Study of $\eta K\bar{K}$ and $\eta' K\bar{K}$ with the fixed center approximation to Faddeev equations

Weihong Liang,[1,2,*] C. W. Xiao,[2] and E. Oset[2]

[1]*Department of Physics, Guangxi Normal University, Guilin 541004, People's Republic of China*
[2]*Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC,*
Institutos de Investigación de Paterna, Apartado 22085, 46071 Valencia, Spain

In the present work we investigate the three-body systems of $\eta K\bar{K}$ and $\eta' K\bar{K}$, by taking the fixed center approximation to Faddeev equations. We find a clear and stable resonance structure around 1490 MeV in the squared $\eta K\bar{K}$ scattering amplitude, which is not sensitive to the renormalization parameters. Conversely, we get only an enhancement effect of the threshold in the $\eta' K\bar{K}$ amplitude that indicates the difficulty to bind the $\eta' K\bar{K}$ system as a consequence of the $\eta' K$ interaction being weaker than the ηK one. We associate the $\eta K\bar{K}$ state found to the $\eta(1475)$.

I. INTRODUCTION

Understanding the nature and structure of hadronic resonances is a main topic in high-energy physics, which attracts the attention of both theory and experiment. With the advent of quantum chromodynamics (QCD) and the standard model, modern hadron physics is developing fast. The traditional picture for the internal structure of hadrons is that a meson is made of $q\bar{q}$ and a baryon of qqq, and quark models describe them well. On the other hand, with the development of the experiments, some states have been found experimentally whose properties cannot be explained by the standard way and may be of more complex structures, like tetraquarks and hybrids including possible glueballs for mesons, pentaquarks and heptaquarks for the baryons, or molecular states (see recent reviews in Refs. [1,2]). For the low-energy region where the abnormal states showed up, nonperturbative QCD should be explored, such as lattice QCD [3–6], QCD sum rule [7–10], effective field theory [11–13], Dyson-Schwinger equations [14–16], the chiral quark model [17,18], the chiral unitary approach [19–23], and so on. Chiral dynamics for meson-meson and meson-baryon interaction has played an important role in understanding the nature and structure of hadronic resonances, and it has shown that many known resonances are generated dynamically as a natural consequence of the hadron-hadron interaction, much as the deuteron appears as a simple bound state of a proton and a neutron.

Following the spirit of the approach of Refs. [19,24], in Ref. [25] the kernel of the interaction (potential) of pseudoscalar mesons was evaluated starting from the chiral Lagrangians [26–30], and then, implementing unitarity in coupled channels, the scalar meson resonances σ [or $f_0(500)$], $f_0(980)$, $a_0(980)$ were dynamically produced, with phase shifts, inelasticities, mass distributions of given channels consistent with the experimental data (the

consistency of the results with QCD sum rules can be seen in Refs. [31,32]). Along the same lines, the unitarity approach with the coupled channels explains successfully both the experimental data for the light scalar mesons [25,33,34] (such as $a_0(980)$, $f_0(980)$, σ, and κ [or $K^*(800)$]) and the light baryons [21,24,35–39], two $\Lambda(1405)$, $\Lambda(1670)$, $N^*(1535)$, $\Delta(1620)$, etc. Extrapolation of this dynamics to the charm sector has also produced many meson states, as the $D_{s0}^*(2317)$, $D_0^*(2400)$, $X(3700)$, $X(3872)$, etc. [40–44], as well as baryon states like the $\Lambda_c(2595)$ [45–47]. More work on the $K\pi$ interaction is done in Refs. [48–50], where starting from the chiral Lagrangian and taking into account unitarity, the S-wave $K\pi$ elastic scattering amplitude is evaluated and good agreement with the experimental phase shifts is obtained. In addition, the scalar resonance κ is generated dynamically, which is also seen in the final state interaction in some reactions [51].

The three-body interaction is another subject in the hadron physics that is also drawing much attention for a long time [52–56]. Combining the three-body Faddeev equations with chiral dynamics, Ref. [57] has reported several S-wave $J^P = \frac{1}{2}^+$ resonances that qualify as two meson-one baryon molecular states. This combination of Faddeev equations and chiral dynamics produces results consistent with QCD sum rules in the investigation of the $DK\bar{K}$ system in the work of Ref. [58]. On the other hand, by taking the fixed center approximation (FCA) [53,59–63] to Faddeev equations, several multi-$\rho(770)$ states are dynamically produced in Ref. [64], in which the resonances $f_2(1270)(2^{++})$, $\rho_3(1690)(3^{--})$, $f_4(2050)(4^{++})$, $\rho_5(2350)(5^{--})$, and $f_6(2510)(6^{++})$ are theoretically found as basically molecules of an increasing number of $\rho(770)$ particles with parallel spins. Analogously, in Ref. [65], the resonances $K_2^*(1430)$, $K_3^*(1780)$, $K_4^*(2045)$, $K_5^*(2380)$ and a new K_6^* could be explained as molecules with the components of an increasing number of $\rho(770)$ and one $K^*(892)$ meson. In the same direction, the work of Ref. [66] predicts several charmed resonances, D_3^*, D_4^*, D_5^* and D_6^*. With the same

*liangwh@gxnu.edu.cn

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approach, the $\Delta_{\frac{1}{2}^+}(2000)$ puzzle is solved in Ref. [67] in the study of the $\pi - (\Delta\rho)$ interaction. The FCA to Faddeev equations is technically simple, and allows one to deal with three-body hadron interactions that would be otherwise rather cumbersome [68–71]. As discussed in Ref. [69], this method is accurate when dealing with bound states, and gets consistent results with the full Faddeev equation evaluation without taking FCA [72], or a variational calculation with a nonrelativistic three-body potential model [73] (more discussions can be seen in Ref. [74]). One should also know the limits of the applicability of the FCA, and one should avoid the case in which the states have enough energy to excite its components in intermediate states [75], which is the case of resonances above threshold. Recently, the results of the FCA to Faddeev equations have been confirmed by the variational method approach with the effective one-channel Hamiltonian in Ref. [76], which uses the two methods to study the DNN system and predicts a narrow quasibound state with the mass of about 3500 MeV.

In our present work we will use the FCA to Faddeev equations to investigate the $\eta K\bar{K}$ and $\eta'K\bar{K}$ systems. When studied in S wave, provided the strength of the interaction allows for it, these systems could give rise to η states. There are many η excited states, the lowest ones the $\eta(1295)$, $\eta(1405)$ and $\eta(1475)$. Since we do not want states too far from threshold, the $\eta(1475)$ could be in principle a candidate for the $\eta K\bar{K}$ system. For the $\eta'K\bar{K}$ system we would have to look for an η state around 1930 MeV. There are two η states around this energy in the Particle Data Group (PDG) [77], the $\eta(1760)$ and the $\eta(2225)$, both far away from the $\eta'K\bar{K}$ threshold mass. There is a peak seen at 1870 MeV in the $J/\psi \rightarrow \eta\pi^+\pi^-$

in Ref. [78], but its quantum numbers are not well determined. Similarly, there is another peak seen in the $J/\psi \rightarrow \eta'\pi^+\pi^-$ reaction that peaks around 1836 MeV [$X(1835)$], with a large width of about 190 MeV [79]. We shall explore the possibility that the $\eta'K\bar{K}$ could be responsible for any of such states, although we anticipate that the interaction is too weak to lead to such strongly bound systems.

