Topological black holes for Einstein-Gauss-Bonnet gravity with a nonminimal scalar field

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We consider the Einstein-Gauss-Bonnet gravity with a negative cosmological constant together with a source given by a scalar field nonminimally coupled in arbitrary dimension D. For a certain election of the cosmological and Gauss-Bonnet coupling constants, we derive two classes of AdS black hole solutions whose horizon is planar. The first family of black holes obtained for a particular value of the nonminimal coupling parameter only depends on a constant M, and the scalar field vanishes as $M = 0$. The second class of solutions corresponds to a two-parametric (with constants M and A) black hole stealth configuration, which is a nontrivial scalar field with a black hole metric such that both sides (gravity and matter parts) of the Einstein equations vanish. In this case, in the vanishing M, the solution reduces to a stealth scalar field on the pure AdS metric. We note that the existence of these two classes of solutions is indicative of the particular choice of the coupling constants, and they cannot be promoted to spherical or hyperboloid black hole solutions in a standard fashion. In the last part, we add to the original action some exact $(D-1)$ forms coupled to the scalar field. The direct benefit of introducing such extra fields is to obtain black hole solutions with a planar horizon for an arbitrary value of the nonminimal coupling parameter.

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I. INTRODUCTION

The anti-de Sitter/conformal field theory (AdS/CFT) correspondence is a power tool that permits us to analyze strongly coupled systems by mapping them into some higher-dimensional gravity theories and establishing a sort of dictionary between both theories [1]. Recently, these ideas have been extended to nonrelativistic physics in order to describe certain condensed matter systems (e.g. see [2,3] for recent reviews) or in order to gain a better understanding of some unconventional superconductors [4–6]. In this last case, the minimal ingredients in the gravity side are given by the Einstein-Hilbert action with a negative cosmological constant, together with a charged self-interacting (complex) scalar field with the Maxwell term. Moreover, in order to reproduce the superconductor phase diagram, the system must admit black holes with scalar hair at low temperatures, and this hair must disappear at high temperatures. However, this problem of finding black hole solutions with a scalar field is rendered difficult by the various no-hair theorems existing in the current literature; see e.g. [7]. Nevertheless, such no-go theorems can be avoided by considering scalar fields nonminimally coupled to gravity [8,9]. By nonminimally coupled scalar fields, we mean a scalar field Φ with its usual kinetic term, together with a term coupled to the Ricci scalar R as $\xi R\Phi^2$ where ξ is the nonminimal coupling parameter. We precisely consider this kind of matter source in order to escape the traditional no-hair theorems.

For the gravity Lagrangian, we are concerned with the Einstein-Gauss-Bonnet action with a negative cosmological constant. This choice is motivated by the recent interest in holographic superconductors in Einstein-Gauss-Bonnet gravity. Indeed, holographic superconductors in such gravity theory have been studied intensively with the purpose of analyzing the effects of the Gauss-Bonnet coupling constant on the critical temperature and on the condensate; see e.g. [10–12].

More specifically, we consider a matter action given by a particular Einstein-Gauss-Bonnet gravity action, together with a self-interacting scalar field nonminimally coupled to gravity, and we look for black holes; for good reviews on Einstein-Gauss-Bonnet black holes, see e.g. [13,14]. For this model, we derive two classes of black hole solutions with a planar base manifold and only for particular values of the nonminimal coupling parameter. Note that the first examples of topological black holes in general relativity (GR) were discussed in [15]. In our case, for $\xi = (D-2)/(4D)$ and $\xi = (D-1)/(4D+4)$, we obtain AdS black holes whose metrics resemble the Schwarzschild-AdS-Tangherlini spacetime. In both cases, and in contrast with the Bocharova-Bronnikov-Melnikov-Bekenstein solution [8,9], the scalar field does not diverge at the horizon. This is due to the presence of the negative cosmological constant as it occurs for the known black hole solutions with scalar fields in four dimensions [16–18]. For the first family of solutions, the scalar field depends on the mass constant M and vanishes identically as $M = 0$, yielding to the purely AdS solution without a source. The second class of solutions is, to our knowledge, the first example of a higher-dimensional black hole stealth configuration. This means a nontrivial scalar field together with a black hole metric such that both sides

