# Role of vector and pseudoscalar mesons in understanding $1 / 2^{-} N^{*}$ and $\Delta$ resonances 

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#### Abstract

A study of nonstrange meson-baryon systems has been made with the idea of understanding the properties of the low-lying $1 / 2^{-} N^{*}$ and $\Delta$ resonances. The coupled channels are built by considering the pseudoscalar and vector mesons together with the octet baryons. The formalism is based on obtaining the interactions from the lowest order chiral Lagrangian when dealing with pseudoscalar mesons and relying on the hidden local symmetry in the case of the vector mesons. The transition between the two systems is obtained by replacing the photon by a vector meson in the Kroll-Ruderman theorem for the photoproduction of pseudoscalar mesons. The subtraction constants, required to calculate the loop function in the scattering equations, are constrained by fitting the available experimental data on some of the reactions with pseudoscalar meson-baryon final states. As a consequence, we find resonances which can be related to $N^{*}(1535), N^{*}(1650)$ (with a double pole structure), $N^{*}(1895)$, and $\Delta(1620)$. We conclude that these resonances can be interpreted, at least partly, as dynamically generated resonances and that the vector mesons play an important role in determining the dynamical origin of the low-lying $N^{*}$ and $\Delta$ states.


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## I. INTRODUCTION

Information on the properties of the excited states of the nonstrange baryons (i.e., those made of quarks $u$ and $d$ ) is highly sought after due to its relevance to nuclear and low energy hadron physics, which can be accessed at several experimental facilities around the world. To state a few examples, these resonances play an important role in understanding $N-N$ interaction [1,2]; in describing cross sections for the reaction with meson-nucleus final states (for instance, see Refs. [3-6]); in approaching some fundamental issues, such as the existence of multiquark states [7], the occurrence of OZI violating processes [8], etc. Such relevance serves as a motivation for the dedicated efforts made by several groups in extracting information related to $N^{*}$ and $\Delta$ resonances through different approaches, such as partial wave analysis of the relevant data [9], quark models [10], unitarized dynamical models (see, for example, Refs. [11-19] and those given in these papers), etc.

The motivation of the present article is in line with the above-mentioned works. To be more specific, we investigate if the meson-baryon dynamics, where pseudoscalar and vector mesons are considered, plays an important role in understanding the properties of isospin $1 / 2$ and $3 / 2$ nonstrange baryon resonances, especially the ones with spin parity $1 / 2^{-}$. This paper can be considered as a continuation of our previous studies of meson-baryon dynamics [20-22]. In Ref. [20], we studied the vector meson-baryon (VB) interaction in detail starting from an

[^0]$\mathrm{SU}(2)$ Lagrangian motivated by the gauge invariance of the hidden local symmetry (which treats vector mesons as gauge bosons) [23]. We found that such a gauge invariance of this Lagrangian compels the consideration of a contact interaction arising from the same Lagrangian. In addition, contrary to the case of the pseudoscalar-baryon (PB) systems, we found that the baryon exchange (in $s$ - and $u$-channels) diagram gives a contribution comparable to the one coming from the $t$-channel diagram (which gives the dominant contribution in the PB case). We showed that the sum of such diagrams leads to a spin-isospin dependent VB interaction. Further, a generalization to the $S U(3)$ case was made, and the results were found to be different from the ones obtained in Ref. [24], where the only $t$-channel diagrams were considered to study VB systems and several spin-degenerate resonances were found to couple strongly to vector mesons. However, we must mention that the work in Ref. [24] has been further extended by including the one pion loop contribution to the VB systems, and some interesting results have been found [25].

Coming back to our works, we further investigated the importance of coupling PB and VB systems with strangeness -1 in Ref. [21], keeping in mind the knowledge that the low-lying strange resonances seem to fit better in a meson-baryon molecular picture [26]. In Ref. [21], we used a simplified VB interaction since we concentrated on low-lying resonances (on which more information is available). We found that low-lying resonances couple strongly to VB systems, implying a large weight of VBB* vertices (where $B^{*}$ represents a baryon resonance) in, for instance, photoproduction processes. This finding motivated a fully coupled PB-VB channel calculation considering detailed VB interaction (as the one used in

Ref. [20]) in order to explore higher mass $\Lambda$ 's and $\Sigma$ 's, which resulted in findings related to several hyperon resonances compatible with information available from experimental studies [22].

As a continuation of Refs. [20-22], and following the formalism developed in these works, here we look at the nonstrange meson-baryon systems. An analysis of mesonbaryon scattering made in Ref. [26] shows that the resonances generated in the nonstrange PB systems [27] do not seem to relate well with dynamically generated states. This is intriguing since producing a light meson, like the pion, requires about 140 MeV of energy only while the first negative parity $N^{*}$ is about 500 MeV heavier than the nucleon, which, intuitively, could have an important contribution from the $\pi N$ interaction and thus be related to a meson-baryon molecular state. In addition, the quark model calculations, for example, of Isgur and Karl [28], did not result in a good reproduction of the properties of $N^{*}(1535)$, indicating that something is missing in their framework. In Ref. [29], it has been suggested that the wave function of $N^{*}(1535)$ may have a large $s \bar{s}$ component and might require five-quark contributions. Yet more studies of PB channels have been made in past, which can reproduce the poles related to $N^{*}(1535), N^{*}(1650)$ by solving Bethe-Salpeter equations as integral ones, by keeping the off-shell nature [30,31] based on chiral Lagrangians and by considering contributions from next-to-leadingorder terms, although such formalisms introduce additional parameters in the model. One could imagine that it might be possible to interpret these additional parameters if one could keep the contributions to the lowest order Lagrangian and add different seeds (intrinsic ones or from other meson-baryon channels) to the formalism.

In view of this situation, obtaining new information in this sector, within our formalism, could be useful. We, thus, try to find an answer to the question: does the inclusion of vector meson-baryon dynamics bring any new information related to the nature of the low-lying nonstrange resonances, like $N^{*}(1535), N^{*}(1650)$ ?

