Axisymmetric Dirac-Nambu-Goto branes on Myers-Perry black hole backgrounds

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Stationary D-dimensional test branes interacting with N-dimensional Myers-Perry bulk black holes are investigated in arbitrary brane and bulk dimensions. The branes are asymptotically flat and axisymmetric around the rotation axis of the black hole with a single angular momentum. They are also spherically symmetric in all other dimensions allowing a total of $O(1) \times O(D-2)$ group of symmetry. It is shown that even though this setup is the most natural extension of the spherical symmetric problem to the simplest rotating case in higher dimensions, the obtained solutions are not compatible with the spherical solutions in the sense that the latter ones are not recovered in the nonrotating limit. The brane configurations are qualitatively different from the spherical problem, except in the special case of a three-dimensional brane. Furthermore, a quasistatic phase transition between the topologically different solutions cannot be studied here, due to the lack of a general, stationary, equatorial solution.

I. INTRODUCTION

Possible interactions between branes and black holes in higher dimensions are interesting and important problems in many fields of modern theoretical physics. One direction, which has been recently introduced by Frolov [1], is a spherically symmetric black hole interacting with Dirac-Nambu-Goto (DNG) test branes [2] in arbitrary brane and bulk dimensions. This brane-black hole system, beyond the interest of its own, has also proven to be very useful as a toy model for various other problems. For example, it possesses striking similarities in its properties to the problem of topology changing and merger transitions between higher-dimensional black solutions [3–5], and also shows a self-similar behavior, very similar to the Choptuik critical collapse phenomenon [6]. Furthermore, it also turned out to be a relevant model in investigating holographic phase transitions in strongly coupled gauge theories [7,8], via the gauge/gravity correspondence [9].

Generalizations to the system, by considering small thickness corrections to the branes, have also been studied lately by Frolov and Gorbonos [10], and more extensively (also within a more general framework) by us [11–13]. The motivation for this extension was to consider higher order, curvature corrections to the thin brane action, which, in the holographic dual picture, correspond to finite 't Hooft coupling corrections, and provide a more realistic description of the phase transition [8].

In the present paper, as a sequel to our previous works on the subject matter [11–13], we provide another generalization of the problem into a different direction. We investigate the brane-black hole system in the rotating case by considering a Myers-Perry (MP) black hole in the background with a single angular momentum. A similar problem has also

been investigated by Kubizňák and Frolov in [14] on a Kerr-NUT-(A)dS background. The motivation of this work is also clear; we would like to understand the role that rotational effects play when a quasistatic, topology changing transition is considered in the system. The problem is interesting not only from the geometrical point of view, but also because it may provide further insights to other topology changing or merger transition problems in higher-dimensional, classical general relativity, or to certain holographic phase transitions in the gauge/gravity dual picture. Furthermore, a quasidynamical approach to the evolution of interacting brane-black hole systems [15,16] provides another interesting direction which also deals with the same geometric setup.

In constructing the model, we follow the method of [1] as closely as possible, and define the DNG branes with the highest possible symmetry properties that the background allows. By this construction the branes possess a total of $O(1) \times O(D-2)$ group of symmetry, and just as in the spherical case, the brane action simplifies radically, resulting in the problem of an ordinary differential equation (highly nonlinear though) for the brane configurations. We present and analyze the general solution of this problem, first analytically in far distances, and later numerically in the near horizon region.

As a result, we obtain that due to the coordinate parametrization of the Myers-Perry metric, this rotating problem is not compatible with the spherical results of [1], in the sense that the latter ones are not recovered in the nonrotating limit. Although the ideal situation would be to provide a rotating solution which is the "corresponding" one to the spherical problem in the above sense, nevertheless we could not find an appropriate coordinate system in which this could be done in a natural way as presented by Frolov in [1], and as we also do it here. Consequently, we conclude that while the construction of the problem is the closest possible to the spherical case, the obtained results are

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qualitatively different, except in the special case of a three-dimensional brane. Furthermore, we also find that stationary equatorial solutions do not generally exist for arbitrary brane dimensions, except again for the case of a three-dimensional brane, and as a result, we cannot study the quasistatic phase transition in the geometric setup as we did in the thickness corrected spherical case [11,12] by following the method of Flachi *et al.* [17].

The plan of the paper is as follows. In Sec. II we define the rotating brane-black hole system analogous to the spherical case. In Sec. III we obtain the brane equation and discuss its incompatibility with the results of [1]. In Sec. IV, first we discuss the analytic properties of the solutions in the near horizon region and derive unique boundary conditions for the topologically different solutions from regularity requirements. Then, we obtain the far distance solution in an analytic form and analyze its properties. In Sec. V we present and illustrate the numerical results in the near horizon region, and finally in Sec. VI we draw our conclusions. In addition, we discuss the problem of the coordinate systems in the Appendix.

