# Analysis of strong decays of the charmed mesons $D_J(2580)$ , $D_J^*(2650)$ , $D_J(2740)$ , $D_I^*(2760)$ , $D_J(3000)$ , $D_I^*(3000)$

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In this article, we tentatively identify the charmed mesons  $D_J(2580)$ ,  $D_J^*(2650)$ ,  $D_J(2740)$ ,  $D_J^*(2760)$ ,  $D_J(3000)$ , and  $D_J^*(3000)$  observed by the LHCb Collaboration according to their spin, parity, and masses. Then we study their strong decays to the ground state charmed mesons plus light pseudoscalar mesons with the heavy meson effective theory in the leading order approximation, and we obtain explicit expressions of the decay widths. The ratios among the decay widths can be used to confirm or reject the assignments of the newly observed charmed mesons. The strong coupling constants in the decay widths can be fitted to the experimental data in the future at the LHCb, BESIII, KEK-B, and PANDA.

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#### **I. INTRODUCTION**

Recently, the LHCb Collaboration studied the  $D^+\pi^-$ .  $D^0\pi^+$ , and  $D^{*+}\pi^-$  final states is pp collisions at a centerof-mass energy of 7 TeV; observed the  $D_1(2420)^0$  in the  $D^{*+}\pi^-$  final state, the  $D_2^*(2460)$  in the  $D^+\pi^-$ ,  $D^0\pi^+$ , and  $D^{*+}\pi^{-}$  final states; and measured their parameters and confirmed their spin-parity assignments [1]. The LHCb Collaboration also observed two natural parity resonances,  $D_I^*(2650)^0$  and  $D_I^*(2760)^0$ , and two unnatural parity resonances,  $D_J(2580)^0$  and  $D_J(2740)^0$ , in the  $D^{*+}\pi^-$  mass spectrum, and tentatively identified the  $D_I(2580)$  as the  $2 S 0^{-}$  state, the  $D_{I}^{*}(2650)$  as the  $2 S 1^{-}$  state, the  $D_{I}(2740)$ as the  $1 D 2^{-}$  state, and the  $D_{I}^{*}(2760)$  as the  $1 D 1^{-}$  state. The  $D_I^*(2760)^0$  observed in the  $D^{*+}\pi^-$  and  $D^+\pi^-$  decay modes have consistent parameters; their charged partner  $D_I^*(2760)^+$  was observed in the  $D^0\pi^+$  final state [1]. Furthermore, the LHCb Collaboration observed one unnatural parity resonance  $D_{I}(3000)^{0}$  in the  $D^{*+}\pi^{-}$  final state, and two structures  $D_I^*(3000)^0$  and  $D_I^*(3000)^+$  in the  $D^+\pi^-$  and  $D^0\pi^+$  mass spectra, respectively [1]. The revelent parameters are presented in Table I.

In 2010, the *BABAR* Collaboration observed four excited charmed mesons D(2550), D(2600), D(2750), and D(2760) in the decays  $D^0(2550) \rightarrow D^{*+}\pi^-$ ;  $D^0(2600) \rightarrow D^{*+}\pi^-$ ;  $D^0(2600) \rightarrow D^{*+}\pi^-$ ;  $D^0(2760) \rightarrow D^{+}\pi^-$ ;  $D^+(2600) \rightarrow D^0\pi^+$ ; and  $D^+(2760) \rightarrow D^0\pi^+$ , respectively, in the inclusive  $e^+e^- \rightarrow c\bar{c}$  interactions [2]. The *BABAR* Collaboration also analyzed the helicity distributions to determine the spin-parity, and tentatively identified the (D(2550), D(2600)) as the 2 S doublet ( $0^-$ ,  $1^-$ ), and the D(2750) and D(2760) as the D-wave states. The revelent parameters are presented in Table II, where we also present the possible correspondences among the particles observed by the LHCb and *BABAR* collaborations.

In Ref. [3], we study the strong decays of the charmed mesons D(2550), D(2600), D(2750), and D(2760) with the

heavy meson effective theory in the leading order approximation, and tentatively identify the (D(2550), D(2600)) as the 2 S doublet  $(0^-, 1^-)$  and the (D(2750), D(2760)) as the 1 D doublet  $(2^-, 3^-)$ , respectively. Other studies lead to similar or slightly different assignments [4,5]. Now, we extend our previous work to study the strong decays of the charmed mesons observed by the LHCb Collaboration with the heavy meson effective theory in the leading order approximation.

Let us take a short digression to discuss how to classify the heavy-light mesons. In the heavy quark limit, the heavy-light mesons  $Q\bar{q}$  can be classified in doublets according to the total angular momentum of the light antiquark  $\vec{s}_{\ell}$ ,  $\vec{s}_{\ell} = \vec{s}_{\bar{q}} + \vec{L}$ , where the  $\vec{s}_{\bar{q}}$  and  $\vec{L}$  are the spin and orbital angular momentum of the light antiquark, respectively [6]. In the case of the radial quantum number n = 1, the doublet  $(P, P^*)$  has the spin-parity  $J^P_{s_\ell} = (0^-, 1^-)_{\frac{1}{2}}$  for L = 0; the two doublets  $(P_0^*, P_1)$  and  $(P_1, P_2^*)$  have the spin-parity  $J_{s_\ell}^P = (0^+, 1^+)_{\frac{1}{2}}$  and  $(1^+, 2^+)_{\frac{3}{2}}$ , respectively, for L = 1; the two doublets  $(P_1^*, P_2)$  and  $(P_2, P_3^*)$  have the spin-parity  $J_{s_{\ell}}^{P} = (1^{-}, 2^{-})_{\frac{3}{2}}$  and  $(2^{-}, 3^{-})_{\frac{5}{2}}$ , respectively, for L = 2; and the two doublets  $(P_2^*, P_3)$  and  $(P_3, P_4^*)$  have the spin-parity  $J_{s_{\ell}}^{P} = (2^{+}, 3^{+})_{\frac{5}{2}}$  and  $(3^{+}, 4^{+})_{\frac{7}{2}}$ , respectively, for L = 3, where the superscript P denotes the parity. The  $n = 2, 3, 4, \dots$  states are clarified by analogous doublets; for example, when n = 2, the doublet  $(P', P^{*'})$  has the spin-parity  $J_{s_{\ell}}^{P} = (0^{-}, 1^{-})_{\frac{1}{2}}$  for L = 0.

The helicity distributions from the *BABAR* Collaboration favor identifying the  $D^0(2550)$  as the 0<sup>-</sup> state; the  $D^0(2600)$ as the 1<sup>-</sup>, 2<sup>+</sup>, 3<sup>-</sup> state; and the  $D^0(2750)$  as the 1<sup>+</sup>, 2<sup>-</sup> state [2], which are compatible with the tentative assignments of the LHCb Collaboration [1]; see Table II. The  $D_J(2580)^0$ ,  $D_J(2740)^0$ , and  $D_J(3000)^0$  have unnatural parity, and their possible spin-parity assignments are  $J^P = 0^-, 1^+, 2^-,$  $3^+, \ldots$  The  $D_J^*(2650)^0$  and  $D_J^*(2760)^0$  have natural parity, and their possible spin-parity assignments are  $J^P = 0^-, 1^+, 2^-,$  $0^+, 1^-, 2^+, 3^-, \ldots$  From the strong decays

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TABLE I. The experimental results from the LHCb Collaboration, where the N and U denote the natural parity and unnatural parity, respectively.

	Mass (MeV)	Width (MeV)	Decay channel	Significance
$D_I^*(2650)^0$ (N)	$2649.2 \pm 3.5 \pm 3.5$	$140.2 \pm 17.1 \pm 18.6$	$D^{*+}\pi^-$	24.5 <i>\sigma</i>
$D_I^*(2760)^0$ (N)	$2761.1 \pm 5.1 \pm 6.5$	$74.4 \pm 3.4 \pm 37.0$	$D^{*+}\pi^-$	$10.2\sigma$
$D_{I}(2580)^{0}$ (U)	$2579.5 \pm 3.4 \pm 5.5$	$177.5 \pm 17.8 \pm 46.0$	$D^{*+}\pi^-$	$18.8\sigma$
$D_J(2740)^0$ (U)	$2737.0 \pm 3.5 \pm 11.2$	$73.2 \pm 13.4 \pm 25.0$	$D^{*+}\pi^-$	$7.2\sigma$
$D_{I}(3000)^{0}$ (U)	$2971.8\pm8.7$	$188.1 \pm 44.8$	$D^{*+}\pi^-$	$9.0\sigma$
$D_I^*(2760)^0$ (N)	$2760.1 \pm 1.1 \pm 3.7$	$74.4 \pm 3.4 \pm 19.1$	$D^+\pi^-$	$17.3\sigma$
$D_{I}^{*}(3000)^{0}$	$3008.1 \pm 4.0$	$110.5 \pm 11.5$	$D^+\pi^-$	$21.2\sigma$
$D_I^*(2760)^+$	$2771.7 \pm 1.7 \pm 3.8$	$66.7 \pm 6.6 \pm 10.5$	$D^0\pi^+$	$18.8\sigma$
$D_J^*(3000)^+$	3008.1 (fixed)	110.5 (fixed)	$D^0\pi^+$	$6.6\sigma$

TABLE II. The experimental results from the *BABAR* Collaboration. The particles in the bracket are the possible corresponding ones observed by the LHCb Collaboration.