The $\eta K\bar{K}$ and $\eta'K\bar{K}$ systems have been investigated before in Ref. [80], following the lines of Ref. [81], where it was concluded that the $\eta K\bar{K}$ system could be the $\eta(1475)$ resonance, and the $\eta'K\bar{K}$ the $X(1835)$. Yet, in Ref. [75] it was discussed that the method of Ref. [81] contains some element of uncertainty which makes it most opportune to perform calculations with a different method and contrast the predictions. Certainly, there are also other options for these resonances using quark models and other approaches and a detailed discussion on it can be found in the Introduction of Ref. [80]. In the present paper we will explore the possible molecular structure of these three-body systems.

II. MULTIBODY INTERACTION FORMALISM

The Faddeev equations under the FCA are an effective tool to deal with multihadron interaction [59–71]. They are particularly suited to study systems in which a pair of particles cluster together and the cluster is not much modified by the third particle. The FCA to Faddeev equations assumes a pair of particles (1 and 2) forming a cluster. Then particle 3 interacts with the components of the cluster, undergoing all possible multiple scattering with those components. This is depicted in Fig. 1. With this basic idea of the FCA, we can write the Faddeev equations
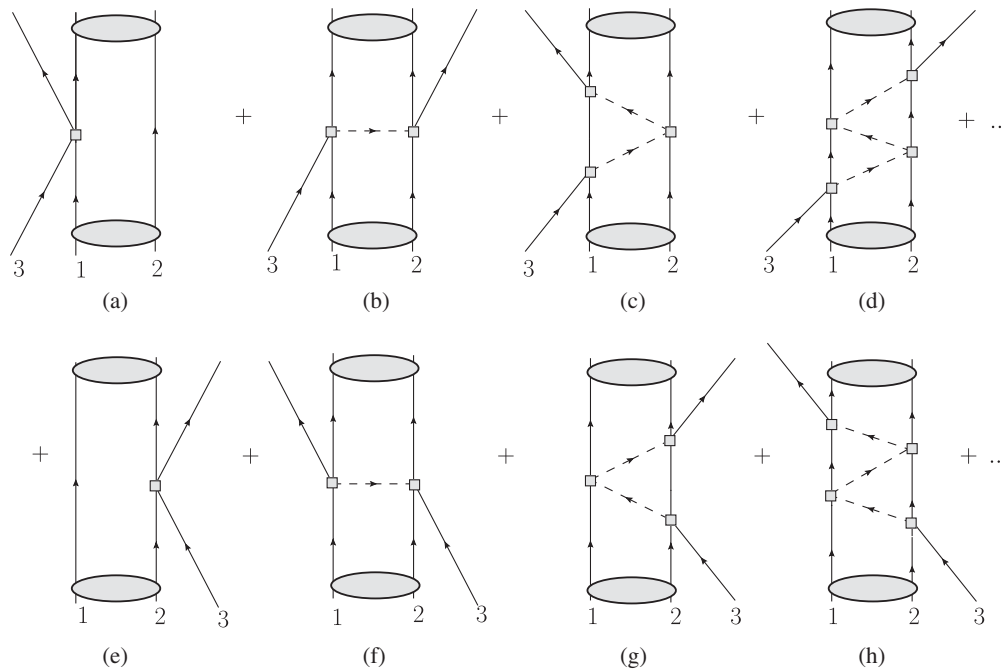

FIG. 1. Diagrammatic representation of the FCA to Faddeev equations.

easily. For this one defines two partition functions T_1 and T_2, which sum all diagrams of the series of Fig. 1 that begin with the interaction of particle 3 with particle 1 of the cluster (T_1), or with the particle 2 (T_2). The equations then read

$$T_1 = t_1 + t_1 G_0 T_2, \tag{1}$$

$$T_2 = t_2 + t_2 G_0 T_1, \tag{2}$$

$$T = T_1 + T_2, \tag{3}$$

where T is the total three-body scattering amplitude that we are looking for. The amplitudes t_1 and t_2 represent the unitary scattering amplitudes with coupled channels for the interactions of particle 3 with particle 1 and 2, respectively. And G_0 is the propagator of particle 3 between the components of the two-body system. In our case we will take the $K\bar{K}$ forming a cluster of the $f_0(980)$ (we shall discuss the $a_0(980)$ also), as appears in the chiral unitary approach [25,82–85] and the η or η' will be the particle 3.

For the unitary amplitudes corresponding to single-scattering contribution, one must take into account the isospin structure of the cluster and write the t_1 and t_2 amplitudes in terms of the isospin amplitudes of the (3,1) and (3,2) systems. But this is trivial in the present case since the η, η' have I = 0 and hence the ηK, $\eta' K$ are in I = 1/2. Besides, because of the normalization of Mandl and Shaw [86], which has different weight factors for the particle fields, we must take into account how these factors appear in the single scattering and double scattering and in the total amplitude. This is easy and is done in detail in Refs. [65,68]. We show below the details for the present case of a meson cluster (also particle 1 and 2) and a meson as scattering particle (the third particle). In this case, following the field normalization of Ref. [86] we find for the S matrix of single scattering,

$$S_1^{(1)} = -i t_1 (2\pi)^4 \delta(k + k_R - k' - k'_R) \times \frac{1}{\mathcal{V}^2} \frac{1}{\sqrt{2\omega_3}}$$
$$\times \frac{1}{\sqrt{2\omega'_3}} \frac{1}{\sqrt{2\omega_1}} \frac{1}{\sqrt{2\omega'_1}}, \tag{4}$$

$$S_2^{(1)} = -i t_2 (2\pi)^4 \delta(k + k_R - k' - k'_R) \times \frac{1}{\mathcal{V}^2} \frac{1}{\sqrt{2\omega_3}}$$
$$\times \frac{1}{\sqrt{2\omega'_3}} \frac{1}{\sqrt{2\omega_2}} \frac{1}{\sqrt{2\omega'_2}}, \tag{5}$$

where, k, k' (k_R, k'_R) refer to the momentum of initial, final scattering particle (R for the cluster), ω_i, ω'_i are the energies of the initial, final particles, $\mathcal{V}$ is the volume of the box where the states are normalized to unity and the subscripts 1, 2 refer to scattering with particle 1 or 2 of the cluster.

