PHYSICAL REVIEW D **88,** 105023 (2013)

Space-time structure of polynomiality and positivity for generalized parton distributions

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(Received 23 June 2013; published 20 November 2013)

We study the space-time structure of polynomiality and positivity—the most important properties which are inherent to the generalized parton distributions (GPDs). In this connection, we reexamine the issue of the time and normal ordering in the operator definition of GPDs. We demonstrate that the contribution of the anticommutator matrix element in the collinear kinematics, which was previously argued to vanish, has to be added in order to satisfy the polynomiality condition. Furthermore, we schematically show that a new contribution due to the anticommutator modifies likewise the so-called positivity constraint, i.e., the Cauchy-Bunyakovsky-Schwarz inequality, which is another important feature of the GPDs.

DOI: 10.1103/PhysRevD.88.105023 PACS numbers: 12.38.Bx, 13.60.Le

I. INTRODUCTION

The space-time structure of the generalized parton distributions (GPDs), together with their polynomiality, is encoded in the matrix elements of the (anti)commutators of the fermion fields. In this connection, the problem of the time ordering and the consistency of the replacement of it by the ordinary ordering in the GPDs has been discussed in the literature for many years (see, e.g., [1,2]). In the cases of the deep inelastic scattering (DIS) and deeply virtual Compton scattering (DVCS) processes, it was argued that the matrix element of the fermion anticommutator vanishes and, therefore, the time ordering in GPDs is "illusory" and can readily be replaced by the ordinary ordering of the corresponding fermion operators. The crucial point of those studies was that the anticommutator contribution is defined by the limit of $1/(k^-)^{n-1}$, where $n \geq 2$ at $k^- \to \infty$ for the Mandelstam variables differ from zero. Furthermore, it was shown in Ref. [2] that the support and spectral properties of the GPDs emerge naturally.

The purpose of this paper is to demonstrate that in the collinear kinematics and within the factorization procedure in the t channel, where the Mandelstam variable t is small compared to s, the matrix element of the fermion anticommutator does not vanish and yields a term necessary to hold the model-independent polynomiality condition for any kind of generalized parton distributions. Moreover, the latter is even valid in the regime where the Mandelstam variables s, u, and t are similarly small, that is, in the so-called totally collinear kinematics. Note that this particular point $s \sim t \sim 0$ in the Mandelstam plane is responsible for the duality regime of the factorization, discussed in detail in Ref. [3], and bridges between the factorizations in the t and s channels. The comprehensive analysis of this very interesting point is forthcoming in [4]. We also demonstrate schematically that the obtained contribution, arising from the matrix element of the fermion anticommutator, modifies evenly another important property of the GPDs, the positivity. We show, moreover, that this modification allows us to relate the GPDs with the nonperturbative fermion condensates.

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II. HEISENBERG AND INTERACTION REPRESENTATIONS

As the first step, let us start with the outline of the main issues of the matching between the Heisenberg and interaction representations. Consider, for instance, the time-ordered product of two fermion fields in the interaction representation with the $\mathbb{S}$ matrix, $\mathbb{S}(t_2 = \infty, t_1 = -\infty) \equiv \mathbb{S}_{\infty,-\infty} \equiv \mathbb{S}$. Using the Wick theorem,

$$\mathrm{T}\psi(x)\bar{\psi}(y)\mathbb{S}_{\infty,-\infty} = G^c(x,y) + \sum_n \frac{(ig)^n}{n!}\int (d^4\xi)_n \times \sum_{\text{pairing}}{}' :\psi(x)\bar{\psi}(y)(\bar{\psi}\hat{A}\psi)_{\xi_1}\dots(\bar{\psi}\hat{A}\psi)_{\xi_n}:, \quad (1)$$

where $:\dots:$ denotes the normal-ordered product of fields. Here $\sum'_{\text{pairing}}$ stands for the sum of all possible sets of contractions (or pairings) between the fields excluding the terms with *all* fields contracted, the latter being accumulated in $G^c(x,y)$.

