent to a perfect Planck scale collider: One would just have to create our

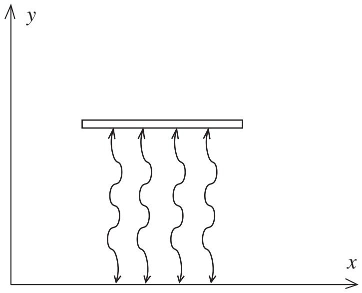

FIG. 2. Mirror supported by a "5D photon" beam above the IR brane.

5D particles with a TeV-scale machine, transport them up the tower using conventional mechanical energy (e.g. in an elevator) and eventually let them decay to UV-brane particles of mass almost equal to m. From a 4D perspective, this corresponds to producing heavy, pointlike particles (since $\mathcal{L}_{UV}$ is supposed to be a weakly coupled local Lagrangian) of mass $m\,\exp(ky_{UV})$ with energy conversion efficiency $\eta_{\rm coll} \sim 1$. (Here we ignore the (in)efficiency of our original 4D collider taking us up to the TeV domain.)

B. Toy model of a suspended mirror

Now it is unfortunately clear that a tower of some particular desired height (e.g. reaching the UV brane) can not be built in general. To understand the limitations, let us first focus on an (at least calculationally) simpler device which is sufficient for suspending an elevator: We add 5D photons to our list of assumptions and let a mirror float above the IR brane, supported by the pressure of photons bouncing back and forth between brane and mirror (cf. Fig. 2). Obviously, to construct the mirror and the elevator, we also have to assume that some form of structured, stable matter exists in 5D.[1] Governed by our 4D experience, we take this matter to consist of some small units ('atoms'). To simplify our analysis, we assume these "atoms" to have mass and inverse size M.[2] We are interested in the lightest possible mirror, which will nevertheless have a thickness of at least a few 'atoms'. The (hyper)surface density of this object will hence be $\rho_s \sim M^4$. [Note that this mirror extends in three spatial dimensions, and hence the corresponding surface density has units of $\text{mass}/(\text{length})^3$]

[1]This is nontrivial since all structures we manipulate every day in 4D rely microscopically on renormalizable gauge theories, which are not available in 5D. In particular, it is well known that the Schrödinger atom is unstable if $d > 4$ (see e.g. [8]). Let us nevertheless assume that some form of structured matter can exist at $d = 5$ (e.g. because a full QFT treatment cures the nonrelativistic instability problem) and press ahead.

[2]Obviously, our familiar 4D atoms have a mass and size which are parametrically different since the former is governed by the mass of the nucleus while the latter depends on electron mass and gauge coupling. In this language, our 5D model of matter corresponds to taking $m_N \sim m_e$ and $\alpha_e \sim 1$ in 4D.

To determine the force required to support such a mirror, consider first a particle with mass m that is stationary at some height y. We assume that no nongravitational field VEVs are present or are at least not relevant in the present context. (This assumption will be removed in Sec. II. C.) The relevant action is

$$S_y = -m \int_{y=\text{const.}} d\tau = -m e^{ky} \int dt, \tag{3}$$

where τ and t are the eigentime and the time at the IR brane, respectively. It is apparent that the same particle, if stationary at height $y + \delta y$, has an action enhanced by a factor $\exp(k\delta y)$. Thus, "lifting" a particle a distance δy costs an energy

$$\delta E = m e^{k(y+\delta y)} - m e^{ky} \simeq m e^{ky} k \delta y \tag{4}$$

from the perspective of the IR brane. Here the factor e^{ky} appears as a blueshift because we took the IR-brane point of view. For a local observer at height y, lifting the same particle by δy costs an energy $\delta E \simeq mk\delta y$. The force required to support a particle m, and now we use the local perspective, is hence km.

