

Nonlinear effects and the behavior of total hadronic and photonic cross sections

A. V. Giannini*

Instituto de Física, Universidade de São Paulo, CEP 05315-970, São Paulo, Brazil

F. O. Durães†

Curso de Física, Escola de Engenharia, Universidade Presbiteriana Mackenzie, CEP 01302-907, São Paulo, Brazil

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In this paper we use an eikonalized minijet model where the effects of the first nonlinear corrections to the DGLAP equations are taken into account. The contributions coming from gluon recombination effects are included in the DGLAP + GLRMQ approach for the free proton in the context of saturation models. The parameters of the model are fixed to fit total pp and $\bar{p}p$ cross sections, including the very recent data from LHC, HiRes, and Pierre Auger collaborations. Glauber and multiple scattering approximations are then used to describe the inclusive inelastic proton-Air cross section. Photoproduction cross sections, without change of parameters fixed before, are also obtained from the model using vector meson dominance and the additive quark model. We show and discuss our main results as well as the implications of saturation effects in the behavior of total hadronic and photonic cross sections at very high energies.

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I. INTRODUCTION

The growth of total hadronic cross sections with energy has been studied for decades and many phenomenological and theoretical efforts have been made to explain it.

One of the most important explanations for this behavior was proposed in the 1970s [1] based on quantum chromodynamics (QCD): the behavior of hadronic cross sections with the center of mass energy (\sqrt{s}) was very similar to the production of jets, indicating that partons would be playing a role in these interactions. This basic idea has led to an approach, called minijet model, which takes into account that the total hadronic cross sections can be decomposed as $\sigma_0 + \sigma_{\text{pQCD}}$, where σ_0 characterizes a nonperturbative contribution to the process (generally taken as energy-independent at high energies) and σ_{pQCD} represents the semihard contributions, calculated in perturbative QCD (pQCD) with use of an arbitrary cutoff at low transverse momenta $p_{T,\text{min}} (> \Lambda_{\text{QCD}})$ [2].

This simple model, however, violates the unitarity of the S matrix for these processes and, consequently, the Froissart bound, which states that total hadronic cross sections cannot grow faster than $\ln^2(s)$ as $s \rightarrow \infty$ [3]. At high energies, the perturbative component of this model is dominated by gluons with very small (Bjorken- x) fractional momentum and pQCD calculations, based on linear QCD evolution (equations developed by Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) [4] and Balitsky-Fadin-Kuraev-Lipatov (BFKL) [5]), show that the minijet cross-section grows very rapidly, dictated by a power-like energy behavior.

To explain the experimental data quantitatively, this idea was reformulated based on the eikonal representation to

ensure unitarity and to contain this strong level of growth. Since then the original eikonalized minijet models [6] have been revisited and several models have been proposed based upon them [7]. Nevertheless, many questions about the dynamics of interaction between partons in the high energies regime still remain open, although, in general, good descriptions of the experimental data have been obtained with these models.

Parallel to these developments, significant progress was achieved in theoretical physics of small x and the results from HERA, with kinematical ranges extended upwards in Q^2 (the four-momentum transfer to the proton) and also downwards in x , changed our view of the structure of the proton [8]. Deep inelastic scattering experiments showed a rapid increase in the density of gluons as x decreases and reinforced the hypothesis that this growth could be related to nonlinear effects in gluon evolution equations. One now knows that for values $x \leq 10^{-4}$, gluons dominate the hadron wave functions but, of course, it is expected that the growth of gluon densities “saturates” at a given time.

The understanding of this expectation is related to the momentum transfer, k_{\perp} , and, therefore, to the transverse size of a gluon ($\propto 1/k_{\perp}$) in semihard interactions. For large momentum transfer, the BFKL evolution predicts a large number of small size gluons per unit of rapidity produced through $g \rightarrow gg$ interactions. For small momentum transfer, on the other hand, the produced gluons overlap themselves in the transverse area and fusion processes, $gg \rightarrow g$, also become important.

This simple scenario shows that a typical scale, Q_s , called “saturation scale,” tells us that these latter processes are small for $k_{\perp}^2 > Q_s^2$. For low enough momentum transfer, $k_{\perp}^2 < Q_s^2$, however, Q_s tell us that the recombination of gluons (fusion processes) cannot be neglected because the gluon density is large and grows with lowering x . At very

*avgiannini@usp.br

†fduraes@if.usp.br; duraes@mackenzie.br

high energies, smaller and smaller values of Bjorken x can be accessed and, under these conditions, the recombination mechanism becomes more and more effective resulting in a decrease in the population of gluons and, therefore, in the idea of “saturation” of partonic distributions mentioned above.

Many studies were made in the latest two decades exploring this subject and, currently, one believes that an effective theory, the color glass condensate [9–11], correctly describes the behavior of very small- x gluons in hadronic wave functions by an infinite hierarchy of coupled evolution equations for the correlators of Wilson lines.

According to this picture, the (highly dense) system formed at these extreme conditions is characterized by the limitation on the maximum phase-space parton density that can be reached in the hadron wave function and an x - or energy-dependent momentum scale, $Q_s(x)$, which separates dense and dilute regimes. In the low density regime the formalism reproduces the BFKL dynamics for partons with transverse momentum much larger than this “saturation momentum.” On the opposite side, the saturation scale becomes large, $Q_s(x) \gg \Lambda_{\text{QCD}}$, and the formalism predicts that partons saturate in the hadron wave function with occupation numbers of order $1/\alpha_s(Q_s)$. In this case the coupling constant becomes weak [$\alpha_s(Q_s) \ll 1$] and the high energy limit of QCD can be studied using weak coupling techniques [12,13].