		Mass (MeV)	Width (MeV)	Decay channel
$D^0(2550)$	$[D_J(2580)^0]$	$2539.4 \pm 4.5 \pm 6.8$	$130 \pm 12 \pm 13$	$D^{*+}\pi^-$
$D^0(2600)$	$[D_I^*(2650)^0]$	$2608.7 \pm 2.4 \pm 2.5$	$93 \pm 6 \pm 13$	$D^{+}  \pi^{-},  D^{*+}  \pi^{-}$
$D^0(2750)$	$[D_{J}(2740)^{0}]$	$2752.4 \pm 1.7 \pm 2.7$	$71 \pm 6 \pm 11$	$D^{*+}\pi^-$
$D^0(2760)$	$[D_I^*(2760)^0]$	$2763.3 \pm 2.3 \pm 2.3$	$60.9 \pm 5.1 \pm 3.6$	$D^+\pi^-$
$D^+(2600)$	Ū.	$2621.3 \pm 3.7 \pm 4.2$	93	$D^0\pi^+$
$D^+(2760)$	$[D_J^*(2760)^+]$	$2769.7 \pm 3.8 \pm 1.5$	60.9	$D^0\pi^+$

$$D_J^*(3000)^0 \to D^+\pi^-, \qquad D_J^*(3000)^+ \to D^0\pi^+, \quad (1)$$

we can conclude that the  $D_J^*(3000)$  have the possible spinparity  $J^P = 0^+, 1^-, 2^+, 3^-, 4^+, \ldots$  The six low-lying states, D,  $D^*$ ,  $D_0(2400)$ ,  $D_1(2430)$ ,  $D_1(2420)$ , and  $D_2(2460)$  are established [7], while the 2 S, 1 D, 1 F, 2 P, and 3 S states are still absent. The newly observed charmed mesons  $D_J(2580)$ ,  $D_J^*(2650)$ ,  $D_J(2740)$ ,  $D_J^*(2760)$ ,  $D_J(3000)$ ,  $D_J^*(3000)$  can be tentatively identified as the missing 2 S, 1 D, 1 F, 2 P, and 3 S states.

The mass is a fundamental parameter in describing a hadron; in Table III, we present the predictions from some theoretical models, such as the relativized quark model based on a universal one-gluon exchange plus linear confinement potential [8], the relativistic quark model that includes the leading order  $1/M_h$  corrections [9], and the QCD-motivated relativistic quark model based on the quasipotential approach [10]. We can identify the  $D_J(2580)$ ,  $D_J^*(2650)$ ,  $D_J(2740)$ ,  $D_J^*(2760)$ ,  $D_J(3000)$ , and  $D_J^*(3000)$  tentatively according to the masses.

In the following, we list out the possible assignments:

$$(D_J(2580), D_J^*(2650)) = (0^-, 1^-)_{\frac{1}{2}}$$
 with  $n = 2, L = 0,$ 
  
(2)

$$(D_J^*(2760), D_J(2740)) = (1^-, 2^-)_{\frac{3}{2}}$$
 with  $n = 1, L = 2,$ 
  
(3)

$$(D_J(2740), D_J^*(2760)) = (2^-, 3^-)_{\frac{5}{2}}$$
 with  $n = 1, L = 2,$ 
  
(4)

$$(D_J^*(3000), D_J(3000)) = (2^+, 3^+)_{\frac{5}{2}}$$
 with  $n = 1, L = 3,$ 
  
(5)

$$(D_J(3000), D_J^*(3000)) = (3^+, 4^+)_{\frac{7}{2}}$$
 with  $n = 1, L = 3,$   
(6)

$$(D_J^*(3000), D_J(3000)) = (0^+, 1^+)_{\frac{1}{2}}$$
 with  $n = 2, L = 1,$ 
(7)

$$(D_J(3000), D_J^*(3000)) = (1^+, 2^+)_{\frac{3}{2}}$$
 with  $n = 2, L = 1,$ 
  
(8)

$$(D_J(3000), D_J^*(3000)) = (0^-, 1^-)_{\frac{1}{2}}$$
 with  $n = 3, L = 0.$ 
  
(9)

In this work, we study the strong decays of the charmed mesons  $D_J(2580)$ ,  $D_J^*(2650)$ ,  $D_J(2740)$ ,  $D_J^*(2760)$ ,  $D_J(3000)$ , and  $D_J^*(3000)$  observed by the LHCb Collaboration with the heavy meson effective theory in the leading order approximation, and we make predictions for the decay widths and the ratios among the decay widths. The ratios can be confronted with the experimental data in the future at the LHCb, BESIII, KEK-B, and PANDA to

TABLE III.	The masses	of the charm	ed mesons fror	n different	quark mode	ls compared	with e	experimental	data,	and the	possible
assignments	of the newly	observed char	rmed mesons. 7	The N and	U denote the	e natural pari	ty and	unnatural pa	arity, r	espectiv	ely.

	n	L	$s_\ell$	$J^P$	Exp [1,7]	GI [1,8]	PE [9]	EFG [10]
D	1	S	$\frac{1}{2}$	0-	1867	1864	1868	1871
$D^*$	1	S	$\frac{1}{2}$	1-	2008	2023	2005	2010
$D_0^*$	1	Р	$\frac{1}{2}$	$0^+$	2400	2380	2377	2406
$D_1^{\circ}$	1	Р	$\frac{1}{2}$	$1^{+}$	2427	2419	2490	2469
$D_1$	1	Р	$\frac{5}{2}$	$1^{+}$	2420	2469	2417	2426
$D_2^*$	1	Р	$\frac{\overline{3}}{2}$	$2^{+}$	2460	2479	2460	2460
$D_1^{\tilde{*}}$	1	D	$\frac{\overline{3}}{2}$	$1^{-}$	? 2760 (N)	2796	2795	2788
$D_2$	1	D	$\frac{\overline{3}}{2}$	$2^{-}$	? 2740 (U)	2801	2833	2850
$\overline{D_2}$	1	D	$\frac{5}{2}$	$2^{-}$	? 2740 (U)	2806	2775	2806
$D_3^*$	1	D	$\frac{5}{2}$	3-	? 2760 (N)	2806	2799	2863
$D_2^*$	1	F	$\frac{5}{2}$	$2^{+}$	? 3000 (N)	3074	3101	3090
$\bar{D_3}$	1	F	<u>5</u>	3+	? 3000 (U)	3074	3123	3145
$D_3$	1	F	$\frac{\overline{7}}{2}$	3+	? 3000 (U)	3079	3074	3129
$D_4^*$	1	F	$\frac{7}{2}$	4+	? 3000 (N)	3084	3091	3187
$D^{'}$	2	S	$\frac{\overline{1}}{2}$	$0^{-}$	? 2580 (U)	2558	2589	2581
$D^*$	2	S	$\frac{1}{2}$	$1^{-}$	? 2650 (N)	2618	2692	2632
$D_0^*$	2	Р	$\frac{\overline{1}}{2}$	$0^+$	? 3000 (N)		2949	2919
$D_1$	2	Р	$\frac{1}{2}$	$1^{+}$	? 3000 (U)		3045	3021
$D_1$	2	Р	$\frac{3}{2}$	$1^{+}$	? 3000 (U)		2995	2932
$D_2^*$	2	Р	$\frac{\overline{3}}{2}$	$2^{+}$	? 3000 (N)		3035	3012
$\tilde{D}$	3	S	$\frac{1}{2}$	$0^{-}$	? 3000 (U)		3141	3062
$D^*$	3	S	$\frac{1}{2}$	1-	? 3000 (N)		3226	3096

distinguish the different assignments. Furthermore, the analytical expressions of the decay widths can be used to determine the strong coupling constants in the heavy mesons' effective Lagrangian in the future. On the other hand, we can also use the  ${}^{3}P_{0}$  model to study those strong decays following Ref. [11]. There have been several works using the heavy meson effective theory to identify the charmed mesons [3,5,12] and to study the radiative, vector-meson, two-pion decays of the heavy quarkonium states [13].

The article is arranged as follows: we study the strong decays of the charmed mesons  $D_J(2580)$ ,  $D_J^*(2650)$ ,

 $D_J(2740)$ ,  $D_J^*(2760)$ ,  $D_J(3000)$ , and  $D_J^*(3000)$  observed by the LHCb Collaboration with the heavy meson effective theory in Sec. II; in Sec. III, we present the numerical results and discussions; and Sec. IV is reserved for our conclusions.

