Phenomenology of space-time imperfection. I. Nonlocal defects

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If space-time is emergent from a fundamentally nongeometric theory it will generically be left with defects. Such defects need not respect the locality that emerges with the background. Here, we develop a phenomenological model that parametrizes the effects of nonlocal defects on the propagation of particles. In this model, Lorentz invariance is preserved on the average. We derive constraints on the density of defects from various experiments.

I. INTRODUCTION

Reductionism has been tremendously successful. It is thus only consequential to question whether the basic ingredients of today's theories are truly fundamental and to devise tests. To make headway on a theory of quantum gravity, it has long been suggested the seemingly smooth space-time we experience might be an emergent feature of an underlying, nongeometric theory that unifies general relativity with quantum mechanics. While the full theory is still unknown, it is nevertheless possible to investigate the phenomenology of specific features it is expected to have. Many of these investigations in the area of quantum gravity phenomenology have been reviewed in [1–3].

We will here focus on the question which observational consequences can be expected from space-time imperfections of nongeometric origin. Since Lorentz-invariance violation is probably the most extensively studied area of Planck-scale physics [4,5], we will here turn our attention to the case where Lorentz invariance is maintained, at least on the average.

If space-time is not fundamental, but emergent from a nongeometric theory described by graphs, then we live today in a period in which the graph approximates to excellent precision a locally connected manifold of dimension four with Lorentzian signature and a metric tensor whose dynamics is determined by Einstein's field equations. This smooth background however will still have defects owing to its nongeometric origin. One expects some defects to prevail simply because in such a scenario perfection begs for additional explanation. This expectation that space-time is left with defects is generic and independent of the details of the underlying theory making it an ideal opportunity to test quantum gravity phenomenology. We will in this paper make as few assumptions as possible about the origin of the defects and work towards a parametrization of possible effects that is as model independent as possible.

Unlike defects one normally deals with (e.g., in crystals), the defects under consideration here occur on space-time points and do not have world lines. One can then distinguish two different types of defects: local ones and nonlocal ones. Local defects respect the emergent locality in the space-time manifold. A particle that encounters a local defect will scatter and change direction, but continue its world line on a continuous curve. Nonlocal defects on the other hand do not respect the emergent locality of the space-time manifold. A particle that encounters a nonlocal defect continues its path in space-time elsewhere, but with the same momentum.

In principle a space-time defect could cause both, a change of position and momentum. But before making things more complicated by combining these two effects, it seems prudent to first study the simpler cases. In this paper, we will develop a model for the nonlocal type of defects. An accompanying paper [6] deals with the local type of defects. A different model for local space-time defects has recently been put forward in [7] (see also the discussion in [6]).

The nonlocal defects that we are interested in here cause a particle to discontinuously jump between two points that have a long distance according to the metric of the approximate manifold.[1] This distance can potentially be macroscopically large. This is not unlike transversable wormholes, except that the connection between the points is not described by geometry itself.

It has been demonstrated explicitly by Markopoulou and Smolin in [8] that spin networks are prone to develop such defects. This finding, aptly dubbed "disordered locality" in [8], was further studied in [9] as to its consequences for the cosmological constant and in [10] for its modifications of the dispersion relation. The approach in these papers however breaks Lorentz invariance by introducing a preferred timelike slicing and is thus not of immediate relevance for the following considerations, though it has served as a motivation.

The picture we will investigate here is what modifications a homogeneous space-time distribution of nonlocal defects causes, depending on the density of the defects and

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[1]This can be given meaning without referral to the metric of the emergent space-time by instead considering the lengths of closed curves that the two points are part of according to some distance measure of the underlying structure.

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the probability of a particle to interact with the defect. We will see that massless particles are stochastically deviated from the light cone. This deviation can be such that the average speed remains the usual speed of light and only stochastic fluctuations around the mean occur. But there are also cases in which the photon's average speed can decrease or increase. These modifications become more relevant for long distances and small energies, in stark contrast to other scenarios where the modifications become relevant for large energies, as in the cases where Lorentz symmetry is broken or deformed. In the scenario with nonlocal defects, the photon acquires an effective mass that can, for the case when the photon's average speed exceeds the usual speed of light, potentially be a tachyonic mass. If we want to include the case in which deviations from the light cone can be spacelike, special attention has to be paid to causality, which we will discuss in Sec. VI.

The effects of nonlocal space-time defects that we consider here are not perturbative in the sense that they do not constitute a small deviation from flat space (or some other background metric). A flat space remains flat in the presence of nonlocal defects. The deviation that the nonlocal defects cause can best be understood as a topology change. Space-time defects give rise to deviations that are small in the sense that we assume the defects are sparse.

We use the unit convention $\hbar = c = 1$. The signature of the metric is $(+, -, -, -)$.

II. THE DISTRIBUTION OF DEFECTS

To develop our model for nonlocal defects, we will assume that the emergent background space-time is flat Minkowski space, i.e., we will here not take into account background curvature. Since violations of Lorentz invariance have been thoroughly studied, and been ruled out to high precision already, we will further only consider the case in which Lorentz invariance is maintained, at least on the average. We first discuss the classical point particles (Sec. III) and include quantum uncertainty later (Sec. IV).

The only presently known probability distribution for points in Minkowski space that preserves Lorentz invariance on the average is the result of a Poisson process developed in [11,12]. With this distribution, the probability of finding N points in a space-time volume V is

$$P_N(V) = \frac{(\beta V)^N \exp(-\beta V)}{N!}, \tag{1}$$

where β is a constant space-time density.

The average value of points that one will find in some volume is the expectation value of the above distribution and given by

$$\langle N(V) \rangle = \sum_{N=0}^{\infty} P_N(V) N = \beta V. \tag{2}$$

The variance that quantifies the typical fluctuations around the mean is $\Delta N \sim \sqrt{\beta V}$, and the corresponding fluctuations in the density of points are $\Delta(N/V) \sim \sqrt{\beta/V}$. In other words, the density fluctuations will be small for large volumes.

We will use the distribution (1) to seed the nonlocal defects with an average density β. In the following, we will not be concerned with fluctuations in the density as our aim here is to first get a general understanding for the size of effects caused by nonlocal defects and using the average will suffice for this purpose. The probability is a density over space-time $\beta = L^{d+1}$, where L is a length scale and d is the number of spatial dimensions. With this, we have introduced a small dimensionless parameter $\epsilon = l_P/L$, where l_P is the Planck length, that one expects to determine the size of effects.

The nonlocal defect causes a jump of the particle to another point. The two points have a nonvanishing distance according to the Minkowski metric. In addition to sprinkling the points, we then also have to identify the orientations at the begin and end of the jump, to prevent that direction from being twisted. We will assume that the tangential space at the entry point of the defect is mapped to the parallel-transported tangential space at the exit point. In flat Minkowski space this means simply that the direction of a particle is not changed and its momentum stays the same while it seems to jump from one point to another.

If the probability distribution of nonlocal defects is constant through space and time, then, to preserve Lorentz invariance, we further have to make sure that the distribution of points to which the particle jumps depends only on Lorentz-invariant quantities. To parametrize this distribution of end points we will now consider one single defect and without loss of generality assume it is located at $(0, 0)$.

The only four-vectors that we have and that can be used to construct a Lorentz-invariant distribution are the particle's momentum before encountering the defect, p^ν, the particle's momentum after encountering the defect, p'^ν, and the space-time distance between the defect and the point where the particle will continue its path, y_ν. By assumption, the momenta before and after encountering the defect are identical $p^\nu = p'^\nu$, so we have only two four-vectors left.