## II. MESON-BARYON INTERACTIONS AND SCATTERING EQUATIONS

We make a brief discussion of the formalism in this section since more detailed information can be obtained from Refs. [20-22]. The aim of this work is to study
meson-baryon systems with total strangeness zero and, as a standard approach, the framework consists of solving the scattering equations in a coupled channel formalism. In the present work, we couple pseudoscalar and vector mesons, which gives nine channels, in the isospin base, with total strangeness zero: $\pi N, \eta N, K \Lambda, K \Sigma, \rho N, \omega N, \phi N, K^{*} \Lambda$, and $K^{*} \Sigma$. To start with the study of these systems, we need amplitudes for the processes: $\mathrm{PB} \rightarrow \mathrm{PB}, \mathrm{VB} \rightarrow \mathrm{VB}$, and $\mathrm{PB} \leftrightarrow \mathrm{VB}$.

The PB $\rightarrow$ VB transition amplitudes are obtained from a Lagrangian (as deduced in Refs. [21,22]) by using the Kroll-Ruderman theorem for the photoproduction of a pion and by introducing the vector meson as the gauge boson of the hidden local symmetry

$$
\begin{align*}
\mathcal{L}_{\mathrm{PBVB}}= & \frac{-i g}{2 f_{\pi}}\left(F\left\langle\bar{B} \gamma_{\mu} \gamma_{5}\left[\left[P, V_{\mu}\right], B\right]\right\rangle\right. \\
& \left.+D\left\langle\bar{B} \gamma_{\mu} \gamma_{5}\left\{\left[P, V_{\mu}\right], B\right\}\right\rangle\right), \tag{1}
\end{align*}
$$

where the trace $\langle\ldots\rangle$ has to be calculated in the flavor space and $F=0.46, D=0.8$ such that $F+D \simeq g_{A}=1.26$, with $g_{A}$ denoting the axial coupling of the nucleon, and the ratio $D /(F+D) \sim 0.63$. The latter ratio is close to the SU(6) quark model value of 0.6 obtained in Ref. [32].

The Lagrangian in Eq. (1) leads to the amplitude

$$
\begin{equation*}
V_{i j}^{\mathrm{PBVB}}=i \sqrt{3} \frac{g_{\mathrm{KR}}}{2 f_{\pi}} C_{i j}^{\mathrm{PBVB}} \tag{2}
\end{equation*}
$$

where, using the Kawarabayashi-Suzuki-RiazuddinFayazuddin relation [33,34], we get

$$
\begin{equation*}
g_{\mathrm{KR}}=m_{\rho} /\left(\sqrt{2} f_{\pi}\right) \sim 6 \tag{3}
\end{equation*}
$$

with the subscript KR on $g$ indicating the Kroll-Ruderman coupling. To obtain this value of the coupling, we have used $f_{\pi}=93 \mathrm{MeV}$ and the mass of the rho meson $m_{\rho}=$ 770 MeV . The coefficients $C_{i j}^{\mathrm{PBVB}}$ in Eq. (2) were not obtained in Refs. [21,22] for the nonstrange meson-baryon systems. We give this information in the present article, in Tables I and II, for isospins $1 / 2$ and $3 / 2$, respectively.

We should mention here that, in our formalism, we can couple PB-VB channels in the spin $1 / 2$ configuration only (in s-wave interaction, which is relevant in the present case since we study dynamical generation of resonances). Thus we study isospin $1 / 2$ and $3 / 2$ meson-baryon systems with total spin $1 / 2$ (which can generate $1 / 2^{-} N^{*}$ 's and $\Delta^{*}$ 's).

TABLE I. $\quad C_{i j}^{\mathrm{PBVB}}$ coefficients of the $\mathrm{PB} \rightarrow \mathrm{VB}$ amplitude [Eq. (2)] in the isospin $1 / 2$ configuration.

|  | $\rho N$ | $\omega N$ | $\phi N$ | $K^{*} \Lambda$ | $K^{*} \Sigma$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $\pi N$ | $-2(D+F)$ | 0 | 0 | $-\frac{1}{2}(D+3 F)$ | $\frac{1}{2}(F-D)$ |
| $\eta N$ | 0 | 0 | 0 | $\frac{1}{2}(D+3 F)$ | $\frac{3}{2}(F-D)$ |
| $K \Lambda$ | $-\frac{1}{2}(D+3 F)$ | $\frac{1}{2 \sqrt{3}}(D+3 F)$ | $-\frac{1}{\sqrt{6}}(D+3 F)$ | $D$ |  |
| $K \Sigma$ | $\frac{1}{2}(F-D)$ | $\frac{\sqrt{3}}{2}(F-D)$ | $\sqrt{\frac{3}{2}}(D-F)$ | $D$ | $D-2 F$ |

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TABLE II. $C_{i j}^{\mathrm{PBVB}}$ coefficients of the $\mathrm{PB} \rightarrow \mathrm{VB}$ amplitude [Eq. (2)] in the isospin 3/2 configuration.

|  | $\rho N$ | $K^{*} \Sigma$ |
| :---: | :---: | :---: |
| $\pi N$ | $(D+F)$ | $(F-D)$ |
| $K \Sigma$ | $(F-D)$ | $(D+F)$ |