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of the Einstein equations (gravity and matter parts) vanish. Up to now, the only known black hole stealth configuration was derived in three dimensions for a conformal scalar field [19] on the Banados-Teitelboim-Zanelli black hole [20]. Finally, in the last part, we add to our original action $(D-2)$ dynamical fields which are exact $(D-1)$ forms coupled with the scalar field in order to relax the restriction on the nonminimal coupling parameter. In this case, we establish the existence of black hole solutions for arbitrary ξ reducing to the pure scalar field solutions as $\xi = (D-2)/(4D)$ or $\xi = (D-1)/(4D+4)$. There also exists a value of the nonminimal parameter $\xi = 1/8$ giving rise to a pure axionic solution, which is a solution where the contribution of the scalar fields disappears.

The paper is organized as follows. In the next section, we present the model and the associated field equations and derive two classes of solutions. In Sec. III, we add to our starting action some axionic fields coupled to the scalar field and obtain black hole solutions, generalizing in some sense those obtained in the previous section. Finally, the last section is devoted to the conclusions and future works.

II. TOPOLOGICAL BLACK HOLES FOR EINSTEIN-GAUSS-BONNET GRAVITY WITH A SCALAR FIELD NONMINIMALLY COUPLED

We consider the following action in arbitrary D dimensions with $D \geq 5$,

$$S = \int d^D x\sqrt{-g}\left[\frac{1}{2}(R - 2\Lambda + \alpha \mathcal{L}_{\rm GB})\right] - \int d^D x\sqrt{-g}\left[\frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi + \frac{\xi}{2}R\Phi^2 + U(\Phi)\right], \quad (1)$$

where $\mathcal{L}_{\rm GB}$ corresponds to the Gauss-Bonnet Lagrangian

$$\mathcal{L}_{\rm GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}.$$

Here, we have normalized the Newton coupling constant G as $8\pi G = 1$ and set the AdS radius l to unity, $l = 1$. The gravity part of (1) corresponds to the Einstein-Gauss-Bonnet action with a cosmological constant Λ, while the matter source is given by a self-interacting scalar field Φ nonminimally coupled to the scalar curvature R through the nonminimal coupling parameter ξ. The potential $U(\Phi)$ is given by a mass term

$$U(\Phi) = \frac{8\xi D(D-1)}{(1-4\xi)^2}(\xi - \xi_D)(\xi - \xi_{D+1})\Phi^2, \quad (2)$$

where ξ_D denotes the conformal coupling in D dimensions,

$$\xi_D = \frac{D-2}{4(D-1)}. \quad (3)$$

The choice of such a potential will be justified in the discussion. We also note that for the conformal couplings in D and $D+1$ dimensions, the potential vanishes identically.

The field equations obtained by varying the action with respect to the metric and the scalar field read

$$G_{\mu\nu} + \Lambda g_{\mu\nu} + \alpha K_{\mu\nu} = T_{\mu\nu}, \quad (4a)$$

$$\Box\Phi = \xi R\Phi + \frac{dU}{d\Phi}, \quad (4b)$$

where the expression of the Gauss-Bonnet tensor $K_{\mu\nu}$ is

$$K_{\mu\nu} = 2(RR_{\mu\nu} - 2R_{\mu\rho}R^{\rho}{}_{\nu} - 2R^{\rho\sigma}R_{\mu\rho\nu\sigma} + R_{\mu}{}^{\rho\sigma\gamma}R_{\nu\rho\sigma\gamma}) - \frac{1}{2}g_{\mu\nu}\mathcal{L}_{\rm GB}, \quad (5)$$

and the stress tensor associated with the variation of the scalar field is given by

$$T_{\mu\nu} = \partial_\mu\Phi\partial_\nu\Phi - g_{\mu\nu}\left(\frac{1}{2}\partial_\sigma\Phi\partial^\sigma\Phi + U(\Phi)\right) + \xi(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu + G_{\mu\nu})\Phi^2. \quad (6)$$

In what follows, we fix the value of the cosmological constant and the Gauss-Bonnet coupling constant α in terms of the dimension D as

$$\Lambda = -\frac{(D-1)(D-2)}{4}, \qquad \alpha = \frac{1}{2(D-3)(D-4)}. \quad (7)$$