## II. THE STRONG DECAYS WITH THE HEAVY MESON EFFECTIVE THEORY

In the heavy meson effective theory, the spin doublets can be described by the effective superfields  $H_a$ ,  $S_a$ ,  $T_a$ ,  $X_a$ ,  $Y_a$ ,  $Z_a$ , and  $R_a$ , respectively [14]:

$$\begin{split} H_{a} &= \frac{1+\dot{\nu}}{2} \{ P_{a\mu}^{*} \gamma^{\mu} - P_{a} \gamma_{5} \}, \quad S_{a} &= \frac{1+\dot{\nu}}{2} \{ P_{1a}^{\mu} \gamma_{\mu} \gamma_{5} - P_{0a}^{*} \}, \quad T_{a}^{\mu} = \frac{1+\dot{\nu}}{2} \left\{ P_{2a}^{*\mu\nu} \gamma_{\nu} - P_{1a\nu} \sqrt{\frac{3}{2}} \gamma_{5} \left[ g^{\mu\nu} - \frac{\gamma^{\nu} (\gamma^{\mu} - \upsilon^{\mu})}{3} \right] \right\}, \\ X_{a}^{\mu} &= \frac{1+\dot{\nu}}{2} \left\{ P_{2a}^{\mu\nu\nu} \gamma_{5} \gamma_{\nu} - P_{1a\nu}^{*} \sqrt{\frac{3}{2}} \left[ g^{\mu\nu} - \frac{\gamma^{\nu} (\gamma^{\mu} + \upsilon^{\mu})}{3} \right] \right\}, \\ Y_{a}^{\mu\nu} &= \frac{1+\dot{\nu}}{2} \left\{ P_{3a}^{*\mu\nu\sigma} \gamma_{\sigma} - P_{2a}^{\alpha\beta} \sqrt{\frac{5}{3}} \gamma_{5} \left[ g^{\mu}_{\alpha} g^{\nu}_{\beta} - \frac{g^{\nu}_{\beta} \gamma_{\alpha} (\gamma^{\mu} - \upsilon^{\mu})}{5} - \frac{g^{\mu}_{\alpha} \gamma_{\beta} (\gamma^{\nu} - \upsilon^{\nu})}{5} \right] \right\}, \\ Z_{a}^{\mu\nu} &= \frac{1+\dot{\nu}}{2} \left\{ P_{3a}^{\mu\nu\sigma} \gamma_{5} \gamma_{\sigma} - P_{2a}^{*\alpha\beta} \sqrt{\frac{5}{3}} \left[ g^{\mu}_{\alpha} g^{\nu}_{\beta} - \frac{g^{\nu}_{\beta} \gamma_{\alpha} (\gamma^{\mu} + \upsilon^{\mu})}{5} - \frac{g^{\mu}_{\alpha} \gamma_{\beta} (\gamma^{\nu} + \upsilon^{\nu})}{5} \right] \right\}, \\ R_{a}^{\mu\nu\rho} &= \frac{1+\dot{\nu}}{2} \left\{ P_{4a}^{*\mu\nu\rho\sigma} \gamma_{5} \gamma_{\sigma} - P_{3a}^{\alpha\beta\tau} \sqrt{\frac{7}{4}} \left[ g^{\mu}_{\alpha} g^{\nu}_{\beta} g^{\rho}_{\tau} - \frac{g^{\nu}_{\beta} g^{\rho}_{\tau} \gamma_{\alpha} (\gamma^{\mu} - \upsilon^{\mu})}{7} - \frac{g^{\mu}_{\alpha} g^{\rho}_{\tau} \gamma_{\beta} (\gamma^{\nu} - \upsilon^{\nu})}{7} - \frac{g^{\mu}_{\alpha} g^{\nu}_{\beta} \gamma_{\tau} (\gamma^{\rho} - \upsilon^{\rho})}{7} \right] \right\}, \tag{10}$$

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where the heavy meson fields  $P^{(*)}$  contain a factor  $\sqrt{M_{P^{(*)}}}$  and have dimension of mass  $\frac{3}{2}$ . The superfields  $H_a$  contain the S-wave mesons;  $S_a$ ,  $T_a$  contain the P-wave mesons;  $X_a$ ,  $Y_a$  contain the D-wave mesons;  $Z_a$  and  $R_a$  contain the F-wave mesons. The n = 1, 2, 3, ... heavy mesons with the same heavy flavor have the same parity, time-reversal, and charge conjunction properties except for the masses, and can be combined into the superfields:  $H_a, H'_a, H''_a, \ldots; S_a, S'_a, S''_a, \ldots;$  $T_a, T'_a, T''_a, \dots$ ; etc, where the superscripts *I*, *II*, and *III* denote the  $n = 2, 3, 4, \dots$  states, respectively. We can replace the heavy meson fields  $P^{(*)}$  with their corresponding radial excited states to obtain the corresponding superfields  $H'_a, S'_a, \ldots$ 

The light pseudoscalar mesons are described by the fields  $\xi = e^{\frac{i\mathcal{M}}{f_{\pi}}}$ , where

$$\mathcal{M} = egin{pmatrix} \sqrt{rac{1}{2}} \pi^0 + \sqrt{rac{1}{6}} \eta & \pi^+ & K^+ \ \pi^- & -\sqrt{rac{1}{2}} \pi^0 + \sqrt{rac{1}{6}} \eta & K^0 \ K^- & ar{K}^0 & -\sqrt{rac{2}{3}} \eta \end{pmatrix},$$

and  $f_{\pi} = 130$  MeV.

At the leading order, the heavy meson chiral Lagrangians  $\mathcal{L}_0$ ,  $\mathcal{L}_H$ ,  $\mathcal{L}_S$ ,  $\mathcal{L}_T$ ,  $\mathcal{L}_X$ ,  $\mathcal{L}_Y$ ,  $\mathcal{L}_Z$ , and  $\mathcal{L}_R$  for the strong decays to the  $D^{(*)}\pi$ ,  $D^{(*)}\eta$ , and  $D_s^{(*)}K$  states can be written as

$$\mathcal{L}_{0} = i \operatorname{Tr}\{\bar{H}_{a}\upsilon \cdot \mathcal{D}_{ab}H_{b}\} + i \operatorname{Tr}\{\bar{S}_{a}\upsilon \cdot \mathcal{D}_{ab}S_{b}\} + i \operatorname{Tr}\{\bar{T}_{a}^{\mu}\upsilon \cdot \mathcal{D}_{ab}T_{\mu b}\} + i \operatorname{Tr}\{\bar{X}_{a}^{\mu}\upsilon \cdot \mathcal{D}_{ab}X_{\mu b}\} + i \operatorname{Tr}\{\bar{Y}_{a}^{\mu\nu}\upsilon \cdot \mathcal{D}_{ab}Y_{\mu\nu b}\}$$

$$+ i \operatorname{Tr}\{\bar{Z}_{a}^{\mu\nu}\upsilon \cdot \mathcal{D}_{ab}Z_{\mu\nu b}\} + i \operatorname{Tr}\{\bar{R}_{a}^{\mu\nu\alpha}\upsilon \cdot \mathcal{D}_{ab}Z_{\mu\nu ab}\} - \delta m_{S}\operatorname{Tr}\{\bar{S}_{a}S_{a}\} - \delta m_{T}\operatorname{Tr}\{\bar{T}_{a}^{\mu}T_{\mu a}\} - \delta m_{X}\operatorname{Tr}\{\bar{X}_{a}^{\mu}X_{\mu a}\}$$

$$- \delta m_{Y}\operatorname{Tr}\{\bar{Y}_{a}^{\mu\nu}\gamma_{\mu\nu a}\} - \delta m_{Z}\operatorname{Tr}\{\bar{Z}_{a}^{\mu\nu}Z_{\mu\nu a}\} - \delta m_{R}\operatorname{Tr}\{\bar{R}_{a}^{\mu\nu\beta}Z_{\mu\nu\beta a}\},$$

$$\mathcal{L}_{H} = g_{H}\operatorname{Tr}\{\bar{H}_{a}H_{b}\gamma_{\mu}\gamma_{5}\mathcal{A}_{ba}^{\mu}\}, \qquad \mathcal{L}_{S} = g_{S}\operatorname{Tr}\{\bar{H}_{a}S_{b}\gamma_{\mu}\gamma_{5}\mathcal{A}_{ba}^{\mu}\} + \operatorname{H.c.},$$

$$\mathcal{L}_{T} = \frac{g_{T}}{\Lambda}\operatorname{Tr}\{\bar{H}_{a}T_{b}^{\mu}(i\mathcal{D}_{\mu}\mathcal{A} + i\mathcal{D}\mathcal{A}_{\mu})_{ba}\gamma_{5}\} + \operatorname{H.c.}, \qquad \mathcal{L}_{X} = \frac{g_{X}}{\Lambda}\operatorname{Tr}\{\bar{H}_{a}X_{b}^{\mu}(i\mathcal{D}_{\mu}\mathcal{A} + i\mathcal{D}\mathcal{A}_{\mu})_{ba}\gamma_{5}\} + \operatorname{H.c.},$$