The field $\psi(x)$ in the interaction representation transforms into the Heisenberg field operator $\psi_H(x)$ as follows: $\psi_H(x) = \mathbb{S}^\dagger_{t,0}\psi(x)\mathbb{S}_{t,0}$. By making use of this transformation, we obtain the relation between the time-ordered products of two fermion fields in the Heisenberg and in the interaction representations, respectively:

$$\mathrm{T}\psi(x)\bar{\psi}(y)\mathbb{S}_{\infty,-\infty} = \mathbb{S}_{\infty,0}\mathrm{T}\psi_H(x)\bar{\psi}_H(y)\mathbb{S}_{0,-\infty}. \quad (2)$$

Calculating the vacuum expectation value of the time-ordered operator product, we get the standard definition of the connected Green function:

$$S^c(x, y) = \frac{\langle 0|\mathrm{T}\psi(x)\bar{\psi}(y)\mathbb{S}_{\infty,-\infty}|0\rangle}{\mathbb{S}_0} = {}^H\langle 0|\mathrm{T}\psi_H(x)\bar{\psi}_H(y)|0\rangle^H, \quad (3)$$

where the normalization condition $\mathbb{S}_0 = \langle 0|\mathbb{S}_{\infty,-\infty}|0\rangle$ cancels all contributions from the disconnected graphs in the interaction representation, while the vacuum state in the Heisenberg picture is defined as ${}^H\langle 0| = \langle 0|\mathbb{S}_{\infty,0}$ and $|0\rangle^H = \mathbb{S}_{0,-\infty}|0\rangle$. In what follows, we shall keep only the superscript H in formulas to indicate the Heisenberg representation.

If we consider now the hadronic matrix element of the time-ordered operator product instead of the vacuum average, we observe (upon application of the Wick theorem) that the terms related to the matrix elements of the normal-ordered operators *do not disappear*. Notice that the same inference is true if our states are the physical or nonperturbative vacuum. At the same time, the fully contracted terms refer to the disconnected matrix elements and, therefore, have to be discarded. Indeed, we have

$$\begin{aligned}&\langle p_2|\mathrm{T}\psi(x)\bar{\psi}(y)\mathbb{S}_{\infty,-\infty}|p_1\rangle\\&= G^c(x, y)\langle p_2|p_1\rangle + \sum_{n;i,j}\int (d^4\xi)_n\langle p_2|{:}\psi(\xi_i)\\&\quad\times C_n(\xi_i, \xi_j; x, y)\bar{\psi}(\xi_j){:}|p_1\rangle\\&\quad + (\text{“}N > 2 \text{ normal-ordered operators”}),\end{aligned} \quad (4)$$

where $C_n(\xi_i, \xi_j; x, y)$ is the corresponding product of different propagators. The first term in the left-hand side of (4), $G^c(x, y)\langle p_2|p_1\rangle$, which is proportional to $\delta^{(4)}(p_2 - p_1)$, describes only the disconnected Feynman diagrams. Thus, we define the connected matrix element of the time-ordered operator product as

$$\begin{aligned}&\langle p_2|\mathrm{T}\psi(x)\bar{\psi}(y)\mathbb{S}_{\infty,-\infty}|p_1\rangle_C\\&= \sum_{n;i,j}\int (d^4\xi)_n\langle p_2|{:}\psi(\xi_i)C_n(\xi_i, \xi_j; x, y)\bar{\psi}(\xi_j){:}|p_1\rangle\\&\quad + (\text{“}N > 2 \text{ normal-ordered operators”}),\end{aligned} \quad (5)$$

where the subscript C points out that we are dealing with the connected matrix elements. On the other hand, the hadron matrix element (5) can be written in compact form in the Heisenberg representation. We have

$$\begin{aligned}&\sum_{n;i,j}\int (d^4\xi)_n\langle p_2|{:}\psi(\xi_i)C_n(\xi_i, \xi_j; x, y)\bar{\psi}(\xi_j){:}|p_1\rangle\\&\quad + (\text{“}N > 2 \text{ normal-ordered operators”})\\&\equiv \langle p_2|{:}\psi(x)\bar{\psi}(y){:}|p_1\rangle_C^H,\end{aligned} \quad (6)$$

or, comparing Eq. (5) with Eq. (6), we conclude that

$$\langle p_2|\mathrm{T}\psi(x)\bar{\psi}(y)\mathbb{S}|p_1\rangle_C = \langle p_2|{:}\psi(x)\bar{\psi}(y){:}|p_1\rangle_C^H. \quad (7)$$

In turn, given that we consider only the connected matrix elements, the normal-ordered operators in the Heisenberg representation can be replaced by the time-ordered operators

$$\langle p_2|{:}\psi(x)\bar{\psi}(y){:}|p_1\rangle_C^H = \langle p_2|\mathrm{T}\psi(x)\bar{\psi}(y)|p_1\rangle_C^H. \quad (8)$$

Let us emphasize that Eqs. (7) and (8) are our principal observations, to which we would like to attract attention of the reader.