Going over to the discussion of VB interactions, it was shown in Ref. [20] that starting with the Lagrangian for the $\rho N$ interaction, which includes the vector and tensor terms

$$
\begin{equation*}
\mathcal{L}=\bar{N}\left(i \not \partial-g F_{1} \gamma_{\mu} \rho^{\mu}\right) N \tag{4}
\end{equation*}
$$

and which is consistent with the gauge invariance of hidden local symmetry, one ends with equally important contributions from $s$-, $t$-, and $u$-channel exchange diagrams together with the contact term (CT) arising from the commutator in the vector meson tensor. Here we should remind the reader that we are considering the exchange of $1 / 2^{+}$ octet baryons in $s$ - and $u$-channel diagrams which, in the case of s-wave meson-baryon interaction, gets contribution only from the negative energy solution of the Dirac equation (giving rise to the corresponding "Z-diagrams"). This has been also explained in Ref. [20] where the contributions for the $s$-, $t$ - and $u$-channels are explicitly obtained in $S U(2)$ first. The $S U(3)$ generalization of Eq. (4) and its application to VB systems showed that all these amplitudes make important contributions to the solution of the BetheSalpeter equations. We, thus, consider here the VB interaction as the sum of the amplitudes obtained from $s$-, $t$-, and $u$-channel diagrams and the contact interaction:

$$
\begin{equation*}
V_{\mathrm{VB}}=V_{t}+V_{\mathrm{CT}}+V_{s}+V_{u} . \tag{5}
\end{equation*}
$$

All these amplitudes are given in Ref. [20] and thus we refer the reader to that article for more details.

Finally, we calculate the PB amplitudes using the Weinberg-Tomozawa theorem and considering the lowest order chiral Lagrangian, exactly as done in Ref. [27], which leads to an amplitude of the form

$$
\begin{align*}
V_{i j}^{\mathrm{PB}}= & -C_{i j}^{\mathrm{PB}} \frac{1}{4 f_{i} f_{j}}\left(2 \sqrt{s}-M_{i}-M_{j}\right) \\
& \times \sqrt{\frac{M_{i}+E_{i}}{2 M_{i}} \sqrt{\frac{M_{j}+E_{j}}{2 M_{j}}}} \tag{6}
\end{align*}
$$

where $E_{i}\left(E_{j}\right)$ and $M_{i}\left(M_{j}\right)$ represent the energy (in the center of mass frame) and mass of the baryon in the initial (final) state, respectively.

Although the $C_{i j}^{\mathrm{PB}}$ coefficients for the PB systems are given in Ref. [27] in the charge basis, we list the corresponding ones projected in the isospin $1 / 2$ and $3 / 2$ basis, which we use in the present article, in Tables III and IV, respectively. With these inputs we solve the Bethe-Salpeter equation

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TABLE III. $\quad C_{i j}^{\mathrm{PB}}$ coefficients of the $\mathrm{PB} \rightarrow \mathrm{PB}$ amplitudes in the isospin $1 / 2$ configuration.

|  | $\pi N$ | $\eta N$ | $K \Lambda$ | $K \Sigma$ |
| :--- | :---: | :---: | :---: | :---: |
| $\pi N$ | 2 | 0 | $\frac{3}{2}$ | $-\frac{1}{2}$ |
| $\eta N$ |  | 0 | $-\frac{3}{2}$ | $-\frac{3}{2}$ |
| $K \Lambda$ |  | 0 | 0 |  |
| $K \Sigma$ |  |  | 2 |  |

TABLE IV. $\quad C_{i j}^{\mathrm{PB}}$ coefficients of the $\mathrm{PB} \rightarrow \mathrm{PB}$ amplitudes in the isospin $3 / 2$ configuration.

|  | $\pi N$ | $K \Sigma$ |
| :--- | :--- | :--- |
| $\pi N$ | -1 | -1 |
| $K \Sigma$ |  | -1 |

$$
\begin{equation*}
T=V+V G T \tag{7}
\end{equation*}
$$

following the method used in Refs. [20-22,24]. In this way, following these previous works, we take care of the fact that some vector mesons have large widths by calculating the loops for the corresponding channels by making a convolution over the varied masses of these mesons.

## III. RESULTS AND DISCUSSIONS

With the background set up in the previous subsections, we could now start discussing the results found in our work. However, before doing that, we need to digress from this idea and briefly discuss the method followed in our work to regularize the loops in the Bethe-Salpeter equations, which are divergent in nature. One usually resorts to using a cutoff or a subtraction constant to calculate the loops in the scattering equations and these parameters are usually fixed by fitting relevant experimental data. This strategy was indeed followed in the previous works where PB and VB systems were studied independently. Thus, in principle, we could use the parameters fixed in those works to study the coupled systems here but, as we discuss below, we come across some difficulties in doing so.

Let us start the discussion with the PB systems. In the earlier study of nonstrange PB systems based on the lowest order chiral Lagrangian [27], the subtraction constants were constrained by fitting the $\pi-N$ amplitudes and as a result a pole in the complex plane was found which was associated to the $N^{*}(1535)$ resonance. In a later work [26] it was analyzed that the subtraction constants used in Ref. [27] indicate that $N^{*}(1535)$ does not seem to fit in the picture of a dynamically generated resonance in PB systems. It was further investigated that the values of the subtraction constants used in Ref. [27] can be interpreted as adding an $s$-channel pole to the formalism (in other words, adding a new particle participating in the scattering), which does not originate from the PB dynamics. This
interpretation can be quickly seen if we consider a single channel two-particle scattering, in which case the BetheSalpeter equation can be rearranged as

$$
\begin{equation*}
T=\frac{1}{V^{-1}-G} \tag{8}
\end{equation*}
$$

As discussed in Ref. [26], the $G$ function, which is a real number at energies below the threshold, must be negative if we assume that there is no contribution coming from any state apart from the two scattering particles. Based on such basic principles of the scattering theory, a different scheme was proposed in Ref. [26] for determining the subtraction constants to calculate $G$. This method ensures no contribution from the $s$-channel poles in the intermediate scattering and it does not require fitting the data. This scheme was named in Ref. [26] as the "natural renormalization scheme." To simplify the further discussion, let us denote the subtraction constants and loop functions obtained in this scheme as $a_{\text {nat }}^{i}$ and $G_{\text {nat }}^{i}$, respectively, where $i$ symbolizes the propagating channel. As in Ref. [26], we call the subtraction constants fixed to reproduce the data the "phenomenological" ones. We label them as $a_{\text {pheno }}^{i}$ and the corresponding loops as $G_{\text {pheno }}^{i}$.