$$\mathcal{L}_{Y} = \frac{1}{\Lambda^{2}}\operatorname{Tr}\{\bar{H}_{a}Y_{b}^{\mu\nu}[k_{1}^{Y}\{\mathcal{D}_{\mu},\mathcal{D}_{\nu}\}\mathcal{A}_{\lambda} + k_{2}^{Y}(\mathcal{D}_{\mu}\mathcal{D}_{\lambda}\mathcal{A}_{\nu} + \mathcal{D}_{\nu}\mathcal{D}_{\lambda}\mathcal{A}_{\mu})]_{ba}\gamma^{\lambda}\gamma_{5}\} + \operatorname{H.c.},$$

$$\mathcal{L}_{R} = \frac{1}{\Lambda^{3}}\operatorname{Tr}\{\bar{H}_{a}R_{b}^{\mu\nu\rho}[k_{1}^{R}\{\mathcal{D}_{\mu},\mathcal{D}_{\nu},\mathcal{D}_{\rho}\}\mathcal{A}_{\lambda} + k_{2}^{Q}(\{\mathcal{D}_{\mu}\mathcal{D}_{\mu}\mathcal{D}_{\mu}\mathcal{A}_{\nu} + \{\mathcal{D}_{\nu},\mathcal{D}_{\rho}\}\mathcal{D}_{\lambda}\mathcal{A}_{\mu} + \{\mathcal{D}_{\mu},\mathcal{D}_{\nu}\}\mathcal{D}_{\lambda}\mathcal{A}_{\rho})]_{ba}\gamma^{\lambda}\gamma_{5}\} + \operatorname{H.c.},$$

$$(11)$$

where

$$\mathcal{D}_{\mu} = \partial_{\mu} + \mathcal{V}_{\mu}, \quad \mathcal{V}_{\mu} = \frac{1}{2} (\xi^{\dagger} \partial_{\mu} \xi + \xi \partial_{\mu} \xi^{\dagger}), \quad \mathcal{A}_{\mu} = \frac{1}{2} (\xi^{\dagger} \partial_{\mu} \xi - \xi \partial_{\mu} \xi^{\dagger}), \quad \{\mathcal{D}_{\mu}, \mathcal{D}_{\nu}\} = \mathcal{D}_{\mu} \mathcal{D}_{\nu} + \mathcal{D}_{\nu} \mathcal{D}_{\mu}, \\ \{\mathcal{D}_{\mu}, \mathcal{D}_{\nu}, \mathcal{D}_{\rho}\} = \mathcal{D}_{\mu} \mathcal{D}_{\nu} \mathcal{D}_{\rho} + \mathcal{D}_{\mu} \mathcal{D}_{\rho} \mathcal{D}_{\nu} + \mathcal{D}_{\nu} \mathcal{D}_{\rho} \mathcal{D}_{\mu} + \mathcal{D}_{\rho} \mathcal{D}_{\mu} \mathcal{D}_{\nu} + \mathcal{D}_{\rho} \mathcal{D}_{\nu} \mathcal{D}_{\mu},$$
(12)

where  $\delta m_S = m_S - m_H$ ,  $\delta m_T = m_T - m_H$ ,  $\delta m_X = m_X - m_H$  $m_H$ ,  $\delta m_Y = m_Y - m_H$ ,  $\delta m_Z = m_Z - m_H$ ,  $\delta m_R =$  $m_R - m_H$ . A is the chiral symmetry-breaking scale and taken as  $\Lambda = 1$  GeV [12]. The strong coupling constants  $g_H$ ,  $g_S$ ,  $g_T$ ,  $g_X$ ,  $g_Y = k_1^Y + k_2^Y$ ,  $g_Z = k_1^Z + k_2^Z$ , and  $g_R = k_1^R + k_2^R$  can be fitted to the experimental data. The heavy meson chiral Lagrangians  $\mathcal{L}_H$ ,  $\mathcal{L}_S$ ,  $\mathcal{L}_T$ ,  $\mathcal{L}_X$ , and  $\mathcal{L}_Y$  are taken from Ref. [15]; the  $\mathcal{L}_Z$  and  $\mathcal{L}_R$  are constructed accordingly in this article. The subscript indexes H, S, T, X, Y, Z, and R denote the interactions between the superfield H and the super-fields H, S, T, X, Y, Z, and R, respectively. We smear the superscripts 1, 11, 111, ... for simplicity. The notation  $g_H$  denotes the strong coupling constants in the vertexes HHA, H'HA, H'HA,  $H''H\mathcal{A},\ldots$ ; the notations  $g_S, g_T, g_X, g_Y, g_Z$ , and  $g_R$ should be understood in the same way. We can also study

the decays to the light vector mesons V besides the pseudoscalar mesons  $\mathcal{P}$  with the replacement  $\mathcal{V}_{\mu} \rightarrow$  $\mathcal{V}_{\mu} + V_{\mu}$ , and we introduce additional phenomenological Lagrangians [16]; therefore, additional unknown coupling constants, which have to be fitted to the precise experimental data in the future, are beyond the present work.

From the heavy meson chiral Lagrangians  $\mathcal{L}_H$ ,  $\mathcal{L}_S$ ,  $\mathcal{L}_T$ ,  $\mathcal{L}_X, \mathcal{L}_Y, \mathcal{L}_Z$ , and  $\mathcal{L}_R$ , we can obtain the widths  $\Gamma$  for strong decays to the final states  $D^{(*)}\pi$ ,  $D^{(*)}\eta$ , and  $D_s^{(*)}K$ ,

$$\Gamma = \frac{1}{2J+1} \sum \frac{p_f}{8\pi M_i^2} |T|^2,$$

$$p_f = \frac{\sqrt{(M_i^2 - (M_f + m_P)^2)(M_i^2 - (M_f - m_P)^2)}}{2M_i},$$
(13)

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where *T* denotes the scattering amplitudes; *i* and *f* denote the initial and final state heavy mesons, respectively; *J* is the total angular momentum of the initial heavy meson;  $\sum$  denotes the summation of all the polarization vectors of the total angular momentum j = 1, 2, 3 or 4; and  $\mathcal{P}$  denotes the light pseudoscalar mesons.

Now we write down the explicit expressions of the decay widths  $\Gamma$  in different channels:

(i)  $(0^{-}, 1^{-})_{\frac{1}{2}} \to (0^{-}, 1^{-})_{\frac{1}{2}} + \mathcal{P},$ 

$$\Gamma(1^{-} \to 1^{-}) = C_{\mathcal{P}} \frac{g_{H}^{2} M_{f} p_{f}^{3}}{3\pi f_{\pi}^{2} M_{i}} \Big[ C_{\mathcal{P}} \frac{g_{H}^{2} (M_{i} + M_{f})^{2} p_{f}^{3}}{12\pi f_{\pi}^{2} M_{i} M_{f}} \Big], \quad (14)$$

$$\Gamma(1^{-} \to 0^{-}) = C_{\mathcal{P}} \frac{g_{H}^{2} M_{f} p_{f}^{3}}{6\pi f_{\pi}^{2} M_{i}} \Big[ C_{\mathcal{P}} \frac{g_{H}^{2} (M_{i} + M_{f})^{2} p_{f}^{3}}{24\pi f_{\pi}^{2} M_{i} M_{f}} \Big], \quad (15)$$

$$\Gamma(0^{-} \to 1^{-}) = C_{\mathcal{P}} \frac{g_{H}^{2} M_{f} p_{f}^{3}}{2\pi f_{\pi}^{2} M_{i}} \Big[ C_{\mathcal{P}} \frac{g_{H}^{2} (M_{i} + M_{f})^{2} p_{f}^{3}}{8\pi f_{\pi}^{2} M_{i} M_{f}} \Big], \quad (16)$$

(ii) 
$$(0^+, 1^+)_{\frac{1}{2}} \rightarrow (0^-, 1^-)_{\frac{1}{2}} + \mathcal{P},$$
  

$$\Gamma(1^+ \rightarrow 1^-) = C_{\mathcal{P}} \frac{g_S^2 M_f (p_f^2 + m_{\mathcal{P}}^2) p_f}{2\pi f_\pi^2 M_i} \left[ C_{\mathcal{P}} \frac{g_S^2 (M_i - M_f)^2 (p_f^2 + m_{\mathcal{P}}^2) p_f}{8\pi f_\pi^2 M_i M_f} \right],$$
(17)

$$\Gamma(0^{+} \to 0^{-}) = C_{\mathcal{P}} \frac{g_{S}^{2} M_{f} (p_{f}^{2} + m_{\mathcal{P}}^{2}) p_{f}}{2 \pi f_{\pi}^{2} M_{i}} \\ \left[ C_{\mathcal{P}} \frac{g_{S}^{2} (M_{i} - M_{f})^{2} (p_{f}^{2} + m_{\mathcal{P}}^{2}) p_{f}}{8 \pi f_{\pi}^{2} M_{i} M_{f}} \right],$$
(18)