II. THE BRANE-BLACK HOLE MODEL

The metric of the N-dimensional Myers-Perry solution [18] in Boyer-Lindquist coordinates with a single angular momentum is given by

$$ds^2 = -\left(1 - \frac{F}{\Sigma}\right)dt^2 + \sin^2\theta\left[r^2 + a^2\left(1 + \frac{F}{\Sigma}\sin^2\theta\right)\right]d\varphi^2$$
$$+ 2a\frac{F}{\Sigma}\sin^2\theta dt d\varphi + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2$$
$$+ r^2\cos^2\theta d\Omega_{N-4}^2, \tag{1}$$

where

$$\Sigma = r^2 + a^2\cos^2\theta, \tag{2}$$

$$\Delta = r^2 + a^2 - F, \tag{3}$$

$$F = \mu r^{5-N}, \tag{4}$$

and $d\Omega_{N-4}^2$ is the line element on an $(N-4)$-dimensional unit sphere. The parameters μ and a are related to the total mass, M, and angular momentum, J, of the black hole as

$$M = \frac{(N-2)A_{N-2}}{16\pi G}\mu, \qquad J = \frac{2}{N-2}Ma, \tag{5}$$

where

$$A_{N-2} = \frac{2\pi^{\frac{N-2}{2}}}{\Gamma(\frac{N-2}{2})} \tag{6}$$

is the area of an $(N-2)$-dimensional unit sphere S^{N-2}. For simplicity, without any loss of generality, we can fix the value of the mass parameter μ to 1.

Test brane configurations in an external gravitational field can be obtained by solving the Euler-Lagrange equation derived from the Dirac-Nambu-Goto action [2]

$$S = \int d^D\zeta\sqrt{-\det\gamma_{\mu\nu}}, \tag{7}$$

where

$$\gamma_{\mu\nu} = g_{ab}\frac{\partial x^a}{\partial\zeta^\mu}\frac{\partial x^b}{\partial\zeta^\nu} \tag{8}$$

is the induced metric on the brane and $\zeta^\mu(\mu = 0, \ldots, D-1)$ are coordinates on the brane world sheet. The brane tension does not enter into the brane equations, thus for simplicity it can also be put equal to 1. We introduce coordinates in the bulk as

$$x^a = \{t, r, \varphi, \theta, \vartheta_1, \ldots, \vartheta_{N-4}\},$$

and it is assumed that the brane is stationary, spherically symmetric in the $\vartheta_i(i = 1, \ldots, n = D-3)$ dimensions, rotationally symmetric in the φ coordinate, and, if $D < N-1$, its surface is chosen to obey the equations

$$\vartheta_{D-2} = \cdots = \vartheta_{N-4} = \pi/2. \tag{9}$$

With the above properties the brane world sheet allows an $O(1) \times O(D-2)$ group of symmetry, and can be completely defined by the single function $\theta = \theta(r)$. We shall use coordinates ζ^μ on the brane as

$$\zeta^\mu = \{t, r, \varphi, \vartheta_1, \ldots, \vartheta_n\},$$

where $n = D-3$. With this parametrization the induced metric on the brane surface is given by

$$\gamma_{\mu\nu}d\zeta^\mu d\zeta^\nu$$
$$= -\left(1 - \frac{F}{\Sigma}\right)dt^2 + \sin^2\theta\left[r^2 + a^2\left(1 + \frac{F}{\Sigma}\sin^2\theta\right)\right]d\varphi^2$$
$$+ 2a\frac{F}{\Sigma}\sin^2\theta dt d\varphi + \Sigma\left(\frac{1}{\Delta} + \dot\theta^2\right)dr^2 + r^2\cos^2\theta d\Omega_n^2, \tag{10}$$

where, and throughout the paper, the overdot denotes the derivative with respect to the radial coordinate, r. The Dirac-Nambu-Goto action (7) reduces to

$$S = 2\pi\Delta t A_n\int\mathcal{L}dr, \tag{11}$$

where Δt is an arbitrary interval of time, A_n is the area of the unit sphere S^n, the 2π factor is obtained from the integration with respect to φ, and the Lagrangian takes the form

$$\mathcal{L} = r^n\cos^n\theta\sin\theta\sqrt{\Sigma(1 + \Delta\dot\theta^2)}. \tag{12}$$

