Higgs sector of the supersymmetric reduced 331 model

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A supersymmetric version of the recently proposed reduced minimal 331 model is considered and its Higgs sector is investigated. We focus on the mass spectrum of the lightest scalars of the model. We show that the Higgs mass of 125 GeV requires substantial radiative corrections. However, stops may develop small mixing and must have a mass around TeV. Moreover, some soft supersymmetry breaking terms may lie at the electroweak scale, which alleviates some tension concerning the fine-tuning of the related parameters. The lightest doubly charged scalar may have a mass of around a few hundreds of GeV, which can be probed at the LHC, while the remaining scalars of the model have masses at the TeV scale.

I. INTRODUCTION

It seems that the ultimate block of the standard model (SM) of particle physics was finally detected in the CERN Large Hadron Collider (LHC) by ATLAS and CMS experiments [1]. Both experiments recently reported the detection of a new particle with a mass of around 125 GeV. Fitting to all available collider data suggests that the discovered particle is, indeed, the elusive Higgs boson.

It is well accepted that the SM is not the final answer in particle physics. A couple of experimental results as, for example, neutrino oscillation [2], dark matter [3], etc., require an extension of the SM. From the theoretical side, the SM also suffers from incompleteness once it is not able to explain problems like the hierarchy problem, family replication, and so on. Thus, we see that at the moment we have experimental and theoretical reasons to go beyond the SM.

Among the alternatives to the SM, there is supersymmetry (SUSY), which has been fascinating physicists for more than three decades. The reasons behind the great interest in SUSY lies in the fact that it is an attractive solution to the hierarchy problem by entangling fermions and bosons, and then justifying the presence of fundamental scalars in the spectrum. Besides, it allows unification of the coupling constants and predicts a light Higgs boson to engender the electroweak symmetry breaking [4]. In the most popular low energy SUSY model, the minimal supersymmetric standard model (MSSM), a 125 GeV Higgs mass requires substantial loop contributions once a SM-like Higgs mass is upper bounded by $M_Z \cos 2\beta$ at tree level. A great number of papers analyzing MSSM's parameter space to reproduce LHC data were published in the last year, showing that, although very restrictive, there

is still enough room in the parameter space for a light Higgs boson [5].

On the other hand, in a class of gauge models based on the $SU(3)_C \times SU(3)_L \times U(1)_N$ (331) gauge symmetry, anomaly cancellation requires the existence of at least three families of fermions [6]. Consequently, the supersymmetric versions of these gauge models would solve the hierarchy problem and family replication altogether. Recently it was shown that the so-called minimal 331 models may be implemented with two Higgs triplets only [7]. The model is very short and predictive in its scalar spectrum, compared to the original version [6]. It was called the reduced 331 model and some of its phenomenology was developed in Ref. [8]. Curiously, the scalar content that survives the reduction has the appropriate quantum numbers to be supersymmetrized, without the need of extra multiplets employed in the first SUSY versions of the minimal 331 model [9,10]. Actually, while we were working on this model, the authors in Ref. [11] also observed this, although their model, as well as results and conclusions, differ in some crucial points from ours as we will remark later. There is an additional issue that motivates the supersymmetrization of this model; namely, the nonsupersymmetric version of minimal 331 has no natural candidate for cold dark matter,[1] while such a candidate is natural in its SUSY version once R parity is present.

In this work we develop the SUSY version of the reduced 331 model. We focus on the scalar spectrum of the model and give particular emphasis on the Higgs boson that arises in its spectrum. Our main result is that the stability of the vacuum imposes a Higgs mass upper bounded <90 GeV at tree level. We calculated the radiative corrections that lift this mass to 125 GeV. We also obtained the masses of the lightest charged scalars.

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[1]Although there is a claim in Ref. [12] that a stable self-interacting dark matter candidate exists in the minimal 331 model, it is easy to show that the lack of a symmetry to guarantee its stability allows us to write down effective operators that are not sufficiently suppressed to avoiding its decay.

II. THE ESSENCE OF THE REDUCED SUSY331 MODEL

In order to implement the supersymmetric version of a certain model, we have to promote its fields to superfields. In this way, the leptons in the reduced SUSY 331 (RSUSY331) model compose three chiral left-handed lepton superfields denoted by

$$\hat{L}_L = \begin{pmatrix} \hat{\nu}_l \\ \hat{l} \\ \hat{l}^c \end{pmatrix}_L \sim (\mathbf{1}, \mathbf{3}, 0), \qquad (1)$$

where $l = e, \mu, \tau$. Notice that the presence of a right-handed component of the charged lepton field in the multiplet allows us to dispose of singlet [under $SU(3)_L$] right-handed leptons.

For the chiral left-handed quark superfields, the first family comes in a triplet representation, while the second and third families come in antitriplet representations, while their right-handed partners are arranged in singlets as denoted below:

$$\hat{Q}_{1L} = \begin{pmatrix} \hat{u}_1 \\ \hat{d}_1 \\ \hat{J}_1 \end{pmatrix}_L \sim \left(\mathbf{3}, \mathbf{3}, \frac{2}{3}\right), \quad \hat{Q}_{iL} = \begin{pmatrix} \hat{d}_i \\ -\hat{u}_i \\ \hat{J}_i \end{pmatrix}_L \sim \left(\mathbf{3}, \mathbf{3}^*, -\frac{1}{3}\right),$$

$$\hat{u}_{1L}^c \sim \left(\mathbf{3}^*, \mathbf{1}, -\frac{2}{3}\right), \quad \hat{d}_{1L}^c \sim \left(\mathbf{3}^*, \mathbf{1}, +\frac{1}{3}\right), \quad \hat{J}_{1L}^c \sim \left(\mathbf{3}^*, \mathbf{1}, -\frac{5}{3}\right),$$

$$\hat{u}_{iL}^c \sim \left(\mathbf{3}^*, \mathbf{1}, -\frac{2}{3}\right), \quad \hat{d}_{iL}^c \sim \left(\mathbf{3}^*, \mathbf{1}, +\frac{1}{3}\right), \quad \hat{J}_{iL}^c \sim \left(\mathbf{3}^*, \mathbf{1}, +\frac{4}{3}\right),$$

$$\qquad (2)$$

with $i = 2, 3$. Here, the extra quarks J_i and J_1 are exotic ones with electric charges $-4/3$ and $+5/3$, respectively.