The double scattering diagram, Fig. 1(b), is given by

$$S^{(2)} = -i(2\pi)^4 \delta(k + k_R - k' - k'_R) \frac{1}{\mathcal{V}^2} \frac{1}{\sqrt{2\omega_3}} \frac{1}{\sqrt{2\omega'_3}} \frac{1}{\sqrt{2\omega_1}}$$
$$\times \frac{1}{\sqrt{2\omega'_1}} \frac{1}{\sqrt{2\omega_2}} \frac{1}{\sqrt{2\omega'_2}} \int \frac{d^3q}{(2\pi)^3} F_R(q)$$
$$\times \frac{1}{q^{0^2} - \vec{q}^2 - m_3^2 + i\epsilon} t_1 t_2, \tag{6}$$

where $F_R(q)$ is the cluster form factor that we shall discuss below. Similarly, the full S matrix for scattering of particle 3 with the cluster will be given by

$$S = -iT(2\pi)^4 \delta(k + k_R - k' - k'_R)$$
$$\times \frac{1}{\mathcal{V}^2} \frac{1}{\sqrt{2\omega_3}} \frac{1}{\sqrt{2\omega'_3}} \frac{1}{\sqrt{2\omega_R}} \frac{1}{\sqrt{2\omega'_R}}. \tag{7}$$

In view of the different normalization of these terms by comparing Eqs. (4)–(7), we can introduce suitable factors in the elementary amplitudes,

$$\tilde{t}_1 = \frac{2M_R}{2m_1} t_1, \qquad \tilde{t}_2 = \frac{2M_R}{2m_2} t_2, \tag{8}$$

with m_1, m_2, M_R the masses of the particles 1, 2 and the cluster, respectively, where we have taken the approximations, suitable for bound states, $\frac{1}{\sqrt{2\omega_i}} = \frac{1}{\sqrt{2m_i}}$, and sum all the diagrams by means of

$$T = T_1 + T_2 = \frac{\tilde{t}_1 + \tilde{t}_2 + 2\tilde{t}_1\tilde{t}_2 G_0}{1 - \tilde{t}_1\tilde{t}_2 G_0^2}. \tag{9}$$

When $\tilde{t}_1 = \tilde{t}_2$, as is the case here for the ηK and $\eta \bar{K}$ interaction, it can be simplified as

$$T = \frac{2\tilde{t}_1}{1 - \tilde{t}_1 G_0}. \tag{10}$$

The function G_0 in Eqs. (9) and (10) is given by

$$G_0(s) = \frac{1}{2M_R} \int \frac{d^3\vec{q}}{(2\pi)^3} F_R(q) \frac{1}{q^{0^2} - \vec{q}^2 - m_3^2 + i\epsilon}, \tag{11}$$

where $F_R(q)$ is the form factor of the cluster of particles 1 and 2. We must use the form factor of the cluster consistently with the theory used to generate the cluster as a dynamically generated resonance. This requires to extend the formalism of the chiral unitary approach, developed for scattering amplitudes, to the wave functions. This work has been done in Refs. [87–89] for S-wave bound states, S-wave resonant states and states with arbitrary angular momentum, respectively. Here we only need the expressions for S-wave bound states, and then the expression for the form factors is given in Sec. 4 of Ref. [88] by

$$F_R(q) = \frac{1}{\mathcal{N}} \int_{|\vec{p}|<\Lambda', |\vec{p}-\vec{q}|<\Lambda'} d^3\vec{p} \frac{1}{2E_1(\vec{p})} \frac{1}{2E_2(\vec{p})}$$
$$\times \frac{1}{M_R - E_1(\vec{p}) - E_2(\vec{p})} \frac{1}{2E_1(\vec{p}-\vec{q})}$$
$$\times \frac{1}{2E_2(\vec{p}-\vec{q})} \frac{1}{M_R - E_1(\vec{p}-\vec{q}) - E_2(\vec{p}-\vec{q})},$$
$$(12)$$

$$\mathcal{N} = \int_{|\vec{p}|<\Lambda'} d^3\vec{p} \left(\frac{1}{2E_1(\vec{p})} \frac{1}{2E_2(\vec{p})} \right.$$
$$\left. \times \frac{1}{M_R - E_1(\vec{p}) - E_2(\vec{p})} \right)^2, \quad (13)$$

where E_1 and E_2 are the energies of the particles 1, 2 and M_R the mass of the cluster. The parameter Λ' is a cutoff that regularizes the integrals of Eqs. (12) and (13). This cutoff is the same that one needs in the regularization of the loop function of the two particle propagators in the study of the interaction of the two particles of the cluster [88]. As done in Refs. [65,68], we take the value of Λ' the same as the cutoff q_{max} used to generate the resonance in the two-body interaction, which is the parameter to produce the cluster of $f_0(980)$ or $a_0(980)$ in our present work and will be discussed in the next section. Thus we do not introduce any free parameters in the present procedure.

In addition, q^0, the energy carried by particle 3 in the rest frame of the three-particle system, is given by

$$q^0(s) = \frac{s + m_3^2 - M_R^2}{2\sqrt{s}}. \quad (14)$$

Note also that the arguments of the amplitudes $T_i(s)$ and $t_i(s_i)$ are different, where s is the total invariant mass of the three-body system, and s_i are the invariant masses in the two-body systems. The value of s_i is given by [65]

$$s_i = m_3^2 + m_i^2 + \frac{(M_R^2 + m_i^2 - m_j^2)(s - m_3^2 - M_R^2)}{2M_R^2},$$
$$(i, j = 1, 2, i \neq j), \quad (15)$$

where m_l ($l = 1, 2, 3$) are the masses of the corresponding particles in the three-body system and M_R the mass of two-body resonance or bound state (cluster).

III. $K\bar{K}$ AND $\eta K(\eta' K)$ TWO-BODY INTERACTIONS

To evaluate the Faddeev equations under the FCA, we need to define the two-body cluster and then let the third particle collide with the cluster. Thus, the starting point of our work is to look for the cluster in the two-body interactions. Following the formalism of Ref. [25], by taking into account the chiral dynamics and the unitary coupled channels approach [21,24,25,33,35,37–39,42,48,50], we should reproduce the resonances $f_0(980)$ and $a_0(980)$ as

the cluster of FCA. We briefly summarize the method of Ref. [25] here.

To calculate the scattering amplitudes with the coupled channels unitary approach, the Bethe-Salpeter equation in coupled channels, with the factorized on-shell potentials [21,25] is used,

$$t = [1 - VG]^{-1}V, \quad (16)$$

where the kernel V is a matrix of the interaction potentials between the channels, given by [25]

$$V_{11}^{I=0}(s) = -\frac{1}{2f_\pi^2}(2s - m_\pi^2),$$
$$V_{12}^{I=0}(s) = -\frac{\sqrt{3}}{4f_\pi^2}s, \quad (17)$$
$$V_{22}^{I=0}(s) = -\frac{3}{4f_\pi^2}s,$$

$$V_{11}^{I=1}(s) = -\frac{1}{3f_\pi^2}m_\pi^2,$$
$$V_{12}^{I=1}(s) = \frac{\sqrt{6}}{36f_\pi^2}(9s - 8m_K^2 - m_\pi^2 - 3m_\eta^2), \quad (18)$$
$$V_{22}^{I=1}(s) = -\frac{1}{4f_\pi^2}s,$$

with f_π the pion decay constant. Note that in isospin $I = 0$, there are two coupled channels, 1 is $\pi\pi$ and 2 is $K\bar{K}$; for $I = 1$, channel 1 denotes $\pi^0\eta$ and 2 as $K\bar{K}$.