(iii) 
$$(1^+, 2^+)_{\frac{3}{2}} \rightarrow (0^-, 1^-)_{\frac{1}{2}} + \mathcal{P},$$
  
 $\Gamma(2^+ \rightarrow 1^-)$   
 $= C_{\mathcal{P}} \frac{2g_T^2 M_f p_f^5}{5\pi f_\pi^2 \Lambda^2 M_i} \Big[ C_{\mathcal{P}} \frac{g_T^2 (M_i + M_f)^2 p_f^5}{10\pi f_\pi^2 \Lambda^2 M_i M_f} \Big],$  (19)

$$\Gamma(2^{+} \to 0^{-}) = C_{\mathcal{P}} \frac{4g_{T}^{2}M_{f}p_{f}^{5}}{15\pi f_{\pi}^{2}\Lambda^{2}M_{i}} \Big[ C_{\mathcal{P}} \frac{g_{T}^{2}(M_{i} + M_{f})^{2}p_{f}^{5}}{15\pi f_{\pi}^{2}\Lambda^{2}M_{i}M_{f}} \Big],$$
(20)

$$\Gamma(1^+ \to 1^-) = C_P \frac{2g_T^2 M_f p_f^5}{3\pi f_\pi^2 \Lambda^2 M_i} \bigg[ C_P \frac{g_T^2 (M_i + M_f)^2 p_f^5}{6\pi f_\pi^2 \Lambda^2 M_i M_f} \bigg], \quad (21)$$

(iv) 
$$(1^{-}, 2^{-})_{\frac{3}{2}} \rightarrow (0^{-}, 1^{-})_{\frac{1}{2}} + \mathcal{P},$$
  

$$\Gamma(2^{-} \rightarrow 1^{-}) = C_{\mathcal{P}} \frac{2g_{X}^{2}M_{f}(p_{f}^{2} + m_{\mathcal{P}}^{2})p_{f}^{3}}{3\pi f_{\pi}^{2}\Lambda^{2}M_{i}} \left[ C_{\mathcal{P}} \frac{g_{X}^{2}(M_{i} - M_{f})^{2}(p_{f}^{2} + m_{\mathcal{P}}^{2})p_{f}^{3}}{6\pi f_{\pi}^{2}\Lambda^{2}M_{i}M_{f}} \right],$$
(22)

$$\Gamma(1^{-} \to 1^{-}) = C_{\mathcal{P}} \frac{2g_{X}^{2}M_{f}(p_{f}^{2} + m_{\mathcal{P}}^{2})p_{f}^{3}}{9\pi f_{\pi}^{2}\Lambda^{2}M_{i}} \\ \left[ C_{\mathcal{P}} \frac{g_{X}^{2}(M_{i} - M_{f})^{2}(p_{f}^{2} + m_{\mathcal{P}}^{2})p_{f}^{3}}{18\pi f_{\pi}^{2}\Lambda^{2}M_{i}M_{f}} \right],$$
(23)

$$\Gamma(1^{-} \to 0^{-}) = C_{\mathcal{P}} \frac{4g_{X}^{2}M_{f}(p_{f}^{2} + m_{\mathcal{P}}^{2})p_{f}^{3}}{9\pi f_{\pi}^{2}\Lambda^{2}M_{i}} \\ \left[ C_{\mathcal{P}} \frac{g_{X}^{2}(M_{i} - M_{f})^{2}(p_{f}^{2} + m_{\mathcal{P}}^{2})p_{f}^{3}}{9\pi f_{\pi}^{2}\Lambda^{2}M_{i}M_{f}} \right],$$
(24)

(v) 
$$(2^{-}, 3^{-})_{\frac{5}{2}} \rightarrow (0^{-}, 1^{-})_{\frac{1}{2}} + \mathcal{P},$$
  

$$\Gamma(3^{-} \rightarrow 1^{-}) = C_{\mathcal{P}} \frac{16g_{Y}^{2}M_{f}p_{f}^{7}}{105\pi f_{\pi}^{2}\Lambda^{4}M_{i}}$$

$$\left[C_{\mathcal{P}} \frac{4g_{Y}^{2}(M_{i} + M_{f})^{2}p_{f}^{7}}{105\pi f_{\pi}^{2}\Lambda^{4}M_{i}M_{f}}\right], \quad (25)$$

$$\Gamma(3^{-} \to 0^{-}) = C_{\mathcal{P}} \frac{4g_{Y}^{2}M_{f}p_{f}^{7}}{35\pi f_{\pi}^{2}\Lambda^{4}M_{i}} \bigg[ C_{\mathcal{P}} \frac{g_{Y}^{2}(M_{i} + M_{f})^{2}p_{f}^{7}}{35\pi f_{\pi}^{2}\Lambda^{4}M_{i}M_{f}} \bigg], \quad (26)$$

$$\Gamma(2^{-} \to 1^{-}) = C_{\mathcal{P}} \frac{4g_{Y}^{2}M_{f}p_{f}^{7}}{15\pi f_{\pi}^{2}\Lambda^{4}M_{i}} \Big[ C_{\mathcal{P}} \frac{g_{Y}^{2}(M_{i} + M_{f})^{2}p_{f}^{7}}{15\pi f_{\pi}^{2}\Lambda^{4}M_{i}M_{f}} \Big],$$
(27)

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(vi) 
$$(2^+, 3^+)_{\frac{5}{2}} \rightarrow (0^-, 1^-)_{\frac{1}{2}} + \mathcal{P},$$
  

$$\Gamma(3^+ \rightarrow 1^-) = C_{\mathcal{P}} \frac{4g_Z^2 M_f(p_f^2 + m_{\mathcal{P}}^2) p_f^5}{15\pi f_\pi^2 \Lambda^4 M_i} \left[ C_{\mathcal{P}} \frac{g_Z^2 (M_i - M_f)^2 (p_f^2 + m_{\mathcal{P}}^2) p_f^5}{15\pi f_\pi^2 \Lambda^4 M_i M_f} \right],$$
(28)

$$\Gamma(2^{+} \to 1^{-}) = C_{\mathcal{P}} \frac{8g_{Z}^{2}M_{f}(p_{f}^{2} + m_{\mathcal{P}}^{2})p_{f}^{5}}{75\pi f_{\pi}^{2}\Lambda^{4}M_{i}} \\ \left[ C_{\mathcal{P}} \frac{2g_{Z}^{2}(M_{i} - M_{f})^{2}(p_{f}^{2} + m_{\mathcal{P}}^{2})p_{f}^{5}}{75\pi f_{\pi}^{2}\Lambda^{4}M_{i}M_{f}} \right],$$
(29)

$$\Gamma(2^{+} \to 0^{-}) = C_{\mathcal{P}} \frac{4g_{Z}^{2}M_{f}(p_{f}^{2} + m_{\mathcal{P}}^{2})p_{f}^{5}}{25\pi f_{\pi}^{2}\Lambda^{4}M_{i}} \\ \left[ C_{\mathcal{P}} \frac{g_{Z}^{2}(M_{i} - M_{f})^{2}(p_{f}^{2} + m_{\mathcal{P}}^{2})p_{f}^{5}}{25\pi f_{\pi}^{2}\Lambda^{4}M_{i}M_{f}} \right],$$
(30)

(vii) 
$$(3^+, 4^+)_{\frac{7}{2}} \to (0^-, 1^-)_{\frac{1}{2}} + \mathcal{P},$$
  
 $\Gamma(4^+ \to 1^-)$   
 $= C_{\mathcal{P}} \frac{4g_R^2 M_f p_f^9}{7\pi f_\pi^2 \Lambda^6 M_i} \Big[ C_{\mathcal{P}} \frac{g_R^2 (M_i + M_f)^2 p_f^9}{7\pi f_\pi^2 \Lambda^6 M_i M_f} \Big],$ 
(31)

$$\Gamma(4^{+} \to 0^{-}) = C_{\mathcal{P}} \frac{16g_{R}^{2}M_{f}p_{f}^{9}}{35\pi f_{\pi}^{2}\Lambda^{6}M_{i}} \left[ C_{\mathcal{P}} \frac{4g_{R}^{2}(M_{i} + M_{f})^{2}p_{f}^{9}}{35\pi f_{\pi}^{2}\Lambda^{6}M_{i}M_{f}} \right], \quad (32)$$

$$\Gamma(3^{+} \to 1^{-}) = C_{\mathcal{P}} \frac{36g_{R}^{2}M_{f}p_{f}^{9}}{35\pi f_{\pi}^{2}\Lambda^{6}M_{i}} \\ \left[ C_{\mathcal{P}} \frac{9g_{R}^{2}(M_{i} + M_{f})^{2}p_{f}^{9}}{35\pi f_{\pi}^{2}\Lambda^{6}M_{i}M_{f}} \right], \quad (33)$$

where the coefficients  $C_{\pi^{\pm}} = C_{K^{\pm}} = C_{K^0} = C_{\bar{K}^0} = 1$ ,  $C_{\pi^0} = \frac{1}{2}$ , and  $C_{\eta} = \frac{2}{3}$ . We obtain the expressions in the brackets by taking into account the different four-velocities of the initial and final state heavy mesons. The on-shell conditions require that  $M_i v_{\mu} = M_f v'_{\mu} + p_{\mu}$  with  $v^2 = v'^2 = 1$ ,  $p^2 = m_P^2$ ,