The scalar sector of the reduced 331 model is composed by two Higgs triplets. Consequently, anomaly cancellation requires that its supersymmetric version possess four chiral left-handed Higgs superfields,

$$\hat{\rho} = \begin{pmatrix} \hat{\rho}^+ \\ \hat{\rho}^0 \\ \hat{\rho}^{++} \end{pmatrix} \sim (\mathbf{1}, \mathbf{3}, +1), \quad \hat{\chi} = \begin{pmatrix} \hat{\chi}^- \\ \hat{\chi}^{--} \\ \hat{\chi}^0 \end{pmatrix} \sim (\mathbf{1}, \mathbf{3}, -1),$$

$$\hat{\rho}' = \begin{pmatrix} \hat{\rho}'^- \\ \hat{\rho}'^0 \\ \hat{\rho}'^{--} \end{pmatrix} \sim (\mathbf{1}, \mathbf{3}^*, -1), \quad \hat{\chi}' = \begin{pmatrix} \hat{\chi}'^+ \\ \hat{\chi}'^{++} \\ \hat{\chi}'^0 \end{pmatrix} \sim (\mathbf{1}, \mathbf{3}^*, +1).$$

$$\qquad (3)$$

These scalar superfields are not enough to render the correct mass pattern for all fermion fields through the superpotential though. Nevertheless, we recall that this class of 331 models possesses a Landau pole around 4 to 5 TeV [13], becoming strongly interacting before that point. Throughout this work we assume that the highest energy scale where the model is found to be perturbatively reliable is $\Lambda = 5$ TeV. This is welcome information since

it allows us to make use of effective operators to complement that part of the mass spectrum not obtained from the renormalizable superpotential. That being said, the superpotential[2] capable of generating the correct masses of the charged fermions in the RSUSY331 model is composed by the following terms:

$$\hat{f} = \hat{f}' + \hat{f}_{\text{E.O.}}, \qquad (4)$$

where

$$\hat{f}' = \lambda_{11}^J \hat{Q}_{1L}\hat{\chi}'\hat{J}_{1L}^c + \lambda_{ij}^J \hat{Q}_{iL}\hat{\chi}\hat{J}_{jL}^c + \lambda_{1a}'\hat{Q}_{1L}\hat{\rho}'\hat{d}_{aL}^c$$
$$+ \lambda_{ia}''\hat{Q}_{iL}\hat{\rho}\hat{u}_{aL}^c + \mu_\rho\hat{\rho}\hat{\rho}' + \mu_\chi\hat{\chi}\hat{\chi}', \qquad (5)$$

and

$$\hat{f}_{\text{E.O.}} = \frac{k_{u_{1a}}}{\Lambda}\varepsilon_{nmp}(\hat{Q}_{1Ln}\hat{\rho}_m\hat{\chi}_p)\hat{u}_{aL}^c$$
$$+ \frac{k_{d_{ia}}}{\Lambda}\varepsilon_{nmp}(\hat{Q}_{iLn}\hat{\rho}'_m\hat{\chi}'_p)\hat{d}_{aL}^c + \frac{k_l}{\Lambda}(\hat{L}_L\hat{\rho}')(\hat{L}_L\hat{\chi}').$$

$$\qquad (6)$$

As before, $i, j = 2, 3$ and $a = 1, 2, 3$ are family index labels.

The soft SUSY breaking terms of the model are given by

$$\mathcal{L}_{\text{soft}} = -\frac{1}{2}[m_{\lambda_c}\lambda_c^a\lambda^a + m_\lambda(\lambda_a\lambda_a) + m'\lambda\lambda + \text{H.c.}]$$
$$- m_L^2\tilde{L}^\dagger\tilde{L} - m_{Q_1}^2\tilde{Q}_1^\dagger\tilde{Q}_1 + m_{u_\alpha}^2\tilde{u}_\alpha^\dagger\tilde{u}_\alpha - m_{d_\alpha}^2\tilde{d}_\alpha^\dagger\tilde{d}_\alpha$$
$$- m_{J_1}^2\tilde{J}_1^\dagger\tilde{J}_1 - m_{J_i}^2\tilde{J}_i^\dagger\tilde{J}_i - m_{Q_i}^2\tilde{Q}_i^\dagger\tilde{Q}_i - m_\chi^2\chi^\dagger\chi$$
$$- m_\rho^2\rho^\dagger\rho - m_{\chi'}^2\chi'^\dagger\chi' - m_{\rho'}^2\rho'^\dagger\rho' + b_\rho\rho^a\rho'^a$$
$$+ b_\chi\chi^a\chi'^a + \tilde{Q}_1A_{1\alpha}^d\rho'\tilde{d}_\alpha^c + \tilde{Q}_1B_{11}^J\chi'\tilde{J}_1^c$$
$$+ \tilde{Q}_iA_{i\alpha}^u\rho\tilde{u}_\alpha^c + \tilde{Q}_iB_{ij}^J\chi\tilde{J}_j^c. \qquad (7)$$

The parameters in the bilinear terms on scalar fields have mass dimension two. Trilinear terms in the soft breaking Lagrangian, bilinear terms in superpotential and gaugino mass terms all have parameters with a mass dimension of one. All the other parameters are dimensionless.

Considering the spontaneous breaking of the gauge symmetries, by supposing that $\langle\chi\rangle, \langle\chi'\rangle \gg \langle\rho\rangle, \langle\rho'\rangle$, we get the following breaking sequence:

$$SU(3)_L \otimes U(1)_X \overset{\langle\chi\rangle,\langle\chi'\rangle}{\Rightarrow} SU(2)_L \otimes U(1)_Y \overset{\langle\rho\rangle,\langle\rho'\rangle}{\Rightarrow} U(1)_{\text{QED}}.$$

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[2]Our superpotential (and soft SUSY breaking terms) is very distinct from that in Ref. [11], mainly because we assume the usual *R* parity as the MSSM, since lepton and baryon numbers are conserved in all interactions due to the association of the leptonic number with some fields, called bileptons. Such bileptons are those scalar and vector fields which connect the first or second components of lepton fields in the triplets to the third one.

This spontaneous symmetry breakdown is appropriate to give masses to the gauge bosons, $V^\pm$, $U^{\pm\pm}$, $W^\pm$, Z', Z, which are encoded in the following expressions:

$$
\begin{aligned}
M_{W^\pm}^2 &= \frac{g^2}{4}(v_\rho^2 + v_{\rho'}^2), \\
M_Z^2 &= \frac{g^2}{4}\frac{(1 + 4t^2)}{(1 + 3t^2)}(v_\rho^2 + v_{\rho'}^2), \\
M_{Z'}^2 &= \frac{g^2}{3}(1 + 3t^2)(v_\chi^2 + v_{\chi'}^2), \\
M_{U^{\pm\pm}}^2 &= \frac{g^2}{4}(v_\rho^2 + v_{\rho'}^2 + v_\chi^2 + v_{\chi'}^2), \\
M_{V^\pm}^2 &= \frac{g^2}{4}(v_\chi^2 + v_{\chi'}^2),
\end{aligned}
\tag{8}
$$

where we have denoted $\langle\chi\rangle = v_\chi$, $\langle\chi'\rangle = v_{\chi'}$, $\langle\rho\rangle = v_\rho$, $\langle\rho'\rangle = v_{\rho'}$, $t = \frac{g_N}{g}$, with g_N being the coupling constant associated to the gauge group $U(1)_N$ and g is the gauge coupling for the $SU(3)_L$ gauge group [and also for the SM $SU(2)_L$, which is embedded in it]. As it should be, one of the gauge bosons remains massless, the photon, A_μ. It turns out that in order to recover the correct electromagnetic coupling in the neutral currents involving the photon, we get the relation between the electric charge e and the 331 model gauge couplings [6],

$$
e = g\frac{t}{\sqrt{1 + 4t^2}} \equiv g\sin\theta_W, \tag{9}
$$

where θ_W is the usual electroweak mixing angle. This promptly leads to the correct phenomenological observation concerning the ratio of the massive weak gauge bosons in Eq. (8),

$$
\frac{M_{W^\pm}^2}{M_Z^2} = \frac{(1 + 3t^2)}{(1 + 4t^2)} = \cos^2\theta_W. \tag{10}
$$