In this work we study the influence of the gluon recombination process assuming that such a system may be formed in hadronic and photonic collisions at very high energies. As mentioned above, at very small x , the parton distribution functions are governed by BFKL dynamics and this mechanism leads to nonlinear power corrections to the DGLAP evolution equations. We also adopt here the first nonlinear (GLRMQ) terms calculated by Gribov, Levin, and Ryskin [14], and after by Mueller and Qiu [15], to describe experimental cross sections and make predictions.

In what follows we briefly present the standard formulations of the eikonalized minijet model for hadronic and photonic cross sections, the inelastic proton-nucleus cross section in the Glauber formalism and describe the main ingredients used in our calculations. Then we present the strategy used to fix the parameters of the model, show and discuss our main results and, in the last section, we outline our conclusions.

II. EIKONALIZED MINIJET MODEL WITH SATURATION EFFECTS

One of the most important contributions to predict the behavior of hadronic cross sections with the energy from the QCD parton model was proposed by Durand and Pi [6] in the late 1980s using a formalism consistent with unitarity constraints. Many QCD-inspired models used today have their origins based on this “eikonal” formulation,

which provides a framework where the minijet cross sections are unitarized via multiple scattering.

In this work we have used a unitarized version of the minijet model [7] where the total, elastic, and inelastic $pp(\bar{p})$ cross sections are given by

$$\begin{aligned}\sigma_{\text{tot}}^{pp(\bar{p})}(s) &= 2 \int d^2\vec{b} \{1 - e^{-\text{Im}\chi(b,s)} \cos[\text{Re}\chi(b,s)]\}, \\ \sigma_{\text{el}}^{pp(\bar{p})}(s) &= \int d^2\vec{b} |1 - e^{i\chi(b,s)}|^2, \\ \sigma_{\text{inel}}^{pp(\bar{p})}(s) &= \int d^2\vec{b} [1 - e^{-2\text{Im}\chi(b,s)}].\end{aligned}\quad (1)$$

The eikonal function $\chi(b,s)$ in the above expressions contains the energy and the transverse momentum dependence of matter distribution in the colliding particles and, through the impact parameter distribution in the b space, it is given by $\chi(b,s) = \text{Re}[\chi(b,s)] + i\text{Im}[\chi(b,s)]$.

The real part of $\chi(b,s)$ represents only about 4% in the ratio of the real to the imaginary part of the forward elastic amplitude for $pp(\bar{p})$ processes and therefore, as a first approximation, we assume $\text{Re}\chi(b,s) = 0$ in this work. We also assume that multiple partonic interactions are Poisson distributed with an average number separated in soft and hard processes in a given inelastic collision, $n(b,s) \equiv 2\text{Im}\chi(b,s) = n_{\text{soft}}(b,s) + n_{\text{hard}}(b,s)$, and can be factorized in b and s as [7]

$$n(b,s) = W(b, \mu_{\text{soft}}) \sigma^{\text{soft}}(s) + \sum_{k,l} W(b, \mu_{\text{hard}}) \sigma_{kl}^{\text{hard}}(s), \quad (2)$$

where $W(b, \mu_{\text{soft}})$ and $W(b, \mu_{\text{hard}})$, which represent the effective overlap functions of the nucleons at impact parameter b , are related to the nucleon form factor in hadronic and partonic levels [normalized such that $\int W(b, \mu) d^2\vec{b} = 1$], and $\sigma^{\text{soft}}(s)$ and $\sigma_{kl}^{\text{hard}}(s)$ represent the behavior of the total cross sections with energy in soft and hard [minijet production (m_j)] regimes in $pp(\bar{p})$ collisions.

At low energies the hard contribution to the eikonal function is very small. In order to describe pp and $p\bar{p}$ scattering at intersecting storage rings energies we parametrize the soft contribution as [7]

$$\begin{aligned}W(b, \mu_{\text{soft}}) &= \frac{\mu_{\text{soft}}^2}{96\pi} (\mu_{\text{soft}} b)^3 K_3(\mu_{\text{soft}} b), \\ \sigma_{\text{soft}}^{pp}(E_{\text{lab}}) &= 47 + \frac{46}{E_{\text{lab}}^{1.39}}, \\ \sigma_{\text{soft}}^{p\bar{p}}(E_{\text{lab}}) &= 47 + \frac{129}{E_{\text{lab}}^{0.661}} + \frac{357}{E_{\text{lab}}^{2.7}},\end{aligned}\quad (3)$$

where μ_{soft} is an adjustable parameter, K_3 is the modified Bessel functions, and E_{lab} is the proton energy in the laboratory system (cross sections are in mb).