As a consequence of this choice, the gravity part of the action (1) becomes proportional to

$$\int d^D x\sqrt{-g}\left(R + \frac{(D-1)(D-2)}{2} + \frac{\mathcal{L}_{\rm GB}}{2(D-3)(D-4)}\right). \quad (8)$$

In five dimensions, this action corresponds to a Chern-Simons action, which is a particular case of the Lovelock action. This latter action can be viewed as a generalization of the Einstein gravity in arbitrary dimensions, yielding at most second-order field equations for the metric. We will come to this point in the discussion when commenting that the solutions derived here in the Einstein-Gauss-Bonnet case can be extended to a particular class of higher-order Lovelock gravity [21].

A. Topological black hole solutions

As shown now for the particular choice of the coupling constants (7), or equivalently for a gravity action given by (8), we derive two classes of topological black hole solutions of Eqs. (4). In both cases, the metric solutions which have a planar base manifold resemble the topological Schwarzschild-AdS-Tangherlini spacetime.

For a value of the nonminimal coupling parameter given by

$$\xi = \xi_D^{\rm b.h} := \frac{D-2}{4D}, \quad (9)$$

which implies that the potential (2) becomes

$$U(\Phi) = \frac{(D-2)^2}{32}\Phi^2, \tag{10}$$

a solution of the field equations (4) in this case is given by

$$ds^2 = -\Big(r^2 - \frac{M}{r^{\frac{D-6}{2}}}\Big)dt^2 + \frac{dr^2}{(r^2 - \frac{M}{r^{\frac{D-6}{2}}})} + r^2 d\vec{x}^2_{D-2},$$
$$\Phi(r) = 2\sqrt{\frac{DM}{D-2}}r^{\frac{2-D}{4}}, \tag{11}$$

where $\vec{x}$ denotes a $(D-2)$-dimensional vector. Various comments can be made concerning this solution. First, the scalar field is real, provided that the constant M is positive, and it is well defined at the horizon while diverging at the singularity $r = 0$. The solution resembles the topological Schwarzschild-AdS solution with a planar horizon. We also stress that, at the vanishing mass limit $M = 0$, the scalar field vanishes and the metric is nothing but the AdS metric written in Poincaré coordinates satisfying the pure gravity equations $G_{\mu\nu} + \Lambda g_{\mu\nu} + \alpha K_{\mu\nu} = 0$ at the point (7). It is also interesting to point out that the spherical or hyperboloid versions of the metric do not accommodate such a source unless the constant $M = 0$. However, in this case, the scalar field vanishes identically and the spacetime geometry is nothing but the AdS metric trivially solving the gravity equation $G_{\mu\nu} + \Lambda g_{\mu\nu} + \alpha K_{\mu\nu} = 0$ with a cosmological constant and Gauss-Bonnet coupling given by (7).

B. Topological black hole stealth solutions

Interestingly enough, there exists another value of the nonminimal parameter that yields an interesting solution, namely, a stealth configuration. By stealth configuration, we mean a nontrivial solution (that is a solution with a nonconstant and nonvanishing scalar field) of the stealth equations

$$G_{\mu\nu} + \Lambda g_{\mu\nu} + \alpha K_{\mu\nu} = 0 = T_{\mu\nu}, \tag{12}$$

where both sides (the gravity and the matter parts) vanish identically. Indeed, for

$$\xi = \xi_D^{\text{stealth}} := \frac{D-1}{4(D+1)}, \tag{13}$$

and hence for a potential (2),

$$U(\Phi) = \frac{(D-1)^2(D-3)}{32(D+1)}\Phi^2, \tag{14}$$

a solution of the stealth equations (12) is given by

$$ds^2 = -\Big(r^2 - \frac{M}{r^{\frac{D-5}{2}}}\Big)dt^2 + \frac{dr^2}{(r^2 - \frac{M}{r^{\frac{D-5}{2}}})} + r^2 d\vec{x}^2_{D-2},$$
$$\Phi(r) = Ar^{\frac{1-D}{4}}, \tag{15}$$