The masses of the charged leptons are obtained strictly from effective operators in the last term of the superpotential, Eq. (6),

$$
m_\ell = \frac{k_l}{2\Lambda}v_{\rho'}v_{\chi'}. \tag{11}
$$

Regarding the quark masses, the superpotential in Eq. (5) along with the first two terms in Eq. (6) provide the following mass matrices for the up-type quarks in the basis (u_1, u_2, u_3):

$$
M_u = \begin{pmatrix}
-\frac{k_{u_{11}}}{\Lambda}v_\rho v_\chi & -\frac{k_{u_{12}}}{\Lambda}v_\rho v_\chi & -\frac{k_{u_{13}}}{\Lambda}v_\rho v_\chi \\
\lambda''_{21}v_\rho & \lambda''_{22}v_\rho & \lambda''_{23}v_\rho \\
\lambda''_{31}v_\rho & \lambda''_{32}v_\rho & \lambda''_{33}v_\rho
\end{pmatrix}, \tag{12}
$$

and for the down-type quarks in the basis (d_1, d_2, d_3),

$$
M_d = \begin{pmatrix}
\lambda'_{11}v_{\rho'} & \lambda'_{12}v_{\rho'} & \lambda'_{13}v_{\rho'} \\
\frac{k_{d_{21}}}{\Lambda}v_{\rho'}v_{\chi'} & \frac{k_{d_{22}}}{\Lambda}v_{\rho'}v_{\chi'} & \frac{k_{d_{23}}}{\Lambda}v_{\rho'}v_{\chi'} \\
\frac{k_{d_{31}}}{\Lambda}v_{\rho'}v_{\chi'} & \frac{k_{d_{32}}}{\Lambda}v_{\rho'}v_{\chi'} & \frac{k_{d_{33}}}{\Lambda}v_{\rho'}v_{\chi'}
\end{pmatrix}. \tag{13}
$$

We can naturally assign the values of Λ, v_χ and $v_{\chi'}$ around TeV scale, while v_ρ and $v_{\rho'}$ lie in the electroweak scale and obey the bound $v_\rho^2 + v_{\rho'}^2 = 246^2$ GeV^2. Thus, for typical values of the Yukawa couplings, we can easily obtain the observed masses for all standard charged fermions.

III. SCALAR SECTOR

In supersymmetric models the scalar sector receives contributions from three different sources that add up to form the scalar potential. These contributions are

$$
V_F = \sum_i \left|\frac{\partial\hat{f}}{\partial S_i}\right|_{\hat{S}=S}^2, \tag{14}
$$

$$
V_D = \frac{1}{2}\sum_{\alpha A}\left(\sum_i S_i^\dagger g_\alpha t_{\alpha A}S_i\right)^2, \tag{15}
$$

and

$$
V_{\text{soft}} = m_1^2\rho^\dagger\rho + m_2^2\rho'^\dagger\rho' + m_3^2\chi^\dagger\chi + m_4^2\chi'^\dagger\chi' - b_\rho\delta_{ab}\rho^a\rho'^b - b_\chi\delta_{ab}\chi^a\chi'^b, \tag{16}
$$

where the summation index i runs over all scalars, α runs through the different symmetry groups, and A through the group generators.

Working out the indices, we have

$$
V_F = \mu_\rho^2|\rho|^2 + \mu_\rho^2|\rho'|^2 + \mu_\chi^2|\chi|^2 + \mu_\chi^2|\chi'|^2 \tag{17}
$$

and

$$
\begin{aligned}
V_D &= \frac{g^2}{2}(\rho^\dagger t_A\rho - \rho'^\dagger t_A^*\rho' + \chi^\dagger t_A\chi - \chi'^\dagger t_A^*\chi')^2 \\
&+ \frac{g_N^2}{2}(\rho^\dagger\rho - \rho'^\dagger\rho' - \chi^\dagger\chi + \chi'^\dagger\chi')^2.
\end{aligned}
\tag{18}
$$

The scalar potential is the sum of the above three contributions:

$$
V = V_F + V_D + V_{\text{soft}}. \tag{19}
$$

By performing the usual shift on the neutral scalars displaced by their respective vacuum expectation values,

$$
\rho^0, \rho'^0, \chi^0, \chi'^0 \rightarrow \frac{1}{\sqrt{2}}(v_{\rho,\rho',\chi,\chi'} + R_{\rho,\rho',\chi,\chi'} + iI_{\rho,\rho',\chi,\chi'}), \tag{20}
$$

the set of minimum conditions is given by

$$\left\langle \frac{\partial V}{\partial \rho^0} \right\rangle_0 = g^2(2v_\rho^2 - 2v_{\rho'}^2 - v_\chi^2 + v_{\chi'}^2) + 6(v_\rho^2 - v_{\rho'}^2 - v_\chi^2 + v_{\chi'}^2)g_N^2 + 12m_1^2 + 12\mu_\rho^2 - 12\frac{v_{\rho'}}{v_\rho}b_\rho = 0,$$

$$\left\langle \frac{\partial V}{\partial \rho'^0} \right\rangle_0 = g^2(-2v_\rho^2 + 2v_{\rho'}^2 + v_\chi^2 - v_{\chi'}^2) - 6(v_\rho^2 - v_{\rho'}^2 - v_\chi^2 + v_{\chi'}^2)g_N^2 + 12m_2^2 + 12\mu_\rho^2 - 12\frac{v_\rho}{v_{\rho'}}b_\rho = 0,$$

$$\left\langle \frac{\partial V}{\partial \chi^0} \right\rangle_0 = g^2(-v_\rho^2 + v_{\rho'}^2 + 2v_\chi^2 - 2v_{\chi'}^2) - 6(v_\rho^2 - v_{\rho'}^2 - v_\chi^2 + v_{\chi'}^2)g_N^2 + 12m_3^2 + 12\mu_\chi^2 - 12\frac{v_{\chi'}}{v_\chi}b_\chi = 0,$$

$$\left\langle \frac{\partial V}{\partial \chi'^0} \right\rangle_0 = g^2(v_\rho^2 - v_{\rho'}^2 - 2v_\chi^2 + 2v_{\chi'}^2) + 6(v_\rho^2 - v_{\rho'}^2 - v_\chi^2 + v_{\chi'}^2)g_N^2 + 12m_4^2 + 12\mu_\chi^2 - 12\frac{v_\chi}{v_{\chi'}}b_\chi = 0. \tag{21}$$

With this set of constraint equations, we are able to obtain the texture of the scalar mass matrices in the model.