The first thing that comes to mind is to distribute the end points for the particle's jump according to the same Lorentz-invariant distribution as the defects. If space is infinitely extended, this means that the particle with probability one jumps infinitely far away and essentially disappears. It also means that it gets replaced with another particle at a rate depending on the probability of any particle to encounter such a defect elsewhere. This then effectively describes a local defect, and we will not further consider this case here; see however the discussion in [6].

To describe the case where the end point of the jump is determined by Lorentz-invariant quantities constructed from the particle's trajectory, we will for now focus on the case with $1 + 1$ dimension and generalize to higher dimension later. In $1 + 1$ dimensions, β has dimensions of

one over length squared. We first consider a massless, right moving particle with momentum $p_\nu p^\nu = 0$, i.e., $p^\nu = (E, E)$.

In $1 + 1$ dimensions, curves of constant distance from a point are hyperbolae in Minkowski space. The first Lorentz-invariant quantity at our disposal is thus the length of y_ν,

$$y_\nu y^\nu = \pm \alpha^2, \tag{3}$$

where α is a (positive, real) constant of dimension length. In $1 + 1$ dimensions, the hyperbolae in quadrant I (see Fig. 1) can be parametrized as

$$y_I^\mu = \left(\sqrt{x^2 + \alpha^2}, x\right). \tag{4}$$

Timelike vectors of length α^2 ending on the hyperbola in quadrant II can be parametrized as

$$y_{II}^\mu = \left(t, \sqrt{t^2 + \alpha^2}\right). \tag{5}$$

(In the following we write y to refer to both y_I and y_{II} together.)

The second Lorentz-invariant quantity that we can construct is the contraction of p^ν with $y_\nu(\alpha)$. We parametrize this contraction as

$$(y_I)_\mu p^\mu = \alpha \Lambda, \tag{6}$$

in quadrant I, and

$$(y_{II})_\mu p^\mu = -\alpha \Lambda \tag{7}$$

in quadrant II. Here, Λ is a (real, positive) parameter of dimension mass. For any given pair of Λ and α, this singles out a point on the hyperbola, which is where the particle that hit the defect at $(0, 0)$ will continue its path. Any normalizable function of α and Λ will be Lorentz invariant and is thus suitable to parametrize the probability distribution for end points of the particle's jump. The vectors y_I and y_{II} are thus functions of α, Λ, and p.
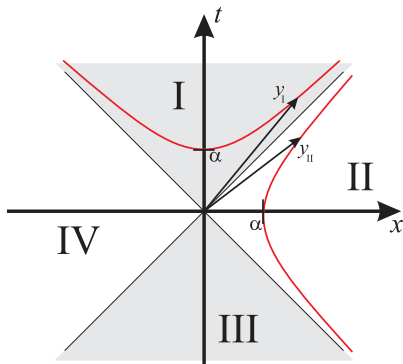

FIG. 1 (color online).   Location of y_I and y_{II} in Minkowski space.

When the particle is deviated from the light cone to the hyperbola in quadrant I it will be slowed down. We will thus refer to this case as subluminal. Correspondingly, the case where the jump goes to quadrant II we will refer to as superluminal.

In the superluminal case we can eliminate the explicit dependence of y_{II} on t and express it through the particle's energy which gives

$$y_{II}(\alpha, \Lambda, p) = \frac{\alpha}{2}\left(\frac{E/\Lambda - \Lambda/E}{E/\Lambda + \Lambda/E}\right). \tag{8}$$

For the subluminal case, one has similarly

$$y_I(\alpha, \Lambda, p) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} y_{II}(\alpha, \Lambda, p). \tag{9}$$

The probability distribution, $P_{NL}(\alpha, \Lambda)$, for the end points of the particles' jumps has to be normalizable to unity, $\int P_{NL}(\alpha, \Lambda) d\alpha d\Lambda = 1$. We will denote average values with respect to this distribution with brackets that carry the index "NL," for example

$$\langle y_I(p) \rangle_{NL} = \int y_I(\alpha, \Lambda, p) P_{NL}(\alpha, \Lambda) d\alpha d\Lambda. \tag{10}$$

We will in the following only roughly quantify the probability distribution by the average values $\langle \alpha \rangle_{NL}$ and $\langle \Lambda \rangle_{NL}$ as well as the corresponding variances $\Delta \alpha$ and $\Delta \Lambda$. The former determine the average distance the particle will jump when it hits the defect, the latter the width of the distribution around that average value.

In summary, for the particle with momentum p_ν, the requirement Eq. (7) singles out one point $y_{II}(\alpha, \Lambda, p)$ on the hyperbola to α, this is the end point of the jump starting in $(0, 0)$. The probability that the particle makes a jump to this point is then $P_{NL}(\alpha, \Lambda)$. This construction is entirely Lorentz invariant. The distribution $P_{NL}(\alpha, \Lambda)$ parametrizes the effects of the nonlocal defects in causing a sudden translation. One may say that the defect resembles a beamer rather than a wormhole.

III. WORLD LINES IN THE PRESENCE OF NONLOCAL DEFECTS

We have now parametrized the distribution of nonlocal defects in the density, β, of the Poisson sprinkling, and the typical deviation from the light cone that the defect causes, quantified in the average values $\langle \alpha \rangle_{NL}$ and $\langle \Lambda \rangle_{NL}$ together with the widths $\Delta \alpha$ and $\Delta \Lambda$. These are the free parameters of the model.

From Eq. (8) we see that the particle is deviated as follows:

(i) In the limit $E \gg \Lambda$, y becomes parallel to $(1, 1)$, i.e., the right forward light cone. In this limit, one jump of the particle skips a distance of $L_{1j} \approx \frac{E\alpha}{\Lambda}$.

(ii) In the limit $E \ll \Lambda$, y_{II} becomes parallel to $(1, -1)$, i.e., the right backward light cone in the superluminal

case, and y_I becomes parallel to $(-1, 1)$, i.e., the left forward light cone in the subluminal case.

(iii) In the case $E \approx \Lambda$, y_{II} becomes parallel to $(0, 1)$ in the superluminal case, and y_I becomes parallel to $(1, 0)$ in the subluminal case.

Since we see that the direction of the jump gets closer to the light cone for large energies and the deviation becomes noticeable for energies close to Λ and below, we conclude that Λ is a modification in the infrared, rather than in the ultraviolet. We thus expect Λ to be a small energy scale. Roughly speaking, Λ is the energy at which a massless particle jumps a distance of α, or $\langle \alpha \rangle_{NL}$ on the average.

Note that the particle's momentum, whenever one measures it, remains (E, E). However, the tangential vector of the *average* trajectory is no longer parallel to the momentum vector. If we parametrize the particle's trajectory x with τ, such that $x(0) = 0$, then the trajectory would now be $x = \tau(1, 1) + \Theta(\tau)y(\alpha, \Lambda, p)$.

This particle can be assigned a pseudomomentum, $\tilde{p}$, and pseudomass, $\tilde{m}$, which is the four-momentum and mass of a particle that moved straight from the defect to the end point of the jump in the background manifold. In the superluminal case this pseudomomentum and pseudomass will be that of a tachyon. Requiring that $\tilde{p}$ be proportional to $y(\alpha, \Lambda, p)$ but not be a function of α itself, and $\tilde{p} \rightarrow p$ for $\Lambda \rightarrow 0$, one finds

$$\tilde{p} = 2\frac{\Lambda}{\alpha}y(\alpha, \Lambda, p), \qquad \tilde{m} = i\Lambda, \qquad (11)$$

in the superluminal case. The speed the particle would have if it would constantly jump with parameters α and Λ is, from Eq. (8),

$$\tilde{c}_{\text{sup}} = \frac{E/\Lambda + \Lambda/E}{E/\Lambda - \Lambda/E}. \qquad (12)$$

For $\Lambda/E \ll 1$, the speed of light with the first correction term is $\tilde{c}_{\text{sup}} \approx 1 + 2(\Lambda/E)^2$. For the subluminal case, one finds the same way $\tilde{m} = \Lambda$, $\tilde{c}_{\text{sub}} = 1/\tilde{c}_{\text{sup}}$, and for large energies it is $\tilde{c}_{\text{sub}} \approx 1 - 2(\Lambda/E)^2$.