III. THE FACTORIZED DVCS AMPLITUDE

Now we concentrate on the DVCS amplitude factorized into the hard and the soft parts. Before the factorization is carried out, the DVCS amplitude in the interaction picture can be expressed as

$$\mathcal{A}_{\mu\nu} = e^2\int d\xi d\eta e^{-iq\cdot\xi + iq'\cdot\eta}\langle p_2|\mathrm{T}J_\nu^{\mathrm{em}}(\eta)J_\mu^{\mathrm{em}}(\xi)\mathbb{S}|p_1\rangle_C,$$

where J_μ^{em} is the electromagnetic current and the $\mathbb{S}$ matrix involves all possible interactions. Expanding the $\mathbb{S}$ matrix in power of the coupling constant (we do not need yet to specify the Lagrangians we are working with) and making use of the Wick theorem, we obtain the standard expression for the amplitude

$$\mathcal{A} \Rightarrow \langle p_2|{:}\bar{\psi}(\eta)\underline{\gamma_\nu S(\eta - \xi)\gamma_\mu}\psi(\xi){:}|p_1\rangle_C + \cdots,$$

where the ellipsis denotes other possible combinations of the normal-ordered operators including the cross terms. We here underlined the combination to stress that it will form the hard part of the amplitude. Notice that the combinations with $N > 2$ normal-ordered operators are not the issues in the present paper.

The factorization of the amplitude in the interaction representation consists in the separation of the hard part (underlined) from the soft part (which will be expressed in what follows in terms of the GPDs):

$$\Phi(x, \xi) = \int d^4k\delta(x - k\cdot n)d^4z e^{i(k-\Delta/2)\cdot z} \times \langle p_2|\tilde{\mathrm{T}}\,\bar{\psi}(0)\psi(z)\mathbb{S}_{\infty,-\infty}|p_1\rangle_C, \quad (9)$$

where $\tilde{\mathrm{T}}$ suggests that we have to hold only two fermion operators as the normal-ordered one. The spinors should be understood as the operators with the free Dirac indices. As has been mentioned above, the Heisenberg representation allows us to rewrite the right-hand side of Eq. (9) in the most compact form as

$$\Phi(x, \xi) = \int d^4k\delta(x - k\cdot n)d^4z e^{i(k-\Delta/2)\cdot z} \times \langle p_2|{:}\bar{\psi}(0)\psi(z){:}|p_1\rangle_C^H. \quad (10)$$

Given that we are again interested in the connected matrix elements only, we may write the time-ordered operators instead of the normal-ordered operators in the Heisenberg representation, i.e.,

$$\Phi(x, \xi) = \int d^4k\delta(x - k\cdot n)d^4z e^{i(k-\Delta/2)\cdot z} \times \langle p_2|\mathrm{T}\bar{\psi}(0)\psi(z)|p_1\rangle_C^H. \quad (11)$$

Alternatively, using the light-cone notations, one has

$$\Phi(x, \xi) = \int dk^- d^2\mathbf{k}_T \Phi(xP^+, k^-, \mathbf{k}_T; \xi). \quad (12)$$

These three representations, Eqs. (10)–(12), are equivalent. Recall that the function Φ possesses the free Dirac indices. If we now project the GPDs (10)–(12) to the γ^+ matrix, we shall obtain the various twist-2 generalized parton distributions, depending on the hadron target:

$$\Phi^{[\gamma^+]} \stackrel{\mathrm{def}}{=} \mathrm{tr}[\gamma^+\Phi] \Rightarrow \{H_1; H, E; \ldots\}. \quad (13)$$

We can thus conclude that, since we deal only with the connected matrix elements, the time ordering and/or the normal ordering occur in the GPDs of any kind in an equivalent way. This is one of our main observations.

Let us now focus on Eq. (11). It is well known that the time-ordered combination of spinors can be expressed through their commutator and anticommutator:

$$\Phi(x) = \Phi^{[\ldots]}(x) + \Phi^{\{\ldots\}}(x), \quad (14)$$

where

$$\Phi^{[\ldots]}(x) = \frac{1}{2}\int d^4k\delta(x - k\cdot n)d^4z e^{i(k-\Delta/2)\cdot z} \times \langle p_2|[\bar{\psi}(0), \psi(z)]|p_1\rangle_C^H \quad (15)$$

and

$$\Phi^{\{\ldots\}}(x) = \frac{1}{2}\int d^4k\delta(x - k\cdot n)d^4z e^{i(k-\Delta/2)\cdot z} \times \varepsilon(z_0) \times \langle p_2|\{\bar{\psi}(0), \psi(z)\}|p_1\rangle_C^H. \quad (16)$$

We would like to emphasize that the presence of $\varepsilon(z_0)$ in Eq. (16) leads to the absence of any $s(u)$-channel cuts in the anticommutator contribution, while the commutator contribution can be related to the $s(u)$-channel cuts. Indeed, consider the first term of the anticommutator contribution [see Eq. (16)]:

$$\frac{1}{2}\int_{-\infty}^{\infty} d^4z\varepsilon(z_0)e^{i(k-\Delta/2)\cdot z}\langle p_2|\bar{\psi}(0)\psi(z)|p_1\rangle_C^H. \quad (17)$$