The hard contribution is described by the minijet production in leading order (LO) pQCD where partons are

produced back to back in the transverse plane according to the differential cross section [16]:

$$\frac{d\sigma_{kl}^{mj}}{dy}(s) = \kappa \int dp_T^2 dy_2 \sum_{i,j} x_1 f_{i/h_1}(x_1, Q^2) x_2 f_{j/h_2}(x_2, Q^2) \times \frac{1}{1 + \delta_{kl}} \left[\delta_{fk} \frac{d\hat{\sigma}^{ij \rightarrow kl}}{d\hat{t}}(\hat{t}, \hat{u}) + \delta_{fl} \frac{d\hat{\sigma}^{ij \rightarrow kl}}{d\hat{t}}(\hat{u}, \hat{t}) \right], \quad (4)$$

where h_1 and h_2 denote the colliding hadrons and $d\hat{\sigma}^{ij \rightarrow kl}/d\hat{t}$ the subprocess cross sections [17].

The rapidities of the final state partons k and l are labeled by y ($\equiv y_1$) and y_2 and the transverse momentum of each parton by p_T ($\geq p_{T_{\min}}$, the smallest transverse momentum allowed for parton scatterings). The fractional momenta of the colliding partons i and j are $x_{1,2} = p_T/\sqrt{s}(e^{\pm y} + e^{\pm y_2})$, i.e., the incoming partons are collinear with the beams. The factor $1/(1 + \delta_{kl})$ is a statistical factor for identical particles in the final state.

In our calculations we have assumed $\kappa = 1$ and only considered the process $gg \rightarrow gg$, $gq(\bar{q}) \rightarrow gq(\bar{q})$ and $qq \rightarrow q\bar{q}$ ($q \equiv u, d, s$). We also have parametrized the ‘‘hard’’ overlap functions in impact parameter space, $W(b, \mu_{gg})$, $W(b, \mu_{gq} \equiv \sqrt{\mu_{qq}\mu_{gg}})$ and $W(b, \mu_{qq})$, as Fourier transforms of a dipole form factor [see Eq. (3)] [18]. The (free) parameters μ_{qq} and μ_{gg} represent masses which describe the ‘‘area’’ occupied by quarks and gluons, respectively, in the colliding protons.

As discussed before, even in conventional eikonalized minijet models the rise of the total $pp(\bar{p})$ cross section with energy is related to the increasing probability of perturbative small- x gluon-gluon collisions: gluon distribution functions governed by DGLAP evolution and contributions of partons with $p_T \geq p_{T_{\min}}$ dominate the integrand of Eq. (4) increasing very fast the rise of total cross section with energy. The numerical evaluation of this partonic contribution strongly depends upon $p_{T_{\min}}$, the chosen set of parton densities [$f_{i,j/h_{1,2}}(x_{1,2}, Q^2)$] and, of course, their evolution in this regime.

The main ingredient of our model is the introduction of nonlinear terms in the evolution of parton densities above. In the context of saturation models, we adopt these corrections and make use of EHKQS parton distribution functions [19,20], where the GLRMQ terms are present. This allows us to test the dynamic responsible for the rise of total $pp(\bar{p})$ cross sections with energy in the presence of saturation effects.

In order to compare these two different regimes, in what follows we shall consider GRV98 (LO) [21] and CTEQ6L [22] parton densities as references. They are governed by DGLAP evolution equations (linear regime), where, therefore, saturation effects are clearly absent.

At this point, we would like to call attention to the latest LHC results [23,24]: they have provided very valuable

information on high-energy multiparticle production, improving our theoretical understanding of soft and semihard parton dynamics, and showed the need for adjustments and even the reformulation of hypotheses employed in models that propose to establish the behavior of hadronic cross sections with energy.

It is also important to note that the recent cosmic ray data from HiRes [25] and Pierre Auger collaborations [26] have allowed a deeper understanding about the nature of produced particles at very high energies and stimulated many discussions between the accelerator and cosmic-ray communities on common issues in these areas. The energy dependence of total hadronic cross sections is probably the most important question for the cosmic-ray community. The phase space regions of relevance to the development of Air showers are not directly accessible in the currently accelerator experiments, and, because of that, descriptions and interpretations of the data in cosmic rays physics at high energies depend crucially on the predictions coming from phenomenological models [27].

For this reason it is very interesting to test the range of the model presented in this work and to verify if it permits a satisfactory description of the data to other processes. Obviously, our first choice is the inelastic proton-Air cross section.

In the Glauber multiple collision model [28] the inelastic proton-nucleus cross section, $\sigma_{\text{inel}}^{pA}(s)$, can be derived in the eikonal limit (straight line trajectories of colliding nucleons) from the corresponding inelastic nucleon-nucleon (NN) cross section, $\sigma_{\text{inel}}^{NN}(s)$.

At the center-of-mass energy \sqrt{s} and the geometry of the pA collision, it is simply determined by the impact parameter of the reaction:

$$\sigma_{\text{inel}}^{pA}(s) = \int d^2\vec{b} [1 - e^{-\sigma_{\text{inel}}^{NN}(s)T_A(b)}]. \quad (5)$$

The usual thickness function $T_A(b) [\equiv \int dz \rho_A(b, z)$, with $\int d^2\vec{b} T_A(b) = A$] gives the number of nucleons in the nucleus A per unit area along the z direction, separated from the center of the nucleus by an impact parameter b . The function $\rho_A(b, z)$ represents the nuclear density of the nucleus A (with radius R_A) and, in what follows, we have used [29]

$$\rho_A(b, z) = \rho_0 \{1 + \exp[(r - R_A)/a_0]\}^{-1}, \quad (6)$$

where $r = \sqrt{b^2 + z^2}$, $R_A = 1.19A^{1/3} - 1.61A^{-1/3}$ (fm), ρ_0 corresponds to the nucleon density in the center of the nucleus A [not important here due the required normalization condition on $T_A(b)$] and a_0 is the so-called ‘‘diffuseness parameter’’ of the Woods-Saxon profile (6), assumed here as a free parameter.