Our mirror is supported by the vertically directed (both up and down) photon stream with energy momentum tensor

$$T_{MN} \sim \text{diag}(\rho, p, p, p, p) = \text{diag}(\rho, 0, 0, 0, \rho), \quad \text{where } M, N \in \{0, 1, 2, 3, 5\}, \tag{5}$$

which is here given in a coordinate system with Minkowski metric in the vicinity of the mirror. To keep the mirror stationary, we need

$$p = \frac{F}{A} = \frac{\rho_s A k}{A} = \rho_s k \sim M^4 k, \tag{6}$$

in self-explanatory notation. This is the pressure (and hence energy density) at the position y of the mirror. Since each photon travels vertically (at constant $\vec{x}$), the number of photons per unit brane surface (in the vicinity of the IR brane) is enhanced by $\exp(3ky)$. Furthermore, due to the gravitational redshift, each photon has an energy enhanced by $\exp(ky)$ when it is reflected by the IR brane. Thus, the energy density of our beam near the brane is

$$\rho_{IR} \sim M^4 k e^{4ky}. \tag{7}$$

Assuming that the reflection of photons both at the IR brane and at our mirror is perfect, we can imagine that this configuration is stationary, without the need of continuous energy input. Nevertheless, the mirror had to be raised to its position y, which required the input of energy into the photon beam near the IR brane. Since we assume that such an energy input can be realized maximally at a scale M, we have the constraint $\rho_{IR} < M^5$. Comparing this with (7), we see that the maximal height $y_{\max}$ which can be achieved is set by

$$e^{ky_{\max}} \sim \left(\frac{M}{k}\right)^{1/4}. \tag{8}$$

In fact, there is an additional constraint arising from the danger of black hole formation (or, more generally, strong deformation of the 5D metric) in the region of high beam density. To see this, note that we actually have a layer of thickness $\sim 1/k$ of an approximate energy density ρ_{IR} directly above the IR brane. We now estimate how large ρ_{IR} can become before black holes are formed in this region. To do so, recall that the mass of a d-dimensional black hole of radius R is (see e.g. [8,9])

$$M_{BH} \sim M_{P,d}^{d-2} R^{d-3}. \tag{9}$$

This has to be compared to the relation between mass and radius of the corresponding smooth energy distribution,

$$M_{BH} \sim R^{d-1}\rho. \tag{10}$$

Eliminating M_{BH} from (9) and (10) and specifying $d = 5$, we determine the critical radius for black hole formation,

$$R_c \sim \sqrt{\frac{M_5^3}{\rho}}, \tag{11}$$

where M_5 is the 5D Planck mass. Now we substitute $R_c \sim 1/k$ and $\rho \equiv \rho_{IR}$ (cf. (7)), in (11)and solve for $\exp(ky)$. This gives us another bound on the achievable height y,

$$e^{ky} \sim \frac{M_5^{3/4} k^{1/4}}{M}, \tag{12}$$

supplementing (8).

One possible interpretation is that (8) remains our basic formula for the maximal height but, due to (12), we in addition need to demand

$$M_5^3 > \frac{M^5}{k^2}, \tag{13}$$

i.e., 5D gravity has to be sufficiently weak. As outlined earlier, we assume that our mirror has been lifted together with an attached 5D elevator, such that we are now in possession of a collider with "energy reach" $(M/k)^{1/4}$. In other words, we can use e.g. photons at energy M to produce particles with mass $M(M/k)^{1/4}$, with 100% energy efficiency (at least in principle). Obviously, we here do not include the one-time energy investment required for the construction of this "collider."

For example, the UV brane or "Planck brane" of [2] could be located at the height $y_{\max}$ given by (8), in which case we could "lift" energy to the Planck brane. Note that, due to the constraint (13), the 4D Planck mass ($M_4^2 \sim M_5^3/k$ using the UV-brane-induced metric) remains higher than M, such that we can never actually reach the 4D Planck scale using this type of "perfect collider."

It is obvious that our construction with a horizontal mirror and a vertical photon beam is far from optimal. It can be improved by making the floating mirror as small (in brane-parallel direction) as possible, curving it appropriately, and supporting it by a tapering photon beam arrangement. This clearly requires an appropriate mirror array at the IR brane. We do not pursue this analysis here but turn, in Sec. III, to the construction of an (also tapering) "real" tower made from solid material.

C. Including bulk fields beyond the metric

A natural objection is that, in 'proper' string-theoretic AdS/CFT [1], a D3-brane can, due to the Bogomol'nyi-Prasad-Sommerfield (BPS) condition, be stationary at any point in the radial direction of Ads_5. Thus, in models with such BPS objects, it appears to be easy to avoid the height restrictions found in the last subsection.