where A is an arbitrary constant. First of all, and contrary to the previous solution, in the zero mass limit $M = 0$, the scalar field does not vanish, and the solution reduces to a stealth configuration on pure AdS spacetime [22]. We also note that the metric solution (15) corresponds to the planar version ($\gamma = 0$) of the solution obtained in Refs. [23,24] for the gravity action given by (8) [25]. In other words, we have derived a black hole stealth configuration for a self-interacting scalar field with the potential (14) nonminimally coupled with the parameter ξ given by (13) on a spacetime geometry which is the planar solution of the particular Einstein-Gauss-Bonnet gravity action [23,24]. The occurrence of such a solution can be explained easily. In fact, it is not difficult to establish that a self-interacting scalar field with potential (2) given by

$$\Phi(r) = Ar^{\frac{2\xi}{4\xi-1}} \tag{16}$$

has a vanishing energy-momentum tensor $T_{\mu\nu} = 0$ on the following ξ-dependent geometry,

$$ds^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2 d\vec{x}^2_{D-2},$$
$$F(r) = \Big(r^2 - \frac{M}{r^{\frac{4(D-2)\xi-(D-3)}{4\xi-1}}}\Big). \tag{17}$$

On the other hand, as shown in [23,24], the metric function $F(r) = r^2 - \frac{M}{r^{\frac{D-5}{2}}}$ satisfies the gravity equation $G_{\mu\nu} + \Lambda g_{\mu\nu} + \alpha K_{\mu\nu} = 0$ for a planar base manifold at the point (7). Hence, requiring that both sides of the stealth equations (12) must vanish, this will fix the value of the parameter ξ to be (13). As a final remark concerning the black hole stealth, it is interesting to note that the stealth metric (17) will correspond to the Schwarzschild-AdS-Tangherlini metric with a planar base manifold only for $\xi = 0$, but this case is of little interest since for a vanishing coupling parameter, the scalar field becomes constant (16).

Hence, we have obtained two classes of solutions for the Einstein-Gauss-Bonnet equations at the point (7) with a matter source composed of a self-interacting nonminimally coupled scalar field. These solutions have been derived for particular values of the nonminimal coupling parameter $\xi = \xi_D^{\text{b.h}}$ or $\xi = \xi_D^{\text{stealth}}$, and in both cases $\xi < 1/4$. It is also intriguing to note that these couplings, as well as the metric function solutions, are related through the dimensions as

$$\xi_{D+1}^{\text{b.h}} = \xi_D^{\text{stealth}}, \qquad F_{D+1}^{\text{b.h}}(r) = F_D^{\text{stealth}}(r).$$

As a final comment, defining $\psi(r) = -\frac{F(r)}{r^2}$ where F is the structural function appearing in the Ansatz metric

$$ds^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2 d\vec{x}^2_{D-2},$$

the integration of the field equations may have a nice form. First, the combination $E_t^t - E_r^r = 0$ where $E_{\mu\nu} = G_{\mu\nu} + \Lambda g_{\mu\nu} + \alpha K_{\mu\nu} - T_{\mu\nu}$ fixes the form of the scalar field as $\Phi(r) = (ar+b)^{\frac{2\xi}{4\xi-1}}$, where for simplicity we only consider the case $b = 0$. For the topological black hole solution, the equation $E_t^t = 0$ yields the following first integral,

$$\frac{d}{dr}\left[r^{D-1}(\psi+1)^2-\frac{D-2}{4D}a^{\frac{2-D}{2}}(\psi+1)r^{\frac{D}{2}}\right]=0. \tag{18}$$