Our second choice to test the model are the γp and $\gamma\gamma$ cross sections. These processes can be derived from the pp forward scattering amplitude using vector meson dominance (VMD) and the additive quark model with the

introduction of a probability ($P_{\text{had}}^{\gamma p(\gamma)}$) that the photon interacts as a hadron (see, for example, articles produced by Block, Pancheri, and Luna with their collaborators on that subject [7,18]).

Assuming, in the spirit of the VMD, that at high energies the photon behaves as a hadronic state composed by two quarks, after the substitutions $\sigma^{s,h} \rightarrow \frac{2}{3}\sigma^{s,h}$ and $\mu_{s,h} \rightarrow \sqrt{\frac{3}{2}}\mu_{s,h}$ in both soft and hard components of Eq. (2), the γp cross section can be written as

$$\sigma_{\text{tot}}^{\gamma p}(s) = 2P_{\text{had}}^{\gamma p} \int d^2\vec{b} \{1 - e^{-\text{Im}\chi^{\gamma p}(b,s)} \cos[\text{Re}\chi^{\gamma p}(b,s)]\}. \quad (7)$$

To obtain the $\gamma\gamma$ cross section we apply the same procedure as before making the substitutions $\sigma^{s,h} \rightarrow \frac{4}{9}\sigma^{s,h}$ and $\mu_{s,h} \rightarrow \frac{3}{2}\mu_{s,h}$:

$$\sigma_{\text{tot}}^{\gamma\gamma}(s) = 2P_{\text{had}}^{\gamma\gamma} \int d^2\vec{b} \{1 - e^{-\text{Im}\chi^{\gamma\gamma}(b,s)} \cos[\text{Re}\chi^{\gamma\gamma}(b,s)]\}, \quad (8)$$

where $P_{\text{had}}^{\gamma\gamma} = (P_{\text{had}}^{\gamma p})^2$.

The simplest VMD formulation only assumes the lightest vector mesons and in this case $P_{\text{had}}^{\gamma p}$ is given by $P_{\text{had}}^{\gamma p} = \sum_{V=\rho,\omega,\phi} \frac{4\pi\alpha_{\text{em}}}{f_V^2}$, where α_{em} is the QED coupling constant and f_V^2 is the $\gamma - V$ coupling. In this work, however, we consider $P_{\text{had}}^{\gamma p}$ as a free parameter fixed by the low energy γp data.

III. RESULTS AND DISCUSSIONS

The relevant experimental data shown in the next figures considers only the most quoted ones in the literature and can be found in the Refs. [18,23–26,30–34].

The strategy adopted in our calculations is as follows: while the soft contribution of the model was parametrized to describe the pp and $p\bar{p}$ scattering at low energies, at higher energies the hard parameters were fixed [for each set of parton distribution functions (PDFs) used] to fit the latest LHC data [23,24] and the experimental results from HiRes [25] and Pierre Auger (converted data) [26] collaborations. Once established the energy dependence of the total and inelastic pp and $p\bar{p}$ cross sections (1), the inelastic proton-Air, the total gamma-proton, and gamma-gamma cross sections were then obtained according to Eqs. (5), (7), and (8).

In all figures below, solid lines represent our results with nonlinear evolution (EHKQS) and hard parameters fixed at $p_{T_{\text{min}}}^2 = 1.51 \text{ GeV}^2$, $\mu_{gg} = 2.00 \text{ GeV}$ and $\mu_{q\bar{q}} = 0.70 \text{ GeV}$. Respectively, dashed and dotted lines show the results in the linear regime of the model with hard parameters fixed at $p_{T_{\text{min}}}^2 = 2.10 \text{ GeV}^2$, $\mu_{gg} = 2.03 \text{ GeV}$ and $\mu_{q\bar{q}} = 0.73 \text{ GeV}$ (CTEQ6L), and $p_{T_{\text{min}}}^2 = 1.32 \text{ GeV}^2$, $\mu_{gg} = 1.88 \text{ GeV}$ and

$\mu_{q\bar{q}} = 1.00 \text{ GeV}$ (GRV98). In all cases we have adopted $\mu_{\text{soft}}^2 = 0.7 \text{ GeV}^2$ [see Eq. (3)].

Figure 1 shows the total pp and $p\bar{p}$ cross sections given by Eq. (1). The experimental data are from Refs. [23–26,30]. As can be seen, the results obtained with nonlinear evolution also produce a good description of the data (as good as the fits shown, for example, in Block *et al.*, Pancheri *et al.*, and Grau *et al.* [7]), in particular the most recent data cited above (LHC, HiRes, and Pierre Auger) at higher energies. Comparing the solid curve with the dashed and dotted ones, we conclude that the rise of these cross sections at high energies is still dictated by the growth of gluon densities at small x . The nonlinear evolution (EHKQS), however, leads to a small softening of this cross section due gluon fusion processes ($gg \rightarrow g$).