For simplicity, we implement the key ingredients directly in our 5D RS setting: Let us assume that the 5D action contains a 5-form field strength $F_5 = dC_4$ and 3-branes charged under C_4. Furthermore, let us supplement the gravitational background of (1) by an F_5 VEV proportional to the volume form. For an appropriately tuned value of this field strength (our RS-model analogue of the type IIB BPS condition), such a brane can rest, in parallel to the IR brane, at any value of y: the gravitional force is precisely compensated by the force of the field strength permeating the bulk.

Before continuing, we note that, from the perspective of "generic" strongly coupled 4D models with 5D gravity dual, the above are rather special requirements: If no such, infinitely extended, brane is present in a given 4D vacuum, it cannot be created by any means (unlike a mirror, which can be assembled from "atoms"). Second, if no 5-form VEV is present or it is not appropriately tuned, it is impossible for any potential experimentalist to create one. Equally, the charge of the 3-brane cannot be adjusted, as the charge of the electron cannot be adjusted in our 4D world.

In principle, one may consider theories in which 3-branes are allowed to have boundaries (this clearly requires further types of charges and gauge fields, etc., but let us assume this can be engineered). Such a finite brane could then be created, but it would not be BPS: it has a finite tension which will in general lead to the shrinking of its area. A detailed technical analysis of whether one could stabilize such a brane and what the energetic cost of "lifting" this stabilized configuration would be goes beyond the goals of this paper.

Let us instead continue with the arguably more "natural" case of an infinitely extended BPS 3-brane, assuming one was present. Indeed, one might imagine 'lifting' such a brane in the (necessarily finite) spatial region accessible to an experimentalist (cf. Fig. 3). In 4D language, this amounts to living in a vacuum with a modulus (the 3-brane position in y direction) and shifting this modulus within a finite spatial domain. It might in principle be interesting to follow this route and see whether one can attach some structure (presumably non-BPS) to this "partially lifted" brane. We do not want to pursue this in the present paper since we already have a clear understanding how this setting avoids the proposed "no-go theorem" in 4D language: one needs a modulus, which can then be displaced through some experimental effort. Clearly, this allows one to explore totally new physics. Whether it can lead to the Planck scale and to a reversible process for transforming macroscopic work to the mass of Planck scale particles remains open at the moment.

Inspired by the above, one may however consider other bulk fields with VEVs in analogy to the 5-form field strength naturally suggested by type IIB string theory. Insisting on 4D Poincare symmetry, the only options are a 4-form or, equivalently, the Hodge-dual 1-form field strength. The latter is just the gradient of a scalar, which anyway has to be present in a complete model because of Goldberger-Wise stabilization [10]. One is now allowed particles (clearly simpler than a 3-brane) coupling to this background scalar field. Thus, one may hope to compensate the gravitational force on those particles through this coupling.

Let us be slightly more specific by considering a 5D fermion ψ,

$$\mathcal{L}_5^{-i\bar{\psi}\gamma^M D_M\psi - \bar{\psi}\psi f(\phi)}, \tag{14}$$

where f is some function of the Goldberger-Wise scalar ϕ. In analogy to the discussion at the beginning of Sec. II. B, one can convince oneself that the 4D energy of such a ψ particle (at rest in x^μ) is

$$E_\psi(y) = m_5(y)e^{ky} = f(\phi(y))e^{ky}. \tag{15}$$

Assuming that the scalar background $\phi(y)$ is a monotonic function, we can without loss of generality [through a field redefinition $\phi \to \phi'(\phi))$] work with the specific background $\phi(y) = cy$. If we furthermore take $f(\phi) = \exp(-k\phi/c)$, we obviously obtain $E_\psi(y) = \text{const}$. As a result, ψ particles can be easily moved to the Planck brane, in analogy to the BPS 3-brane discussed earlier. Equally obviously, however, a machine doing this is *not* a perfect collider since the particles do not become extremely massive (in 4D language) when moved to the UV.