Inserting of the full set and making use of the translation invariance, one presents this expression in the following form:

$$\sum\!\!\!\!\!\!\int_X \frac{i}{\pi}\mathcal{P}\frac{1}{k_0 - P_0 + P_0^X}\delta^{(3)}(\vec{k} - \vec{P} + \vec{P}_X) \times \langle p_2|\bar{\psi}(0)|P_X\rangle_C^H\langle P_X|\psi(0)|p_1\rangle_C^H. \quad (18)$$

One can see that the four-dimensional δ function, needed for the appearance of the cut in the $s(u)$ channel, is absent. A similar result is valid for the second term of Eq. (16).

It is obvious that if the anticommutator were vanishing for some reason (see, e.g., [1,2]), it would be permitted to replace the time ordering by the ordinary product of operators. That is to say, the time ordering gets illusory.

However, we here present an alternative approach to show that the contribution of the anticommutator, $\Phi^{\{\ldots\}}(x)$, does not vanish in the case of factorization in the t channel, using the collinear kinematics (see below), where the Mandelstam variable t is small compared to s. One of our main evidences is that the contribution of the anticommutator matrix element is necessary to obey the model-independent *polynomiality condition* for the GPDs, which arises from the requirement of the Lorentz covariance of the corresponding matrix element. We will demonstrate this by taking as an example the box diagram within a toy model which was very useful to the introduction of GPDs [5].

IV. A TOY MODEL FOR THE BOX DIAGRAM

Consider first the box diagram contribution to the DVCS amplitude:

$$\gamma^*(q) + A(p_1) \to \gamma(q') + A(p_2) \quad (19)$$

in the perturbation theory. The box diagram is the most illustrative object to reveal the main features of the factorization approach involving the GPDs; see [5]. Because the factorization procedure is extensively described in the literature, we will skip the details of this procedure. We begin with the definition of the light-cone kinematics, which we will use in what follows:

$$\begin{aligned} &n^2 = p^2 = 0, \quad p\cdot n = 1, \quad g^T_{\mu\nu} = g_{\mu\nu} - p_\mu n_\nu - p_\nu n_\mu, \\ &p_2 = (1-\xi)p + (1+\xi)\frac{\bar{M}^2}{2}n + \Delta_T/2, \\ &p_1 = (1+\xi)p + (1-\xi)\frac{\bar{M}^2}{2}n - \Delta_T/2, \quad q' = P.q'n, \\ &\bar{Q} = (q+q')/2, \quad P = (p_1+p_2)/2, \quad \Delta = p_2 - p_1, \\ &P^2 = \bar{M}^2 = \frac{\Delta_T^2 - t}{4\xi^2}, \quad \Delta^2 = t. \end{aligned} \quad (20)$$

Without loss of generality, we may use the *collinear* kinematics which corresponds to the case when $\Delta_T \approx 0$. We now approach the factorized amplitude in the perturbation theory, so that we can write in the twist-2 level

$$\mathcal{A}_{\mu\nu} = \int_{-1}^{1} dx\, \mathrm{tr}[\gamma_\nu S(xP + \bar{Q})\gamma_\mu \gamma^-] \times \int d^4k\, \delta(x - k\cdot n)\Phi^{[\gamma^+]}(k) + \text{“crossed.”} \quad (21)$$

We identify the initial and final states in the corresponding matrix elements with the electron and quark states. In this case, the soft part of this amplitude takes the following form (in the Feynman gauge) (see Fig. 1):

$$\begin{aligned}\Phi^{[\gamma^+]}(x,\xi) &= \int (d^4k)\delta(x - k\cdot n)\Phi^{[\gamma^+]}(k) \\ &\overset{g^2}{=} ig^2 \int (d^4k)\delta(x - k\cdot n)D(k-P) \\ &\quad \times [\bar{u}(p_2)\gamma_\alpha S(k+\Delta/2) \\ &\quad \times \gamma^+ S(k-\Delta/2)\gamma_\alpha u(p_1)]. \end{aligned} \quad (22)$$

Making use of Eq. (20), we obtain that

$$\begin{aligned}(k-\Delta/2)^2 &= 2k^- p^+(x+\xi) - (x+\xi)\xi\bar{M}^2 - \mathbf{k}_T^2, \\ (k+\Delta/2)^2 &= 2k^- p^+(x-\xi) + (x+\xi)\xi\bar{M}^2 - \mathbf{k}_T^2. \end{aligned} \quad (23)$$