Now that we have described one jump caused by a nonlocal defect, let us join several. Consider the photon makes a sequence of N_I timelike translations with α_i, $i \in 1, \ldots, N_I$, and N_{II} spacelike translations with α_j, $j \in 1, \ldots, N_{II}$. Measured in the background manifold the trajectory of the photon, instead of having zero proper length, will appear to have length $\sum_i \alpha_i + \sum_j \alpha_j$. The corresponding pseudomomentum, $\langle \tilde{p} \rangle$, assigned to the photon encountering the sequence of defects is the average over the joined jumps. Thus,

$$\langle \tilde{p} \rangle = 2\frac{\Lambda}{N}\left(\sum_i^{N_I}\frac{1}{\alpha_i}y_I(\alpha_i, p, \Lambda) + \sum_j^{N_{II}}\frac{1}{\alpha_j}y_{II}(\alpha_j, p, \Lambda)\right), \quad (13)$$

$$\langle \tilde{m} \rangle = \sqrt{\langle \tilde{p}^2 \rangle}, \qquad (14)$$

where $N = N_I + N_{II}$,[2] and the sums are taken only over those end points in the respective quadrant. The average speed the photon would appear to have moved with is

$$\langle \tilde{c} \rangle = \frac{\langle \tilde{p}_x \rangle}{\langle \tilde{p}_t \rangle}. \qquad (15)$$

The average speed can be equal to one if there are lightlike and spacelike translations, and will be strictly larger (smaller) than one if there are only spacelike (timelike) translations.

The pseudomomentum is proportional to a sum of four-vectors and thus Lorentz covariant. Thus, the speed assigned to the particle transforms under the usual special relativistic laws. We note that since the pseudomomentum does not depend on α, the average pseudomomentum does also not depend on the probability distribution for α. Inserting Eqs. (8) and (9), one finds

$$\langle \tilde{p} \rangle_{NL} = E\left(1 + \frac{\langle \Lambda^2 \rangle_{NL}}{E^2}\frac{N_{II} - N_I}{N_I + N_{II}}, 1 + \frac{\langle \Lambda^2 \rangle_{NL}}{E^2}\frac{N_I - N_{II}}{N_I + N_{II}}\right). \qquad (16)$$

As one expects, if we have a distribution of nonlocal defects that is the same for timelike and spacelike translations ($N_I = N_{II}$) then one will initially have some fluctuations, but for a large number of jumps $N_{II} \approx N_I$ and so $\langle \tilde{p} \rangle_{NL} \rightarrow p$. Note that the number of jumps between two nodes on the curve is an invariant.

In the case $N_I \neq N_{II}$, for example, when jumps happen exclusively into one quadrant, the average of the path deviates from the light cone. If $N_I = N_{II} \equiv N$, the average of the path remains the light cone, but the variance is still nonvanishing. In other words, even if the average remains on the light cone, a representative path will deviate from it. Since the particle's jumps constitute a random walk in one dimension, we can estimate that the average deviation in this case scales like

$$\Delta y \sim \frac{\sqrt{N}}{E}\Delta(\alpha\Lambda) \qquad (17)$$

in the direction perpendicular to the direction of propagation, i.e., $y_0 - y_x$. Assuming that the variables α and Λ are uncorrelated, we could further rewrite

$$\Delta(\alpha\Lambda) = \sqrt{\langle \alpha \rangle_{NL}^2(\Delta\Lambda)^2 + \langle \Lambda \rangle_{NL}^2(\Delta\alpha)^2}. \qquad (18)$$

However, we are not interested in a direct sequence of translations, but in a sequence of translations interrupted through propagation on the light cone in the background manifold that respects locality. To model this local process we have to understand the background as discretized, even though the details will not matter in the following. The reason is that a locally finite distribution of points,[3] as we

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[2]This is not the same N as in (1).
[3]This means finitely many points in all causal diamonds.

are using here for the nonlocal defects, has measure zero in a continuous background, and the probability of any particle's world line to hit such a defect in a continuous background would be zero.

We will thus consider the local propagation as a sequence of jumps, just that these are shorter (in the sense that it takes many of them to achieve the same as one nonlocal translation). For illustration, see Fig. 2. We will here not attempt to develop an equation of motion for this local process of normal propagation; this is territory of the underlying theory of quantum gravity. Instead, we just use our existing knowledge and assume that the massless particle to good precision moves on the light cone, and whatever its equation of motion looks like, it does not deviate too far from it. It is possible that the fundamental discreteness has other observable effects besides those caused by defects, but the aim of this paper is to focus on the nonlocal defects, exactly because evidence for a perfect fundamentally discrete structure is hard to come by.

However, a concrete example for how such a propagation might look like has been developed in the causal sets approach [13–15] in which the dynamical process is described by "chains" (sequences of points) that are generated iteratively [16]. In this case the momentum undergoes random fluctuations, called "swerves." These swerves are however unobservably small if the fundamental length scale of discretization is the Planck length [17,18].

One may have this concrete example in mind, but the details do not matter in the following. The only thing we need is that the probability to encounter a defect is not zero as it would be in a continuous background which contains an uncountably infinite amount of points.

We then describe the motion of the photon as a sequence of n short, local jumps with α_k, $k \in 1, \ldots, n_\mathrm{I}$ for quadrant I, α_l, $l \in 1, \ldots, n_\mathrm{II}$ for quadrant II, and $n = n_\mathrm{I} + n_\mathrm{II}$. The probability distribution is $P_\mathrm{L}(\alpha, \lambda)$ and the same in both quadrants. Here, λ plays the same role for the local jumps

that Λ plays for the nonlocal jumps. The parameters are the average values taken with the distribution $P_\mathrm{L}(\alpha, \Lambda)$ and its variances. We will assume henceforth that this short-distance structure is at the Planck length $\langle \alpha \rangle_\mathrm{L} \sim 1/\langle \lambda \rangle_\mathrm{L} \sim l_\mathrm{P}$.

In the large n limit with the symmetric distribution of α, one then consistently has $\langle \tilde{p} \rangle = p$. Now combining both local and nonlocal translations, one gets for the pseudo-momentum

$$\langle \tilde{p} \rangle = \frac{2}{N+n} \left(\sum_i^{N_\mathrm{I}} \frac{\Lambda}{\alpha_i} y_\mathrm{I}(\alpha_i, p, \Lambda) + \sum_j^{N_\mathrm{II}} \frac{\Lambda}{\alpha_j} y_\mathrm{II}(\alpha_j, p, \Lambda) \right.$$
$$\left. + \sum_i^{n_\mathrm{I}} \frac{\lambda}{\alpha_i} y_\mathrm{I}(\alpha_i, p, \lambda) + \sum_j^{n_\mathrm{II}} \frac{\lambda}{\alpha_j} y_\mathrm{II}(\alpha_j, p, \lambda) \right)$$
$$= E \left(1 + \frac{\Lambda^2}{E^2} \frac{N_\mathrm{II} - N_\mathrm{I}}{N+n} + \frac{\lambda^2}{E^2} \frac{n_\mathrm{II} - n_\mathrm{I}}{N+n}, \right.$$
$$\left. 1 + \frac{\Lambda^2}{E^2} \frac{N_\mathrm{I} - N_\mathrm{II}}{N+n} + \frac{\lambda^2}{E^2} \frac{n_\mathrm{I} - n_\mathrm{II}}{N+n} \right). \tag{19}$$

In the case $n_I = n_{II} \equiv n$, the average deviation from the lightlike path remains the same as (17) because the short jumps do not contribute by assumption.