III. THE BRANE EQUATION

Test brane configurations are solutions of the Euler-Lagrange equation

$$\frac{d}{dr}\left(\frac{\partial\mathcal{L}}{\partial\dot\theta}\right) - \frac{\partial\mathcal{L}}{\partial\theta} = 0, \tag{13}$$

which for the Lagrangian (12) reads as

$$\ddot{\theta} + \left(\alpha\Delta + \frac{\dot{\Delta}}{2}\right)\dot{\theta}^3 + \beta\dot{\theta}^2 + \left(\alpha + \frac{\dot{\Delta}}{\Delta}\right)\dot{\theta} + \frac{\beta}{\Delta} = 0, \qquad (14)$$

where α and β are

$$\alpha = \frac{n}{r} + \frac{r}{\Sigma}, \qquad (15)$$

$$\beta = n\tan\theta - \cot\theta + \frac{a^2\sin\theta\cos\theta}{\Sigma}. \qquad (16)$$

The horizon of the black hole is defined as the largest solution of $\Delta = 0$, and one can consider the nonrotating problem by taking the $a \to 0$ limit.

In the case of the nonrotating limit, however, one notices that the Euler-Lagrange equation obtained from (14) is not identical to the one that has been obtained and analyzed by Frolov in [1], and what we also investigated in the presence of thickness corrections in the spherically symmetric case [11–13]. After some analysis one can show that the difference stems from the different coordinate systems used by the Myers-Perry and Schwarzschild-Tangherlini (ST) solutions [19], and which disappears in standard four dimensions in the $a \to 0$ limit, but remains present in higher dimensions, even after taking the nonrotating limit. The detailed calculation to show this is a bit lengthy, therefore we present it as an appendix at the end of the paper.

As a consequence, it is very important to emphasize that the DNG brane defined as $\theta(r)$ in the previous section is *not* the "corresponding" brane to the one that we investigated in the Schwarzschild-Tangherlini case, in the sense that it does *not* reproduce the spherical results of [1] in the non-rotating limit. This is because the angular coordinate θ, through which the brane is defined in the Myers-Perry metric, is different from the one (denoted with the same letter θ) in the Schwarzschild-Tangherlini solution, even after taking the nonrotating limit. They correspond trivially only in four dimensions, where we are accustomed to obtain the Schwarzschild coordinates in the nonrotating limit of the Kerr solution.

It may be also worth mentioning that we have been trying to find an appropriate coordinate system for the rotating case, where those "corresponding" branes, which would reproduce the solutions of [1] as their nonrotating limit, could be defined naturally. The problem, however, turns out to be very difficult, because in those systems where the limit in the bulk is automatic, either the defini-tion of the rotationally symmetric brane is problematic, or the coordinate transformations involve angles from the extra dimensions of the metric, which cannot be integrated out from the action in the simple way as we did in (11). Although we believe that the problem should ultimately be resolved in one way or another, nevertheless, we were not able to obtain a satisfactory resolution so far.

Accordingly, in the present paper we are focusing on those DNG branes which are defined in Sec. II and are the solutions of the Euler-Lagrange equation (14). The problem is, of course, interesting in its own right, being the most naturally defined DNG-brane problem on a rotating black hole background in arbitrary dimensions, and also the most natural extension of the spherical problem to the simplest rotating case. However, we have to keep in mind that it is essentially different from the one that would provide back the Schwarzschild-Tangherlini solution of [1] in the nonrotating limit.

IV. ASYMPTOTIC AND REGULARITY ANALYSIS

In this section we present the near horizon and far distance asymptotic solutions of the brane equation. From regularity requirements in the near horizon region, we obtain unique boundary conditions for the problem which will be used for the numerical solution in the following section.

A. Near horizon behavior

For a brane crossing the horizon (black hole embedding case or supercritical branch in Frolov's terminology [1]), (14) has a regular singular point on the horizon, $r = r_0$. A regular solution at this point has the following expansion near the horizon:

$$\theta = \theta_0 + \dot{\theta}_0(r - r_0) + \cdots, \qquad (17)$$

where the regularity requirement imposes the condition

$$\dot{\theta}_0 = -\frac{\beta}{\dot{\Delta}}\bigg|_{r_0}. \qquad (18)$$

Consequently, supercritical solutions are all uniquely determined by their boundary value θ_0.