In Eq. (16) G is a diagonal matrix of the loop function of two mesons in the i channel, given by

$$G_i(s) = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{(P-q)^2 - m_1^2 + i\varepsilon} \frac{1}{q^2 - m_2^2 + i\varepsilon}, \quad (19)$$

where m_1, m_2 are the masses of the mesons in the i channel, q is the four-momentum of one meson, and P is the total four-momentum of the system, thus, $s = P^2$. Note that the integral of Eq. (19) is logarithmically divergent. Then, using a cutoff momentum to regularize it, we have

$$G_i(s) = \int_0^{q_{max}} \frac{d^3\vec{q}}{(2\pi)^3} \frac{\omega_1 + \omega_2}{2\omega_1\omega_2} \frac{1}{P^{02} - (\omega_1 + \omega_2)^2 + i\varepsilon}, \quad (20)$$

where $\omega_i = \sqrt{\vec{q}^2 + m_i^2}$, $(i = 1, 2)$, and q_{max} is the cutoff of the three-momentum, the free parameter. Also the analytic formula of Eq. (20) can be seen in Refs. [33,50]. On the other hand, the analytic expression of the dimensional regularization for Eq. (19) can be seen in Ref. [21] (more discussions about G_i are also seen in Refs. [90–92]) with a scale μ fixed *a priori* and the subtraction constant $a(\mu)$ as free parameter.

Taking $\Lambda = 1.03$ GeV and $\Lambda = \sqrt{q_{max}^2 + m_K^2}$ as done in Ref. [25], we get $q_{max} = 903$ MeV. Our results are shown in Fig. 2. In Fig. 2, we produce the resonances of

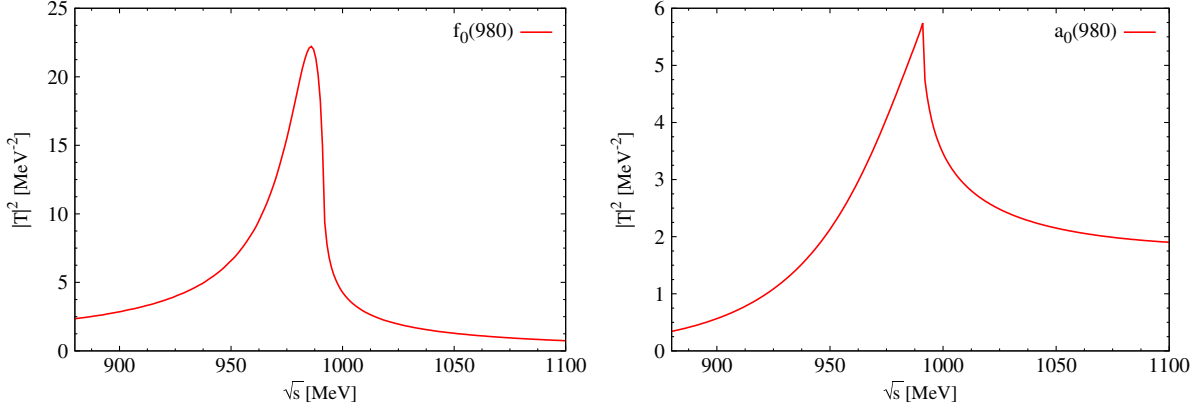FIG. 2 (color online). Modulus squared of the scattering amplitudes. Left: $|t_{12}^{I=0}|^2/10^3$, $f_0(980)$; Right: $|t_{22}^{I=1}|^2/10^3$, $a_0(980)$.

$f_0(980)$ and $a_0(980)$, which are consistent with Ref. [25] and form the clusters of the $\eta K\bar{K}$ and $\eta' K\bar{K}$ three-body interactions in our present work.

As discussed in the former section, to perform the evaluation of Faddeev equations under the FCA, we need the calculation of the two-body interaction amplitudes ($\tilde{t}_1$ and $\tilde{t}_2$) of ηK and $\eta\bar{K}$ for the $\eta K\bar{K}$ system ($\eta' K$ and $\eta'\bar{K}$ for the $\eta' K\bar{K}$ system), which are investigated in Refs. [48–50] as mentioned before. The former input is needed to construct the form factor of the cluster entering Eq. (11).

Next we address the ηK, $\eta\bar{K}$ and $\eta' K$, $\eta'\bar{K}$ interaction. Since we are involving the η', it is convenient to take the three coupled channels πK, ηK and $\eta' K$, labeled by channel 1, 2 and 3, respectively. Thus, the potentials are [50]

$$V_{11}^{I=1/2}(s) = -\frac{1}{4f_\pi^2}(4s + 3t - 4m_\pi^2 - 4m_K^2), \quad (21)$$

$$V_{12}^{I=1/2}(s) = -\frac{\sqrt{2}}{6f_\pi^2}(-3t + 2m_K^2 + m_\eta^2), \quad (22)$$

$$V_{13}^{I=1/2}(s) = \frac{1}{12f_\pi^2}(-3t + 3m_\pi^2 + 8m_K^2 + m_{\eta'}^2), \quad (23)$$

$$V_{22}^{I=1/2}(s) = -\frac{2}{9f_\pi^2}(3t - m_K^2 - 2m_\eta^2), \quad (24)$$

$$V_{23}^{I=1/2}(s) = \frac{\sqrt{2}}{18f_\pi^2}(3t - 3m_\pi^2 + 2m_K^2 - m_\eta^2 - m_{\eta'}^2), \quad (25)$$

$$V_{33}^{I=1/2}(s) = -\frac{1}{36f_\pi^2}(3t - 6m_\pi^2 + 32m_K^2 - 2m_{\eta'}^2), \quad (26)$$

where there is a minus sign difference with Refs. [48,80] in some nondiagonal matrix elements resulting from taking different phase conventions.[1] As done in Ref. [50], we take

$$\mu = m_K, \qquad a(\mu) = -1.383, \quad (27)$$

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[1]The final scattering amplitudes are the same, as pointed out by J. A. Oller.

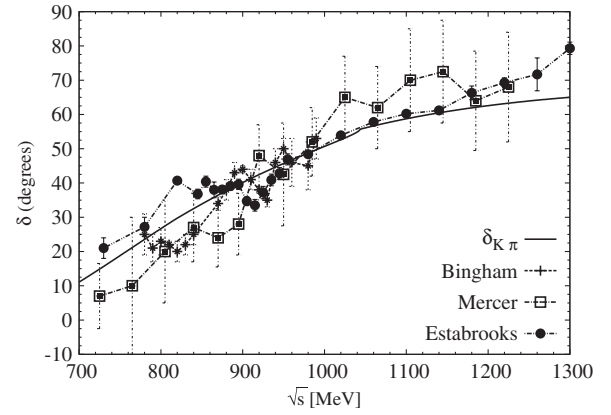
FIG. 3.   The S-wave $K\pi$ phase shifts in isospin $I = 1/2$. The experimental data are taken from Mercer [93], Bingham [94], Estabrooks [95].

in the loop function for all channels, and we obtain the same results as in Ref. [50], seen in Fig. 3, which agree fairly well with the data except at the higher energies.