We start with the *CP*-odd scalars, that lead to two 2×2 mass matrices. The first one, in the basis $(I_\chi, I_{\chi'})^T$, takes the form

$$\begin{pmatrix} \frac{v_{\chi'}b_\chi}{2v_\chi} & \frac{b_\chi}{2} \\ \frac{b_\chi}{2} & \frac{v_\chi b_\chi}{2v_{\chi'}} \end{pmatrix}, \tag{22}$$

while the second one, in the basis $(I_\rho, I_{\rho'})^T$, takes the form,

$$\begin{pmatrix} \frac{v_{\rho'}b_\rho}{2v_\rho} & \frac{b_\rho}{2} \\ \frac{b_\rho}{2} & \frac{v_\rho b_\rho}{2v_{\rho'}} \end{pmatrix}. \tag{23}$$

Both matrices in Eqs. (22) and (23) have the same pattern and are easily diagonalized, providing the following eigenvalues:

$$M_A^2 = \frac{(v_{\rho'}^2 + v_\rho^2)b_\rho}{v_\rho v_{\rho'}},$$

$$M_{A'}^2 = \frac{(v_{\chi'}^2 + v_\chi^2)b_\chi}{v_\chi v_{\chi'}}, \tag{24}$$

$$M_G^2 = M_{G'}^2 = 0,$$

whose eigenstates are respectively given by

$$A = \frac{v_{\rho'}}{\sqrt{v_\rho^2 + v_{\rho'}^2}}I_\rho + \frac{v_\rho}{\sqrt{v_\rho^2 + v_{\rho'}^2}}I_{\rho'},$$

$$A' = \frac{v_{\chi'}}{\sqrt{v_\chi^2 + v_{\chi'}^2}}I_\chi + \frac{v_\chi}{\sqrt{v_\chi^2 + v_{\chi'}^2}}I_{\chi'},$$

$$G = -\frac{v_\rho}{\sqrt{v_\rho^2 + v_{\rho'}^2}}I_\rho + \frac{v_{\rho'}}{\sqrt{v_\rho^2 + v_{\rho'}^2}}I_{\rho'}, \tag{25}$$

$$G' = -\frac{v_\chi}{\sqrt{v_\chi^2 + v_{\chi'}^2}}I_\chi + \frac{v_{\chi'}}{\sqrt{v_\chi^2 + v_{\chi'}^2}}I_{\chi'},$$

where A and A' are the massive *CP*-odd states and G and G' are the Goldstone bosons eaten by the neutral gauge bosons Z and Z'.

For the singly charged scalars we also have two 2×2 mass matrix. The first one, in the basis $(\rho^+, \rho'^+)^T$, takes the form,

$$\begin{pmatrix} \frac{g^2 v_{\rho'}^2}{4} + \frac{b_\rho v_{\rho'}}{v_\rho} & -\left(\frac{1}{4}v_\rho v_{\rho'}g^2 + b_\rho\right) \\ -\left(\frac{1}{4}v_\rho v_{\rho'}g^2 + b_\rho\right) & \frac{g^2 v_\rho^2}{4} + \frac{b_\rho v_\rho}{v_{\rho'}} \end{pmatrix}, \tag{26}$$

while the second one, in the basis $(\chi^+, \chi'^+)^T$ is

$$\begin{pmatrix} \frac{g^2 v_{\chi'}^2}{4} + \frac{b_\chi v_{\chi'}}{v_\chi} & -\left(\frac{1}{4}v_\chi v_{\chi'}g^2 + b_\chi\right) \\ -\left(\frac{1}{4}v_\chi v_{\chi'}g^2 + b_\chi\right) & \frac{g^2 v_\chi^2}{4} + \frac{b_\chi v_\chi}{v_{\chi'}} \end{pmatrix}. \tag{27}$$

These two matrices have eigenvalues,

$$M_{H^+}^2 = M_A^2 + M_W^2,$$

$$M_{H'^+}^2 = M_{A'}^2 + M_V^2, \tag{28}$$

$$M_{G^+}^2 = M_{G'^+}^2 = 0,$$

with the respective eigenvectors,

$$H^+ = -\frac{v_{\rho'}}{\sqrt{v_\rho^2 + v_{\rho'}^2}}\rho^+ + \frac{v_\rho}{\sqrt{v_\rho^2 + v_{\rho'}^2}}\rho'^+,$$

$$H'^+ = -\frac{v_{\chi'}}{\sqrt{v_\chi^2 + v_{\chi'}^2}}\chi^+ + \frac{v_\chi}{\sqrt{v_\chi^2 + v_{\chi'}^2}}\chi'^+,$$

$$G^+ = \frac{v_\rho}{\sqrt{v_\rho^2 + v_{\rho'}^2}}\rho^+ + \frac{v_{\rho'}}{\sqrt{v_\rho^2 + v_{\rho'}^2}}\rho'^+, \tag{29}$$

$$G'^+ = \frac{v_\chi}{\sqrt{v_\chi^2 + v_{\chi'}^2}}\chi^+ + \frac{v_{\chi'}}{\sqrt{v_\chi^2 + v_{\chi'}^2}}\chi'^+,$$

where M_V and M_W stand for the gauge boson masses in Eq. (8). As can be seen, there are two Goldstone bosons, those eaten by the two singly charged gauge bosons.