TABLE IV. The strong decay widths of the newly observed charmed mesons with possible assignments.

	n	L	$s_{\ell}$	$J^P$	Decay channels	Widths (GeV)	Decay channels	Widths (GeV)
$D_{I}(2580)$	2	S	1/2	0-	$D^{*+}\pi^-$	$0.86744g_{H}^{2}$		
<b>J</b> × <i>j</i> ×			2		$D_{s}^{*+}K^{-}$	0		
					$D^{*0}\pi^0$	$0.44284g_{H}^{2}$		
					$D^{*0}\eta$	$0.01545g_{H}^{2}$		
$D_{I}^{*}(2650)$	2	S	$\frac{1}{2}$	1-	$D^{*+}\pi^-$	$0.78436g_{H}^{2}$	$D^+\pi^-$	$0.61923g_H^2$
3			2		$D_{s}^{*+}K^{-}$	$0.03361g_{H}^{2}$	$D_s^+ K^-$	$0.15626g_H^2$
					$D^{*0}\pi^0$	$0.39903g_H^2$	$D^0\pi^0$	$0.31475g_{H}^{2}$
					$D^{*0}\eta$	$0.07934g_H^2$	$D^0\eta$	$0.15758g_H^2$
$D_{J}^{*}(2760)$	1	D	$\frac{3}{2}$	1-	$D^{*+}\pi^-$	$0.33541g_X^2$	$D^+\pi^-$	$1.28546g_X^2$
			2		$D_{s}^{*+}K^{-}$	$0.06149g_X^2$	$D_s^+ K^-$	$0.45974g_X^2$
					$D^{*0}\pi^0$	$0.17110g_X^2$	$D^0\pi^0$	$0.65652g_X^2$
					$D^{*0}\eta$	$0.08867g_X^2$	$D^0\eta$	$0.50190g_X^2$
$D_J(2740)$	1	D	$\frac{3}{2}$	$2^{-}$	$D^{*+}\pi^-$	$0.87560g_X^2$		
			2		$D_{s}^{*+}K^{-}$	$0.13301g_X^2$		
					$D^{*0}\pi^0$	$0.44709g_X^2$		
					$D^{*0}\eta$	$0.20791g_X^2$		
$D_J(2740)$	1	D	$\frac{5}{2}$	$2^{-}$	$D^{*+}\pi^-$	$0.12739g_Y^2$		
			2		$D_{s}^{*+}K^{-}$	$0.00193g_Y^2$		
					$D^{*0}\pi^0$	$0.06597g_Y^2$		
					$D^{*0}\eta$	$0.00524g_Y^2$		
$D_J^*(2760)$	1	D	$\frac{5}{2}$	3-	$D^{*+}\pi^-$	$0.08910g_Y^2$	$D^+\pi^-$	$0.17382g_Y^2$
			-		$D_{s}^{*+}K^{-}$	$0.00207g_Y^2$	$D_s^+ K^-$	$0.01760g_Y^2$
					$D^{*0}\pi^0$	$0.04607g_Y^2$	$D^0 \pi^0$	$0.08995g_Y^2$
					$D^{*0}\eta$	$0.00477g_Y^2$	$D^0\eta$	$0.02413g_Y^2$

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and  $v_{\mu} \neq v'_{\mu}$ . In some multiplets, the conditions  $v_{\mu} = v'_{\mu}$  and  $v_{\mu} \neq v'_{\mu}$  lead to quite different decay widths but similar ratios among the decay widths. The ratios among the decay widths can be used to identify the heavy mesons; we expect that the expressions in the brackets cannot lead to different conclusions. In calculations, we take the approximation  $\mathcal{A}_{\mu} \approx i \frac{\partial_{\mu} \mathcal{M}}{f_{\pi}}$  and neglect the intermediate loops of light pseudoscalar mesons. Furthermore, we neglect the flavor and spin violation corrections of order  $\mathcal{O}(1/m_{Q})$  to avoid introducing new unknown coupling constants, and we expect that

the corrections would not be larger than (or as large as) the leading order contributions.

### **III. NUMERICAL RESULTS**

The input parameters are taken as  $M_{\pi^+} = 139.57$  MeV,  $M_{\pi^0} = 134.9766$  MeV,  $M_{K^+} = 493.677$  MeV,  $M_{\eta} = 547.853$  MeV,  $M_{D^+} = 1869.60$  MeV,  $M_{D^0} = 1864.83$  MeV,  $M_{D_s^+} = 1968.47$  MeV,  $M_{D^{*+}} = 2010.25$  MeV,  $M_{D^{*0}} = 2006.96$  MeV,  $M_{D_s^{*+}} = 2112.3$  MeV [7],  $M_{D(2420)} = 2419.6$  MeV, and  $M_{D_s^*(2460)} = 2460.4$  MeV [1].

The numerical values of the widths of the strong decays of the charmed mesons  $D_J(2580)$ ,  $D_J^*(2650)$ ,  $D_J(2740)$ ,

TABLE V. The strong decay widths of the newly observed charmed mesons with possible assignments.

	п	L	s <sub>l</sub>	$J^P$	Decay channels	Widths (GeV)	Decay channels	Widths (GeV)
$D_J^*(3000)$	1	F	$\frac{5}{2}$	2+	$D^{*+}\pi^-$	$0.35576g_Z^2$	$D^+\pi^-$	$1.03474g_Z^2$
					$D_{s}^{*+}K^{-}$	$0.09260g_Z^2$	$D_s^+ K^-$	$0.38738g_Z^2$
					$D^{*0}\pi^0$	$0.18146g_Z^2$	$D^0 \pi^0$	$0.52903g_Z^2$
			-		$D^{*0}\eta$	$0.11279g_Z^2$	$D^0 \eta$	$0.40754g_Z^2$
$D_J(3000)$	1	F	<u>5</u> 2	3+	$D^{*+}\pi^-$	$0.71619g_Z^2$		
					$D_{s}^{*+}K^{-}$	$0.16586g_Z^2$		
					$D^{*0}\pi^0$	$0.36571g_Z^2$		
		_	7	<b>a</b> +	$D^{*0}\eta$	$0.21080g_Z^2$		
$D_J(3000)$	1	F	$\frac{7}{2}$	3+	$D^{*+}\pi^-$	$1.70315g_R^2$		
					$D_{s}^{*+}K^{-}$	$0.13668g_R^2$		
					$D^{*0}\pi^0$	$0.87774g_R^2$		
		-	7	. +	$D^{*0}\eta$	$0.21081g_R^2$	<b>n</b> + -	
$D_J^*(3000)$	1	F	$\frac{7}{2}$	4+	$D^{*+}\pi^-$	$1.25746g_R^2$	$D^+\pi^-$	$2.42276g_R^2$
					$D_{s}^{*+}K^{-}$	$0.12371g_R^2$	$D_s^+ K^-$	$0.43906g_R^2$
					$D^{*0}\pi^{0}$	$0.64701g_R^2$	$D^0\pi^0$	$1.25012g_R^2$
D*(2000)		P	1	0	$D^{*0}\eta$	$0.17952g_R^2$	$D^0 \eta$	$0.52012g_R^2$
$D_J^*(3000)$	2	P	$\frac{1}{2}$	$0^+$			$D^+\pi^-$	$4.59878g_{S}^{2}$
							$D_s^{\dagger} K$	$3.76337g_S^2$
							$D^0\pi^0$	$2.31591g_{S}^{2}$
D (2000)		P	1	a ⊥	<b>⊳</b> *⊥ _	2 22552 2	$D^{0}\eta$	$2.99368g_S^2$
$D_J(3000)$	2	P	$\frac{1}{2}$	1'	$D^{*+}\pi$	$3.32572g_{S}^{2}$		
					$D_s^{*+}K$	$2.41575g_{S}^{2}$		
					$D^{*0}\pi^0$	$1.6/433g_{S}^{2}$		
D (2000)	2	D	3	1 +	$D^{*0}\eta$	$2.06/38g_{S}^{2}$		
$D_J(3000)$	2	P	$\frac{3}{2}$	1'	$D^{*+}\pi$	$2.73390g_T^2$		
					$D_s^+ K$	$0.08819g_{T}^{2}$		
					$D^{\circ}\pi^{\circ}$	$1.38912g_T^2$		
$D^{*}(2000)$	2	ת	3	$\mathbf{a}^+$	$D^{\circ}\eta$	$0.71409g_{\tilde{T}}^2$	n+ -	$20000 c^{-2}$
$D_{J}(3000)$	Z	P	$\frac{1}{2}$	Ζ.	$D^+\pi^-$	$1.9107/g_T^2$	$D^+\pi^-$	$2.00990g_T$
					$D_s^* \Lambda$ $D^{*0} - 0$	$0.55802g_T^2$	$D_s \mathbf{K}$ $D_s^0 - 0$	$0.79622g_T$ 1.02155 $c^2$
					$D^*\pi^*$	$0.97002g_T^2$	$D^{\circ}\pi^{\circ}$	$1.02133g_T$
ת (2000)	2	c	1	0-	$D^*\eta$ $D^{*+}-$	$0.540/2g_T^2$	$D^*\eta$	$0.71319g_T$
$D_{J}(5000)$	3	3	$\overline{2}$	0	$D \pi D^{*+} V^{-}$	$5.22080g_H$		
					$D_s \mathbf{K}$ $D^{*0} - 0$	$1.43637g_H$ 1.62708 $a^2$		
					$D^*\pi^*$	$1.02/98g_{H}^{2}$		
אַר (2000)	2	c	1	1-	$D^*\eta$ $D^{*+}-$	$1.22389g_{H}^{-}$	$D^+$ ––	1 40912 2
$D_{J}(5000)$	3	3	2	1	$D^+\pi^-$	$2.54003g_{H}^{-1}$	$D^+\pi^-$	$1.49013g_{H}^{2}$
					$D_{s}^{*0} - 0$	$1.11939g_{H}^{-1}$	$D_s \Lambda$	$0.07742g_{H}^{2}$
					$D^*\pi^*$	$1.18299g_{H}^{-}$	$D^{*}\pi^{*}$ $D^{0}\pi^{*}$	$0.73308g_{H}^{2}$
					$D^{-}\eta$	$0.954/0g_{H}^{-}$	$D^*\eta$	$0.08341g_{H}^{2}$