Let us then turn our attention to the superluminal case where $P_\mathrm{NL} \equiv 0$ in quadrant I, and take the average of the distribution of nonlocal and local jumps. We take the limit of an infinitely long curve with $N, n \to \infty$, $n_\mathrm{I} - n_\mathrm{II} \to 0$, while the ratio $N/n := \mathcal{J} \ll 1$ is held fixed. One then gets

$$\langle \tilde{p} \rangle_\mathrm{NL} = E \left(1 + \mathcal{J} \frac{\langle \Lambda^2 \rangle_\mathrm{NL}}{E^2}, 1 - \mathcal{J} \frac{\langle \Lambda^2 \rangle_\mathrm{NL}}{E^2} \right). \tag{20}$$

In other words, the particle resembles one with effective mass $\langle \tilde{m} \rangle = i\sqrt{\langle \Lambda^2 \rangle_\mathrm{NL} \mathcal{J}}$. In the subluminal case, one gets similarly $\langle \tilde{m} \rangle = \sqrt{\langle \Lambda^2 \rangle_\mathrm{NL} \mathcal{J}}$.

Note however that for the case in which the photon travels only a short distance, it might be $N = 0$, and as
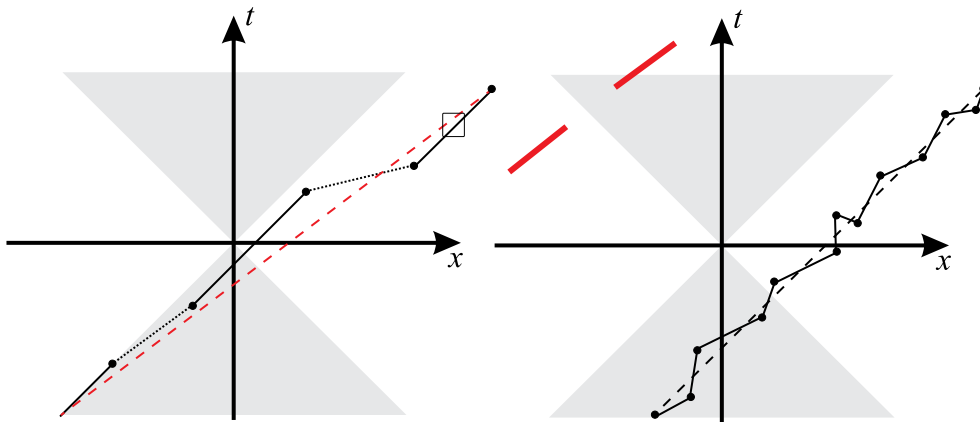

FIG. 2 (color online). Averaging of the particle's trajectory. Left: The solid lines are on the light cone, the dotted lines indicate the translations caused by the nonlocal defects. The long-dashed (red) line is the effective speed of the particle. Right: Zoom in on the box in the left picture. Looking close, it turns out that the particle's trajectory is a series of short translations (solid) that average to a lightlike curve (dashed).

long as N is small the effective mass will make discontinuous jumps. The question is then what is "short distance?"

IV. PARTICLES WITH SPATIAL WIDTH

To quantify short distance, we leave the image of the point particle and instead consider a massless particle that covers some volume in space-time, a natural image when taking into account quantum uncertainty (see Fig. 3, left). This particle has a typical width Δx and travels for a time T. The width and time depend on the reference frame, but the space-time volume covered $V = T\Delta x$ does not. Instead of using the spatial width of the world line and its temporal extension, we could consider a causal diamond (see Fig. 3, right). The former is more intuitive to think of, the latter is more convenient to analyze Lorentz invariance.

The average number of defects the quantum particle encounters on its travel is $\beta T \Delta x$. Now we need an assumption for the interaction probability, P_{int}, that cannot follow from our previous, entirely kinematic, considerations. We will assume that the strength of the interaction is determined (as in the local case, see [6]) by the typical volume of the defect over the typical volume in which to find the defect. In two dimensions, this means just $P_{\text{int}} = \epsilon^2$.

Then in two dimensions the average number of defects the particle interacts with scales as

$$\bar{N}_2 = P_{\text{int}} \beta T \Delta x = \frac{l_P^2 T \Delta x}{L^4}. \qquad (21)$$

This means that the average time it takes until the particle encounters a defect is

$$\langle T_{1d} \rangle \sim \frac{L^4}{l_P^2 \Delta x}. \qquad (22)$$

The limit $T \ll \langle T_{1d} \rangle$ is then, for a massless particle, the "short" distance we are looking for, in which the particle is unlikely to have encountered any defect.

A word of caution: Δx is here the length of the baseline of the parallelogram in Fig. 3, left. It is only identical to the width of the particle if the baseline is at constant time, which is not a Lorentz-invariant statement. This means when interpreting Δx in terms of the width of the wave function, we best do so in a rest frame where the baseline of the parallelogram is parallel to the x axis, or otherwise first have to derive the relation between both. We will later, in Sec. V, identify the width from the experimental setup.

If $\langle T_{1d} \rangle \gg T$ the particle is unlikely to have made a jump. If $T \gg \langle T_{1d} \rangle$, on the other hand, the particle is likely to have encountered many nonlocal defects already. If that is so, then the time T that enters the volume that the particle sweeps out in space-time is no longer the same as the apparent time $\tilde{T}$ that one would measure upon arrival, because the volume does not increase while the particle jumps from one point to the other.

The relation between the apparent time $\tilde{T}$ and T is

$$\tilde{T} = T\left(\frac{\langle T_{1d} \rangle + \langle T_{1j} \rangle}{\langle T_{1d} \rangle}\right), \qquad (23)$$

where $\langle T_{1j} \rangle_{\text{NL}}$ is the average time the particle jumps at each defect. Note that $\tilde{T}$ could be zero or negative in the superluminal case. From Eq. (8) one finds

$$\langle T_{1j} \rangle_{\text{II}} = \frac{1}{2}\left(E\langle \alpha/\Lambda \rangle_{\text{NL}} - \frac{\langle \alpha\Lambda \rangle_{\text{NL}}}{E}\right) \qquad (24)$$

for the superluminal case and

$$\langle T_{1j} \rangle_{\text{I}} = \frac{1}{2}\left(E\langle \alpha/\Lambda \rangle_{\text{NL}} + \frac{\langle \alpha\Lambda \rangle_{\text{NL}}}{E}\right) \qquad (25)$$

for the subluminal case. The corresponding distances that the particles jump are $\langle L_{1j} \rangle_{\text{II}} = \langle T_{1j} \rangle_{\text{I}}$ for the superluminal case and $\langle L_{1j} \rangle_{\text{I}} = \langle T_{1j} \rangle_{\text{II}}$ for the subluminal case. Since the particle is massless $\langle L_{1d} \rangle = \langle T_{1d} \rangle$.

This means if the measured time $\tilde{T}$ is large enough so that $T \gg \langle T_{1d} \rangle$, then the particle jumps over a fraction
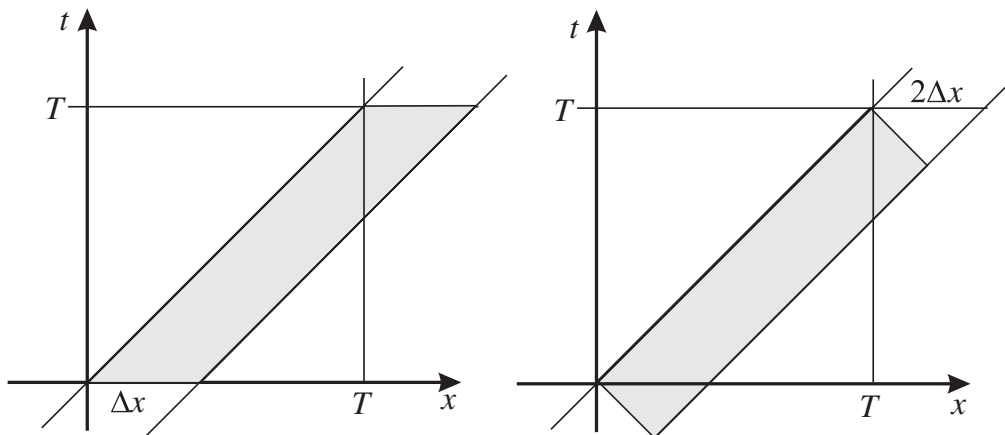

FIG. 3. To describe quantum particles, we consider world lines of finite width. While the width and the propagation time depend on the rest frame, the space-time volume that is swept out by the quantum particle is Lorentz invariant.