For the doubly charged scalars, we are going to have a 4×4 mass matrix which, in the basis $(\rho^{++}, \rho'^{++}, \chi^{++}, \chi'^{++})^T$, takes the form

$$\begin{pmatrix}
\frac{g^2 v_\rho(v_{\rho'}^2+v_\chi^2-v_{\chi'}^2)+4v_{\rho'}b_\rho}{4v_\rho} & -\frac{g^2 v_\rho v_{\rho'}}{4} - b_\rho & \frac{g^2 v_\rho v_\chi}{4} & -\frac{g^2 v_\rho v_{\chi'}}{4} \\[2mm]
-\frac{g^2 v_\rho v_{\rho'}}{4} - b_\rho & \frac{g^2 v_{\rho'}(v_\rho^2-v_\chi^2+v_{\chi'}^2)+4v_\rho b_\rho}{4v_{\rho'}} & -\frac{g^2 v_{\rho'} v_\chi}{4} & \frac{g^2 v_{\rho'} v_{\chi'}}{4} \\[2mm]
\frac{g^2 v_\rho v_\chi}{4} & -\frac{g^2 v_{\rho'} v_\chi}{4} & \frac{g^2 v_\chi(v_\rho^2-v_{\rho'}^2+v_{\chi'}^2)+4v_{\chi'}b_\chi}{4v_\chi} & -\frac{g^2 v_\chi v_{\chi'}}{4} - b_\chi \\[2mm]
-\frac{g^2 v_\rho v_{\chi'}}{4} & \frac{g^2 v_{\rho'} v_{\chi'}}{4} & -\frac{g^2 v_\chi v_{\chi'}}{4} - b_\chi & \frac{g^2 v_{\chi'}(-v_\rho^2+v_{\rho'}^2+v_\chi^2)+4v_\chi b_\chi}{4v_{\chi'}}
\end{pmatrix}. \tag{30}$$

To obtain analytical eigenvalues and eigenstates for this matrix is a somewhat cumbersome task that we will not follow. However, its determinant is equal to zero, which provides (after a thorough numerical analysis of all the eigenvalues) only one null eigenvalue that will be the Goldstone eaten by the gauge boson U^{++}. Later, in the next section, when we have specified some of the model parameters, we will present the range of mass values for the lightest doubly charged scalar, which will be around some few hundreds of GeV.

Let us finally focus on the CP-even scalars. Considering the basis $(R_\rho, R_{\rho'}, R_\chi, R_{\chi'})^T$, its mass matrix takes the following form:

$$\begin{pmatrix}
\left(\frac{g^2}{3}+g_N^2\right)v^2\sin^2\beta + \frac{b_\rho}{\tan\beta} & -\left(\frac{g^2}{3}+g_N^2\right)v^2\cos\beta\sin\beta - b_\rho & -\left(\frac{g^2}{6}+g_N^2\right)vv_\chi\sin\beta & \left(\frac{g^2}{6}+g_N^2\right)vv_{\chi'}\sin\beta \\[2mm]
-\left(\frac{g^2}{3}+g_N^2\right)v^2\cos\beta\sin\beta - b_\rho & \left(\frac{g^2}{3}+g_N^2\right)v^2\cos^2\beta + b_\rho\tan\beta & \left(\frac{g^2}{6}+g_N^2\right)vv_\chi\cos\beta & -\left(\frac{g^2}{6}+g_N^2\right)vv_{\chi'}\cos\beta \\[2mm]
-\left(\frac{g^2}{6}+g_N^2\right)vv_\chi\sin\beta & \left(\frac{g^2}{6}+g_N^2\right)vv_\chi\cos\beta & \left(\frac{g^2}{3}+g_N^2\right)v_\chi^2 + b_\chi\frac{v_{\chi'}}{v_\chi} & -\left(\frac{g^2}{3}+g_N^2\right)v_\chi v_{\chi'} - b_\chi \\[2mm]
\left(\frac{g^2}{6}+g_N^2\right)vv_{\chi'}\sin\beta & -\left(\frac{g^2}{6}+g_N^2\right)vv_{\chi'}\cos\beta & -\left(\frac{g^2}{3}+g_N^2\right)v_\chi v_{\chi'} - b_\chi & \left(\frac{g^2}{3}+g_N^2\right)v_{\chi'}^2 + b_\chi\frac{v_\chi}{v_{\chi'}}
\end{pmatrix}, \tag{31}$$

where

$$\tan\beta = \frac{v_\rho}{v_{\rho'}}, \tag{32}$$

and v is the SM electroweak symmetry breaking scale given by $v^2 = v_\rho^2 + v_{\rho'}^2$. From this mass matrix we are going to have four massive scalars, from which the lightest one reproduces the properties of the SM Higgs, which we assume as the scalar boson recently found in the LHC. In order to guarantee that such a scalar plays the role of a Higgs boson, we are going to demand throughout this work that its eigenstate is at least 95% composed of the real part of ρ^0. This is mandatory to certify that such a CP-even scalar behaves very much like the SM Higgs boson, since the two first components of the triplet ρ mimics the SM Higgs doublet in the context of the minimal 331 model. Besides, this choice assures us that its branching ratios to SM particles are basically the same as those computed in Refs. [14,15], except for some minor corrections coming from the extra particles in the SUSY spectrum (an analysis we are going to pursue somewhere else). In the next section we study the behavior of the lightest scalars of the model.

IV. HIGGS PHENOMENOLOGY AND SCALAR SECTOR

To assess the capability of this model to reproduce the results of the ATLAS and CMS Collaborations [1], it will be necessary to define the parameter space responsible for the Higgs mass. As can be seen from the mass matrices of all scalars of the model, there are five free parameters $(\beta, v_\chi, v_{\chi'}, b_\rho, b_\chi)$ that define their eigenvalues. Before proceeding further, it is necessary to call the attention to some features of the RSUSY331 model: First of all, charged lepton masses, through Eq. (11), impose restrictions to $v_{\rho'}$ and $v_{\chi'}$ in order to avoid entering some nonperturbative regime where $k_l > \sqrt{4\pi}$, a worry only justified for the case of the tau lepton. Second, we establish that $v_\chi^2 + v_{\chi'}^2 = v_{331}^2$, where v_{331} is the energy scale characteristic of the 331 spontaneous symmetry breakdown. It is important to notice that v_{331} should not exceed the cutoff scale (Λ), otherwise the theory would fall in the nonperturbative regime concerning the $U(1)_N$ gauge coupling. Third, the soft SUSY breaking parameters, b_ρ and b_χ, have a mass dimension of two and we can think of it as a product of two mass scales of the order of SUSY breaking, that is, $b_\rho, b_\chi \sim M_{\text{SUSY}}^2$. Assuming that the SUSY breaking scale is, roughly speaking, of the order of a few TeV, the range of these two parameters can be set, without loss of generality, to

$$10^5 \text{ GeV}^2 < b_\rho, \qquad b_\chi \lesssim 10^6 \text{ GeV}^2. \tag{33}$$

Lastly, we will require that the squared masses of all scalars of the model be real and positive. This condition restricts b_ρ and b_χ in a way that tachyons are not present in the scalar spectrum, a problem that had to be circumvented in Ref. [11] at the expense of having two massless doubly
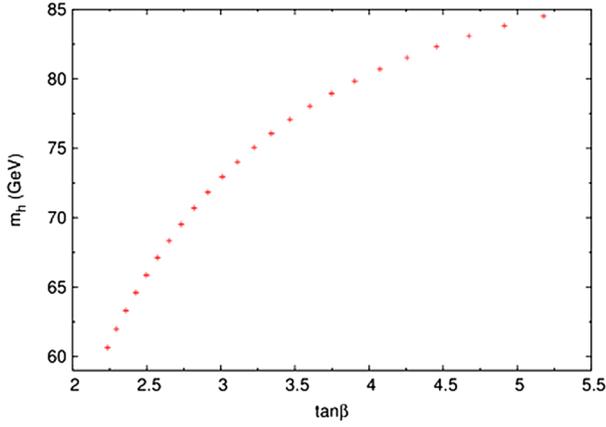
FIG. 1 (color online).   Values of $\tan\beta$ compatible with tree-level Higgs mass $m_h > 60$. As can be seen, there is a maximum value of m_h given the assumptions made, where $\tan\beta \approx 5$.