Figures 2(a) and 2(b) show, respectively, the ratios between the cross sections presented in Fig. 1 and the parametrizations from the Particle Data Group (PDG) [30] and from the Block-Halzen's analysis (BH) [18,35]:

$$\sigma_{\text{PDG}}^{\mp}(s) = a_0 + a_1 A_1^{b_1} \mp a_2 A_1^{b_2} + a_3 \ln^{b_3}(A_2), \quad (9)$$

$$\sigma_{\text{BH}}^{\pm}(\nu) = c_0 + c_1 C^{d_1} \pm c_2 C^{d_2} + c_3 \ln(C) + c_4 \ln^{d_3}(C), \quad (10)$$

where the upper (lower) sign is for pp ($p\bar{p}$) scattering, $A_1 \equiv s/s_l$, $A_2 \equiv s/s_h$, $C \equiv \nu/m$ [$\approx s/2m^2$, ν and m represent, respectively, the laboratory energy of the incoming proton (antiproton) and the proton mass]. The values of the corresponding parameters are displayed in Table I.

It should be noted from these figures that, above $\sim 10 \text{ TeV}$, *all* curves fall rapidly: at 10^3 TeV , the ratios with PDG parametrization are ~ 0.86 for the calculations with CTEQ6L or GRV98 and ~ 0.79 for EHKQS, and the

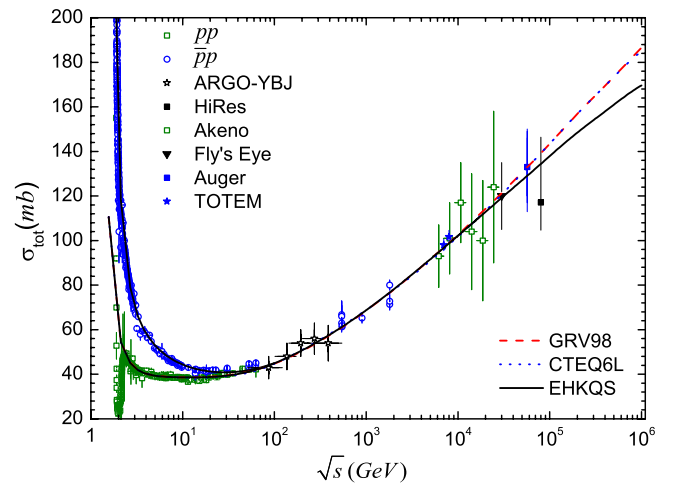


FIG. 1 (color online). Total pp and $p\bar{p}$ cross sections. Solid, dashed, and dotted lines represent the results of the model [Eq. (1)] in the nonlinear and linear regimes. The experimental data are from Refs. [23–26,30].

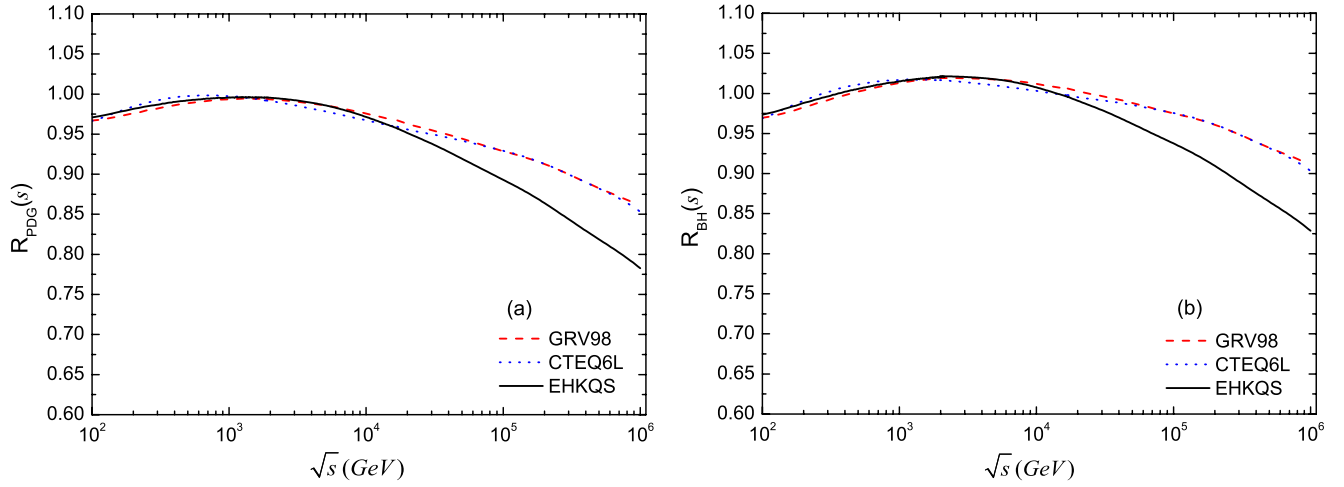


FIG. 2 (color online). Ratios between total cross sections showed in the Fig. 1 and the parametrizations given by Eqs. (9) and (10).

ratios with BH parametrization are ~ 0.91 for the calculations with CTEQ6L or GRV98 and ~ 0.84 for EHKQS.