In the Minkowski embedding (subcritical) case, the brane does not cross the horizon, and its surface reaches its minimal distance from the black hole at $r_1 > r_0$, which, for symmetry reasons, occurs at $\theta = 0$. A regular (but not smooth or even differentiable, see [11–13]) solution of (14) near this point has the asymptotic behavior

$$\theta = \eta\sqrt{r - r_1} + \sigma(r - r_1)^{3/2} + \cdots, \qquad (19)$$

where the regularity requirement on the axis of rotation imposes the conditions

$$\eta = \frac{2}{\sqrt{\kappa\Delta + \frac{\dot{\Delta}}{2}\big|_{r_1}}}, \qquad (20)$$

and

$$\sigma = \frac{16}{8 - 9\eta^2(\dot{\Delta} + 2\kappa\Delta)}\left[\frac{\eta}{2}\left(\kappa + \frac{\dot{\Delta}}{\Delta}\right) - \frac{1}{\eta\Delta}\right.$$

$$+ \frac{\eta^3}{4}\left(n + \frac{1}{3} + \frac{a^2}{a^2 + r_1^2} + \frac{\Delta}{2}\left[\frac{a^2(1 + \eta^2 r_1) - r_1^2}{(a^2 + r_1^2)^2} - \frac{n}{r_1^2}\right]\right.$$

$$\left.\left.+ \frac{2\kappa\dot{\Delta} + \ddot{\Delta}}{4}\right)\right]_{r_1}, \tag{21}$$

with

$$\kappa = \frac{n}{r_1} + \frac{r_1}{a^2 + r_1^2}. $$

Hence, all subcritical solutions are also uniquely determined by the single parameter, r_1.

B. Far distance solution

Since the Myers-Perry solution is asymptotically flat, the brane function $\theta(r)$ has to converge to a constant value, θ_∞, as $r \to \infty$. The explicit value of θ_∞ is not known for the moment (in contrast with the spherical case where it was $\pi/2$ for all dimensions), rather it can be obtained by the following consideration. The far distance solution of (14) can be searched in a perturbative form

$$\theta(r) = \theta_\infty + \nu(r), \tag{22}$$

where $\nu(r)$ is a first-order small function compared to θ_∞, and we require that

$$\lim_{r \to \infty} \nu(r) = 0. \tag{23}$$

We shall only keep the linear terms of ν in (14) which yields the asymptotic equation

$$\ddot{\nu} + \frac{n + 3}{r}\dot{\nu} + \frac{1 + n + n\tan^2\theta_\infty + \cot^2\theta_\infty}{r^2}\nu$$

$$+ \frac{n\tan\theta_\infty - \cot\theta_\infty}{r^2} + \frac{a^2\sin\theta_\infty\cos\theta_\infty}{r^4} = 0. \tag{24}$$

The general solution of (24) reads as

$$\nu(r) = -\frac{B}{1 + n + A} - \frac{C}{(1 - n + A)r^2}$$

$$+ r^{-1-\frac{n}{2}-\frac{i}{2}\sqrt{4A-n^2}}\left[p + p'r^{i\sqrt{4A-n^2}}\right] \tag{25}$$

with

$$A = n\tan^2\theta_\infty + \cot^2\theta_\infty, \qquad B = n\tan\theta_\infty - \cot\theta_\infty,$$

$$C = a^2\sin\theta_\infty\cos\theta_\infty.$$

Before running into the analysis of the complex powers in the solution, we notice that the first term of (25) is a constant. Thus (25) can only be a good solution of (24) if $B = 0$, due to the requirement (23). This implies the asymptotic constraint

$$n\tan\theta_\infty - \cot\theta_\infty = 0, $$

and yields the asymptotic value

$$\theta_\infty = \arctan\left[\frac{1}{\sqrt{n}}\right]. \tag{26}$$

According to this result, we can conclude that for each brane dimension, n, the solutions have different asymptotic behavior, and the asymptotic value θ_∞ coincides with the Schwarzschild value, $\pi/2$, only in the case of $n = 0$, that is, when the brane is three dimensional.

It is interesting to note here that three-dimensional branes tend to behave differently from their higher-dimensional counterparts in other aspects too. In our previous works [12,13], we also found that three-dimensional branes had exceptional analytic properties in the near horizon region when thickness corrections had been considered in the nonrotating case.

Another interesting feature to note is that the asymptotic value does not depend on the rotation parameter of the black hole; it is determined solely by the number of inner dimensions of the brane in which it is spherically symmetric.