charged scalars in the spectrum, something we definitely do not desire to happen in our model.[3] The choice of the range of parameters made in Eq. (33) already satisfies this condition. For the other parameters of the model, we consider that they vary inside the following range of values:

$$1000 \text{ GeV} \leq v_{331} \leq 4500 \text{ GeV}, \tag{34}$$

$$0.1 \leq k_\tau \leq 0.9, \tag{35}$$

$$0 < \beta < \frac{\pi}{2}. \tag{36}$$

In Figs. 1 and 2 we show the behavior of the lightest scalar mass m_h at tree level with the free parameters of the model.

Perceive that we have the upper bound of $m_h < 90$ GeV at tree level. This recovers, in part, the behavior of the Higgs in the MSSM and is consistent with the previous estimate in an enlarged version of this model [9]. Thus, we conclude that this scalar should play the role of the Higgs in the RSUSY331 model. Hence, a Higgs with mass of 125 GeV, as measured by CMS and ATLAS, demands substantial contribution from radiative corrections, as is the case for the MSSM. We stress that classical conditions for the stability of the potential and absence of tachyons in our model refrain the mass of the Higgs in the RSUSY331 model from going beyond 90 GeV at tree level. In this sense, it turns imperative to analyze the main loop contributions to the mass of the Higgs in the RSUSY31 model. As is widely known, the stop gives the main contribution to the Higgs mass. In our case, the stop loop correction to the Higgs mass will be calculated using the effective potential approach [16]. For this we first have to evaluate the mass matrix of the stops. This is given by

$$\begin{pmatrix} m_t^2 + m_{Q_3}^2 + \frac{1}{3}\left(\frac{g^2}{2} + g_N^2\right)\Delta v_\chi & m_t X_t \\ m_t X_t & m_t^2 + m_{u_3}^2 + \frac{2}{3}g_N^2\Delta v_\chi \end{pmatrix}, \tag{37}$$

where D-terms which contribute minimally were neglected. Its eigenvalues are

$$m_{\tilde{t}_1}^2 = m_t^2 + \frac{1}{2}\left(m_{Q_3}^2 + \frac{1}{3}\left(\frac{g^2}{2} + g_N^2\right)\Delta v_\chi\right) + \frac{1}{2}\left(m_{u_3}^2 + \frac{2}{3}g_N^2\Delta v_\chi\right)$$
$$- \frac{1}{2}\sqrt{\left(\left(m_{Q_3}^2 + \frac{1}{3}\left(\frac{g^2}{2} + g_N^2\right)\Delta v_\chi\right) - \left(m_{u_3}^2 + \frac{2}{3}g_N^2\Delta v_\chi\right)\right)^2 + 4m_t^2 X_t^2}, \tag{38}$$

$$m_{\tilde{t}_2}^2 = m_t^2 + \frac{1}{2}\left(m_{Q_3}^2 + \frac{1}{3}\left(\frac{g^2}{2} + g_N^2\right)\Delta v_\chi\right) + \frac{1}{2}\left(m_{u_3}^2 + \frac{2}{3}g_N^2\Delta v_\chi\right)$$
$$+ \frac{1}{2}\sqrt{\left(\left(m_{Q_3}^2 + \frac{1}{3}\left(\frac{g^2}{2} + g_N^2\right)\Delta v_\chi\right) - \left(m_{u_3}^2 + \frac{2}{3}g_N^2\Delta v_\chi\right)\right)^2 + 4m_t^2 X_t^2}, \tag{39}$$

where m_{Q_3} and m_{u_3} are soft SUSY breaking terms given in Eq. (7), $\Delta v_\chi = \frac{1}{2}(v_\chi^2 - v_{\chi'}^2)$ and $X_t = A_t + \mu_\rho \cot\beta$ is the mixing parameter between $\tilde{t}_L$ and $\tilde{t}_R$ [we have identified $A_t \equiv A_{33}^u$ from Eq. (7)]. It is opportune to remark that Eq. (40) dictates that the lightest stop is $\tilde{t}_1$.

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[3]We guess that such differences between these models may be related to the different *R*-parity symmetries chosen, besides some terms in the SUSY soft breaking Lagrangian as well as in the superpotential.
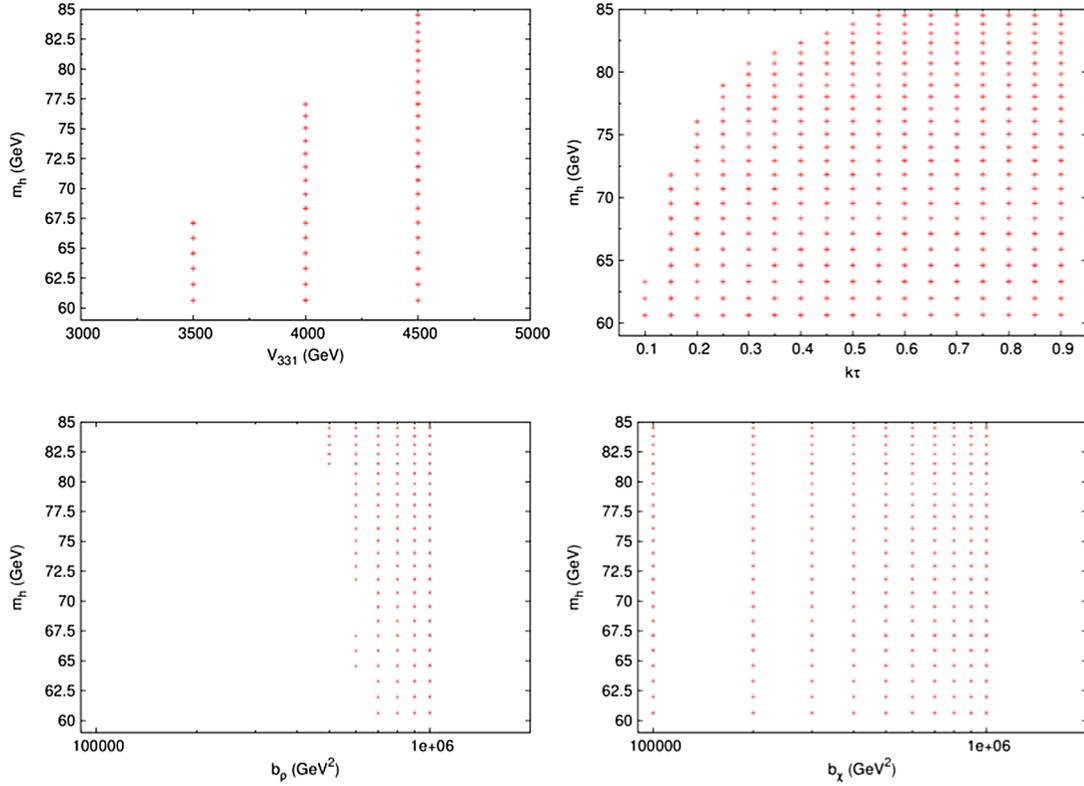
FIG. 2 (color online). Behavior of the free parameters compared with the Higgs mass m_h. The 331 breaking scale v_{331} must have the highest possible value to achieve the highest tree-level mass for the Higgs boson (top left); The τ lepton Yukawa coupling k_τ must be greater than 0.5 to reach the maximum value of the Higgs mass (top right);. Although b_ρ must be greater than 5×10^5 GeV^2 to have the highest Higgs mass (bottom left), b_χ is insensitive to such mass limit (bottom right).