As can be seen, apparently, our results at high energies, in all cases, seem to be not compatible with the Froissart-type behavior contained in both parametrizations above. However, we have checked (in pp collisions, for example) that above $\sqrt{s} \sim 6$ GeV, our curves GRV98, CTEQ6L and EHKQS (dashed, dotted, and solid lines, shown in Fig. 1), can also be fitted by the following ‘‘inspired’’ (Froissart-type) parametrizations *à la* PDG [Eq. (9), with $s_l = s_h = 1$ GeV 2] and *à la* BH [Eq. (10), with $m = 0.938$ GeV], respectively:

$$\tilde{\sigma}_{\text{CTEQ6L}}^{\text{GRV98}}(s) = (27.9 \pm 0.3) + a_1 s^{b_1} - a_2 s^{b_2} + (0.2152 \pm 0.0015) \ln^2(s), \quad (11)$$

$$\tilde{\sigma}^{\text{EHKQS}}(s) = (30.5 \pm 0.8) + a_1 s^{b_1} - a_2 s^{b_2} + (0.2014 \pm 0.0035) \ln^2(s), \quad (12)$$

$$\tilde{\sigma}_{\text{CTEQ6L}}^{\text{GRV98}}(s) = (29.74 \pm 0.11) + 49.2143 s^{d_1} - 39.7501 s^{d_2} - (0.252 \pm 0.004) \ln(s) + (0.2230 \pm 0.0004) \ln^2(s), \quad (13)$$

TABLE I. The fitted parameters from the quoted references through parametrizations (9) and (10).

PDG	[30]	BH	[18,35]
$a_0(\text{mb})$	35.35 ± 0.48	$c_0(\text{mb})$	37.32
$a_1(\text{mb})$	42.53 ± 1.35	$c_1(\text{mb})$	37.10
$a_2(\text{mb})$	33.34 ± 1.04	$c_2(\text{mb})$	-28.56
$a_3(\text{mb})$	0.308 ± 0.010	$c_3(\text{mb})$	-1.440 ± 0.070
b_1	-0.458 ± 0.017	$c_4(\text{mb})$	0.2817 ± 0.0064
b_2	-0.545 ± 0.007	d_1	-0.5
b_3	2	d_2	-0.585
$s_l(\text{GeV}^2)$	1.0	d_3	2
$s_h(\text{GeV}^2)$	28.9 ± 5.4		

$$\tilde{\sigma}^{\text{EHKQS}}(s) = (18.2 \pm 0.5) + 49.2143 s^{d_1} - 39.7501 s^{d_2} + (1.80 \pm 0.08) \ln(s) + (0.1445 \pm 0.0026) \ln^2(s), \quad (14)$$

where the parameters a_1, b_1, a_2, b_2, d_1 , and d_2 are the same shown in Table I.

Despite that, even within the uncertainties of parameters shown in Table I, our parametrized results present a very different behavior of our cross sections than those predicted by Eqs. (9) and (10). They suggest that it may be much more modest with the energy, especially if nonlinear effects are taken into account. At high energies the coefficients of $\ln(s)$ and $\ln^2(s)$ in all parametrizations above are very important and determine the growth of the hadronic cross sections with energy. Of course, the rise of the total hadronic cross sections at the highest energies still constitutes an open problem [36], demanding further and detailed investigations (theoretical and experimental), and also, our model and results will be tested in the next high energies experiments.

The results for the inelastic pp and $\bar{p}p$ cross sections [Eq. (1)] are shown in Fig. 3(a). The experimental data include the most recent results from the Pierre Auger collaboration [26], LHC [23,24,31] and the oldest ones from the Particle Data Group [30], where was considered $\sigma_{\text{inel}}^{pp(\bar{p})}(s) = \sigma_{\text{tot}}^{pp(\bar{p})}(s) - \sigma_{\text{el}}^{pp(\bar{p})}(s)$.

In all the cases the description of the data is not good. As discussed above, the parameters of the present model were fixed *only* through the *total* experimental pp and $\bar{p}p$ cross sections. They determine the behavior of the imaginary part of the eikonal function with energy and do not provide a satisfactory description of the experimental inelastic pp and $\bar{p}p$ cross sections. It is important to note, however, that conventional eikonal models possess the same problem and also cannot be able to produce a reasonable description of these data.

In order to modify this behavior, some alternative QCD-inspired eikonal models have been proposed. For example,