After deriving the value for the asymptotic constants, we can obtain the corresponding asymptotic solutions by plugging back θ_∞ into (24), which results in the asymptotic equation

$$\ddot{\nu} + \frac{n + 3}{r}\dot{\nu} + \frac{2(n + 1)}{r^2}\nu + \frac{a^2\sqrt{n}}{(n + 1)r^4} = 0, \tag{27}$$

or plugging it directly into (25), and take a bit of time with the power analysis. In either case, the asymptotic solution takes the form

$$\nu(r) = \begin{cases} \frac{p\sin[\delta(r)] + p'\cos[\delta(r)]}{r^{1+\frac{n}{2}}} - \frac{a^2\sqrt{n}}{2(n+1)r^2}, & \text{if } n \le 4, \\ \frac{p + p'r^{\sqrt{-\gamma}}}{r^{1+\frac{n}{2}+\frac{\sqrt{\gamma}}{2}}} - \frac{a^2\sqrt{n}}{2(n+1)r^2}, & \text{if } n \ge 5, \end{cases} \tag{28}$$

where

$$\delta(r) = \frac{\sqrt{\gamma}}{2}\ln(r), \qquad \gamma = -n^2 + 4n + 4. \tag{29}$$

It may seem, at first sight, that branes with $n \le 4$ dimensions have different far distance asymptotics than the ones with dimensions $n \ge 5$. The real change occurs, however, at $n = 3$, as we can see from the following analysis.

In the $n = 0$ case, the rotation of the black hole does not seem to affect the asymptotic behavior directly, and, as we mentioned earlier, this is the exceptional case of the three-dimensional brane, when the asymptotic value θ_∞ is $\pi/2$, just as in the Schwarzschild problem. When $n = 1$, the first term dominates the solution, because the second one, which is controlled by the rotation parameter, decays faster. In the case of $n = 2$, both terms decay essentially as r^{-2}. In all other cases, starting from $n \ge 3$, the first terms in the solution decay much faster than the second

one, which results that all the branes with $D = 6$ or more dimensions have an almost uniform convergence to the asymptotic value in the far distance region, and this is controlled by the rotation parameter of the black hole.

The coefficients p and p' in the solutions are continuous functions of the θ_0 or r_1 boundary parameters that we obtained previously from regularity requirements in the near horizon region. On the other hand, because of the complicated asymptotic behavior, the interpretation of p or p' is not so clear as it was in the Schwarzschild case (being the distance of the brane from the asymptotic value at infinity [1]).

V. NUMERICAL RESULTS

After obtaining the far distance solution of the problem in analytic form and also deriving boundary conditions from regularity requirements in the near horizon region, we can consider the numerical solution of (14). As it was shown earlier, the boundary value θ_0, or the radial coordinate r_1, uniquely determines the corresponding super- or subcritical solutions, respectively. The numerical solution itself does not require very advanced techniques; we have performed it by using the MATHEMATICA NDSOLVE function.

On Fig. 1 we are plotting a sequence of $D = 3$ ($n = 0$) brane solutions from both topologies in the near horizon region. The asymptotic constant in this special case is $\pi/2$ and we have chosen the value 0.4 for the rotation parameter, a. As a result (just as we expect from the far distance analysis), the brane configurations are very similar to what we had before in the spherical case [1,11].

By increasing the brane dimension from $D = 3$ ($n = 0$) to $D = 4$ ($n = 1$), and keeping the bulk dimension fixed ($N = 6$), we can see the interesting new result on the asymptotic behavior. We plotted this situation on Fig. 2. In this case, the asymptotic value θ_∞ is $\pi/4$, and it can be
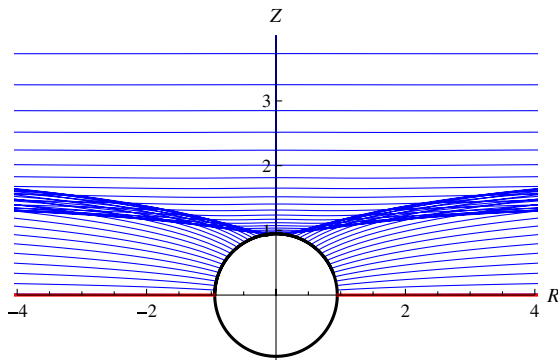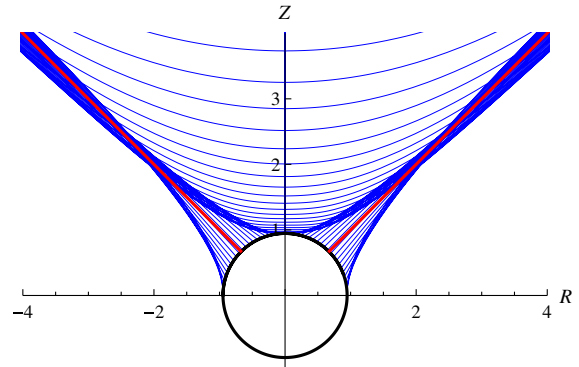
FIG. 2 (color online). The picture shows a sequence of ($D = 4$)-dimensional branes with varying boundary values embedded in a bulk with $N = 6$ dimensions. R and Z are standard cylindrical coordinates, and the thick, red lines represent the value θ_∞, to which the solutions asymptotically converge, $\pi/4$ for the present case. The value of the rotation parameter $a = 0.4$.

seen that all solutions tend asymptotically to this value (in good agreement with the far distance analysis) independent of the near horizon boundary values.