This means that the expression in brackets must be a constant m; however, the remaining independent field equation $E_i^i=0$ imposes that $m=0$, and the solution reduces to the one found previously. Concerning the black hole stealth equation, something amusing occurs where both sides (gravity and matter parts) are identical and the stealth equation reduces to

$$-\frac{1}{4}(2r\psi'+\psi(D-1))(\psi+1)(D-2)=0.$$

III. TURNING ON THE NONMINIMAL PARAMETER WITH EXACT p FORMS

For both classes of solutions derived previously, the value of the nonminimal coupling parameter is unique and fixed in terms of the dimension D; see (9) and (13). This feature is not a novelty, and it also occurs in Einstein gravity (eventually with a cosmological constant) with a scalar field nonminimally coupled to gravity. Indeed, in this case, the only known black hole solutions are those obtained in four dimensions for the conformal coupling parameter $\xi=1/6$ and whose horizon topology is either spherical or hyperbolic; see [8,9,16–18]. Recently, it has been shown that the inclusion of multiple exact p forms homogeneously distributed permits the construction of black holes with a planar horizon [28,29] without any restrictions on the dimension or on the value of the nonminimal parameter [30]. Indeed, on one side, an appropriate coupling between the scalar field and the exact p forms permits us to relax the condition on the nonminimal parameter as well as the dimension. On the other side, the p forms being homogeneously distributed causes the horizon topology to be planar. Since our working hypothesis is concerned with black hole solutions with a planar base manifold, we propose to appropriately introduce some exact p forms to obtain topological black hole solutions with an arbitrary nonminimal coupling parameter. In order to achieve this task, we consider the following action in arbitrary D dimensions,

$$S-\int d^Dx\sqrt{-g}\left[\frac{\epsilon(\Phi)}{2(D-1)!}\sum_{i=1}^{D-2}\mathcal{H}^{(i)}_{\alpha_1\ldots\alpha_{D-1}}\mathcal{H}^{(i)\alpha_1\ldots\alpha_{D-1}}\right].$$

Here S denotes our original action (1) with the coupling constants chosen as in (7), to which we have added $(D-2)$ fields which are exact $(D-1)$ forms $\mathcal{H}^{(i)}$. The coupling function between the scalar field and the $(D-1)$ forms denoted by $\epsilon(\Phi)$ depends on the scalar field Φ as

$$\epsilon(\Phi)=\sigma\Phi^{\frac{1-8\xi}{\xi}} \tag{19}$$

where σ is a coupling constant. The field equations obtained by varying the action with the different dynamical fields $g_{\mu\nu}$, Φ and $\mathcal{H}^{(i)}$ read

$$G_{\mu\nu}+\Lambda g_{\mu\nu}+\alpha K_{\mu\nu}=T_{\mu\nu}+T_{\mu\nu}^{\text{extra}},\qquad \nabla_\alpha(\epsilon\mathcal{H}^{(i)\alpha\alpha_1\ldots\alpha_{D-2}})=0, \tag{20a}$$

$$\Box\Phi=\xi R\Phi+\frac{dU}{d\Phi}+\frac{1}{2}\frac{d\epsilon}{d\Phi}\left[\sum_{i=1}^{D-2}\frac{1}{(D-1)!}\mathcal{H}^{(i)}_{\alpha_1\ldots\alpha_{D-1}}\mathcal{H}^{(i)\alpha_1\ldots\alpha_{D-1}}\right]=0, \tag{20b}$$

where the extra piece in the energy-momentum tensor reads

$$T_{\mu\nu}^{\text{extra}}=\epsilon\sum_{i=1}^{D-2}\left[\frac{1}{(D-2)!}\mathcal{H}^{(i)}_{\mu\alpha_2\ldots\alpha_{D-1}}\mathcal{H}_{\nu}^{(i)\alpha_2\ldots\alpha_{D-1}}-\frac{g_{\mu\nu}}{2(D-1)!}\mathcal{H}^{(i)}_{\alpha_1\ldots\alpha_{D-1}}\mathcal{H}^{(i)\alpha_1\ldots\alpha_{D-1}}\right]. \tag{21}$$

Searching for a purely electrically homogeneous Ansatz for the $(D-1)$ forms, we get

$$\mathcal{H}^{(i)}_{trx_1\ldots x_{i-1}x_{i+1}\ldots x_{D-2}}(r)dtdr\ldots dx^{i-1}dx^{i+1}\ldots dx^{D-2},$$

where the wedge product is understood. A solution of the field equations (20) with a purely electric Ansatz is given by

$$ds^2=-\left(r^2-\frac{M}{r^{\frac{2(6\xi-1)}{1-4\xi}}}\right)dt^2+\frac{dr^2}{\left(r^2-\frac{M}{r^{\frac{2(6\xi-1)}{1-4\xi}}}\right)}+r^2d\vec{x}^2_{D-2},$$