With these parameters, we also find the pole of κ [or $K^*(800)$], $(743.72 - i275.36)$ MeV, which is consistent with the result of Ref. [50], $(0.742 - i0.273)$ GeV. Then, using these parameters, we can get the ηK and $\eta' K$ scattering amplitudes. Because of charge conjugation symmetry, the amplitude for $\eta\bar{K}$ ($\eta'\bar{K}$) is the same as that for ηK ($\eta' K$).

IV. $\eta K\bar{K}$ AND $\eta' K\bar{K}$ THREE-BODY INTERACTIONS

As discussed in the former section, we calculate the ηK and $\eta\bar{K}$ ($\eta' K$ and $\eta'\bar{K}$) amplitudes using the same parameters, and then we use Eq. (10) to evaluate the three-body amplitude of the $\eta K\bar{K}$ ($\eta' K\bar{K}$) system. Also, as discussed in the former section, the Λ' of Eq. (12) can be taken as $q_{max} = 903$ MeV for the cluster of $f_0(980)$ or $a_0(980)$.

In Fig. 4 (left), we can see a clear resonance structure in the modulus squared of the $\eta K\bar{K}$ scattering amplitude, which is around 1490 MeV, with the width of about
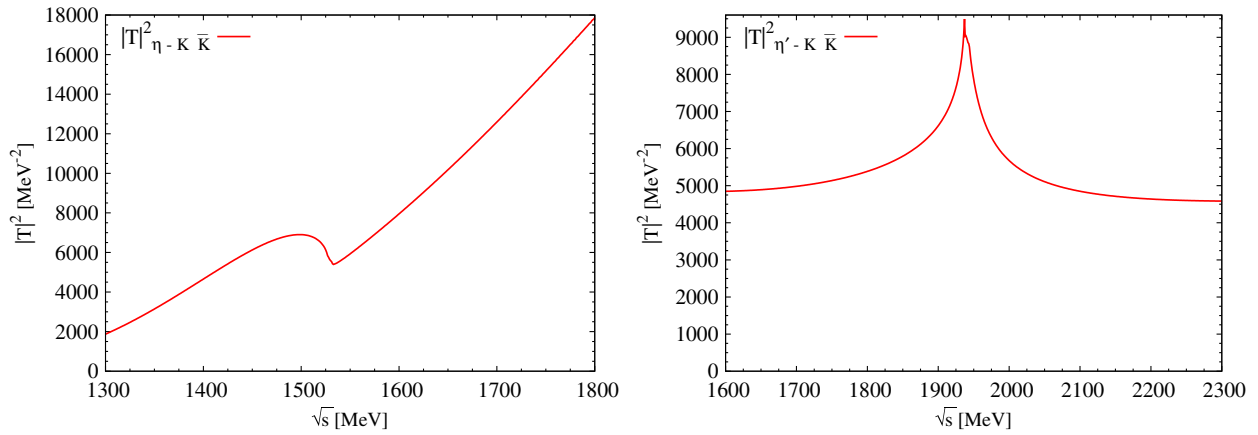
FIG. 4 (color online).   Modulus squared of the three-body interaction amplitudes. Left: $|T_{\eta K \bar{K}}|^2$; Right: $|T_{\eta' K \bar{K}}|^2$.

100 MeV, and about 38 MeV below the threshold of $\eta f_0(980)$. This result is consistent with the one found in Ref. [80]. From the PDG [77], this resonance may be the $\eta(1475)$ of $I = 0$, with mass 1476 ± 4 MeV and width 85 ± 9 MeV. Comparing our results with the PDG, both the mass and the width are consistent with the experimental values if we assume 10–15 MeV uncertainties in our calculated results.

Since the masses of the $K\bar{K}$ bound states $f_0(980)$ and $a_0(980)$ are the same and only their isospins are different, the three-body amplitudes of $\eta K \bar{K}$ and $\eta' K \bar{K}$ in our formalism are degenerated in isospin $I = 0$ and $I = 1$. This means that if we predict a bound state for the $\eta f_0(980)$ system, we also have the same for $\eta a_0(980)$. This is so, assuming that the $f_0(980)$ and $a_0(980)$ resonances are predominantly $K\bar{K}$ molecules. But, as we have discussed, in the construction of the $f_0(980)$ resonance we need the $\pi\pi$ and $K\bar{K}$ channels, and the $\pi\pi$ is marginal in the structure of the resonance; it simply provides a decay mode. However, this is not the case for the $a_0(980)$ where the $\pi\eta$ channel already plays an important role in the buildup of the resonance. Then a more elaborate, and technically complex, study of the η, η' interacting with this system, would have much contribution from $\eta\eta$, which only comes from coupled channels and is very weak, and $\eta\pi$ which is also weak. The signal that we get in Fig. 4 would be much diluted and we do not expect an $I = 1$ state.

We also see an obvious peak in Fig. 4 (right) for the $\eta' K \bar{K}$ interaction. But the mass position of the peak is about 1940 MeV, which is very close to threshold, 1942 MeV. Therefore, this peak should be an enhancement effect of the threshold, a cusp effect, and we will check it further in the next section.

V. FURTHER DISCUSSIONS

We showed the results of our investigation of the $\eta K \bar{K}$ and $\eta' K \bar{K}$ systems in the former section. For the $\eta K \bar{K}$ scattering, we find one resonance structure in the modulus

squared of amplitude. But, for the other one, the clear peak of the $\eta K \bar{K}$ amplitude turns into an enhancement effect at the threshold in the $\eta' K \bar{K}$ amplitude, a cusp effect reflecting the cusp of the $\tilde{t}_1$ amplitude, used in Eq. (10), at threshold. In all these results we did not take into account the width of $f_0(980)$ as done in Ref. [66]. In the PDG, the width of the $f_0(980)$ is 40 to 100 MeV, which is not small compared to the binding energy found.

Following Ref. [66], we can take into account the width of the $f_0(980)$ in the three-body scattering amplitudes, just by replacing M_R in Eqs. (12) and (13) by $M_R - i\frac{\Gamma_R}{2}$. The new results are given in Fig. 5, where we just take the width as 60 MeV. For the $\eta K \bar{K}$ amplitude we can see in Fig. 5 (left), comparing with Fig. 4 (left), that the strength of the amplitude is reduced and the peak position is still not changed, but the width becomes a little larger (around 120 MeV), which is in line with the finding in Ref. [66]. For the $\eta' K \bar{K}$ amplitude, shown in Fig. 5 (right), by comparing to Fig. 4 (right), we can see that the strength at the peak is a bit increased and the shape changes a bit when considering the contribution of the width $f_0(980)$. The important thing, however, is that the shape of the $\eta' K \bar{K}$ amplitude continues to be that of a cusp effect. In summary, as discussed in Ref. [66], we can conclude that the effects of the contribution of the cluster's width are small and do not change the relevant features found before.