By increasing the number of brane dimensions, n, the value of the asymptotic constant θ_∞ changes according to (26), but the qualitative picture of the solutions remains essentially similar to what we see on Fig. 2. For the sake of illustration, on Fig. 3, we also plot the $D = 5$ ($n = 2$)-dimensional case with $N = 6$.

By changing the value of the rotation parameter, the near horizon configurations are also changing together with the asymptotic convergence to θ_∞ that we discussed earlier in the far distance solution. In order to illustrate this change, we define the function

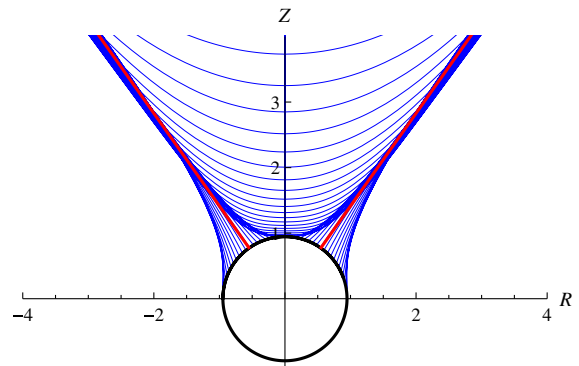$$\Delta\theta(r) = \theta(r) - \theta_\infty, \tag{30}$$

FIG. 3 (color online). The picture shows a sequence of ($D = 5$)-dimensional branes with varying boundary values embedded in a bulk with $N = 6$ dimensions. R and Z are standard cylindrical coordinates, and the thick, red lines represent the value θ_∞, to which the solutions asymptotically converge, $\theta_\infty = \arctan[1/\sqrt{2}]$ for the present case. The value of the rotation parameter $a = 0.4$.
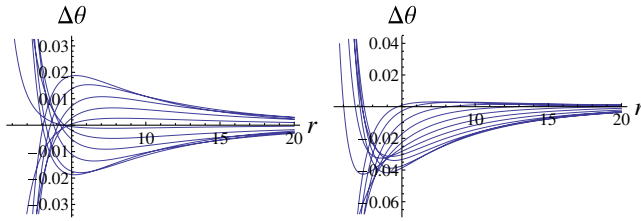
FIG. 1 (color online). The picture shows a sequence of ($D = 3$)-dimensional branes with varying boundary values embedded in a bulk with $N = 6$ dimensions. R and Z are standard cylindrical coordinates, and the thick, red lines represent the value θ_∞ which is $\pi/2$ for the present case. The value of the rotation parameter $a = 0.4$.

FIG. 4 (color online). The picture shows a sequence of $\Delta\theta(r)$ functions of $(D = 4)$-dimensional branes embedded in an $(N = 6)$-dimensional bulk. The boundary values are equally distributed around $\theta_\infty = \pi/4$ in the $\theta_0 \in (0, \pi/2)$ region. The left picture belongs to a slow rotation, $a = 0.1$, while the right picture belongs to the $a = 0.9$ value.

and compute the $\Delta\theta(r)$ functions for a sequence of brane solutions with different boundary values, θ_0, equally distributed around the $\theta_\infty = \pi/4$ value in the $\theta_0 \in (0, \pi/2)$ region, just as on Figs. 2 and 3. The corresponding curves are plotted on Fig. 4 for two different rotation parameter values, $a = 0.1$ (left picture) and $a = 0.9$ (right picture).

In the case of slow rotation ($a = 0.1$), the $\Delta\theta(r)$ functions have an almost "mirror symmetric" amplitude distribution around the $\theta_\infty = \pi/4$ value (right picture on Fig. 4), while in the case of a large rotation parameter ($a = 0.9$), the picture becomes very asymmetric. The branes with boundary values $\theta_0 \in (\pi/4, \pi/2)$ deviate strongly from θ_∞ in the near horizon region, while the branes with $\theta_0 \in (0, \pi/4)$ approach the asymptotic value very quickly.