For the parton subprocess, we also introduce the corresponding Mandelstam variables:

$$\begin{aligned}\hat{s} &= (k+P)^2 = 2k^- p^+(x+1) + (x+1)\bar{M}^2 - \mathbf{k}_T^2, \\ \hat{u} &= (k-P)^2 = 2k^- p^+(x-1) + (1-x)\bar{M}^2 - \mathbf{k}_T^2. \end{aligned} \quad (24)$$

Notice that within the collinear kinematics, $\bar{M} \approx \sqrt{-t}/(2\xi)$, and, therefore, it can be discarded with respect to the large p^+. At the same time, keeping the terms which are proportional to t will never allow the poles to jump from the upper plane to the lower one.

For the sake of simplicity, we extract the following structure integral:

$$\Phi^{[\gamma^+]}(x,\xi) = \bar{u}(p_2)\mathcal{I}^{[\gamma^+]}(x,\xi)u(p_1), \quad (25)$$

where

$$\mathcal{I}^{[\gamma^+]}(x,\xi) \overset{\mathrm{def}}{=} \int d\mu(\mathbf{k}_T)\int dk^- \frac{\phi^+(k,\Delta)}{D_1 D_2 D_3}\bigg|_{k^+ = xP^+} \quad (26)$$

with

$$\begin{aligned}\phi^+ &= \gamma_\alpha(\not{k} + \not{\Delta}/2)\gamma^+(\not{k} - \not{\Delta}/2)\gamma_\alpha \approx -\mathbf{k}_T^2\gamma^+, \\ D_{1,3} &= 2k^- P^+(x \mp \xi) - \mathbf{k}_T^2 + i\epsilon, \\ D_2 &= 2k^- P^+(x-1) - \mathbf{k}_T^2 + i\epsilon. \end{aligned} \quad (27)$$

We introduced the effective integration measure $d\mu(\mathbf{k}_T)$ in Eq. (26) in order to ensure the convergence of the corresponding integration. Let us emphasize that this modification of the measure will not affect the results of our study. Indeed, our reasoning is also valid for the GPDs in the toy scalar model, considered, e.g., in Refs. [5,6], because the numerator $\phi^+(k,\Delta)$ contains only $\mathbf{k}_T^2$ in the collinear kinematics.

Let us first carry out the integration over k^- in (26) in the complex plane. To this end, we will analyze the analytical properties on the integrand, namely, the position of the poles in the complex plane of the variable k^-. We have (cf. [7])

$$k_1^- = -\frac{\mathbf{k}_T^2}{2P^+(\xi - x)} + i\epsilon, \qquad k_2^- = -\frac{\mathbf{k}_T^2}{2P^+(1-x)} + i\epsilon, \qquad k_3^- = -\frac{\mathbf{k}_T^2}{2P^+(\xi + x)} - i\epsilon, \quad (28)$$

for $0 < x < \xi$, and

$$k_2^- = -\frac{\mathbf{k}_T^2}{2P^+(1-x)} + i\epsilon, \qquad k_1^- = \frac{\mathbf{k}_T^2}{2P^+(x-\xi)} - i\epsilon, \qquad k_3^- = \frac{\mathbf{k}_T^2}{2P^+(x+\xi)} - i\epsilon, \quad (29)$$

for $x > \xi > 0$. For the negative fraction x, especially for the interval $-\xi < x < 0$, the poles are situated similarly to the case of $0 < x < \xi$; while for the interval $x < -\xi$ all poles lie in the same semiplane, and, therefore, this region of the fraction does not contribute. In (28) and (29), $k_{1,3}^-$ correspond to the quark poles while k_2^- to the gluon pole.

Thus, integrating over k^- in its complex plane, we obtain

$$\mathcal{I}^{[\gamma^+]}(x,\xi) = \gamma^+ \int (d\mathbf{k}_T^2)\frac{\Psi^2(\mathbf{k}_T^2)}{\mathbf{k}_T^2 + \Lambda^2} H(x,\xi), \quad (30)$$

where

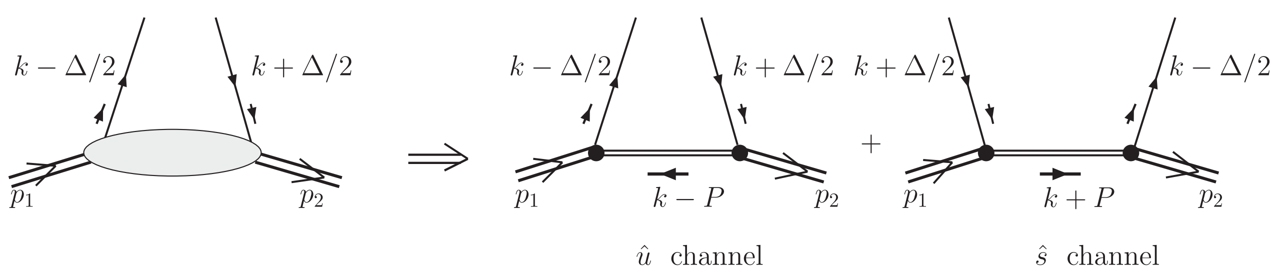

FIG. 1. GPDs within a toy model.