$\langle T_{1j}\rangle/\langle T_{1d}\rangle$ of this propagation time $\tilde{T}$ and the corresponding distance. This can reduce the probability of it interacting with other particles. We will examine the observational consequences of this in Sec. V.

For a plane wave, the stochastic deviations of the world line (17) become fluctuations in the phase of order $\Delta\phi \sim E\Delta y$. This can be calculated from (8), or (9) respectively, and comes out to be just $\Delta\phi \sim \sqrt{\bar{N}}\Delta(\alpha\Lambda)$. This means that the wave acquires a random phase shift of size $\Delta(\alpha\Lambda)$ at each defect, as one could have expected already from (6) and (7), and the phase thus acquires the blurring from the distribution $P_{NL}(\alpha,\Lambda)$. It is only the part of the jump perpendicular to the direction of propagation that blurs the phase. With the understanding that Δx is identified as explained above, we can then express the random phase shift for a plane wave as

$$\Delta\phi = \epsilon\sqrt{\frac{T\Delta x}{L^2}}\Delta(\alpha\Lambda), \tag{26}$$

where the variance can further be rewritten by Eq. (18) so that the phase shift is expressed in the five parameters we had chosen initially.

We can also use the above considerations to take the limit to classical particles and to connect to the discussion in the previous section. The average speed of light is, in the superluminal case, given by $\tilde{c} = (\langle T_{1d}\rangle + \langle L_{1j}\rangle_{II})/(\langle T_{1d}\rangle + \langle T_{1j}\rangle_{II})$. In the limit where the deviation is small, $\langle L_{1j}\rangle_{II}/\langle T_{1d}\rangle \ll 1$, one finds $\tilde{c} \approx 1 + \langle\alpha\Lambda\rangle_{NL}/(E\langle T_{1d}\rangle)$. With $\Delta x \sim 1/E$, we can then identify

$$\mathcal{J} = \epsilon^2 \frac{\langle\alpha\rangle_{NL}}{L} \frac{1}{\langle\Lambda\rangle_{NL}L} \frac{\langle\Lambda\rangle_{NL}^2}{\langle\Lambda^2\rangle_{NL}}. \tag{27}$$

The first factor in this expression is the density of the defects over the density of the fundamental structure. The second factor is the ratio of the typical distance the particle jumps to the typical distance of defects. The larger this factor, the more the jumps matter, though we would expect it to be of order 1. The third factor is the ratio of $\hbar$ (set to one) over ΛL. It takes into account that when the particle jumps it misses a certain amount of steps of the fundamental structure, which is why the ratio N/n is not just l_P/L. The last factor tells us that the point particle is more strongly affected than a particle with spatial spread if the distribution is wide, which enters through the difference between the square of the average and the average of the square. Note that this identification of $\mathcal{J}$ however only can be made this way in the limit of small jumps. One could turn these steps around and start with generalizing $\mathcal{J}$ for quantum particles, and then obtain P_{int} from it.

Of course describing a quantum particle means more than giving a particle a finite width and a quantum field theoretical description of the nonlocal defects would be desirable. However, in this present work we aim at first estimating the effects of nonlocal spacetime defects to see

if they are of interest and if they justify the development of a more sophisticated model.

A. Massless particles in $3 + 1$ dimensions

For massless particles, the extension of the previously constructed model to $3 + 1$ dimensions, while maintaining Lorentz invariance, necessitates that we use additional information about the particle that encounters the defect. Naively, what we want is that on the average the spatial direction of propagation remains unchanged, i.e., deviations from the direction are uniformly distributed and do not single out a preferred direction.

However, to make sense of this requirement (i.e., to make sense of the meaning of direction of travel for a massless particle), we need to take into account the finite spatial width of the photon's wave function in two directions perpendicular to the (mean) momentum. For simplicity we will take the width to be the same in both directions and denote them with $\Delta x_\perp := \Delta x_1 = \Delta x_2$. We then assume that these spatial widths remains unchanged when the particle jumps, i.e., the mean value of y in the perpendicular direction $y_\perp(\alpha)$ is zero, $\langle y_\perp\rangle = 0$ and $\Delta y_\perp = \Delta x_\perp$.

The need to use additional information about the incident massless particle goes back to the structure of the Lorentz group. This is explained in more detail in Sec. 3.3 of [6], but can also be seen as follows.

The reason we cannot introduce jumps into directions transverse to the direction of propagation without taking into account the width of the photon's wave function is that we would have only two quantities y^ν and p_ν at our disposal to construct the end point of the jump. If the total number of dimensions is higher than two, fixing the length of y^ν and the product of y^ν and p_ν will not single out a point, but a hypersurface. This hypersurface will include arbitrarily large distances. Lorentz invariance would then require a homogeneous probability distribution on an infinite interval, which is no longer normalizable. We essentially assume that this non-normalizable distribution arises because a particle with sharply defined momentum is indeed spatially infinitely extended. Thus, taking into account that in reality the wave function has a finite extension takes care of the infinite volume.

In $3 + 1$ dimensions we do moreover expect the scaling of the densities to have different powers than in $1 + 1$ dimensions. The volume that is swept out by a particle is now $T\Delta x_1\Delta x_\perp^2$, and the average number of defects the particle encounters is $T\Delta x_1\Delta x_\perp^2/L^4$. P_{int} now scales with the fourth power of epsilon. This means that in $3 + 1$ dimensions the average number of defects that a photon moving in direction x_1 interacts with is

$$\bar{N} \sim \epsilon^4 \frac{T\Delta x_1\Delta x_\perp^2}{L^4}, \tag{28}$$

i.e., the same as (21) except for a prefactor of ϵ^2 and a volume factor stemming from the two additional perpendicular dimensions.

Consequently in $3 + 1$ dimensions the average time the particle travels until encountering a defect [compare to (22) in $1 + 1$ dimensions] is

$$\langle T_{1d} \rangle \sim \frac{L^8}{l_P^4 \Delta x_1 \Delta x_\perp^2}, \qquad (29)$$

and the acquired phase shift is [compare to (26) in $1 + 1$ dimensions]

$$\Delta \phi \sim \frac{\epsilon^2}{L^2} \sqrt{T \Delta x_1 \Delta x_\perp^2} \Delta(\alpha \Lambda). \qquad (30)$$

Note that the distribution in the transverse direction does not contribute to the phase uncertainty because all points on the hypersurface cut out by $y_\nu p^\nu$ for a given value of α have the same phase.

Retracing our earlier steps, we can identify $\mathcal{J}$ in $3 + 1$ dimensions as (compare to (27))

$$\mathcal{J} = \epsilon^4 \frac{\langle \alpha \rangle_{NL}}{L} \frac{1}{\Lambda L} \frac{\langle \Lambda \rangle_{NL}^2}{\langle \Lambda^2 \rangle_{NL}} \frac{\Delta x_\perp^2}{L^2}, \qquad (31)$$

i.e., we gained a factor ϵ per each additional spatial dimension and a factor that compares the perpendicular extension to the spacing of defects; the larger the perpendicular spread, the more defects can be hit. This expression is not Lorentz invariant because taking the limit $E \gg \Lambda$ is not.