In our previous works [11,12], we also investigated the problem of a quasistatic evolution of a brane from the equatorial plane in black hole embeddings, to a Minkowski embedding topology, through a topology change transition. The question was very natural there, following the method developed in [17], because the equatorial configuration was a general solution in every dimension of the spherical problem. In the present rotating case, however, as we saw above, the equatorial configuration is a solution of the problem only in the exceptional case of the three-dimensional brane, and we cannot use this method for a general discussion. Although we could analyze the topological phase transition in this special case, nevertheless we believe that it would be misleading since the relevant problems are usually obtained from higher dimensions, like the case of the holographic dual phase transition. As a consequence, the question remains open in the present rotating case.

VI. CONCLUSIONS

In the present work, we studied the problem of rotationally symmetric, stationary, Dirac-Nambu-Goto branes on the background of a Myers-Perry black hole with a single angular momentum. In defining the interacting brane-black hole system, we strongly followed the spherical problem given by Frolov [1]. Although this model is the most

natural extension of the spherical setup to the simplest rotating case, we found that due to the nonequivalent coordinate parametrization, the obtained solutions are not compatible with the spherical solutions in the sense that the latter ones are not recovered in the nonrotating limit. Our efforts to find an appropriate coordinate system in which the rotating problem could be formulated naturally, in a way that the spherical case could also be reproduced in the $a \rightarrow 0$ limit, has not succeeded so far. It is an open question whether it can be done at all.

After clarifying the above situation, we analyzed the properties of the obtained problem, and presented its solution both analytically, at far distances, and numerically, in the near horizon region. In the latter case, we found that the analytic properties of the test brane solutions, both on the axis of rotation and on the horizon, are very similar to what we saw in the spherical case. From regularity requirements we could obtain unique numerical solutions for each freely chosen, boundary value in both topologies.

By analyzing the far distance solutions, we obtained a new interesting result that the asymptotic behavior of the rotating solutions are qualitatively different from the spherical problem, except in the special case of a three-dimensional brane. This difference changes the entire structure of the brane configurations in the near horizon region too, because all solutions are attracted asymptotically to the same constant value, independent of the near horizon boundary conditions. Furthermore, the asymptotic value is different for every brane dimension, and it is controlled solely by the dimension parameter of the brane.

Another interesting result is that the rotation of the black hole has a direct effect only on how the solutions tend to the asymptotic value, and we illustrated this phenomenon in the cases of a small and a large rotation parameter.

One of the motivations of this work was to understand the role that rotational effects may play in a quasistatic, topology changing phase transition of the system. As a negative result, we obtained that the problem cannot be studied here in the geometrical way that we applied in the thickness corrected spherical problem [11,12], due to the lack of a general, stationary, equatorial solution for arbitrary dimensions. Consequently, we could not obtain general results on the phase transition in this paper, so the question remains open for the rotating case.

The lack of the equatorial solution has another consequence which is connected to the stability of the rotating brane-black hole system. It has been shown by Hioki *et al.* [20] that equatorial solutions are stable against small perturbations in the spherical case. Stability is an important issue in higher dimensions, and it would be also important to know whether similar results may hold for the present axisymmetric case too. Unfortunately, because of the lack of the equatorial solution, the question of stability cannot be studied here by using the method of Hioki *et al.* for the general case.

As another stability issue, in this paper we have not considered the cases of extremal and ultraspinning black holes. The reason for this is the fact that ultraspinning black holes are expected to be unstable [21], and the instability limit occurs at a surprisingly low value of the angular momentum, i.e. not far in the ultraspinning regime. In fact, the magnitude of the critical rotation parameter a has been estimated by Emparan and Myers for several dimensions [21], and it turned out that a typical value is around $a \approx 1.3$. According to this, in the present paper we constrained ourselves to keep the value of the rotation parameter small enough to stay away from the presumably unstable regime.

ACKNOWLEDGMENTS

APPENDIX: THE SCHWARZSCHILD-TANGHERLINI LIMIT OF THE MYERS-PERRY SOLUTION

The N-dimensional Myers-Perry metric with a single angular momentum in Boyer-Lindquist coordinates is given in (1), while the Schwarzschild-Tangherlini solution of the same dimension is given by

$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 d\Omega_{N-2}^2, \qquad \text{(A1)}$$

where

$$f = 1 - \frac{\mu}{r^{N-3}}. \qquad \text{(A2)}$$

In both formulas $d\Omega_k^2$ is the metric of a k-dimensional unit sphere S^k, which is parametrized with the polar coordinates ξ_k defined by the following recursive relation:

$$d\Omega_{k+1}^2 = d\xi_{k+1}^2 + \sin^2\xi_{k+1} d\Omega_k^2. \qquad \text{(A3)}$$