$$H(x,\xi) = \theta(-\xi < x < \xi)\left[\frac{\xi - x}{2\xi(1-\xi)} - \frac{1-x}{1-\xi^2}\right] - \theta(\xi < x < 1)\frac{1-x}{1-\xi^2}. \tag{31}$$

We here introduce the effective UV and IR regularizations following Ref. [6]. Equations (31) can be split into the contributions of the quark and gluon poles separately:

$$\begin{aligned} H^{[\ldots]}(x,\xi) &= -\theta(-\xi < x < 1)\frac{1-x}{1-\xi^2},\\ H^{\{\ldots\}}(x,\xi) &= \theta(-\xi < x < \xi)\frac{\xi - x}{2\xi(1-\xi)}, \end{aligned} \tag{32}$$

where the "anticommutator part" of the GPDs, $H^{\{\ldots\}}(x,\xi)$, is related to the quark pole contributions and the "commutator part," $H^{[\ldots]}(x,\xi)$ to the gluon pole contribution. Indeed, consider the commutator contribution written in the following form [see (10)–(12)]:

$$\begin{aligned} H^{[\ldots]}(x,\xi) &= \int d^4k\,\delta(x - k\cdot n)\mathcal{A}^{[\ldots]}(k),\\ \mathcal{A}^{[\ldots]}(k) &= \frac{1}{2}\int d^4z e^{i(k-\Delta/2)\cdot z}\langle p_2|[\bar\psi(0)\gamma^+, \psi(z)]|p_1\rangle^H_C. \end{aligned} \tag{33}$$

As before, we identify the initial and final states in Eq. (33) with the electrons and quarks. Hence, we insert in Eq. (33) the full set of the intermediate states $\sum_X |P_X\rangle^{HH}\langle P_X| = 1$ and obtain

$$\mathcal{A}^{[\ldots]}(k) = \frac{1}{2}\sum\!\!\!\!\!\!\int_X \delta^{(4)}(k - P + P_X) \times \langle p_2|\bar\psi(0)\gamma^+|P_X\rangle^H\langle P_X|\psi(0)|p_1\rangle^H_C. \tag{34}$$

In order to be able to make use of the perturbation theory, we transform to the interaction representation and keep the terms up to the g^2 order:

$$\begin{aligned} \mathcal{A}^{[\ldots]}(k) &= \frac{1}{2}\sum\!\!\!\!\!\!\int_X \delta^{(4)}(k - P + P_X)\langle p_2|\mathrm{T}(\bar\psi(0)\gamma^+\mathbb{S})|P_X\rangle\\ &\quad\times\langle P_X|\mathrm{T}(\psi(0)\mathbb{S})|p_1\rangle_C\\ &\overset{g^2\mathrm{PT}}{\Rightarrow} \delta((P-k)^2)\bar u(p_2)\gamma\cdot\varepsilon^* S(k+\Delta/2)\gamma^+\\ &\quad\times S(k-\Delta/2)\gamma\cdot\varepsilon u(p_1), \end{aligned} \tag{35}$$

where we have used the one-particle states $|p_1\rangle = b^+(p_1)|0\rangle$ and $\langle p_2| = \langle 0|b^-(p_2)$ and we have chosen the one-boson (photon and gluon) state as the intermediate state. Therefore, we obtain

$$\begin{aligned} H^{[\ldots]}(x,\xi) &= \frac{1}{2}\int d\mathbf{k}_T^2 dk^-\,\delta(2k^-P^+(x-1) - \mathbf{k}_T^2)\bar u(p_2)\\ &\quad\times\gamma\cdot\varepsilon^* S(k+\Delta/2)\gamma^+ S(k-\Delta/2)\gamma\cdot\varepsilon u(p_1), \end{aligned}$$

where we assume that $k^+ = xP^+$. This expression can be rewritten in the Heisenberg representation:

$$\begin{aligned} H^{[\ldots]}(x,\xi) &= \frac{1}{2}\int d\mathbf{k}_T^2 dk^-\,\delta(2k^-P^+(x-1) - \mathbf{k}_T^2)\\ &\quad\times\langle p_2|\bar\psi(0)\gamma^+|P-k\rangle^H\langle P-k|\psi(0)|p_1\rangle^H_C. \end{aligned} \tag{36}$$

One can easily see that this expression is nothing else but the cut of the amplitude (25) in the photon and gluon propagator. To say the same thing in a different way, this contribution comes from the diagrams where the photon and gluon propagator is replaced by its imaginary part (that is to say, it yields the gluon pole contribution). In the same way, we can show that the anticommutator contribution is given by the quark pole contribution or by picking up the cut in the quark propagator with the momentum $k + \Delta/2$.