B. Massive particles in $3 + 1$ dimensions

The notion of a direction transverse to that of propagation is meaningless for massive particles, but for massive particles one can instead use reference to the particle's rest frame. The requirement that the particle jumps spatially equally far in each spatial direction leads for a massive particle to a homogeneous distribution on a compact hypersurface. The requirement "equally far" however is not Lorentz invariant and valid only in one reference frame. For the massive particle, it is natural to choose this reference frame to be the one in which the particle is in rest. Since the distribution of end points $P_{NL}(\alpha, \Lambda)$ depends on the properties of the ingoing particle, it could be different for massive particles.

There is however some ambiguity in how to generalize the model to massive particles because massive particles endow us with an additional scale, the mass of the particle, m. We could either use this scale instead of Λ, or use it in addition to Λ. This leads to two different possibilities for massive particles that we will discuss below, referred to as massive case 1 and massive case 2, respectively.

In massive case 1 the parametrization (6) for the subluminal jumps reads

$$(y_I)_\mu(\alpha, \Lambda, p) p^\mu = \alpha m, \qquad (32)$$

with (3) unchanged. Going into the rest frame of the particle, one notes that $y_0 = \alpha$. There is only one such

point on the hyperbola with α, that with $y_1 = y_2 = y_3 = 0$. Thus, the particle continues its path in the same direction while being absent for part of its travel, and there is no spatial deviation. If we apply a boost with γ-factor E/m in the y_1 direction, the world line of the massive particle will tend towards $\alpha E/m(1, 1, 0, 0)$ and fit with the massless case when identifying m with the parameter Λ.

For the superluminal jumps

$$(y_{II})_\mu(\alpha, \Lambda, p) p^\mu = -\alpha m \qquad (33)$$

leads, in the rest frame of the particle, to $y_0 = -\alpha$ and $|\vec{y}| = \sqrt{2}\alpha$. This too defines a compact two-dimensional submanifold, and assigning a uniform probability in the rest frame is possible. This case will however not tend towards the massless case under large boosts, unless one alters the normalization of y^2 to absorb the $\sqrt{2}$.

In massive case 2, we can require that in the limit of large boosts, when the massive particle becomes ultrarelativistic, we find in the direction of the boost the same average jumps as for the massless particle. This fixes the requirement for subluminal case 2 to

$$(y_I)_\mu(\alpha, \Lambda, p) p^\mu = m^2 \frac{\alpha}{\Lambda}. \qquad (34)$$

In the particle's rest frame, this means $y_0 = m\alpha/\Lambda$ and $|\vec{y}| = \alpha \sqrt{(m/\Lambda)^2 - 1}$. Again, this is a compact space on which we can normalize a probability distribution. We note that the particle can only be translated by the defect if its mass $m \geq \Lambda$. The average of the distribution in the rest frame is at $(y_0, 0, 0, 0)$. A boost with γ-factor E/m in the y_1 direction then moves the average to $\alpha E/\Lambda(1, 1, 0, 0)$, i.e., the same as for the massless particle that moves in the y_1 direction, provided $m \gg \Lambda$. This determines the factors in (34), though there could be terms in higher order of Λ/m.

In superluminal case 2 we have similarly

$$(y_{II})_\mu(\alpha, \Lambda, p) p^\mu = -m^2 \frac{\alpha}{\Lambda}. \qquad (35)$$

In the particle's rest frame, this means $y_0 = -m\alpha/\Lambda$ and $|\vec{y}| = \alpha \sqrt{(m/\Lambda)^2 + 1}$. The average is again at $(y_0, 0, 0, 0)$ and a boost in the y_1 direction will on the average reproduce the massless case for the particle moving in this direction. As in the case of massless particles, we will assume that the transverse width of the particle is preserved as it jumps, which means $\Delta y_1 = \Delta y_2 = \Delta y_3 = \Delta x_\perp$, where $\Delta x_\perp$ is the spatial width of the massive particle in its rest frame as it encounters the defect (for simplicity assumed to be the same in all three directions). In other words, we assume that the nonlocal defects do not cause additional spatial dispersion.

We will in the following only consider massive case 2, where the average longitudinal jump fits well with the massless case in the ultrarelativistic limit.

V. OBSERVATIONAL CONSEQUENCES
AND CONSTRAINTS

In this section we will look at constraints on nonlocal defects from existing experimental data. To that end, we will take the so-far anonymous massless particle to be a photon.

There is no direct way to apply to this model existing constraints on the photon mass because these constraints depend on the way the mass is generated. In the model discussed here the photon does not actually acquire a mass. It is just that, if the average trajectory of the photon deviates from the light cone because of nonlocal defects, then we can assign an effective mass to the photon, which is the mass that would have caused the same deviation as the scattering on the defects. So we will have to look for other constraints.

At this point we will reduce the number of parameters in this model by assuming that it contains only one new typical length scale, $\langle \alpha \rangle_{\mathrm{NL}} \sim \Delta \alpha \sim L$ in both quadrants. We will also assume that there is only one new mass scale and thus $\langle \Lambda \rangle_{\mathrm{NL}} \sim \Delta \Lambda$. We are then left with two parameters, $\langle \Lambda \rangle_{\mathrm{NL}}$ and L. For the rest of this section we will drop the brackets and denote the average value with just Λ. In the following, we further only consider the pure superluminal and subluminal cases. The purpose of this section is to gauge the promise of various observables to roughly constrain the abundance of nonlocal defects and factors of order 1 will be omitted. Since we have considered only the case of a flat background state, the estimates in the following will be good only so long as the gravitational effects can be neglected. This is a suitable approximation for Earth based laboratories and for intergalactic propagation as long as redshift is negligible.

A. Constraints from single photons in a cavity

An experiment that delivers good constraints on this model is the tracking of single photons in an ultrahigh-finesse optical cavity [19]. In this truly amazing experiment, a single photon with a frequency of $\nu \sim 1/\mathrm{mm}$ (in the deep infrared) is kept bouncing between mirrors that are approximately 3 cm apart. The photon can be kept a typical time of 0.5 seconds, which means it travels in total more than 100 000 km back and forth between the mirrors.

From this we can derive constraints by noting that a photon of this energy jumps a distance given by (25) in the superluminal case and (24) in the subluminal case, and if it jumps then that distance better be less than 3 cm. For $\Delta x_1 \sim 1/\nu$ and $\Delta x_\perp \sim$ cm we have $T \Delta x_1 \Delta x_\perp^2 \sim 10 \mathrm{~m}^4$, and the allowed parameter range is either

$$L \gtrsim 10^{-18} \mathrm{~m}, \tag{36}$$

or, if $L \lesssim 10^{-18}$ m, then

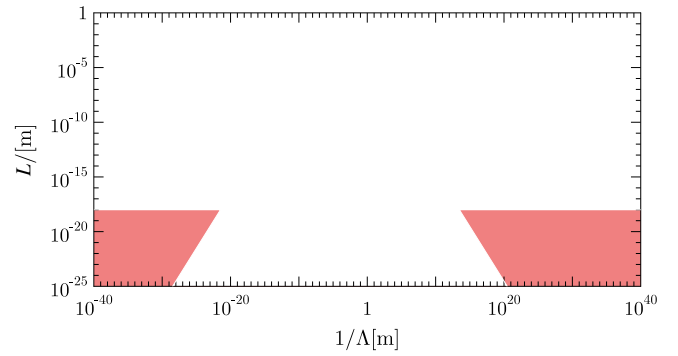$$\frac{L}{2} \left| \frac{\nu}{\Lambda} \pm \frac{\Lambda}{\nu} \right| < 3 \mathrm{~cm}, \tag{37}$$

FIG. 4 (color online). Constraints from single photons in a cavity. Shaded regions are excluded.

where the upper sign stands for the subluminal case and the lower sign for the superluminal case. These constraints are summarized in Fig. 4.