By taking the limit of $a \to 0$ in the MP metric, the ST solution has to be reproduced. In order to check this, after taking the limit in the coefficient functions, one arrives at the following equation for the metric on the $(N-2)$-dimensional unit sphere,

$$d\Omega_{N-2}^2 = d\theta^2 + \sin^2\theta d\varphi^2 + \cos^2\theta d\Omega_{N-4}^2. \qquad \text{(A4)}$$

Applying the recursive relation given in (A3) we can rewrite (A4) into the form

$$d\xi_1^2 + \sin^2\xi_1 d\xi_2^2 + \sin^2\xi_1 \sin^2\xi_2 d\Omega_{N-4}^2$$
$$= d\theta^2 + \sin^2\theta d\varphi^2 + \cos^2\theta d\Omega_{N-4}^2. \qquad \text{(A5)}$$

From (A5) it is clear that if $N > 4$, the angular parametrization of the two-sphere in question is different from that of the ST metric of the same dimension. This difference, however, disappears in standard four dimensions since the last terms are zero on both sides yielding the equivalence

$$\theta = \xi_1, \qquad \varphi = \xi_2. \qquad \text{(A6)}$$

In order to see the ST limit of the MP metric for $N > 4$, one needs to verify that the angular parametrization given in (1) is equivalent with the Schwarzschild parametrization. To show this, we need to find the transformation laws from the spherical coordinates defined by the polar angles ξ_1 and ξ_2, to the coordinate system defined by the angles θ and φ. The transformation rules are the solution of the following system of equations obtained from (A5),

$$\left(\frac{\partial\theta}{\partial\xi_1}\right)^2 + \sin^2\theta\left(\frac{\partial\varphi}{\partial\xi_1}\right)^2 = 1, \qquad \text{(A7)}$$

$$\left(\frac{\partial\theta}{\partial\xi_2}\right)^2 + \sin^2\theta\left(\frac{\partial\varphi}{\partial\xi_2}\right)^2 = \sin^2\xi_1, \qquad \text{(A8)}$$

$$\frac{\partial\theta}{\partial\xi_1}\frac{\partial\theta}{\partial\xi_2} + \sin^2\theta\frac{\partial\varphi}{\partial\xi_1}\frac{\partial\varphi}{\partial\xi_2} = 0, \qquad \text{(A9)}$$

$$\sin^2\xi_1\sin^2\xi_2 = \cos^2\theta, \qquad \text{(A10)}$$

where $\theta = \theta(\xi_1, \xi_2)$ and $\varphi = \varphi(\xi_1, \xi_2)$. This system can be integrated in a closed form with the solution

$$\theta = \arccos[\sin\xi_1 \sin\xi_2], \qquad \text{(A11)}$$

$$\varphi = \arctan[\cos\xi_2 \tan\xi_1], \qquad \text{(A12)}$$

or equivalently the inverse transformations are

$$\xi_1 = \arcsin\left[\sqrt{1 - \cos^2\varphi\sin^2\theta}\right], \qquad \text{(A13)}$$

$$\xi_2 = \arcsin\left[\frac{\cos\theta}{\sqrt{1 - \cos^2\varphi\sin^2\theta}}\right]. \qquad \text{(A14)}$$

To see how the angles θ and φ parametrize the unit two-sphere let us utilize the standard Cartesian coordinates x, y, z given by

$$x = \sin\xi_1 \cos\xi_2, \quad y = \sin\xi_1 \sin\xi_2, \quad z = \cos\xi_1. \quad \text{(A15)}$$

Here the polar angle $\xi_1 \in [0, \pi]$ is measured from the positive z direction, and the azimuthal angle $\xi_2 \in [0, 2\pi]$ runs in the x-y plane measured from the positive x direction. Expressing now x, y and z as functions of θ and φ through the transformation formulas (A13) and (A14) we get

$$x = \sin\varphi\sin\theta, \quad y = \cos\theta, \quad z = \cos\varphi\sin\theta. \quad \text{(A16)}$$

It is thus clear that θ and φ are also spherical polar coordinates of the unit two-sphere in a way that the polar angle $\theta \in [0, \pi]$ is measured from the positive y direction,

and the azimuthal angle $\varphi \in [0, 2\pi]$ runs in the z-x plane measured from the positive z direction.

According to this, we observe that in standard four dimensions the axis of rotation of the Kerr black hole is orthogonal to the x-y plane (corresponding to the labeling of the standard Cartesian coordinates above), however, in dimensions $N > 4$, the axis of rotation "switches" to be orthogonal to the z-x plane instead. One has to be thus very careful in taking the Schwarzschild-Tangherlini limit of the Myers-Perry solution in higher dimensions, because the angular parametrization of the two solutions remains different even in the nonrotating limit.

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