V. POLYNOMIALITY AND POSITIVITY FOR GPDS

We are now in a position to address the polynomiality condition for (31). To this end, we calculate the corresponding moments of (31) by the straightforward integration of (31) weighted by x^{2n} and x^{2n+1}. We have

$$\begin{aligned} \int_{-1}^1 dx x^{2n} H(x,\xi) &= -\frac{2(1-\xi^{2n+2})}{(2n+1)(2n+2)(1-\xi^2)}\\ &= c_0 + c_2\xi^2 + \cdots + c_{2n}\xi^{2n},\\ \int_{-1}^1 dx x^{2n+1} H(x,\xi) &= -\frac{2(1-\xi^{2n+2})}{(2n+2)(2n+3)(1-\xi^2)}\\ &= d_0 + d_2\xi^2 + \cdots + d_{2n}\xi^{2n}. \end{aligned} \tag{37}$$

Let us stress that the box diagram itself cannot ensure the so-called D-term contribution which describes the resonance exchange diagram (see, e.g., Refs. [6,8]). We will therefore treat, for a moment, the polynomiality of the GPDs as the expression of the corresponding moments through the finite series with only even orders of ξ; see (37). By making use of the splitting (32), we can verify the polynomiality for each of the commutator and anticommutator contributions. We have the following:

$$\int_{-1}^1 dx x^n H^{[\ldots]}(x,\xi) = \frac{c_{-1}}{1-\xi} + \sum_{k=0}^n a_k\xi^k, \tag{38}$$

$$\int_{-1}^1 dx x^n H^{\{\ldots\}}(x,\xi) = -\frac{c_{-1}}{1-\xi} + \sum_{k=0}^n b_k\xi^k, \tag{39}$$

where $a_{2k-1} = -b_{2k-1}$. One can see that neither the commutator contribution nor the anticommutator contribution obeys the polynomiality separately. In other words, we have the polynomiality only after summation of these two terms. We conclude, therefore, that the anticommutator contribution is necessary to satisfy the model-independent polynomiality condition and, therefore, cannot be discarded by default. This is our principal result.

Now let us present the scheme for how the new contribution arising from the $H^{\{\ldots\}}$ term, Eq. (32), affects the positivity constraint, Ref. [9]. The full and comprehensive analysis will be presented in the forthcoming paper [4]. The structure of the photon and gluon and the quark pole contributions in the factorized box diagram amplitude, where the soft part has been calculated perturbatively, helps us to write down the Cauchy-Bunyakovsky-Schwarz inequality. We have

$$\int d^4k\delta(x-k\cdot n)\delta((P-k)^2) \times|\lambda\langle P-k|\psi_+(0)|p_2\rangle^H+\langle P-k|\psi_+(0)|p_1\rangle^H|^2 + \int d^4k\delta(x-k\cdot n)\delta((k+\Delta/2)^2) \times\left|\lambda\langle k+\frac{\Delta}{2},p_1|\psi_+^\dagger(0)|p_2\rangle^H+\langle k+\frac{\Delta}{2}|\psi_+^\dagger(0)|0\rangle^H\right|^2\geq 0.$$

Here, the light-cone components of the fermion fields are given by $\psi_\pm = 1/2\gamma^\mp\gamma^\pm\psi$. The characteristic equation of Eq. (40) takes the following form: $\lambda^2 A + \lambda B + C \geq 0$, where (using the crossing where needed)

$$A = \int d^4k\delta(x-k\cdot n)\delta((P-k)^2) \times\langle p_2|\psi_+^\dagger(0)|P-k\rangle\langle P-k|\psi_+(0)|p_2\rangle^H + \int d^4k\delta(x-k\cdot n)\delta((k+\Delta/2)^2) \times\langle p_2,-p_1|\psi_+(0)\left|k+\frac{\Delta}{2}\right\rangle\left\langle k+\frac{\Delta}{2}\right| \times\psi_+^\dagger(0)|-p_1,p_2\rangle^H, \tag{40}$$