In this way, when the $G_{\text {pheno }}$ function differs from $G_{\text {nat }}$ by a constant, say, $\Delta a$, we can write $G_{\text {pheno }}=G_{\text {nat }}+\Delta a$, in which case Eq. (8) becomes

$$
\begin{equation*}
T=\frac{1}{V^{-1}-G_{\mathrm{pheno}}}=\frac{1}{V^{-1}-\left(G_{\mathrm{nat}}+\Delta a\right)} \tag{9}
\end{equation*}
$$

which can be rearranged as

$$
\begin{equation*}
T=\frac{1}{\left(V^{-1}-\Delta a\right)-G_{\mathrm{nat}}} \tag{10}
\end{equation*}
$$

The term $\left(V^{-1}-\Delta a\right)$ acts like a redefined kernel of the Bethe-Salpeter equation. Thus, a deviation of $G_{\text {pheno }}$ from $G_{\text {nat }}($ for $\Delta a>0)$ can be interpreted [26] as a modification of the two-particle interaction. Considering the standard form of the Weinberg-Tomozawa meson-baryon interaction, $V \propto(\sqrt{s}-M) / f^{2}$, where $M$ and $f$ represent the mass of the baryon and the decay constant of the meson, respectively, it can be easily shown that the modified kernel can contain a Castillejo-Dalitz-Dyson ( $s$-channel) pole [26]. This seems to be precisely the case in the PB study of Ref. [27].

Since our purpose is to study the contribution of the vector mesons in understanding the nonstrange resonances, it would not be very useful to work in a formalism which already requires Castillejo-Dalitz-Dyson poles to explain these resonances. One alternative way would be to start by calculating the PB loops in the natural renormalization scheme of Ref. [26]. Let us see what results we obtain in this case.

Now, in the case of the VB systems, for the sake of uniformity we also stick to the scheme of Ref. [26] instead of using the subtraction constants of our (and other)
previous works $[20,24]$ on nonstrange VB systems, although their values $(a=-2)$ are not very different from the "natural $a$ " values (given in Table V for both PB and VB channels). We should remind the reader that in the natural scheme of Ref. [26], the regularization scale, present in the loop function, is set to the mass of the baryon.

Let us now discuss the results we obtain by solving the Bethe-Salpeter equations with the interaction kernels of Eq. (5) and loops obtained with the subtraction constants listed in Table V. We give the poles obtained in our study for total isospin $1 / 2$, in two cases in Table VI: (1) when PB-VB systems are not coupled (labeled by $g_{\text {KR }}=0$ coupling) and (2) when they are coupled [labeled by $g_{\text {KR }}=6$ as given by Eq. (3)].

As can be seen from Table VI, we find a wide pole around 1650 MeV in the PB channels and a very narrow pole in the VB channels in a close vicinity when the two systems are uncoupled. None of these poles can be related to known resonances.

When the coupling between the PB and VB channels is switched on [by allowing $g_{\text {KR }}=6$ in Eq. (2)], these two closely spaced poles move in the complex plane to new positions: one of them ends up at $1548-i 101 \mathrm{MeV}$ and another at 1563 - i17 MeV. The former of these two new poles could be related to the $N^{*}(1535)$, but the latter one cannot be identified with the next known nucleon resonance with spin-parity $1 / 2^{-}, N^{*}(1650)$. Moreover, we find that the known experimental data related to the $\pi N$ amplitude cannot be well reproduced, although the resulting cross sections on the $\pi^{-} p \rightarrow \eta n$ and $\pi^{-} p \rightarrow K^{0} \Lambda$ reactions are relatively closer to the data. The discrepancy between the experimental data and our results obtained within the natural renormalization scheme implies that some information is missing in our formalism.

The question now arises if the discrepancy between our results and the experimentally known facts can be reduced by allowing the subtraction constants to vary and if, by doing that, an $s$-channel pole would appear in the formalism. To check this, we treat the subtraction constants for all nine channels as free parameters which are to be fixed by requiring a fit to the experimental data. We shall later see if it is possible to make an interpretation of those parameters (in line with Ref. [26]).

TABLE V. The "natural" subtraction constants for PB and VB channels within the conditions explained in Ref. [26].

|  | Subtraction <br> constant $(a)$ | VB channel | Subtraction <br> constant $(a)$ |
| :--- | :---: | :---: | :---: |
| $\pi N$ | -0.3976 | $\rho N$ | -1.5843 |
| $\eta N$ | -1.239 | $\omega N$ | -1.60145 |
| $K \Lambda$ | -1.143 | $\phi N$ | -1.91566 |
| $K \Sigma$ | -1.138 | $K^{*} \Lambda$ | -1.63265 |
|  |  | $K^{*} \Sigma$ | -1.59025 |

TABLE VI. Poles and their couplings to PB and VB channels when the Bethe-Salpeter equations are solved with loops calculated within the dimensional regularization method with the subtraction constants listed in Table V.

| PB-VB coupling | $g_{\mathrm{KR}}=0$ |  |  | $g_{\mathrm{KR}}=6$ |  |
| :--- | ---: | ---: | ---: | ---: | :---: |
| Pole $(\mathrm{MeV}) \longrightarrow$ | $1581-i 2$ | $1649-i 130$ |  | $1548-i 101$ |  |
| Channels (threshold) $\downarrow$ |  |  | Coupling |  |  |
| $\pi N(1076)$ | $0.0+i 0.0$ | $-1.0+i 0.7$ | $-1.2+i 0.2$ | $0.1-i 0.4$ |  |
| $\eta N(1486)$ | $0.0+i 0.0$ | $-3.2-i 0.1$ | $-1.3-i 3.0$ | $1.1-i 0.8$ |  |
| $K \Lambda(1612)$ | $0.0+i 0.0$ | $1.5+i 0.8$ | $-0.4+i 3.6$ | $-1.0-i 0.3$ |  |
| $K \Sigma(1689)$ | $0.0+i 0.0$ | $4.7+i 0.3$ | $1.3+i 1.5$ | $-0.6+i 1.7$ |  |
| $\rho N(1709)$ | $-0.3-i 0.0$ | $0.0+i 0.0$ | $0.4+i 0.1$ | $-0.6+i 0.2$ |  |
| $\omega N(1721)$ | $-2.1-i 0.0$ | $0.0+i 0.0$ | $1.0+i 1.0$ | $-2.5+i 0.3$ |  |
| $\phi N(1959)$ | $3.2+i 0.0$ | $0.0+i 0.0$ | $-0.7-i 1.5$ | $3.7-i 0.3$ |  |
| $K^{*} \Lambda(2008)$ | $1.4+i 0.0$ | $0.0+i 0.0$ | $-3.6-i 2.3$ | $1.9-i 1.1$ |  |
| $K^{*} \Sigma(2085)$ | $5.9+i 0.0$ | $0.0+i 0.0$ | $3.0-i 2.8$ | $6.0+i 0.9$ |  |