$$\Phi(r)=\sqrt{\frac{M(8\xi-1)(D-2)}{2\xi[2\xi(3D-4)-(D-2)]}}r^{\frac{2\xi}{4\xi-1}}, \tag{22}$$

$$\mathcal{H}^{(i)}_{trx_1\ldots x_{i-1}x_{i+1}\ldots x_{D-2}}=\frac{p}{\epsilon(\Phi)}r^{D-4},$$

where the constant p is given by

$$p=\frac{4M^{\frac{1-4\xi}{4\xi}}}{(4\xi-1)}\left[\frac{2\xi}{(8\xi-1)(D-2)}\right]^{\frac{8\xi-1}{4\xi}}\times[2\xi(3D-4)-(D-2)]^{\frac{6\xi-1}{4\xi}}\times\sqrt{-\sigma D(D+1)\xi(\xi-\xi_D^{\text{b.h}})(\xi-\xi_D^{\text{stealth}})},$$

and where the constants $\xi=\xi_D^{\text{b.h}}$ and $\xi=\xi_D^{\text{stealth}}$ are the particular values given by (9)–(13).

As expected, for $\xi=\xi_D^{\text{b.h}}$ or $\xi=\xi_D^{\text{stealth}}$, the constant $p=0$, and the resulting solutions are those derived previously, with a source only given by a scalar field. Another interesting value is $\xi = 1/8$ since in this case the scalar field vanishes and the coupling function ϵ becomes constant (19). As a consequence, for $\xi = 1/8$, we end with a pure axionic solution (which is a solution without a scalar field),

$$G_{\mu\nu} + \Lambda g_{\mu\nu} + \alpha K_{\mu\nu} = T_{\mu\nu}^{\text{extra}},$$

given by

$$ds^2 = -(r^2 - Mr)dt^2 + \frac{dr^2}{(r^2 - Mr)} + r^2 d\vec{x}_{D-2}^2,$$
$$\mathcal{H}^{(i)}_{\text{tr}x_1\ldots x_{i-1}x_{i+1}\ldots x_{D-2}} = -\sqrt{\frac{D-3}{2\sigma}} M r^{D-4}.$$

Note that this last solution is obtained from the generic solution (22), taking the well-defined limit $\xi \longrightarrow 1/8$.

Finally, it is interesting to mention that solutions of the field equations (20) can be obtained without imposing the values of the cosmological and Gauss-Bonnet coupling constants as given by (7) but rather by considering the following relation between them:

$$\alpha = \frac{2\Lambda + (D-1)(D-2)}{(D-1)(D-2)(D-3)(D-4)}. \tag{23}$$

Of course, the restrictions (7) are a particular case of this last constraint. In fact, the relation (23) is obtained by requiring that the pure AdS metric solves the gravity equations without a source. For a value of the parameter ξ given by the conformal one in $(D+1)$ dimensions, $\xi = \xi_{D+1}$, which in turn implies that the potential vanishes (2), a solution of the field equations (20) can be obtained for a coupling ϵ given by

$$\epsilon(\Phi) = \frac{\sigma}{\Phi^{\frac{4(D-2)}{D-1}}}.$$

In this case, the metric function is the Schwarzschild-AdS-Tangherlini spacetime

$$ds^2 = -\left(r^2 - \frac{M}{r^{D-3}}\right)dt^2 + \frac{dr^2}{(r^2 - \frac{M}{r^{D-3}})} + r^2 d\vec{x}_{D-2}^2,$$
$$\Phi(r) = \sqrt{\frac{8MD(D-2)(2\Lambda + (D-1)(D-2))}{(D^2 - 3D + 4)(D-1)^2}} r^{\frac{1-D}{2}},$$
$$\mathcal{H}^{(i)}_{\text{tr}x_1\ldots x_{i-1}x_{i+1}\ldots x_{D-2}} = \frac{p}{\epsilon(\Phi)} r^{D-4}, \tag{24}$$

where the constant p is given by

$$p = \frac{\sqrt{-2M\sigma}}{4(D-2)}(D-1)^{\frac{5D-9}{2(D-1)}} \times \left(\frac{(D^2 - 3D + 4)}{8MD(D-2)(2\Lambda + (D-1)(D-2))}\right)^{\frac{D-3}{2(D-1)}}.$$