Next, we want to check the uncertainties in Eq. (10) when we make a small change in the parameters in the evaluation of $\tilde{t}_1$. Following Ref. [50], we can only change $a(\mu)$. This parameter was chosen in Ref. [50] to fit the experimental data of the $K\pi$ phase shifts. Then, we change 50% up and down the parameter $a(\mu)$ of Eq. (27), to a point where the $K\pi$ phase shifts are not too good, as shown in Fig. 6 (left). From Fig. 6 (right), we can see that the resonance structure in $\eta K \bar{K}$ scattering is not changed so much even with these extreme changes in the input, and both the peak position and the width have practically not changed. This gives us confidence that the results that we get are rather solid and do not change with small variations
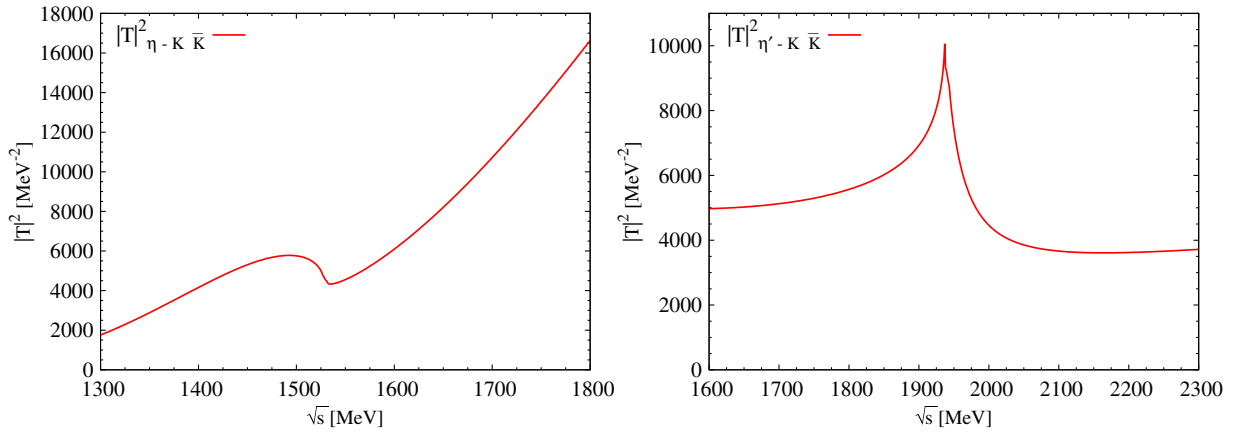
FIG. 5 (color online). Modulus squared of the three-body interaction amplitudes with the contribution of the width of $f_0(980)$ [or $a_0(980)$]. Left: $|T_{\eta K\bar{K}}|^2$; Right: $|T_{\eta' K\bar{K}}|^2$.
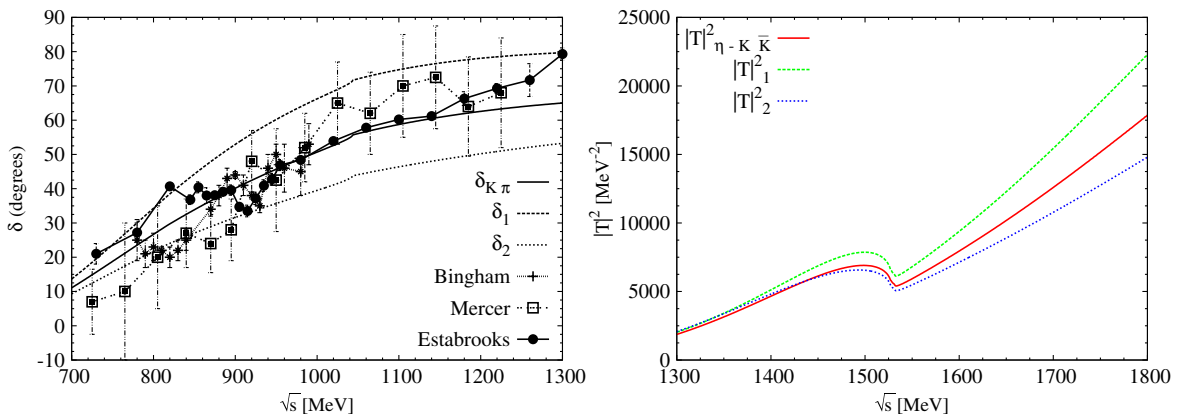

FIG. 6 (color online). The results for the change of the parameter in Eq. (27). Left: the $K\pi$ phase shifts, solid line is the fit one, the two dash line of δ_1, δ_2 are with 50% changes; Right: $|T_{\eta K\bar{K}}|^2$, solid line is with fit parameter, the two dash line of T_1, T_2 are with 50% changes of the fit.

of the parameters. The same changes only affect in a minor way the $\eta' K\bar{K}$ amplitude, and the cusp effect at threshold is the only relevant feature of the amplitude.

At this point one must comment on the results of Ref. [80]. In that work the interaction of the η with the $K\bar{K}$ cluster is done as here, although the formalism seems rather different. There the primary amplitude for the η interaction with the components of the cluster is evaluated and then the η and the cluster propagate similarly to the propagation of the meson components in the G function. The caveat is that while the regularization parameters are fitted to data on meson scattering, here one does not have this information for the scattering of η and $f_0(980)$ and one must make assumptions on how this new loop is regularized. As a consequence, there is an element of uncertainty and usually what one assumes is that the interaction gives rise to a certain resonance to fix the parameters, although they are kept within a natural range; there is, hence, not a genuine prediction. In that work, the $\eta f_0(980)$ gives rise to the $\eta(1475)$ as we have also claimed here. But the

$\eta' f_0(980)$ is claimed to produce the $X(1835)$ resonance, something that our approach does not give.

The difference between the $\eta K\bar{K}$ and $\eta' K\bar{K}$ systems could be qualitatively understood by recalling that the ηK, together with the πK system, generate the broad κ resonance, but the $\eta' K$ amplitude has no structure around the $\eta' K$ energies (up to the unavoidable cusp at threshold) and is small and smooth around these energies. In order to see how far we are from creating a resonance structure in the $\eta' K\bar{K}$ system, we artificially multiply V_{33} of Eq. (26) by a factor and look at the $\eta' K\bar{K}$ amplitude. We must multiply by a factor of 4 the V_{33} potential to see the peak move a bit (by about 6 MeV) below the threshold. Since the uncertainties of the model are by no means that large (we can accept about 20% uncertainty in the potentials), the former exercise tells us that the cusp character of the $\eta' K\bar{K}$ amplitude is quite a stable result and we cannot associate a physical η state to it.