B. Constraints from the GZK cutoff

The Pierre Auger collaboration [20,21] has found a correlation between the directions of ultrahigh energetic cosmic rays (UHECRs), with energies $E_{\mathrm{GZK}} \gtrsim 6 \times 10^{19}$ eV, and active galactic nuclei (AGN) up to a distance of about 75 Mpc. This correlation decreases when AGNs at larger distances are included; beyond 100 Mpc the correlation is no longer statistically significant. This is evidence for the Greisen-Zatsepin-Kuzmin cutoff, which predicts that the mean-free path of protons in the background of the photons from the cosmic microwave background (CMB) is of the order of 100 Mpc. Protons of energy high enough to produce pion pairs when scattering at the CMB photons should no longer reach us. These results from Pierre Auger thus confirm the predictions of the standard model.[4]

We can use this to derive constraints on the density of nonlocal defects as follows. If the UHECRs protons would jump over a significant part of their path from the source to Earth, then their cross section with the CMB photons would decrease, and we should have noticed a correlation with AGNs over longer distances than predicted by standard model interactions. This means that either the protons are unlikely to encounter nonlocal defects or, if they do, they do not jump very far.

Deriving a constraint necessitates to find an estimate for the world volume swept out by the particles we measure. On the one hand, the protons are massive particles and experience dispersion during their travel from the source to Earth. Standard quantum mechanical time evolution leads to an increase of the spatial width of their wave function. On the other hand, if the particle is spread out over a

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[4]A word of caution: The statistical significance of these results has decreased with more recent events taken into account. It will probably take more time for the situation to become entirely clear.
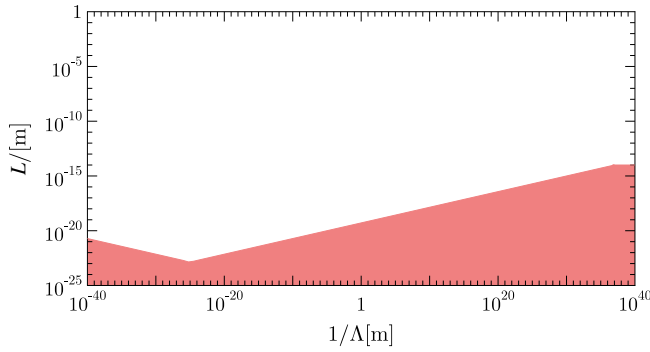
FIG. 5 (color online). Constraints from the correlation of UHECRs with AGNs. The shaded region is excluded.

distance much larger than the detector or telescope, one will only measure the fraction that is collected in the measurement device. Increasing the spatial width of the wave function beyond the size of the detector means the particle will encounter more defects, but at the same time one is less likely to measure anything at all. This would mean that the world volume of the particles one observes is effectively only TV_D, where V_D is the three volume of the detector.

For the case of the UHECRs we can take the detector's volume to be the volume of the upper atmosphere where the protons scatter, which means a transverse extension of about 10^{14} m^2. To estimate the width of the proton's wave function upon arrival on Earth, we will assume that it was emitted at the AGN with a spatial width σ_0 determined by the inverse of the temperature of photoionization equilibrium, which is about 1 eV [22]. In the limit of large times, the width upon arrival is then $\sigma(T) \sim T/(m\sigma_0)$ in the proton rest frame, where $m \sim$ GeV is the proton rest mass. Boosting by a γ factor of $\sim 10^{10}$, one finds $\Delta x_\perp \sim 10^5$ m and $\Delta x_1 \sim 10^{-5}$ m. Thus, the particles fit well inside the detector and the world volume is about $T\Delta x_1 \Delta x_\perp^2 \sim 10^{29}$ m^4.

This means that either the defects are sparse,

$$L \gtrsim 10^{-14} \text{ m}, \tag{38}$$

or, if $L \lesssim 10^{-14}$, then $\langle L_{1j}\rangle/\langle L_{1d}\rangle \lesssim 1/10$, where we allow that there is a 10% modification to the protons' mean-free path, which the correlation analysis by the Auger collaboration would not have been sensitive to. This leads to the bounds

$$l_P^4 10^5 \text{ m}^3 \left| \frac{1}{\Lambda} \mp \frac{\Lambda}{E_{GZK}^2} \right| \lesssim L^7, \tag{39}$$

where again the upper sign stands for the subluminal case and the lower sign for the superluminal case. These constraints are summarized in Fig. 5.

C. Constraints from Airy rings of quasars

Random phase shifts in the propagation of photons have previously been studied as possible signatures of quantum

gravity in the models proposed in [23–26]. Constraints on these phase shifts can be derived by noting that images from distant quasars show interference effects, which would be washed out if random phase shifts were substantial, $\Delta\phi \sim 1$, at that distance and energy [27,28]. It was pointed out already in [27] however that modern telescopes allow one to set much tighter bounds because a telescope focuses a larger part of the light's wave front than just one wavelength $1/\nu$. Since the telescope is sensitive to phase smearing all over its opening diameter D, the constraints on loss of Airy rings have more recently been improved to $\Delta\phi \lesssim 1/(\nu D) \sim 10^{-8}$ [29].

These constraints on the model [23] can also be used for the here discussed propagation in the presence of nonlocal defects, but first it is worthwhile to point out the differences.

In the model going back to [23], the phase shifts also accumulate by a random walk, but the number of steps of that walk is proportional to the distance. Here instead, by the requirement of Lorentz invariance, it is the space-time volume swept out by the wave front that determines the number of steps in the random walk, reflected in Eq. (28).

Another difference is that the T which determines $\bar{N}$ is here not necessarily equal to the distance (time) inferred from the measurement, $\tilde{T}$. If the particle jumps over part of its path, T is instead only the distance (time) the photon propagated in the background and generally related to $\tilde{T}$ by (23). Unlike in the case of UHECR it is difficult to tell how much of its path a photon from a distant quasar might have omitted due to nonlocal defects. One may speculate that if it jumps over a significant distance, it would experience less redshift than normally expected. Alas, to take into account background expansion, a more sophisticated model would first be needed (see also the discussion in Sec. VI). The source would also appear less bright because at any given time, including the one we take the image, a certain fraction of photons is missing from space-time. Since this will generally be the case though, regardless of the distance, we lack a reference to quantify the relative lack.

Thus, we will consider three cases here. Either the photons do not encounter any defects at all or they do and we have $\langle T_{1j}\rangle \gg \langle T_{1d}\rangle$ (jumps are short) or $\langle T_{1j}\rangle \ll \langle T_{1d}\rangle$ (jumps are long) but too small to blur the interference rings. Assuming that Δx is the coherence time of the light received from the quasar and typically a few wavelengths, we have $T/\nu \sim 10^{19}$ m^2 for a typical distance of some Gpc and $1/\nu \sim 500$ nm as used in [29]. The perpendicular width is determined by the opening diameter of the telescope $D \sim$ m (see discussion in Sec. V B).