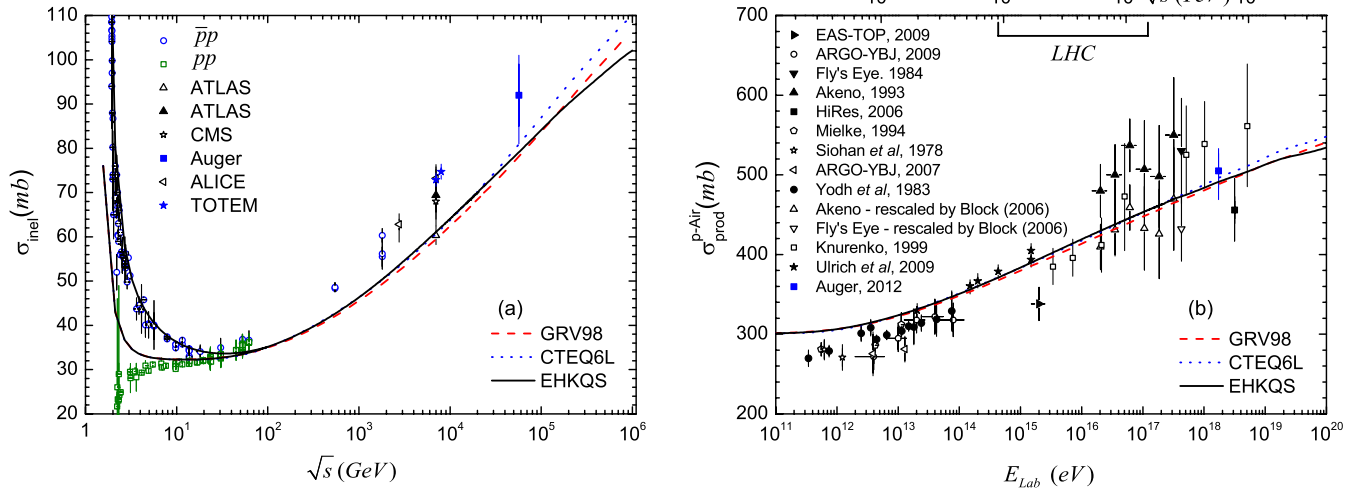


FIG. 3 (color online). (a) Total inelastic pp and $\bar{p}p$ cross sections. Solid, dashed, and dotted lines represent the results of the model [Eq. (1)] in the nonlinear and linear regimes. The experimental data are from Refs. [23,24,26,30,31]. (b) Inelastic proton-Air cross sections. Solid, dashed, and dotted lines represent the results of the model [Eq. (5)]. The experimental data are from Refs. [18,26,32].

Luna and collaborators [7] have introduced *ad hoc* an infrared dynamical gluon mass scale in the calculations of pp and $p\bar{p}$ forward scattering quantities, which (as claimed by the authors) allows one to describe successfully the forward scattering quantities, $\sigma_{\text{tot}}^{pp(\bar{p})}$, ρ (the real to the imaginary part of the forward scattering amplitude), the “nuclear slope” B and differential cross section $d\sigma_{\text{el}}^{pp(\bar{p})}(s, t)/dt$, in excellent agreement with the available experimental data (at least up to $\sqrt{s} = 1.8$ TeV). Gribov and collaborators [14], on the other hand, have proposed an energy dependent cutoff at low transverse momentum, which, effectively, mimics saturation effects. Another interesting approach assumes a resummation of soft gluon emission (down to zero momentum) to soften the rise of the total cross section due to the increasing number of gluon-gluon collisions at low x (Grau *et al.* [7]). The model presented in this work, as can be noted, does not include any of these approaches.

As in other eikonal models cited before, on the other hand, our model also does not include diffractive processes. As pointed out in [37], the recently published measurements of pp inelastic diffraction cross sections at LHC indicate that the rates of diffractive events in inelastic collisions, estimated from the pseudorapidity distributions of charged particles, are $\sigma_{\text{SD}}/\sigma_{\text{inel}} \approx 0.20$ for single diffraction processes and $\sigma_{\text{DD}}/\sigma_{\text{inel}} \approx 0.12$ for double diffraction processes (diffractive mass $M_X^2 < 200 \text{ GeV}^2$ and collisions at $\sqrt{s} \sim 1\text{--}7$ TeV). Of course, an inelastic diffractive component would be desirable and necessary in our model in order to reconcile the estimates of $\sigma_{\text{inel}}^{pp}$ with all experimental data (σ_{diff}), but its inclusion in a consistent way would require a more complex framework than that used here.

Albeit complementary, this figure [Fig. 3(a)] shows the role played by the saturation effects at very high energies,

and tells us that the inclusion of other approaches or diffractive processes may be really needed.

Figure 3(b) shows the results for the inelastic proton-Air cross sections given by Eq. (5). They take into account only the geometry of the p -Air collision [we have fixed $A \equiv A_{\text{Air}} = 14.5$ and $a_0 = 0.75$ fm, see Eq. (6)] and the semi-hard dynamics of the model, via the imaginary part of the eikonal function contained in the inelastic cross sections (determined by the parameters fixed to describe total pp and $\bar{p}p$ data cross sections).

As discussed above, the description of the experimental pp and $\bar{p}p$ inelastic cross sections is not good. Nevertheless, the results obtained with this simple approach (even with nonlinear evolution) at higher energies have the same quality of Monte Carlo predictions [38,39], even though at lower energies, of course, the description is worse than that of such models. But it is interesting to note that our results favor a moderately slow rise of the proton-Air cross section towards higher energies, as indicated by the Pierre Auger measurement [26]. These results will have implications about future measurements at the LHC (whose first analysis also indicates slightly smaller hadronic cross sections than expected within many models) and, certainly, will test the hypothesis, dynamics, and predictions of the models.

Figures 4(a) and 4(b) show our results for the total γp and $\gamma\gamma$ cross sections, respectively given by Eqs. (7) and (8). The experimental data are from Refs. [30,33,34]. The probability that in these collisions the photon interacts as a hadron was fixed at $P_{\text{had}}^{\gamma p} = 1/221$.

Considering that these cross sections are determined by the same parameters used in pp and $\bar{p}p$ collisions and by the changes *only* coming from the “weights” introduced by the VMD and additive quark model, in all the cases (including in the nonlinear regime), the agreement with data points is reasonable.