We should comment on the paper [96], where using the Faddeev approach in the version of Ref. [57], one peak in

$|T|^2$ for $\pi K\bar{K}$ is found around 1400 MeV, which is associated to the $\pi(1300)$. It is mentioned there that the $\eta K\bar{K}$ system is also investigated and no clear signal is seen. The coupled channels approach used there contains πK and ηK but not $\eta'K$. We have checked that removing the $\eta'K$ channel does not change qualitatively the $\eta K\bar{K}$ amplitude, although the distribution of $|T|^2$ in energy has a broader shape. Consideration of the $\eta'K$ channel makes the energy distribution a little sharper. The fact that no clear peak for the $\eta K\bar{K}$ amplitude appears is somewhat unexpected, since one usually gets qualitative agreement between the FCA and the Faddeev calculations for bound states. For instance, the three-body $\bar{K}NK$ scattering amplitude was calculated using the FCA to the Faddeev equations in Ref. [67], and the results of that work are in good agreement with the other theoretical works [72,73] evaluated using variational and Faddeev approaches, respectively. The same can be said when one studies the $\bar{K}NN$ system in the FCA [71] or in Faddeev calculations [97], or variational calculations [98]. The DNN system is another case of agreement between the FCA and variational calculation [76]. We state the present situation and call for further calculations of the $\eta K\bar{K}$ system using different approaches in order to clarify the situation.

VI. CONCLUSIONS

In our work, we study the three-body systems of $\eta K\bar{K}$ and $\eta'K\bar{K}$, by using the fixed center approximation to the Faddeev equations. The clusters of $f_0(980)$ for the fixed center approximation are successfully reproduced by the chiral unitary approach. With this approach, the experimental S-wave $K\pi$ phase shifts of isospin $I = 1/2$ are also well fitted. For the three-body scattering we find a resonant structure in the $\eta K\bar{K}$ scattering amplitude, which may correspond to the $\eta(1475)$ state for $I = 0$. This finding is consistent with the result of Ref. [80]. We also make an estimation of our theory uncertainties for this state by taking into account the contribution of the cluster's width and reasonable changes in the free parameters, and we get stable results. As for the $\eta'K\bar{K}$ scattering, we only get an enhancement effect at the threshold in the modulus squared of the interaction amplitude, and we cannot claim that this can be associated to any resonance.

[1] V. Crede and W. Roberts, Rep. Prog. Phys. **76**, 076301 (2013).
[2] E. Klempt and J.-M. Richard, Rev. Mod. Phys. **82**, 1095 (2010).
[3] J. B. Kogut, Rev. Mod. Phys. **55**, 775 (1983).
[4] M. Luscher, S. Sint, R. Sommer, and P. Weisz, Nucl. Phys. **B478**, 365 (1996).
[5] M. Luscher, S. Sint, R. Sommer, P. Weisz, and U. Wolff, Nucl. Phys. **B491**, 323 (1997).
[6] D. Mohler, S. Prelovsek, and R. M. Woloshyn, Phys. Rev. D **87**, 034501 (2013).
[7] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979).
[8] L. J. Reinders, H. Rubinstein, and S. Yazaki, Phys. Rep. **127**, 1 (1985).
[9] J. M. Dias, R. M. Albuquerque, M. Nielsen, and C. M. Zanetti, Phys. Rev. D **86**, 116012 (2012).
[10] C. Hidalgo-Duque, J. Nieves, A. Ozpineci, and V. Zamiralov, arXiv:1305.4487.
[11] H. D. Politzer and M. B. Wise, Phys. Lett. B **208**, 504 (1988).

[12] H. Georgi, Phys. Lett. B **240**, 447 (1990).
[13] E. Epelbaum, H.-W. Hammer, and U.-G. Meissner, Rev. Mod. Phys. **81**, 1773 (2009).
[14] C. D. Roberts and A. G. Williams, Prog. Part. Nucl. Phys. **33**, 477 (1994).
[15] P. Maris and C. D. Roberts, Int. J. Mod. Phys. E **12**, 297 (2003).
[16] C. S. Fischer, J. Phys. G **32**, R253 (2006).
[17] A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984).
[18] Z. Y. Zhang, Y. W. Yu, P. N. Shen, L. R. Dai, A. Faessler, and U. Straub, Nucl. Phys. **A625**, 59 (1997).
[19] N. Kaiser, P. B. Siegel, and W. Weise, Phys. Lett. B **362**, 23 (1995).
[20] J. A. Oller, E. Oset, and A. Ramos, Prog. Part. Nucl. Phys. **45**, 157 (2000).
[21] J. A. Oller and U. G. Meissner, Phys. Lett. B **500**, 263 (2001).
[22] T. Hyodo, D. Jido, and A. Hosaka, Phys. Rev. C **78**, 025203 (2008).
[23] E. Oset, L. S. Geng, D. Gamermann, M. J. Vicente Vacas, D. Strottman, K. P. Khemchandani, A. Martinez Torres,

J. A. Oller, L. Roca, and M. Napsuciale, Int. J. Mod. Phys. E **18**, 1389 (2009).

[24] N. Kaiser, P. B. Siegel, and W. Weise, Nucl. Phys. **A594**, 325 (1995).

[25] J. A. Oller and E. Oset, Nucl. Phys. **A620**, 438 (1997); **A652**, 407(E) (1999).

[26] J. Gasser and H. Leutwyler, Nucl. Phys. **B250**, 465 (1985).

[27] U. G. Meissner, Rep. Prog. Phys. **56**, 903 (1993).

[28] A. Pich, Rep. Prog. Phys. **58**, 563 (1995).

[29] G. Ecker, Prog. Part. Nucl. Phys. **35**, 1 (1995).

[30] V. Bernard, N. Kaiser, and U.-G. Meissner, Int. J. Mod. Phys. E **04**, 193 (1995).

[31] H.-X. Chen, A. Hosaka, and S.-L. Zhu, Phys. Rev. D **76**, 094025 (2007).

[32] Y.-R. Liu and S.-L. Zhu, Phys. Rev. D **75**, 034003 (2007).

[33] J. A. Oller, E. Oset, and J. R. Pelaez, Phys. Rev. D **59**, 074001 (1999); **60**, 099906(E) (1999); **75**, 099903(E) (2007).

[34] L. Y. Dai, X. G. Wang and H. Q. Zheng, Commun. Theor. Phys. **57**, 841 (2012).

[35] E. Oset and A. Ramos, Nucl. Phys. **A635**, 99 (1998).

[36] T. Inoue, E. Oset, and M. J. Vicente Vacas, Phys. Rev. C **65**, 035204 (2002).

[37] D. Jido, J. A. Oller, E. Oset, A. Ramos, and U. G. Meissner, Nucl. Phys. **A725**, 181 (2003).

[38] C. Garcia-Recio, J. Nieves, E. Ruiz Arriola, and M. J. Vicente Vacas, Phys. Rev. D **67**, 076009 (2003).

[39] T. Hyodo, S. I. Nam, D. Jido, and A. Hosaka, Phys. Rev. C **68**, 018201 (2003).

[40] E. E. Kolomeitsev and M. F. M. Lutz, Phys. Lett. B **582**, 39 (2004).

[41] J. Hofmann and M. F. M. Lutz, Nucl. Phys. **A733**, 142 (2004).

[42] F.-K. Guo, P.-N. Shen, H.-C. Chiang, R.-G. Ping, and B.-S. Zou, Phys. Lett. B **641**, 278 (2006).