In order to find the new subtraction constants, we look for the best $\chi^{2}$ fit to the data set consisting of the isospin $1 / 2$ and $3 / 2 \pi N$ amplitudes, and the $\pi^{-} p \rightarrow \eta n$ and $\pi^{-} p \rightarrow K^{0} \Lambda$ cross sections in the energy region of the low-lying resonances. In doing so, we stick to using the mass of the baryon of each channel as the regularization scale, such that we can conveniently check if we depart from the basic idea of the natural renormalization scheme of Ref. [26]. To make this fit, we consider PB and VB as coupled systems; i.e., we fix $g_{\text {KR }}=6$. In this way, we constrain the VB amplitudes too, although the data set consists of reactions involving the PB channels only.

The best fit is obtained for the subtraction constants given in Table VII. Although these values differ from the ones given in Table V , it is interesting to notice that all of them are negative numbers, which means that we get the loop functions with negative values below the respective thresholds (at least in the neighborhood of the threshold region), as required in the scheme of Ref. [26]. This finding indicates that the generation of the resonances in this formalism can be partly attributed to the meson-baryon dynamics.

Let us now discuss the results obtained with these subtraction constants. We begin by showing in Fig. 1 the real (imaginary) parts of the $\pi N$ amplitudes, obtained by

TABLE VII. The subtraction constants which give the best $\chi^{2}$ fit to the experimental data on the $\pi N$ amplitudes in isospin $1 / 2$ as well as $3 / 2$ and on the $\pi^{-} p \rightarrow \eta n, \pi^{-} p \rightarrow K^{0} \Lambda$ reactions. The corresponding regularization scales are the baryon masses.

|  | Subtraction <br> constant $(a)$ | VB channel | Subtraction <br> constant $(a)$ |
| :--- | :---: | :---: | :---: |
| $\pi N$ | -1.955 | $\rho N$ | -0.45 |
| $\eta N$ | -0.777 | $\omega N$ | -0.955 |
| $K \Lambda$ | -4.476 | $\phi N$ | -2.972 |
| $K \Sigma$ | -1.945 | $K^{*} \Lambda$ | -0.184 |
|  |  | $K^{*} \Sigma$ | -1.152 |

solving the coupled channel Bethe-Salpeter equations, as solid (dashed) lines for isospins $1 / 2$ and $3 / 2$. Figure 1 also shows the data [35] on the real (imaginary) part of these amplitudes by the dotted (dash-dotted) lines. The dimensionless amplitudes (denoted by $\tilde{T}$ ) shown in Fig. 1 are related to the amplitudes obtained in our formalism $(T)$ through

$$
\begin{equation*}
\tilde{T}_{i f}(\sqrt{s})=-T_{i f}(\sqrt{s}) \sqrt{\frac{M_{i} q_{i}}{4 \pi \sqrt{s}}} \sqrt{\frac{M_{f} q_{f}}{4 \pi \sqrt{s}}} \tag{11}
\end{equation*}
$$

where $M_{i}\left(M_{f}\right)$ and $q_{i}\left(q_{f}\right)$ represent the mass of the baryon, and the center of mass momentum, in the initial (final) state. As can be seen from Fig. 1, the behavior of the $\pi N$ amplitudes gets reasonably reproduced up to about 2 GeV . The important point to be mentioned here is that we could not reproduce the behavior of the data beyond 1550 MeV by considering the PB dynamics alone (decoupled to VB systems). This is in agreement with the previous study of PB systems involving the lowest order chiral Lagrangian [27], where a reasonable fit to the data was obtained only up to a total energy of 1550 MeV . Our work shows that the coupling of the vector mesons to the low-lying resonances plays an important role in obtaining a better agreement with the experimental data.

Next, we show the total cross sections of the $\pi^{-} p \rightarrow \eta n$ and $\pi^{-} p \rightarrow K^{0} \Lambda$ reactions as a function of the beam momentum (denoted by $\mathrm{P}_{\text {Lab }}$ ) in Fig. 2. For the $\chi^{2}$ fitting we have used the experimental data on the $\pi^{-} p \rightarrow \eta n$ reaction up to $\sqrt{s} \sim 1550 \mathrm{MeV}$, which corresponds to the beam momentum (shown in Fig. 2) of about 865 MeV . It can be seen that the data near the threshold are well reproduced. In addition, a bump structure beyond that energy gets developed around the beam momentum of 1600 MeV , which corresponds to a total energy of $\sim 1930 \mathrm{MeV}$. We shall later discuss the poles found in the complex plane, which could help in understanding if this bump can be related to any of the known resonances.


FIG. 1 (color online). Spin half $\pi N$ amplitudes for the isospin $1 / 2$ (left panel) and $3 / 2$ (right panel) configurations. The dotted (dash-dotted) lines represent experimental data from Ref. [35] on the real (imaginary) part of the $\pi N$ amplitudes. The corresponding amplitudes obtained in our work [renormalized through Eq. (11)], using the subtraction constants given in Table VII, are shown as the solid and dashed lines.