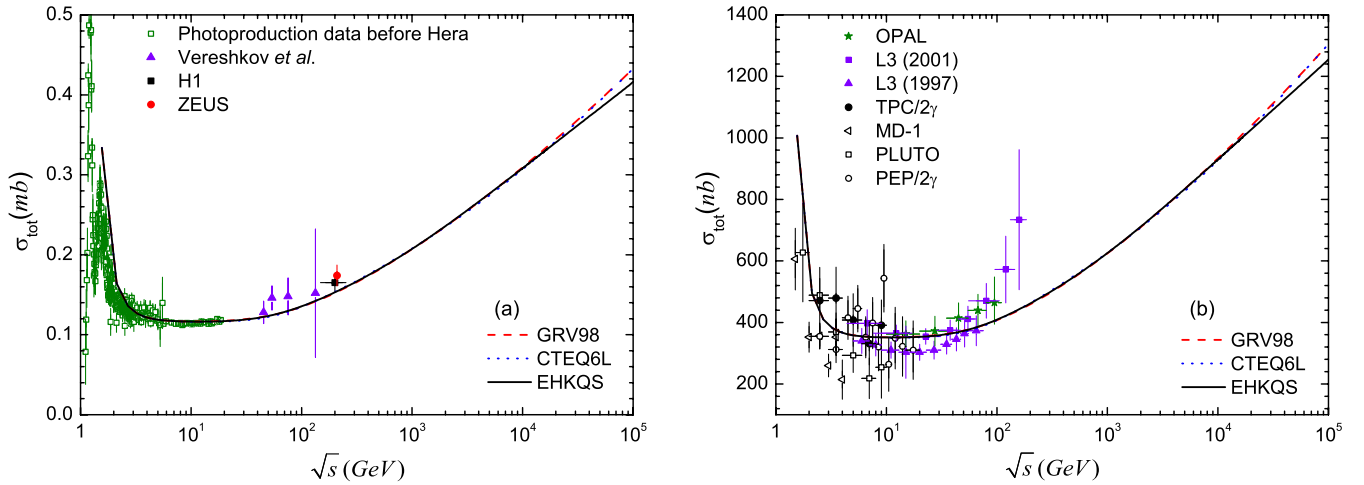


FIG. 4 (color online). (a) Total gamma-proton cross sections. Solid, dashed, and dotted lines represent the results of the model [Eq. (7)]. The experimental data are from Refs. [30,33]. (b) Total gamma-gamma cross sections. Solid, dashed, and dotted lines represent the results of the model [Eq. (8)]. The experimental data are from Ref. [34].

The model underestimates the latest data points for total γp cross sections and cannot describe the data above $\sqrt{s} \sim 100$ GeV for $\gamma\gamma$ cross sections. But, as discussed above, this is also a problem for other models in the literature. To circumvent this problem, for example, Luna and Natale [7] suggest that the probability that a photon interacts as a hadron increases logarithmically with \sqrt{s} [like $P_{\text{had}} = a + b \ln(s)$]. Even then, they only get a successful description with this hypothesis if their results are compared to the (OPAL and L3) data, handled with the PYTHIA and PHOJET Monte Carlo generators. The Bloch-Nordsieck formalism to these collisions (Grau *et al.* [7]), on the other hand, is obviously more robust. It basically depends on the choice of minimum hard cutoff ($p_{T_{\text{min}}}$, related to the chosen PDF set) and on the infrared parameter (p), which controls the quenching of the (minijet cross section) rise at high energy and, consequently, the absolute value of n^{hard} [see Eq. (2)]. Models constructed from that describe more satisfactorily the experimental data at energies around 100 GeV but are strongly dependent on the infrared parameter above.

IV. CONCLUSIONS

In this work we have used an eikonalized minijet model, where the effects of the first nonlinear corrections to the DGLAP equations are taken into account, to investigate (simultaneously) the energy behavior of total hadronic and photonic cross sections.

First of all, we call attention that the main dynamical ingredients of our model are completely determined by the choices of the PDFs used and, consequently, by the hard parameters [presented in Eq. (4)], which have been fixed to fit the latest LHC data and the experimental results from HiRes and Pierre Auger (converted data) collaborations, for pp and $p\bar{p}$ collisions at high energies. The behavior

with the energy of other hadronic and photonic cross sections studied here is, therefore, dictated by these conditions (especially on the hard parameters).

We do not have a good description of inelastic pp and $p\bar{p}$ data cross sections in any case, which seems to indicate the need for different dynamical ingredients into the semi-hard component of the model. Despite that, even though the procedure adopted might be considered “particular,” the introduction of nonlinear corrections into the model allows a satisfactory description of experimental cross sections investigated in this work and should not be discarded *a priori* for this processes. The corrections are relatively small and only manifest themselves at extremely high energies, but our results show that the saturation effects attenuate more strongly the growth of total hadronic and photonic cross sections than those obtained by conventional eikonal models governed by the linear regime.

We also call attention that, as discussed above, our results can be described through a Froissart-type behavior (dictated by the PDG and BH parametrizations quoted in the literature). But, when the nonlinear effects are included in the calculations, our EHKQS results exhibit a very moderated behavior with energy coming from the gluon recombination effects. This is the main result of our study.

The fact that we have achieved reasonable success in using this simple model encourages us to test it to try to describe another observable where, perhaps, nonlinear effects of QCD can be more evidently present (works are in progress).

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