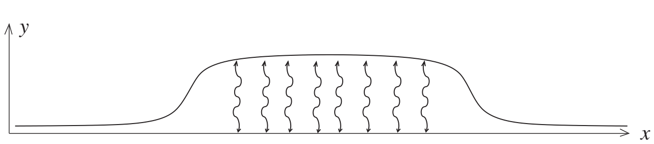

FIG. 3. Infinitely extended 3-brane lifted in a finite spatial region. Note that the "photon beam" in the figure is only meant to symbolize some mechanism for displacing the brane. It is unclear whether such a fundamental 3-brane can play the role of an actual mirror for some form of radiation.

Nevertheless, it is interesting to consider the 4D analogue of the above situation: In 4D, the ψ particles are composites characterized my some mass m_4 and size l_4. Given the right choice of f, they possess a flat a direction (the analogue of the y-position of the elementary 5D particle ψ) on which l_4 (but not m_4) depends. As y increases, l_4 becomes tiny, making the particle pointlike in 4D language. To ensure that this particle in addition has a Planck-scale mass, m_4 would need to be $\sim M_4$ from the start. In other words, we need a theory with very heavy, extended objects (e.g. some type of soliton) which can be 'manufactured' using macroscopic work. These objects would also need to possess a flat direction along which their size changes. Clearly, this is rather exotic (even more so than the rest of the paper) and we choose not to pursue this line of thinking for now.

III. MAXIMAL-HEIGHT 5D TOWERS

We return to the simplest possible setting without any nonmetric VEVs in 5D. An optimal tower will use the strongest 5D material available, i.e., that with the largest ratio p/ρ.[3] We will henceforth assume that this ratio is maximized for one particular substance, which we will use to build our tower. Most naively, one would try to adapt Weisskopf's famous argument [11] for the maximal height of mountains (expressed in terms of fundamental constants) to our situation. While his argument is energetic (sinking of the mountain vs. melting of the rock at the bottom of the mountain), we make the following essentially equivalent force-based estimate:

First, as a warm up, let $ky \ll 1$ such that $\exp(ky) \simeq 1 + ky$. A rectangular 5D mountain with (constant) cross section A and height y has mass $Ay\rho$ and exerts a force $Ay\rho k$ on its base. The base can provide a force Ap. Hence, for a given constant p/ρ the maximal height is

$$y_{\max} = \frac{1}{k} \cdot \frac{p}{\rho}. \tag{16}$$

While self-consistent with our linearization (since $p/\rho < 1$), this is clearly not interesting. The crucial energy reach of

[3]Presumably $p/\rho \ll 1$ holds even for the strongest available material, at least if this material is made from pointlike weakly interacting particles, as in our 4D world. Note, however, that our world is not weakly coupled throughout and that much stronger materials, such as the neutron star crust, appear to exist [12].

our "collider" is $\exp(ky_{\max})$, which can hence not become large in our toy model with constant cross section.

An optimal tower will taper towards its tip, such that each cross section is just large enough (assuming maximal vertical pressure at each point of the cross section) to support the part of the tower above. This clearly can be cast in the form of a differential equation for the cross section $A(y)$, and we will do so shortly. The solution then determines the shape of the tower and, as we will see, its maximal height.

Naively, one might expect to find a complete solution of this simple and fundamental problem in engineering textbooks or papers. However, in real-world towers, wind pressure is the most important issue and (unlike our case) the gravitational field can be treated either as linear ($\exp(ky) \to gy$) or, if one considers extremely high towers, according to the $1/r^2$ force-law. The closest related ideas and calculations in the literature appear to be related to either the Tsiolkovsky tower or the space elevator [13] suspended from a point in geostationary orbit (in the latter case, the tapering is towards the bottom for obvious reasons). In any case, we were not able to find a treatment of a situation exactly equivalent to ours.

Fortunately, the corresponding equations are simple even in our exotic case. Everything can be derived from an equation relating the vertical forces at heights y and $y + \delta y$,

$$F(y) = F(y + \delta y) \cdot (1 + k\delta y) + k\rho A(y)\delta y. \tag{17}$$

Except for the factor $(1 + k\delta y)$, this is self-evident: going down the tower by a distance δy, the force grows by the weight of an additional layer of material. The factor $(1 + k\delta y)$ comes from the warping. As explained earlier, raising a mass from y to $y + \delta y$ costs an energy $mk\delta y \ \exp(ky)$ from the perspective of $y = 0$. This means that this mass exerts a force $mk \ \exp(ky)$ at any support at $y = 0$, while it obviously only exerts a force mk at any support at its own height. In other words, vertical forces are subject to warping in the very same way as energies. Thus, the weight of all the tower material above $y + \delta y$ exerts a force on the surface at height y which is enhanced by a factor $\exp(k\delta y) \simeq (1 + k\delta y)$. This is the content of the first term on the rhs of (17).