FIG. 2. Total cross sections for the $\pi^{-} p \rightarrow \eta n$ (left panel) and $\pi^{-} p \rightarrow K^{0} \Lambda$ (right panel) reactions as a function of the beam momentum ( $\mathrm{P}_{\text {Lab }}$ ). The experimental data have been taken from Refs. [46-50].

In case of the $\pi^{-} p \rightarrow K^{0} \Lambda$ reaction, we considered data up to the total energy of 1760 MeV (which corresponds to the beam momentum of 1160 MeV ) in the $\chi^{2}$ fitting, which lead to finding the subtraction constants given in Table VII. In this case too, a peak is seen beyond the threshold energy region, near the beam momentum of 1450 MeV (or $\sqrt{s}=$ 1900 MeV ), which seems to be in good agreement with the data. It should be mentioned in this context that in some of the recent works [36,37], an important contribution from the $1 / 2^{+} N^{*}(1710)$ resonance to the $\pi^{-} p \rightarrow$ $K^{0} \Lambda$ reaction cross section has been found. However, our formalism is restricted to s-wave meson-baryon interactions (which generates $1 / 2^{-}$resonances). We also miss the $\pi \pi N$ system where the $N^{*}(1710)$ has been found to get dynamically generated [38]. The addition of the $\pi \pi N$ channel may help in better reproducing the data. This possibility should be explored in future.

In order to better understand the results shown in Figs. 1 and 2, we should look for poles in the complex plane. The poles obtained in our study and their couplings to the different channels are given in Table VIII, which also shows the known resonances to which these poles can be related. We find evidence for three isospin $1 / 2$ and one isospin $3 / 2$ resonances. Let us discuss how the properties of the poles found in the present work compare with those of the known resonances.
(i) Isospin $1 / 2$
(1) Let us begin with the pole found at $1504-$ $i 55 \mathrm{MeV}$. The properties of this state are in good agreement with those of the negative parity $S_{11}$ $N^{*}(1535)$ resonance [39], which is known to correspond to a pole with the mass between 1490 and 1530 MeV and full width between 90 and 250 MeV [39]. Also, the experimental studies find that this resonance has similar branching ratios to the channels $\pi N$ and $\eta N$ [39]. This is compatible with our finding of the coupling of $\pi N$ to this state, whose strength is about half of the one obtained for the $\eta N$ channel. In fact, we calculate the branching ratios of our state to these decay channels by using the imaginary part of the corresponding amplitude in order to take the width of the resonance into account [40]. As a result, we obtain a branching ratio of $43 \%$ for the $\pi N$ and $55 \%$ for the $\eta N$ channel. These results are in excellent agreement with those given in Ref. [39].
(2) Next we find a twin pole with positions 1668 $i 28 \mathrm{MeV}$ and $1673-i 67 \mathrm{MeV}$ (see Fig. 3). We find that the appearance of a twin pole is unavoidable in this energy region while minimizing the $\chi^{2}$. We relate these states to the next low-lying $S_{11}$ resonance, $N^{*}(1650)$, which, according to

TABLE VIII. Poles and their couplings to PB and VB channels when the Bethe-Salpeter equations are solved with loops calculated within the dimensional regularization method with the subtraction constants given in Table VII. The $\times$ symbols signify no coupling of the resonance to the channel due to isospin violation. The \#\# superscript indicates that the mass and the width of the state have been found from the amplitudes obtained on the real axis.

| Poles (MeV) $\longrightarrow$ <br> Resonances associated | Isospin 1/2 |  |  |  |  | Isospin 3/2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $1504-i 55$ | 1668 - i28 | $1673-i 67$ | 1801-i96 | 1912-i54 | 1689-i56\#\# |
|  | $N^{*}(1535)$ | $N^{*}(1650)$ | $N^{*}(1895)$ |  |  | $\Delta(1600)$ |
| Channels (threshold) $\downarrow$ | Couplings |  |  |  |  |  |
| $\pi N(1076)$ | $0.9-i 0.3$ | $-0.5-i 0.5$ | $1.3-i 0.6$ | $0.5+i 0.3$ | $0.1-i 0.5$ | $-0.6+i 0.3$ |
| $\eta N(1486)$ | $1.3-i 1.2$ | $-0.2+i 1.2$ | $-1.3-i 0.4$ | $-0.3-i 0.2$ | $-0.2-i 0.7$ | $\times$ |
| $K \Lambda(1612)$ | $-0.7-i 0.6$ | $-1.0+i 0.7$ | $-1.2-i 1.1$ | $-0.5-i 0.6$ | $-0.7+i 0.3$ | $\times$ |
| $K \Sigma(1689)$ | $-2.5+i 0.7$ | $-0.2-i 0.7$ | $0.7+i 0.6$ | $0.1+i 0.2$ | $0.7-i 0.6$ | $-0.9+i 0.2$ |
| $\rho N(1709)$ | $-1.2-i 1.1$ | $3.8+i 1.5$ | $-2.7+i 1.9$ | $0.2+i 0.5$ | $0.4+i 0.2$ | $-3.0-i 0.1$ |
| $\omega N(1721)$ | $-0.8-i 1.4$ | $-2.2+i 1.7$ | $-2.8-i 3.0$ | $-0.9-i 1.3$ | $-0.5+i 0.1$ | $\times$ |
| $\phi N(1959)$ | $1.4+i 2.2$ | $4.1-i 2.7$ | $4.5+i 5.2$ | $2.1+i 1.8$ | $0.9-i 0.2$ | $\times$ |
| $K^{*} \Lambda$ (2008) | $1.1+i 1.2$ | $4.1-i 2.6$ | $4.5+i 4.0$ | $2.5+i 3.0$ | $4.0+i 0.4$ | $\times$ |
| $K^{*} \Sigma(2085)$ | $-1.2+i 0.6$ | $2.3-i 2.8$ | $4.9+i 1.8$ | $4.6+i 0.3$ | $-3.0-i 1.0$ | $3.3+i 0.0$ |