In a first approximation, the mass matrix of the *CP*-even scalars given in Eq. (31) gets one-loop corrections only in the entries 11, 12, and 22, which are

$$\delta M_{11} = \frac{3 G_F m_t^4 \mathrm{cosec}^2 \beta \left(A_t (A_t + \mu_\rho \cot \beta) \ln \left(\frac{m_{\tilde{t}_2}^4}{m_{\tilde{t}_1}^4} \right) - (m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2) \ln \left(\frac{m_t^4}{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2} \right) \right)}{2\sqrt{2}\pi^2 (m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)}$$
$$+ \frac{3 G_F m_t^4 A_t^2 (A_t + \mu_\rho \cot \beta)^2 \mathrm{cosec}^2 \beta \left(2m_{\tilde{t}_2}^2 - 2m_{\tilde{t}_1}^2 - (m_{\tilde{t}_2}^2 + m_{\tilde{t}_1}^2) \ln \left(\frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2} \right) \right)}{2\sqrt{2}\pi^2 (m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)^3}, \tag{40}$$

$$\delta M_{12} = \frac{3 G_F m_t^4 \mu_\rho^2 (A_t + \mu_\rho \cot \beta)^2 \mathrm{cosec}^2 \beta \ln \left(\frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2} \right)}{2\sqrt{2}\pi^2 (m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)}$$
$$+ \frac{3 G_F m_t^4 A_t \mu_\rho (A_t + \mu_\rho \cot \beta)^2 \mathrm{cosec}^2 \beta \left(2m_{\tilde{t}_2}^2 - 2m_{\tilde{t}_1}^2 - (m_{\tilde{t}_2}^2 + m_{\tilde{t}_1}^2) \ln \left(\frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2} \right) \right)}{2\sqrt{2}\pi^2 (m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)^3}, \tag{41}$$

$$\delta M_{22} = \frac{3 G_F m_t^4 \mu_\rho^2 (A_t + \mu_\rho \cot \beta)^2 \mathrm{cosec}^2 \beta \left(2m_{\tilde{t}_2}^2 - 2m_{\tilde{t}_1}^2 - (m_{\tilde{t}_2}^2 + m_{\tilde{t}_1}^2) \ln \left(\frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2} \right) \right)}{2\sqrt{2}\pi^2 (m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)^3}. \tag{42}$$
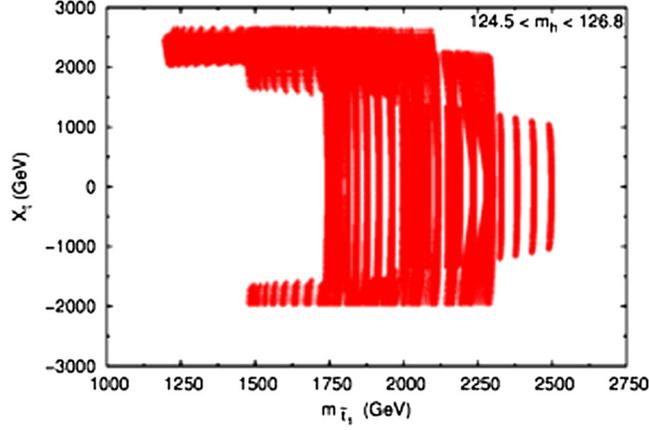
FIG. 3 (color online).   Stop mass and mixing necessary to obtain a Higgs mass in the region 124.5 GeV $\leq m_h \leq$ 126.8 GeV.
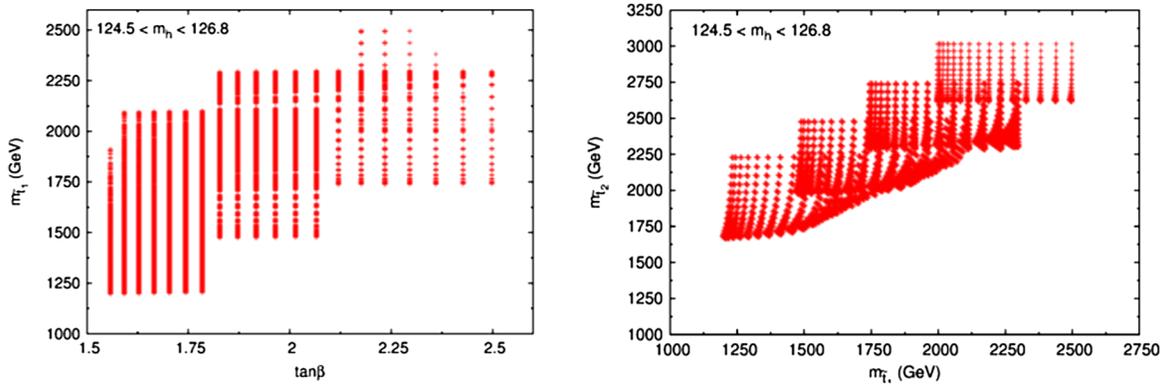
FIG. 4 (color online).   Comparison of the lightest stop mass with $\tan \beta$ and the heaviest stop mass. (left) $\tan \beta$ must lie between 1.5 and 2.5 to be in agreement with the Higgs mass constraint. (right) The heaviest stop must be heavier than $m_{\tilde{t}_2} \approx 1625$ GeV, for the smallest light stop mass $m_{\tilde{t}_1} \approx 1180$ GeV.