With $F(y) = pA(y)$ and $p = \text{const}$ (an optimal tower will have maximal pressure at any layer), one then immediately derives a differential equation for A,

$$-A'(y) = A(y)k(1 + \rho/p), \tag{18}$$

where ρ is constant by assumption. The solution is

$$A(y) = A_0 e^{-(1+\rho/p)ky}. \tag{19}$$

Just to prevent any possible confusion: as should be clear from the derivation, this function $A(y)$ characterizes the y dependence of the cross section of our tower as a locally well-defined 5D physical quantity. For example, it could be the cross section in 5D Planck units. It is very different from the cross section as measured in the coordinates x^μ of (1).

In our analysis of the shape of the tower, we have neglected any horizontal force components. This is only justified as long as the tower is a "thin object," i.e., $A(y)$ does not change too rapidly with y. Quantitatively, this will certainly hold if the angle between the tower surface and the vertical axis is small. Most naively, one would estimate this angle as (minus) the derivative of the tower radius with respect to the height, $-[A^{1/3}(y)]'$. However, due to warping, this derivative is nonzero even for a vertical tower, i.e. for a tower the surface of which is made from lines at $\vec{x} = \text{const}$ In fact, the cross section of such a vertical tower is given by $A_v(y) = A_0 \exp(3ky)$. Thus, when estimating the angle at the base of the tower and requiring it to be parametrically small, we have to do so relative to vertical tower,

$$-\{[A^{1/3}(y)]' - [A_v^{1/3}(y)]'\} = \left\{\frac{(1 + \rho/p)k}{3} + k\right\} A_0 \ll 1. \tag{20}$$

This translates into an estimate of the maximal A_0 allowed,

$$A_0^{1/3} \sim \frac{3}{(4 + \rho/p)k}. \tag{21}$$

At its tip, our tower can certainly not become thinner than $1/M$. Thus, substituting $A_0^{1/3}$ from (21) and $A(y)^{1/3} \sim 1/M$ in (19), we eventually find that the maximal height $y_{\max}$ is determined by

$$e^{ky_{\max}} \sim \left(\frac{3M}{(4 + \rho/p)k}\right)^{\frac{3}{1+\rho/p}}. \tag{22}$$

Note that this is rather similar to our "floating mirror" result of (8). Since we did not keep track of $\mathcal{O}(1)$ factors, the prefactor $3/(4 + \rho/p)$ accompanying the ratio M/k is most probably irrelevant. The only difference is then in the exponent. For an isotropic 5D radiation gas, which is presumably close to the stiffest possible matter, we have $p = \rho/4$ and hence an exponent $3/5$. This is better than the $1/4$ of (8), although we have to remember that we did not try to optimize the shape of the beam in Sec. II. Thus, the competition between the two "perfect collider technologies" of Secs. II and II cannot be decided at this level of precision.

It is interesting to note that the approximate agreement arises in spite of the two configurations being distinctly different: the tower we are presently constructing becomes wider towards its base. By contrast, the region of the IR brane from which the photon beam of Sec. II is reflected is much smaller than the floating mirror.

Finally, we expect a bound on M_5 arising from the danger of black hole formation at the base of the tower. It is easy to obtain by requiring that the critical radius of (11) is smaller than the width of the tower at its base, given by (21). One finds

$$M_5^3 > \frac{9\rho}{(4+\rho/p)^2 k^2}, \tag{23}$$

which, for the natural value $\rho \sim M^5$, once again becomes extremely similar to the analogous bound of (13).