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 $D_{I}^{*}(2760), D_{I}(3000), \text{ and } D_{I}^{*}(3000) \text{ observed by the LHCb}$ Collaboration are presented in Tables IV and V, where we retain the strong coupling constants  $g_H$ ,  $g_X$ ,  $g_Y$ ,  $g_Z$ , and  $g_R$ . The strong coupling constants can be fitted to the experimental data in the future at the LHCb, BESIII, KEK-B, and PANDA, and taken as basic input parameters in studying the interactions among the heavy mesons. The strong coupling constant  $g_H$  for n = 1 varies in a large range  $g_H = 0.1-0.6$  from different theoretical approaches; it is difficult to choose the optimal value [17]. We usually fit the  $g_H$  to the decay width  $\Gamma(D^{*+} \rightarrow D^0 \pi^+)$  from the CLEO Collaboration [18,19]. The strong coupling constants  $g_H$ (with n = 2, 3),  $g_S$  (with n = 2),  $g_T$  (with n = 2),  $g_X, g_Y$ ,  $g_Z$ , and  $g_R$  involve the radial excited S-wave and P-wave heavy mesons and ground state D-wave and F-wave heavy mesons; it is impossible to determine their values with the heavy quark (or meson) effective theory itself without enough experimental data. The existing theoretical works focus on the strong coupling constants  $g_H$ ,  $g_S$ , and  $g_T$  for the ground state S-wave and P-wave heavy mesons [16,17,20], while works on other strong coupling constants are rare [21].

In Tables VI and VII, we present the ratios  $\hat{\Gamma} = \frac{\Gamma}{\Gamma(D_j^{(*)} \to D^{*+}\pi^{-})}$  of the strong decays of the charmed

TABLE VI. The ratios  $\hat{\Gamma} = \frac{\Gamma}{\Gamma(D_j^{(i)} \to D^{i+}\pi^{-})}$  of the strong decays of the newly observed charmed mesons with possible assignments.

	n	L	$s_\ell$	$J^P$	Decay channels	Γ	Decay channels	Γ
$D_{I}(2580)$	2	S	$\frac{1}{2}$	0-	$D^{*+}\pi^-$	1		
J 、 /			2		$D_{s}^{*+}K^{-}$	0		
					$D^{*0}\pi^0$	0.51		
					$D^{*0}\eta$	0.02		
$D_{I}^{*}(2650)$	2	S	$\frac{1}{2}$	$1^{-}$	$D^{*+}\pi^-$	1	$D^+  \pi^-$	0.79
0			2		$D_s^{*+}K^-$	0.04	$D_s^+ K^-$	0.20
					$D^{*0}\pi^0$	0.51	$D^0\pi^0$	0.40
					$D^{*0}\eta$	0.10	$D^0\eta$	0.20
$D_J^*(2760)$	1	D	$\frac{3}{2}$	$1^{-}$	$D^{*+}\pi^-$	1	$D^+\pi^-$	3.83
			2		$D_{s}^{*+}K^{-}$	0.18	$D_s^+ K^-$	1.37
					$D^{*0}\pi^0$	0.51	$D^0\pi^0$	1.96
					$D^{*0}\eta$	0.26	$D^0\eta$	1.50
$D_J(2740)$	1	D	$\frac{3}{2}$	$2^{-}$	$D^{*+}\pi^-$	1		
			2		$D_{s}^{*+}K^{-}$	0.15		
					$D^{*0}\pi^0$	0.51		
					$D^{*0}\eta$	0.24		
$D_J(2740)$	1	D	<u>5</u> 2	$2^{-}$	$D^{*+}\pi^-$	1		
			-		$D_{s}^{*+}K^{-}$	0.02		
					$D^{*0}\pi^0$	0.52		
					$D^{*0}\eta$	0.04		
$D_J^*(2760)$	1	D	$\frac{5}{2}$	3-	$D^{*+}\pi^-$	1	$D^+ \pi^-$	1.95
			-		$D_s^{*+}K^-$	0.02	$D_s^+ K^-$	0.20
					$D^{*0}\pi^0$	0.52	$D^0\pi^0$	1.01
					$D^{*0}\eta$	0.05	$D^0\eta$	0.27

mesons  $D_J(2580)$ ,  $D_J^*(2650)$ ,  $D_J(2740)$ ,  $D_J^*(2760)$ ,  $D_J(3000)$ , and  $D_J^*(3000)$  observed by the LHCb Collaboration, which can be used to identify the charmed mesons by confronting them with the experimental data in the future. In a previous work [3], we tentatively identify the (D(2550), D(2600)) as the doublet  $(0^-, 1^-)_{\frac{1}{2}}$  with n=2and the (D(2750), D(2750)) as the doublet  $(2^-, 3^-)_{\frac{5}{2}}$  with n = 1 via analyzing the ratios of the branching fractions:

TABLE VII. The ratios  $\hat{\Gamma} = \frac{\Gamma}{\Gamma(D_j^{(*)} \to D^{*+} \pi^{-})}$  of the strong decays of the newly observed charmed mesons with possible assignments. In the decays  $0_1^+ \to 0_1^- + \mathcal{P}$ ,  $\hat{\Gamma} = \frac{\Gamma}{\Gamma(D_j^* \to D^+ \pi^{-})}$ .