We would like to stress that this solution is valid even in the vanishing cosmological constant, but on the other hand, the GR limit $\Lambda = -(D-1)(D-2)/2$ (23) is not well defined because the constant p will blow up. This clearly emphasizes the importance of the higher-order curvature terms present in the Einstein-Hilbert-Gauss-Bonnet Lagrangian. As before, defining $\psi(r) = -\frac{F(r)}{r^2}$ where F is the structural function, we get the same expression for the scalar field $\Phi(r) = (ar)^{\frac{2\xi}{4\xi-1}}$, where ξ is the conformal coupling in $(D+1)$ dimensions as well as the solution for the axionic field as $\mathcal{H}^{(i)} = p\epsilon^{-1} r^{D-4}$. The equation $E_t^t = 0$ yields the following first integral,

$$\frac{d}{dr}\left[-\frac{\tilde{\Lambda}}{2} r^{D-1}(1+\psi)^2 + \bar{\Lambda} r^{D-1}(1+\psi) - \sigma(1+\psi) - \beta r^{1-D}\right] = 0, \tag{25}$$

where we have defined $\tilde{\Lambda} = 2\Lambda + (D-1)(D-2)$, $\bar{\Lambda} = [2\tilde{\Lambda} - (D-1)(D-2)]/2$, σ is a constant that depends only on the integration constant a, while β depends on a and p. The expression in brackets must be a constant m, and in contrast to the pure scalar field case, this constant m can be nonzero in order to cancel the contribution proportional to $\bar{\Lambda}$. Note that this latter part vanishes at the point (7) but not at the point that we are considering now (23). The last remaining independent equation $E_i^i = 0$ fixes the relation between the constants and yields the solution obtained in (24).

IV. CONCLUSIONS AND FURTHER WORKS

Here, the gravity action we have considered is a particular combination of the Einstein-Hilbert action with a negative cosmological constant, together with the Gauss-Bonnet density. In five dimensions, this action corresponds to a Chern-Simons action which is a particular case of the so-called Lovelock action. This latter action can be viewed as a generalization of Einstein gravity in arbitrary dimensions, yielding at most second-order field equations for the metric. The resulting theory is described by a D form constructed with the vielbein e^a, the spin connection ω^{ab}, and their exterior derivatives without using the Hodge dual. The Lovelock action is a polynomial of degree $[D/2]$ (where $[x]$ denotes the integer part of x) in the curvature two-form, $R^{ab} = d\omega^{ab} + \omega^a{}_c \wedge \omega^{cb}$ as

$$\int \sum_{p=0}^{[D/2]} \alpha_p L^{(p)},$$
$$L^{(p)} = \epsilon_{a_1\ldots a_d} R^{a_1 a_2} \ldots R^{a_{2p-1}a_{2p}} e^{a_{2p+1}} \ldots e^{a_d}, \tag{26}$$

where the α_p are arbitrary dimensionful coupling constants and where wedge products between forms are understood. Here $L^{(0)}$ and $L^{(1)}$ are, respectively, the well-known cosmological term and the Einstein-Hilbert Lagrangian. As shown in Ref. [23], requiring the Lovelock action to have a unique cosmological constant fixes the α_p, yielding a series of actions indexed by an integer k given by

$$I_k = -\int \sum_{p=0}^{k} \frac{C_p^k}{(D-2p)} L^{(p)}, \quad 1 \le k \le \left[\frac{D-1}{2}\right], \tag{27}$$

where C_p^k corresponds to the combinatorial factors. The gravity action we have considered here (8) is nothing but the action I_2 given by the expression (27). We believe that the class of solutions derived here for I_2 can be generalized for an arbitrary gravity action I_k with $k \ge 2$ [21]. For example, the stealth black hole solution obtained for I_2 can easily be extended for an arbitrary action I_k by adjusting the value of the nonminimal coupling parameter appearing in the stealth metric solution (17) in order for the resulting metric to match with the pure gravity solution of I_k [23,24]. Moreover, to our knowledge, there do not exist black hole solutions with a planar base manifold for standard general relativity (with or without a cosmological constant) with a source given by a nonminimally coupled scalar field [31]. This reinforces our conviction that the existence of the solutions derived for a gravity action given by I_2 is strongly indicative of the presence of the higher-order curvature terms [21].