Ref. [39], corresponds to a pole with the real part ranging from 1640 to 1670 MeV and the full width varying from 100 to 170 MeV . In fact, as pointed out in Ref. [39], the two-pole nature of this resonance has earlier been discussed in Ref. [35]. Our results are similar to the poles found for this resonance in Ref. [35]: $1673-i 41,1689-i 96 \mathrm{MeV}$.
It should be emphasized here that a dynamically generated nature for both $N^{*}(1535)$ and $N^{*}(1650)$ has been found within a formalism based on the lowest order chiral Lagrangian which requires fixing the minimum number of parameters. We would like to mention again here that a good fit beyond 1600 MeV is not found when considering PB channels only (which is similar to the findings of Ref. [27]). In this sense, we can consider the finding of poles corresponding to $N^{*}(1650)$, together with a


FIG. 3 (color online). Double pole structure related to $N^{*}(1650)$.
reasonable fit obtained to the data on $\pi N$ amplitudes beyond 1600 MeV , as a success of our model. Although, as can be seen from Figs. 1 and 2, our amplitudes are not in perfect agreement with the experimental data, and the subtraction constants used here (Table VII) differ from the natural $a$ given in Table V. This implies that these resonances cannot be interpreted as purely dynamically generated ones. There is some information missing in our formalism. However, our findings do indicate that adding the vector mesons to the coupled channel space improves the compatibility with the experimental data.
(3) Further, we find a resonance at $1801-i 96 \mathrm{MeV}$ and another one at $1912-i 54 \mathrm{MeV}$. Little is known about $1 / 2^{-}$nucleon resonances beyond $N^{*}(1650)$. This becomes evident if one looks at the note in Ref. [39] under the next, and the only other, $1 / 2^{-}$nucleon resonance, $N^{*}(1895)$, which says that any structure in the $S_{11}$ wave found above 1800 MeV is listed together. Although we find two resonance poles beyond 1800 MeV , it might be difficult to identify them in the experimental data due to their large overlapping widths, and a single peak might be seen as a result of the interference of the two. In fact, our results on the $\pi^{-} p \rightarrow K^{0} \Lambda$ reaction show only one resonance peak around 1900 MeV (which corresponds to the beam momentum of about 1400 MeV ) in the total cross section plot shown in the right panel of Fig. 2. As can be seen, the cross section near the peak position is compatible with the data. Also, a bump is seen in the $\pi^{-} p \rightarrow \eta N$ cross sections around this energy. We have also calculated the total cross sections for the $\pi^{-} p \rightarrow \omega n$ and $\pi^{-} p \rightarrow \phi n$ reactions to check



FIG. 4. Total cross section on the pion induced omega (left panel) and phi (right panel) production on a nucleon. The left panel shows the cross section as a function of the center of mass momentum in the final state, while on the right side we show the cross section as a function of the beam momentum. This is done to show our results in comparison with the experimental data as available from Refs. [51,52].
the contribution of the resonances at 1801 $i 96 \mathrm{MeV}$ and $1912-i 54 \mathrm{MeV}$ found in our work. The results on the $\phi n$ production, to which only the tail of these resonances should contribute, are in agreement with the data (see Fig. 4). The data on the $\pi^{-} p \rightarrow \omega n$ reaction are not well reproduced by our results, but we are studying only $1 / 2^{-}$resonances here. It is known that several resonances with different spin-parities contribute to the omega production [41,42].
(ii) Isospin 3/2

Finally, let us discuss the pole obtained at 1689 $i 56 \mathrm{MeV}$ in the isospin $3 / 2$ configuration. The mass and width of this state have been extracted using the amplitudes obtained on the real axis. The corresponding pole in the complex plane lies very close to the threshold of the $\rho N$ channel, and since we need to calculate the loop function for this channel by convoluting over the width of the $\rho$ meson, it gets complicated to make a reliable pole analysis in the complex plane. This pole can be associated with the well-known $\Delta(1620)$ resonance, for which the mass and the width are given in the ranges of $1600-$ 1660 MeV and $130-150 \mathrm{MeV}$, respectively [39]. The range for the pole position listed in Ref. [39] is $(1590-1610)-i(120-140) \mathrm{MeV}$, although our state has a slightly higher mass. We should notice that the $\Delta(1620)$ resonance is known to decay with a large branching ratio to the $\Delta \pi$ channel, which is not included in our present formalism. However, a significant branching ratio for $\Delta(1620)$ to the $\rho N$ decay channel has also been found, which is compatible with the large coupling shown in Table VIII.

It is also important to notice that our work shows that a $S_{31}$ resonance with negative parity, and mass near 1690 MeV , couples relatively weakly to the PB channels. This is also in agreement with the small branching ratio to the $\pi N$ channel $[39,43]$ in spite of the presence of a large phase space.
We have also calculated the scattering lengths for the different channels, which are summarized in Table IX for the isospin $1 / 2$ configuration and in Table X for the isospin $3 / 2$ case. As can be seen from these tables, we obtain $a_{\eta N}=0.4+i 0.2 \mathrm{fm}, a_{\pi N}^{I=1 / 2}=0.22 \mathrm{fm}$, which are in reasonable agreement with known values. The $\eta N$ scattering length is known to vary in a wide range: $a_{\eta N ; \text { known }}=$ $(0.18+i 0.16) \mathrm{fm}$ to $(1.03+i 0.49) \mathrm{fm}$ (for a recent review on $\eta$-nuclear interaction, see Ref. [44]). The isospin $1 / 2 \pi N$ scattering length is known from the experimental study [45] to be $a_{\pi N ; \exp }^{I=1 / 2}=0.25 \pm 0.005 \mathrm{fm}$. On the other hand, the $\pi N$ scattering length found here, in the isospin $3 / 2$ case, $a_{\pi N}^{I=3 / 2}=-0.27 \mathrm{fm}$, is higher as compared to the experimentally known value $a_{\pi N ; \text { exp }}^{I=3 / 2}=-0.132 \pm$ 0.0132 fm [45]. As mentioned earlier, the discrepancy between our results and the data indicates that the current formalism lacks some ingredient.