We will reanalyze the mass matrix in Eq. (31) but are now taking into account the above corrections. However, we select values of the free parameters that maximize the value of m_h at tree level, as well as impose that the Higgs is more than 95% composed of ρ^0, meaning that our analysis is now restricted to a narrower range of values of the parameters that enter in the calculations, namely,

$$2500 \text{ TeV} \leq v_{331} \leq 4500 \text{ TeV}, \qquad (43)$$

$$1 < \beta < 1.56, \qquad (44)$$

$$k_\tau = 0.5, \qquad (45)$$

$$10^5 \text{ GeV}^2 < b_\rho, b_\chi \leq 10^6 \text{ GeV}^2. \qquad (46)$$

We have also chosen the soft breaking stop mass, m_{Q_3} and m_{u_3}, between 100 GeV and 1.5 TeV, and the trilinear soft breaking parameter -2 TeV $< A_t < 2$ TeV.

On considering all these assumptions, we are able to present the stop mass and mixing necessary to fit the recent data reported by the CMS and ATLAS experiments. In Fig. 3 we present the behavior of the lightest stop in function of their mixing.

With all the restrictions imposed by the model to determine the parameter space in Eqs. (43) through (46), the lightest stop cannot be as light as 1180 GeV and may explain why no sign of such particles was discovered in the accelerators so far, differently from the MSSM where it can be as light as a few hundreds of GeV and, therefore, capable of being detected nowadays. However, the stop mixing can be small and even negligible if the stop mass is higher than 1750 GeV. If we focus on Eq. (40), we can see that, even if the soft breaking parameters were of the order of hundreds of GeV, we would have a stop mass above TeV because the Δv_χ term will drive the stop masses to the TeV scale. Moreover, the stop soft mass parameters can be as low as 100 GeV. This is interesting because it requires less fine-tuning compared to the MSSM [17]. For the sake of completeness, in Fig. 4 we present the behavior of the lightest stop mass in function of $\tan \beta$ and compare it with the mass of the heaviest stop.
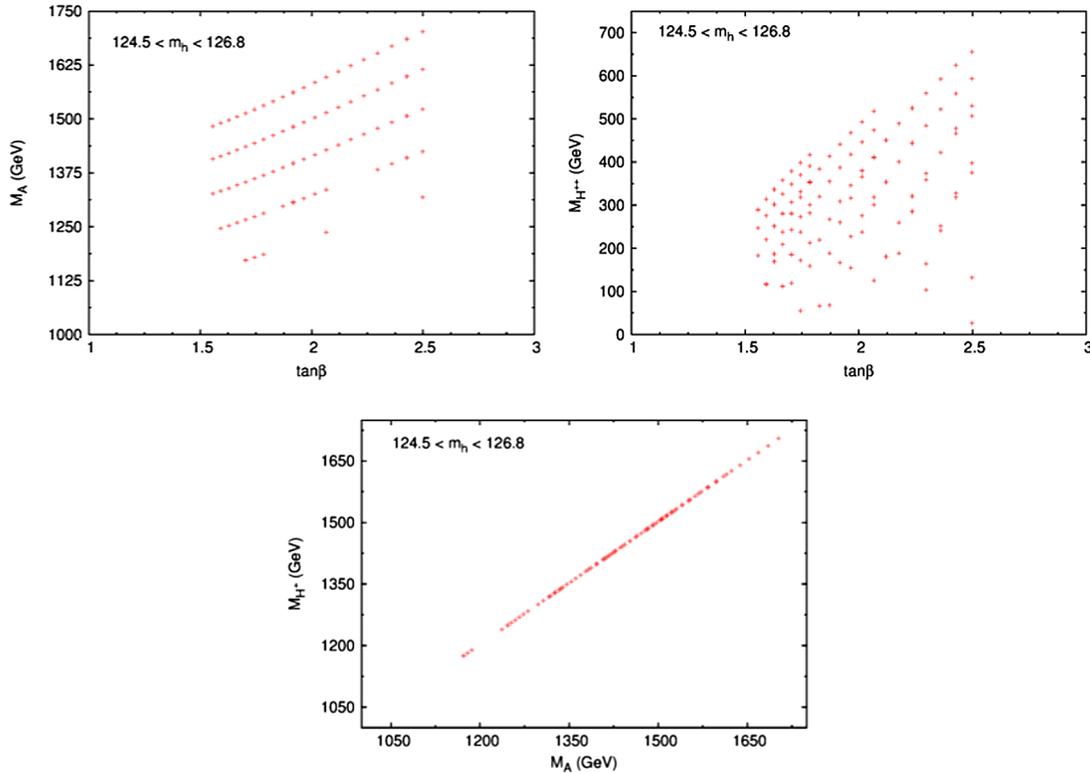
FIG. 5 (color online). *CP*-odd, singly and doubly charged scalars masses. (top left) The *CP*-odd scalar mass must lie between 1125 and 1700 GeV for $1.5 < \tan\beta < 2.5$. (top right) For $\tan\beta$ in the same range, the doubly charged Higgs mass lies between 25 and 650 GeV. (bottom) As is expected in a decoupled Higgs sector, the singly charged scalar is almost perfectly degenerate with the *CP*-odd one as predicted by Eq. (28).

On regarding the other scalars of the model, we present the behavior of the lightest singly and doubly charged scalars and the *CP*-odd one. Figure 5 tells us that these scalars, with the exception of the doubly charged one, lie at the TeV scale. As a nice result, observe that the mass of the doubly charged scalar may be as low as 250 GeV, which can be probed in the LHC. Such a probe can be made analyzing its decay, given by last term of Eq. (6), which contributes with a coupling proportional to $\overline{e_{i_R}} H^{--} (e^c)_{i_L}$ + H.c. (*i* being the family index), therefore mainly decaying into two same-sign taus as in Ref. [15].[4]

V. CONCLUSIONS

In this work we have developed the scalar sector of the recently proposed RSUSY331 model, concentrating on the mass of the Higgs boson, which we enforced to almost match the SM one. We have shown that, similarly to the MSSM case, a Higgs with a mass of 125 GeV requires robust radiative corrections. However, differently from the MSSM case, the radiative corrections require stops with the mass at the TeV scale, but with small mixing. Moreover, the soft breaking mass terms are free to be as

light as hundreds of GeV. This is nice because we can avoid substantial fine-tuning, differently from the MSSM.

The model predicts that the lightest doubly charged scalar has a mass at the electroweak scale which can be probed at the LHC. The remaining scalars of the model must have masses at the TeV scale. Most importantly, our framework naturally avoid tachyons or unwanted massless charged scalars in its spectrum compared to Ref. [11]. Another interesting phenomenological aspect of this model not discussed here is the decay of the exotic quarks into SM quarks and vector bileptons (see Ref. [19]). Finally, this SUSY version of the minimal 331 has a reduced scalar sector suitable to perform further phenomenological analysis and the lightest supersymmetric particle can be the dark matter candidate, not present in the nonsupersymmetric version, which is an issue to be investigated somewhere else.

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[4]For more references on bilpetons and their possible couplings, see Ref. [18].

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