We now turn to the choice of the mass term potential (2) in our starting action. First of all, we may note that this kind of potential has been considered in the dual description of unconventional superconductor [2,4–6]. Moreover, in our case, because of the presence of the nonminimal coupling term $\xi R\Phi^2$ in the action, in the case of constant scalar curvature solutions $R = -D(D-1)$, one may define an effective square mass

$$m_{\text{eff}}^2 = -\xi D(D-1)\left[1 - \frac{16}{(1-4\xi)^2}(\xi - \xi_D)(\xi - \xi_{D+1})\right].$$

In the solutions obtained here, the only ones of constant scalar curvature (apart from the trivial situation of taking $M = 0$) are those with the p-form fields (22) for $\xi = D/(4D+4)$ and $\xi = (D-1)/(4D)$. It is interesting to note that the square effective mass m_{eff}^2 precisely saturates the Breitenlohner-Freedman bound for $\xi = (D-1)/(4D)$. To close the chapter concerning the potential, we stress that is a particular case of potentials allowing the existence of a self-interacting scalar field Φ nonminimally coupled to the vanishing stress tensor

$$T_{\mu\nu} := \partial_\mu\Phi\partial_\nu\Phi - g_{\mu\nu}\left(\frac{1}{2}\partial_\sigma\Phi\partial^\sigma\Phi + U(\Phi)\right) + \xi(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu + G_{\mu\nu})\Phi^2 = 0$$

on the AdS background

$$ds^2 = -r^2dt^2 + \frac{dr^2}{r^2} + r^2 d\vec{x}_{D-2}^2.$$

Indeed, as shown in [22], the stealth solution is given by the following configuration,

$$U(\Phi) = \frac{\xi}{(1-4\xi)^2}[2\xi b^2 \Phi^{\frac{1-2\xi}{\xi}} - 8(D-1)(\xi - \xi_D) \times (2\xi b \Phi^{\frac{1}{2\xi}} - D(\xi - \xi_{D+1})\Phi^2)], \tag{28a}$$

$$\Phi(r) = (Ar + b)^{\frac{2\xi}{4\xi-1}}. \tag{28b}$$

Note that this kind of potential also appears when looking for AdS wave solutions for a nonminimally coupled scalar field [32]. The scalar field solutions obtained in this paper, as well as the potential considered, correspond to the $b = 0$ limit of this stealth configuration (28). It has been shown recently that, in the context of standard general relativity, self-interacting scalar fields nonminimally coupled with the potential given by (28a) and extra axionic fields admit black hole solutions [30]. The derivation of these solutions was precisely operated from the stealth configuration (28) through a Kerr-Schild transformation [30]. We believe that there may exist more general black hole solutions than those derived here for this more general class of potentials (28a). To conclude with the stealth origin, we would like to stress that the horizon topology of our solutions is planar, and their extension to spherical or hyperboloid black hole solutions in a standard fashion is not possible. This may be related to the fact that a static stealth scalar field on the AdS background requires the base space to be flat [22] for dimensions $D \ge 4$.

An interesting task to realize will be the study of the thermodynamics properties of the solutions derived here for I_2 as well as those for general I_k, and to compare them with the pure gravity solutions [24].

In Ref. [33], the authors constructed a conformal coupling to arbitrary higher-order Euler densities. It will be interesting to see whether such a matter source can accommodate the kind of solutions derived here.

As a final remark, we note that the coupling with the Maxwell electromagnetic field is an open problem even in the black hole stealth case. Indeed, even if the pure gravity solution with the Maxwell source

$$G_{\mu\nu} + \Lambda g_{\mu\nu} + \alpha K_{\mu\nu} = T_{\mu\nu}^{\text{Maxwell}}$$

is known [23,24], it seems that this solution cannot be promoted to a black hole stealth configuration with a nonminimal scalar field as was possible in the neutral case.

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