The photons are unlikely to have encountered any defects so long as

$$10^{-15} \text{ m} \lesssim L. \tag{40}$$

Jumps are long if $L \lesssim 10^{-15}$ m and

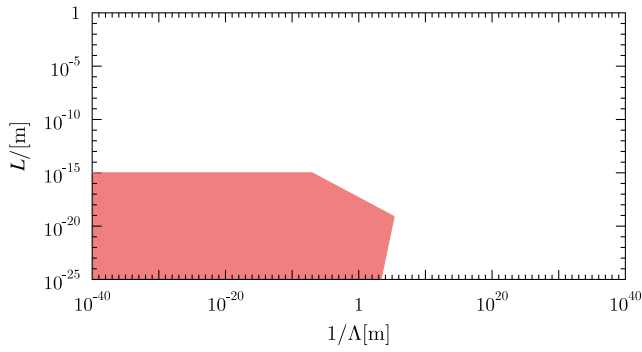$$l_P^4 \frac{D^2}{2\nu} \left| \frac{1}{\Lambda} \pm \frac{\Lambda}{\nu^2} \right| \gg L^7, \tag{41}$$

FIG. 6 (color online). Constraints from the observation of interference patters in images of distant quasars. The red shaded region is excluded.
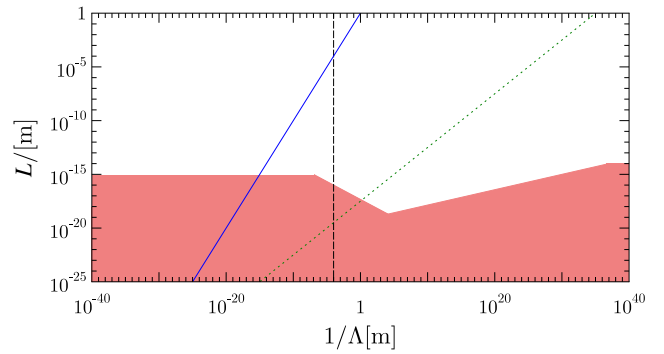

FIG. 7 (color online). Summary of constraints. The red shaded region is excluded. The dashed (black) line indicates the value of the cosmological constant. The dotted (green) line depicts the case $\Lambda L = \epsilon$ and the solid (blue) line the case $\Lambda L = 1$.

where the upper sign stands for the subluminal case and the lower sign for the superluminal case. If $L \leq 10^{-15}$ m and jumps are short, the requirement that $\Delta \phi \leq 10^{-8}$ leads to

$$10^{-52} \text{ m}^4 \leq L^3 \Lambda^{-1}. \tag{42}$$

If the jumps are long one finds

$$10^{-35} \text{ m}^{-2} \left| \frac{1}{\Lambda^3} \pm \frac{1}{\Lambda^2 \nu} \right| \geq L. \tag{43}$$

These constraints are summarized in Fig. 6.

One can try to improve the constraints on phase shifts by looking for interference rings at longer wavelengths because this will increase the effective world volume. The most distant radio pulsar [30] is at a distance of some Gpc and emits at a frequency of $\nu \sim 1/m$. Astrophysical radio signals are measured in very large arrays with a baseline of the order of $\sim 10^4$ m. Since coherence over the wave front is necessary to reconstruct the signal, we can estimate $\Delta \phi \leq 10^{-4}$. However, the dependence on the space-time volume enters only with the square root, so one gains a factor $\sim 10^7$ from the volume, while losing about $\sim 10^4$ from the accuracy. This means that the constraint from blurring of interference rings could potentially be improved with large area telescopes currently under construction, but the details require further investigation of achievable experimental precision and accuracy of image reproduction.

D. Summary of constraints

Finally, we can summarize all the above constraints in Fig. 7. Roughly speaking, we can take away that nonlocal defects cannot be denser than one per femtometer.

As noted earlier, we expect Λ to be a scale in the infrared, and so we can speculate that it is of the same order of magnitude as the cosmological constant. This value is shown in Fig. 7 as the black, dashed line. The diagonal, dotted (green) line shows the relation $\Lambda L = \epsilon$ and the solid (black) line shows $\Lambda L = 1$, which seem the most natural cases. As we can see the case $\Lambda L = \epsilon$ with L being comparable to the cosmological constant is not

compatible with the data. The relevant constraint in this case comes from the blurring of quasar interference rings.

We see that the constraints considered here are about 10 orders of magnitude too weak to be sensitive to a density of nonlocal links comparable to the cosmological constant, which is a natural parameter range in the cosmological setting. However, the bounds considered here can most likely be improved with a more sophisticated model which incorporates background curvature, and by exploiting technological advances in radio astronomy. It thus may be possible to reach the interesting parameter range. In fact it could be that presently existing data harbors so far unrecognized evidence for space-time defects.

VI. DISCUSSION

Let us first summarize the assumptions we have made. We have restricted our examination by requiring Lorentz invariance to be preserved on the average.

We have further fixed the typical length scale for the underlying microscopic discrete structure to be the Planck length, and later assumed that the typical distance for the nonlocal jumps is similar to the length scale of the probability distribution $\langle \alpha \rangle_{NL} = L$. While we did not use a particular ansatz for the distribution we used its typical width and assumed that the widths are of the same order of magnitude as the average values $\langle \alpha \rangle_{NL} \sim \Delta \alpha$ and $\langle \Lambda \rangle_{NL} \sim \Delta \Lambda$. These are plausible assumptions, but they could in principle be relaxed.

As mentioned in Sec. II we have here not discussed the possibility that the end points of nonlocal jumps are distributed the same way as the entry points because this would appear like a local defect. The difference between the two cases can also be understood as the difference between nonlocal connections that can be passed one way only and nonlocal connections that can be passed both ways. The case discussed here is the one-way case. An alternative interpretation of nonlocal defects that suggests itself is that they cause a translation with a stochastic element.

Since the nonlocal defects in the superluminal case open the possibility of particles moving backwards in time in some rest frames, some words on the issue of causality are in order. The possibility for curves that are effectively time-like and closed, even though parts of it are jumped over, necessitates that we require consistency for the time evolution in order to prevent causal paradoxa. This is possible, but only seems natural in the presence of an emergent arrow of time. For such an arrow of time we have to consider a background filled with quantum fields of increasing entropy. The Friedmann-Robertson-Walker (FRW) background then provides a preferred slicing according to which one can require jumps to be "forward."

The conservative approach to the causality problem is of course to just restrict the translations to happen exclusively into quadrant I, which is a Lorentz-invariant requirement and does entirely circumvent the issue because then particles cannot travel over spacelike distances, as normally. Note that this restriction is a restriction on the distribution of translations, i.e., on $P_{NL}(\alpha)$ and independent of the distribution of start points ρ_{NL}.

That having been said, it is clearly desirable to develop this model to a stage where space-time curvature and quantum fields can be taken into account. Then it would be possible to use constraints from cosmology, in particular from the time evolution of the Universe itself. To take into account a FRW background, one needs to make a plausible assumption for how the density of defects evolves with time. Ideally of course, this time dependence will at some point be derived from a candidate theory for quantum gravity. We hope that the here studied case of nonlocal defects in Minkowski space serves as a good starting point for such further developments.

VII. SUMMARY

We have developed a phenomenological model for non-local space-time defects that maintains Lorentz invariance on the average. We parametrized the effects with the space-time density of defects and the average distance a particle jumps when it encounters a defect, which depends on the ratio of the particle's energy over a mass scale in the infrared. The smaller the energy of the particle, the more pronounced the effect. We then calculated how the average deviation from the light cone scales with the propagation distance and used this to derive constraints on the model from existing data. We have seen that the nonlocal defects become more relevant in the infrared for two reasons. First, they create a small effective mass and second, a particle with large spatial width is more likely to encounter a defect.

We found that bounds from various available observations exclude approximately more than one nonlocal defect in a world volume of a femtometer to the fourth power. These constraints could be improved by studying the interference patterns of distant radio sources in large array telescopes. The vanishing of interference rings could signal a phase distortion by scattering on nonlocal defects. Finally, we noted that it would be desirable to further develop the model so that it can be applied to a Friedmann-Robertson-Walker background which would open the possibility to analyze cosmological precision measurements for possible evidence of space-time defects.

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