To sum up, we have now seen in a second, independent way that a perfect collider with energy reach $(M/k)^\alpha$ (with $\alpha \sim \mathcal{O}(1)$) can be built in principle. Since we are using a weak-coupling analysis on the gravity side, the corresponding 4D theory has to be strongly coupled. This can be seen most explicitly in "proper" AdS/CFT [1], i.e. in the duality between 4D $\mathcal{N}=4$ super Yang Mills theory with gauge group $SU(N)$ and type IIB string theory on $\mathrm{AdS}_5 \times S^5$. In this setting, one has

$$\lambda \sim g_{YM}^2 N \sim \left(\frac{M_s}{k}\right)^4, \tag{24}$$

where λ is the 't Hooft coupling, i.e. the actual control parameter of perturbation theory on the 4D side, and $M_s \sim 1/l_s$ is the string scale. We see that λ becomes large as $k \to 0$. This is believed to be a more general feature of AdS/CFT: small k corresponds to large 4D coupling. Small k also corresponds to a large energy reach $(M/k)^\alpha$ of our "perfect collider," which indeed scales very similarly to the coupling λ of (24). In other words, this type of perfect collider exists *precisely* because the 4D theory is strongly coupled. Clearly, this does not exclude the existence of perfect colliders of some totally different type in the weak coupling domain, but it is a hint to the contrary.

IV. CONCLUSIONS

We have presented some, admittedly rather speculative, ideas concerning the (im)pos-sibility of a perfect collider. Our main technical point was very simple: for theories having a 5D gravity dual, reaching for UV energy scales corresponds to building 5D towers based on the IR brane and pointing to the UV brane. In many theories, the height of such towers appears to be limited at a rather fundamental level (quite analogously to the limited height of mountains, given the limited strength of granite). We estimated this maximal height and conjectured (given the parametric behavior of our result) that at least in generic 4D weakly coupled theories, it is completely impossible to build a perfect machine (i.e. a machine with energetic efficiency near unity) which transforms energy, starting from the "structure scale" of our theory, towards the UV.

Clearly, in our holographic approach, accelerator physicists are "tower builders," struggling with the 5D gravitational potential.[4] The 4D weak-coupling analogue of their problem is apparent: the tendency of massive objects to fall translates into the tendency of energy to transfer from the UV to the IR in conventional QFT.

In specific models, perfect or near-perfect "colliders" might nevertheless exist. Two classes of potential examples are discussed in Sec. II C: these are theories with completely flat directions (moduli) in field space and theories with solitonic objects possessing such flat directions. The naive intuition about the overwhelming force of 5D gravity fails in such settings and it is conceivable that a perfect machine transforming energy from IR to UV can be built. Another class of examples can be constructed as follows: if scale invariance in the energy regime between the TeV and Planck scale is strongly broken by many intermediate branes [15],[5] reaching the Planck scale is at least much easier (e.g. by a cascade of smaller towers). The weak-coupling dual of this class of examples is obvious: assume that many stable charged particle species with masses spread throughout this energy range exist. One can then imagine a cascade of conventional storage-ring colliders, each filled with one type of particles and producing the next-heavier species, working their way up to the Planck scale.

Thus, we cannot expect a fundamental no-go theorem that is completely general. The assumptions of such a possible theorem have to include details of the relevant model (i.e. of the concrete 4D QFT). Obviously, counterexamples which are as close to the real world as possible would be much more exciting than the proof of a no-go theorem.

Many interesting questions are still open. For example, it has to be clarified whether a tower is really the only or at least the best way to transfer energy reversibly (with efficiency near unity) from the IR brane to an arbitrarily high position above it. Furthermore, it would be crucial to understand possible limits directly in weakly coupled 4D theories. Indeed, while there appear to be no fundamental obstructions[6] to building an efficient linear accelerator reaching some very high energy scale M_{UV}, its efficiency in transforming energy into mass of heavy particles might be limited *in principle*. This is suggested by the scaling of cross sections as $1/M_{UV}^2$ and possible fundamental limitations on the quality of beam focusing. We have to leave these questions to further research.

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[4]If an entropic understanding of gravity [14] could be established, one might thus hope that the efficiency limitations discussed in the present paper are related to the Carnot efficiency limit. Unfortunately, we are unable to make this more precise at present.

[5]To be stabilized by the Goldberger-Wise mechanism [10] at certain 5D positions y_i.

[6]Here we ignore 4D gravity, i.e. we work in the limit $M_{P,4} \to \infty$. Possible fundamental obstructions involving gravitational effects are discussed in [3].

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