$$B = \int d^4k\delta(x-k\cdot n)\delta((P-k)^2) \times\langle p_2|\psi_+^\dagger(0)|P-k\rangle\langle P-k|\psi_+(0)|p_1\rangle^H + \int d^4k\delta(x-k\cdot n)\delta((k+\Delta/2)^2) \times\langle p_2,-p_1|\psi_+(0)\left|k+\frac{\Delta}{2}\right\rangle\left\langle k+\frac{\Delta}{2}\right| \times\psi_+^\dagger(0)|0\rangle^H+(p_1\leftrightarrow p_2), \tag{41}$$

and

$$C = \int d^4k\delta(x-k\cdot n)\delta((P-k)^2) \times\langle p_1|\psi_+^\dagger(0)|P-k\rangle\langle P-k|\psi_+(0)|p_1\rangle^H + \int d^4k\delta(x-k\cdot n)\delta((k+\Delta/2)^2) \times\langle 0|\psi_+(0)\left|k+\frac{\Delta}{2}\right\rangle\left\langle k+\frac{\Delta}{2}\right|\psi_+^\dagger(0)|0\rangle^H. \tag{42}$$

We now see that the first and the second terms of Eq. (41) produce the "commutator" and "anticommutator" GPDs, respectively, while the first and the second terms of Eqs. (40) and (42) correspond to the forward distributions and the vacuum expectations (the quark condensate), respectively. To satisfy the above-mentioned characteristic equation we have to demand that $D = B^2 - 4AC \leq 0$, which is equivalent to the following inequality [the corresponding normalization of $q(x)$ is implied]:

$$[H^{[\ldots]}_{S(A)}(x,\xi)+H^{\{\ldots\}}_{S(A)}(x,\xi)]^2 \leq [q(x_2)+D(x_2)][q(x_1)+C(x_1)], \tag{43}$$

where we introduced the symmetrized and antisymmetrized in $x \leftrightarrow -x$ combinations of the corresponding GPDs and performed the rescaling of the fractions; see [9]. After the summation over the intermediate states, the functions $D(x)$ and $C(x)$ [Eq. (43)] take the following form:

$$D(x) = \int d^4k\delta(x-k\cdot n)d^4z e^{i(k-\Delta/2)\cdot z} \times\langle p_2,p_1|\psi_+(z)\psi_+^\dagger(0)|p_2,p_1\rangle^H,$$
$$C(x) = \int d^4k\delta(x-k\cdot n)d^4z e^{i(k-\Delta/2)\cdot z} \times\langle 0|\psi_+(z)\psi_+^\dagger(0)|0\rangle^H. \tag{44}$$

VI. CONCLUSIONS

In conclusion, we have found that, in the collinear kinematics and in the factorization regime with $t \approx 0$, the matrix element of the fermion anticommutator does not vanish. We have demonstrated, moreover, that the existence of this contribution is dictated by the polynomiality condition for the GPDs. Furthermore, we have obtained a new possible constraint for the GPDs wherein the new contributions from the forward distribution and the quark condensate are included.

Let us also emphasize that the model we have used in our analysis do not assume the existence of any semi-disconnected graphs due to the absence of the two-hadron vertices. Moreover, being taken into account in another framework (e.g., with the resonance exchange in the "t"channel or with the D term included in the corresponding GPDs), they will not affect our conclusions but rather extend and generalized them.

ACKNOWLEDGMENTS

The authors thank A. V. Efremov, L. Szymanowski, O. V. Teryaev, and S. Wallon for useful discussions and comments. This work is supported in part by the RFBR (Grant No. 12-02-00613).

[1] R. L. Jaffe, Nucl. Phys. **B229**, 205 (1983).
[2] M. Diehl and T. Gousset, Phys. Lett. B **428**, 359 (1998).
[3] I. V. Anikin, I. O. Cherednikov, N. G. Stefanis, and O. V. Teryaev, Eur. Phys. J. C **61**, 357 (2009).
[4] I. V. Anikin, I. O. Cherednikov, and O. V. Teryaev (unpublished).
[5] A. V. Radyushkin, Phys. Rev. D **56**, 5524 (1997).
[6] A. V. Radyushkin, Phys. Rev. D **83**, 076006 (2011).
[7] S. J. Brodsky and F. J. Llanes-Estrada, Eur. Phys. J. C **46**, 751 (2006); S. J. Brodsky, F. J. Llanes-Estrada, and A. P. Szczepaniak, Phys. Rev. D **79**, 033012 (2009).
[8] M. V. Polyakov and C. Weiss, Phys. Rev. D **60**, 114017 (1999).
[9] B. Pire, J. Soffer, and O. Teryaev, Eur. Phys. J. C **8**, 103 (1999).