	n	L	Se	$J^P$	Decay channels	Γ	Decay channels	Γ
$\overline{D_{I}^{*}(3000)}$	1	F	5	2+	$D^{*+}\pi^-$	1	$D^+\pi^-$	2.91
<b>J</b> ()			2		$D_s^{*+}K^-$	0.26	$D_s^+ K^-$	1.09
					$\overset{{}_{\!\!\!}}{D^{*0}}\pi^0$	0.51	$D^{0}\pi^{0}$	1.49
					$D^{*0}\eta$	0.32	$D^0\eta$	1.15
$D_{I}(3000)$	1	F	5	3+	$D^{*+}\pi^-$	1	,	
3			2		$D_{s}^{*+}K^{-}$	0.23		
					$\overset{{}_{*0}}{D^{*0}}\pi^0$	0.51		
					$D^{*0}n$	0.29		
$D_{I}(3000)$	1	F	7	3+	$D^{*+}\pi^-$	1		
J ( /			2		$D_{s}^{*+}K^{-}$	0.08		
					$D^{*0}\pi^0$	0.52		
					$D^{*0}n$	0.12		
$D_{I}^{*}(3000)$	1	F	7	$4^{+}$	$D^{*+}\pi^-$	1	$D^+\pi^-$	1.93
J()			2		$D_{*}^{*+}K^{-}$	0.10	$D^+_{s}K^-$	0.35
					$D^{*0}\pi^{0}$	0.51	$D^0\pi^0$	0.99
					$D^{*0}n$	0.14	$D^0 n$	0.41
$D_{I}^{*}(3000)$	2	Р	$\frac{1}{2}$	$0^+$	,		$D^+\pi^-$	1
J()			2				$D_{s}^{+}K^{-}$	0.82
							$D^{0}\pi^{0}$	0.50
							$D^0 \eta$	0.65
$D_{I}(3000)$	2	Р	$\frac{1}{2}$	$1^{+}$	$D^{*+}\pi^-$	1	,	
J ( /			2		$D_s^{*+}K^-$	0.73		
					$\overset{{}_{\!\!\!}}{D^{*0}}\pi^0$	0.50		
					$D^{*0}\eta$	0.62		
$D_{I}(3000)$	2	Р	3	$1^{+}$	$D^{*+}\pi^-$	1		
J ( /			2		$D_{s}^{*+}K^{-}$	0.25		
					$D^{*0}\pi^0$	0.51		
					$D^{*0}\eta$	0.26		
$D_{I}^{*}(3000)$	2	Р	3	$2^{+}$	$D^{*+}\pi^-$	1	$D^+\pi^-$	1.05
J			2		$D_s^{*+}K^-$	0.28	$D_{s}^{+}K^{-}$	0.42
					$D^{*0}\pi^0$	0.51	$D^{0}\pi^{0}$	0.53
					$D^{*0}n$	0.28	$D^0 n$	0.37
$D_{I}(3000)$	3	S	$\frac{1}{2}$	$0^{-}$	$D^{*+}\pi^-$	1	,	
J ( /			2		$D_{s}^{*+}K^{-}$	0.45		
					$D^{*0}\pi^{0}$	0.50		
					$D^{*0}\eta$	0.38		
$D_{I}^{*}(3000)$	3	S	$\frac{1}{2}$	1-	$D^{*+}\pi^{-}$	1	$D^+\pi^-$	0.64
J )		-	2		$D_s^{*+}K^-$	0.48	$D_s^+ K^-$	0.37
					$\overset{{}_{}}{D^{*0}}\pi^0$	0.50	$\dot{D^0}\pi^0$	0.32
					$D^{*0}n$	0.40	$D^0 n$	0.29

TABLE VIII. The experimental values of the ratio  $\frac{\Gamma(D_2^*(2460)^0 \rightarrow D^+ \pi^-)}{\Gamma(D_2^*(2460)^0 \rightarrow D^{*+} \pi^-)}$  compared to the prediction of the leading order heavy meson effective theory.

BABAR	CLEO	CLEO	ARGUS	ZEUS	This work
$1.47 \pm 0.03 \pm 0.16$	$2.2 \pm 0.7 \pm 0.6$	$2.3 \pm 0.8$	$3.0 \pm 1.1 \pm 1.5$	$2.8\pm0.8^{+0.5}_{-0.6}$	2.29

$$\frac{\operatorname{Br}(D_{2}^{*}(2460)^{0} \to D^{+}\pi^{-})}{\operatorname{Br}(D_{2}^{*}(2460)^{0} \to D^{*+}\pi^{-})}, \qquad \frac{\operatorname{Br}(D(2600)^{0} \to D^{+}\pi^{-})}{\operatorname{Br}(D(2600)^{0} \to D^{*+}\pi^{-})}, 
\frac{\operatorname{Br}(D(2760)^{0} \to D^{+}\pi^{-})}{\operatorname{Br}(D(2750)^{0} \to D^{*+}\pi^{-})}, \qquad (34)$$

with the heavy meson effective theory in the leading order approximation. The measurement of the LHCb Collaboration also favors the assignment  $(D_J(2580), D_J^*(2650)) = (0^-, 1^-)_{\frac{1}{2}}$  with n = 2 [1]. The helicity distribution from the *BABAR* Collaboration disfavors identifying the D(2750) as the 3<sup>-</sup> state [2], which is compatible with the measurement of the LHCb Collaboration that the  $D_J(2740)$  has unnatural parity [1]. The measurement of the LHCb Collaboration favors two possible assignments:

$$(D_J^*(2760), D_J(2740)) = (1^-, 2^-)_{\frac{3}{2}}$$
 with  $n = 1, L = 2,$   
(35)

$$(D_J(2740), D_J^*(2760)) = (2^-, 3^-)_{\frac{5}{2}}$$
 with  $n = 1, L = 2.$ 
  
(36)

We tentatively identify the  $D_J(2740)$  as the 1 D state with  $J^P = 2^-$ ; however, the assignments  $1 D \frac{3}{2}2^-$  and  $1 D \frac{5}{2}2^-$  lead to quite different ratios among the decay widths. The  $D_J^*(2760)$  have natural parity, the assignments  $1 D \frac{3}{2}1^-$  and  $1 D \frac{5}{2}3^-$  also lead to quite different ratios among the decay widths. We can confront the present predictions with the experimental data in the future to identify the newly observed charmed mesons.

In Table VIII, we present the experimental data on the ratio  $\frac{\Gamma(D_2^*(2460) \rightarrow D^+ \pi^-)}{\Gamma(D_2^*(2460) \rightarrow D^{**} \pi^-)}$  for the well-established meson  $D_2^*(2460)$  from the *BABAR* [2], CLEO [22,23], ARGUS [24], and ZEUS [25] collaborations; the present prediction 2.29, based on the heavy meson effective theory in the leading order approximation, is in excellent agreement with the average experimental value 2.35. The heavy meson effective theory in the leading order approximation works well.

If we saturate the total widths of the doublet  $(D_J(2580), D_J^*(2650))$  or (D(2550), D(2600)) with the two-body decays to the ground states  $(D, D^*)_{\frac{1}{2}} + \mathcal{P}$  and the total widths  $\Gamma_{D_J(2580)} \approx 1.3g_H^2$  GeV,  $\Gamma_{D_J^*(2650)} \approx 2.5g_H^2$  GeV,  $\Gamma_{D(2550)} \approx 1.7g_H^2$  GeV,  $\Gamma_{D(2600)} \approx 2.0g_H^2$  GeV, then the ratio  $\frac{\Gamma_{D_J(2580)}}{\Gamma_{D_J^*(2650)}} \approx 0.52$  is smaller than the experimental data

 $\frac{\Gamma_{D_I(2580)}}{\Gamma_{1}} \approx 1.27$  from the LHCb Collaboration [1], and the  $\overline{\Gamma}_{D_{r}^{*}(2650)}$ ratio  $\frac{\Gamma_{D(2550)}}{\Gamma_{D(2600)}} \approx 0.85$  is also smaller than the experimental data  $\frac{\Gamma_{D(2500)}}{\Gamma_{D(2500)}} \approx 1.40$  from the *BABAR* Collaboration [2]. The 2 S states  $(D_1(2580), D_1^*(2650))$  or (D(2550), D(2600))can decay to the P-wave states  $(0^+, 1^+)_{\frac{1}{2}} + \mathcal{P}$  and  $(1^+, 2^+)_{\frac{3}{2}} + \mathcal{P}$ , which also contribute to the total decay widths. The decays to the P-wave states  $(0^+, 1^+)_{\frac{1}{2}} + V$ are kinematically forbidden, where the V denotes the light vector mesons  $\rho$ ,  $K^*$ , and  $\omega$ . If those contributions are taken into account properly, the discrepancy may be smeared. The ratio  $\frac{\Gamma_{D_1(2420)}}{\Gamma_{D_2^{*}(2460)}} \approx 0.31$  from the heavy meson effective theory in the leading order approximation is also smaller than the experimental data 0.81 [1]. For the charmed mesons, the flavor and spin violation corrections of order  $O(1/m_0)$  may be sizable, and we may have to introduce new unknown coupling constants. The discrepancy may be smeared with the optimal parameters [26]; furthermore, more precise measurements are needed to make a reliable comparison.

#### **IV. CONCLUSION**

In this article, we tentatively identify the charmed mesons  $D_J(2580)$ ,  $D_J^*(2650)$ ,  $D_J(2740)$ ,  $D_J^*(2760)$ ,  $D_J(3000)$ , and  $D_{I}^{*}(3000)$  observed by the LHCb Collaboration according to their spin, parity, and masses. Then we study their strong decays to the ground state charmed mesons plus light pseudoscalar mesons with the heavy meson effective theory in the leading order approximation, and we obtain explicit expressions of the decay widths. The strong coupling constants  $g_H$ ,  $g_S$ ,  $g_T$ ,  $g_X$ ,  $g_Y$ ,  $g_Z$ , and  $g_R$  in the heavy meson chiral Lagrangians can be fitted to the experimental data in the future at the LHCb, BESIII, KEK-B, and PANDA, and can be taken as basic input parameters in studying the interactions among the heavy mesons. Also, the ratios among the decay widths can be used to confirm or reject the assignments of the newly observed charmed mesons.

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