[43] D. Gamermann, E. Oset, D. Strottman, and M. J. Vicente Vacas, Phys. Rev. D **76**, 074016 (2007).

[44] D. Gamermann and E. Oset, Eur. Phys. J. A **33**, 119 (2007).

[45] J. Hofmann and M. F. M. Lutz, Nucl. Phys. **A763**, 90 (2005).

[46] T. Mizutani and A. Ramos, Phys. Rev. C **74**, 065201 (2006).

[47] L. Tolos, A. Ramos, and T. Mizutani, Phys. Rev. C **77**, 015207 (2008).

[48] J. A. Oller and E. Oset, Phys. Rev. D **60**, 074023 (1999).

[49] L. Li, B.-S. Zou, and G.-L. Li, Phys. Rev. D **67**, 034025 (2003).

[50] F. K. Guo, R. G. Ping, P. N. Shen, H. C. Chiang, and B. S. Zou, Nucl. Phys. **A773**, 78 (2006).

[51] J. A. Oller, Phys. Rev. D **71**, 054030 (2005).

[52] J. Fujita and H. Miyazawa, Prog. Theor. Phys. **17**, 360 (1957).

[53] L. D. Faddeev, Zh. Eksp. Teor. Fiz. **39**, 1459 (1960) [Sov. Phys. JETP **12**, 1014 (1961)].

[54] W. Glockle, T. S. H. Lee, and F. Coester, Phys. Rev. C **33**, 709 (1986).

[55] S. Weinberg, Phys. Lett. B **295**, 114 (1992).

[56] J. M. Richard, Phys. Rep. **212**, 1 (1992).

[57] A. Martinez Torres, K. P. Khemchandani, and E. Oset, Phys. Rev. C **77**, 042203 (2008).

[58] A. Martinez Torres, K. P. Khemchandani, M. Nielsen, and F. S. Navarra, Phys. Rev. D **87**, 034025 (2013).

[59] G. Toker, A. Gal, and J. M. Eisenberg, Nucl. Phys. **A362**, 405 (1981).

[60] R. C. Barrett and A. Deloff, Phys. Rev. C **60**, 025201 (1999).

[61] A. Deloff, Phys. Rev. C **61**, 024004 (2000).

[62] S. S. Kamalov, E. Oset, and A. Ramos, Nucl. Phys. **A690**, 494 (2001).

[63] A. Gal, Int. J. Mod. Phys. A **22**, 226 (2007).

[64] L. Roca and E. Oset, Phys. Rev. D **82**, 054013 (2010).

[65] J. Yamagata-Sekihara, L. Roca, and E. Oset, Phys. Rev. D **82**, 094017 (2010).

[66] C. W. Xiao, M. Bayar, and E. Oset, Phys. Rev. D **86**, 094019 (2012).

[67] J.-J. Xie, A. Martinez Torres, E. Oset, and P. Gonzalez, Phys. Rev. C **83**, 055204 (2011).

[68] J.-J. Xie, A. Martinez Torres, and E. Oset, Phys. Rev. C **83**, 065207 (2011).

[69] M. Bayar, J. Yamagata-Sekihara, and E. Oset, Phys. Rev. C **84**, 015209 (2011).

[70] C. W. Xiao, M. Bayar, and E. Oset, Phys. Rev. D **84**, 034037 (2011).

[71] M. Bayar and E. Oset, Nucl. Phys. **A883**, 57 (2012).

[72] A. Martinez Torres, K. P. Khemchandani, and E. Oset, Phys. Rev. C **79**, 065207 (2009).

[73] D. Jido and Y. Kanada-En'yo, Phys. Rev. C **78**, 035203 (2008).

[74] E. Oset, D. Jido, T. Sekihara, A. Martinez Torres, K. P. Khemchandani, M. Bayar, and J. Yamagata-Sekihara, Nucl. Phys. **A881**, 127 (2012).

[75] A. Martinez Torres, E. J. Garzon, E. Oset, and L. R. Dai, Phys. Rev. D **83**, 116002 (2011).

[76] M. Bayar, C. W. Xiao, T. Hyodo, A. Dote, M. Oka, and E. Oset, Phys. Rev. C **86**, 044004 (2012).

[77] J. Beringer *et al.* (Particle Data Group Collaboration), Phys. Rev. D **86**, 010001 (2012).

[78] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **107**, 182001 (2011).

[79] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **106**, 072002 (2011).

[80] M. Albaladejo, J. A. Oller, and L. Roca, Phys. Rev. D **82**, 094019 (2010).

[81] L. Alvarez-Ruso, J. A. Oller, and J. M. Alarcon, Phys. Rev. D **80**, 054011 (2009).

[82] N. Kaiser, Eur. Phys. J. A **3**, 307 (1998).

[83] M. P. Locher, V. E. Markushin, and H. Q. Zheng, Eur. Phys. J. C **4**, 317 (1998).

[84] J. Nieves and E. Ruiz Arriola, Nucl. Phys. **A679**, 57 (2000).

[85] J. R. Pelaez and G. Rios, Phys. Rev. Lett. **97**, 242002 (2006).

[86] F. Mandl and G. Shaw, *Quantum Field Theory* (Wiley, New York, 1984).

[87] D. Gamermann, J. Nieves, E. Oset, and E. Ruiz Arriola, Phys. Rev. D **81**, 014029 (2010).

[88] J. Yamagata-Sekihara, J. Nieves, and E. Oset, Phys. Rev. D **83**, 014003 (2011).

[89] F. Aceti and E. Oset, Phys. Rev. D **86**, 014012 (2012).

[90] C. Garcia-Recio, L. S. Geng, J. Nieves, and L. L. Salcedo, Phys. Rev. D **83**, 016007 (2011).

[91] J.-J. Wu, L. Zhao, and B. S. Zou, Phys. Lett. B **709**, 70 (2012).

[92] C. W. Xiao and E. Oset, Eur. Phys. J. A **49**, 139 (2013).

[93] R. Mercer, P. Antich, A. Callahan, C. Y. Chien, B. Cox, R. Carson, D. Denegri, L. Ettlinger *et al.*, Nucl. Phys. **B32**, 381 (1971).

[94] H. H. Bingham, W. M. Dunwoodie, D. Drijard, D. Linglin, Y. Goldschmidt-Clermont, F. Muller, T. G. Trippe, F. Grard *et al.*, Nucl. Phys. **B41**, 1 (1972).

[95] P. Estabrooks, R. K. Carnegie, A. D. Martin, W. M. Dunwoodie, T. A. Lasinski, and D. W. G. S. Leith, Nucl. Phys. **B133**, 490 (1978).

[96] A. Martinez Torres, K. P. Khemchandani, D. Jido, and A. Hosaka, Phys. Rev. D **84**, 074027 (2011).

[97] Y. Ikeda, H. Kamano, and T. Sato, Prog. Theor. Phys. **124**, 533 (2010).

[98] A. Dote, T. Hyodo, and W. Weise, Phys. Rev. C **79**, 014003 (2009).