One possible extension of the present formalism could be the inclusion of the decuplet baryons. An elaborate work on meson-baryon systems with strangeness $0,-1,-2$, and -3 , involving pseudoscalar and vector mesons together with octet and decuplet baryons, has been done in Ref. [19], where dynamical generation of several $1 / 2^{-}$resonances, including $N^{*}(1535), N^{*}(1650)$, has been reported. While $s$ - and $u$-channel interactions were not considered in Ref. [19], the authors have extended the

TABLE IX. Isospin $1 / 2$ scattering lengths obtained in the present work for different PB, VB channels.

| PB channels | $\pi N$ | $\eta N$ | $K \Lambda$ | $K \Sigma$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $a_{i}^{I=1 / 2}(\mathrm{fm})$ | 0.22 | $0.4+i 0.2$ | $-0.05+i 0.26$ | $-0.43+i 0.05$ |  |
| VB channels | $\rho N$ | $\omega N$ | $\phi N$ | $K^{*} \Lambda$ | $K^{*} \Sigma$ |
| $a_{i}^{I=1 / 2}(\mathrm{fm})$ | $-0.62+i 1.2$ | $-0.61+i 0.37$ | $-0.22+i 0.09$ | $-0.66+i 0.4$ | $-0.56+i 0.2$ |

TABLE X. Isospin $3 / 2$ scattering lengths obtained in the present work for different PB, VB channels.

|  | $\pi N$ | $K \Sigma$ | $\rho N$ | $K^{*} \Sigma$ |
| :---: | :---: | :---: | :---: | :---: |
| $a_{i}^{I=3 / 2}(\mathrm{fm})$ | -0.27 | $-0.1+i 0.08$ | $0.4+i 1.13$ | $-0.32+i 0.13$ |

Weinberg-Tomozawa interaction to include both vector mesons and decuplet baryons by assuming $\mathrm{SU}(6)$ symmetry. Such a symmetry consideration is useful, and the results found in Ref. [19] should serve as a point of reference to make comparisons in the future. It would be interesting, thus, to extend our present formalism by including decuplet baryons, like in Refs. [18,19], in the future.

Before ending this section, we would like to mention a limitation of the present formalism. This is related to the energy range in which the present formalism is reliable. We find that the isospin $1 / 2 T$ matrices obtained in this work are not reliable when going far below the threshold region. There, peaks corresponding to unphysical states might be found, although the interaction potential is repulsive in nature. Of course, our amplitudes cannot also bring reliable information beyond 2.2 GeV , i.e., while going far from the threshold of the heaviest VB channel, since only s-wave meson-baryon interaction is considered here, which is suitable to study the dynamical generation of baryon resonances in these systems.

## IV. SUMMARY

We have studied meson-baryon systems as coupled channels to investigate the dynamical generation of resonances. The systems under consideration have total isospins $1 / 2$ and $3 / 2$ and spin $1 / 2$. The mesons and baryons interact in an $s$ wave, which implies that the possible resonances generated in the system can have spin-parity $1 / 2^{-}$. The formalism consists of solving Bethe-Salpeter equations, for which the interaction kernels are obtained from the Lagrangians based on the chiral and hidden local symmetries. In order to calculate the divergent loop functions, we use the dimensional regularization scheme. We first attempt to strictly follow the natural renormalization scheme of Ref. [26] to get the subtraction constants needed to compute the loop functions. The advantage of this scheme lies in the possibility of getting contributions from the dynamically generated resonances exclusively. As a result, we find two poles near 1550 MeV which cannot be related to known resonances. Further, we cannot reproduce the available experimental data with the corresponding amplitudes.

Next, we obtain the subtraction constants by making a $\chi^{2}$ fit to the available experimental data on the $\pi N$
amplitudes (till $\sim 2 \mathrm{GeV}$ ) and on the reactions: $\pi^{-} p \rightarrow$ $\eta n$ (up to 1550 MeV ) and $\pi^{-} p \rightarrow K^{0} \Lambda$ (up to 1760 MeV ). Although the subtraction constants found in this way differ from the ones within the natural renormalization scheme, we find that their values (and hence the loop functions) are such that the states obtained in this work can be interpreted as partly dynamically generated ones.

Consequently, we find poles in the complex plane whose properties are in good agreement with those of some known resonances. To be specific, we find evidence for the $N^{*}(1535)$ and $N^{*}(1650)$ resonances, with a double pole associated to the latter one. Since the information on all the $1 / 2^{-}$states with mass beyond 1800 MeV is put together under the resonance $N^{*}(1895)$ in Ref. [39], it appears that there is only one known $S_{11}$ resonance beyond 1800 MeV . Our work provides evidence for the existence of two $N^{*}$ 's, one with mass 1800 eV and another with 1912 MeV , though the large overlapping widths found for these two resonances show that it could be difficult to identify two distinct states in the cross sections in the $1800-1900 \mathrm{MeV}$ region.

Finally, we find a resonance with isospin $3 / 2$ at 1689 - i56 MeV. This state can be related to $\Delta(1620)$.

We can conclude this work by answering the question raised in the beginning of this article: does vector mesonbaryon dynamics bring new information toward understanding the nature of the low-lying nonstrange resonances like $N^{*}(1535), N^{*}(1650)$. Our work indicates that with the addition of vector mesons to build the coupled channels, we seem to move in the direction of understanding the lowlying $N^{*}$ and $\Delta$ resonances as dynamically generated states, at least partly. The next question which might be raised now is which information can help in getting a clearer picture about the origin of the low-lying nonstrange resonances. The answer may be found by adding the decuplet baryons to the formalism, since some resonances decay to meson and decuplet baryon channels with significant branching ratios. The results found here should serve as a motivation for such an extension of the formalism in future works.

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