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Reconciliation of modified Newtonian dynamics and dark matter theory

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We show that modified Newtonian dynamics (MOND) is equivalent to assuming an isothermal dark matter density profile, with its density related to the enclosed total baryonic mass. This density profile can be deduced by physical laws if a dark matter core exists and if the baryonic component is spherically symmetric, isotropic, and isothermal. All the usual predictions of MOND, as well as the universal constant a_0, can be derived in this model. Since the effects of baryonic matter are larger in galaxies than in galaxy clusters, this result may explain why MOND appears to work well for galaxies but poorly for clusters. As a consequence of the results presented here, MOND can be regarded as a misinterpretation of a particular dark matter density profile.

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I. INTRODUCTION

The dark matter problem is one of the key issues in modern astrophysics, with cold dark matter (CDM) particles being the generally accepted model. The CDM model can provide excellent fits on large-scale structure observations, such as the Lyα spectrum [1,2], the 2dF Galaxy Redshift Survey [3], and the cosmic microwave background (CMB) [4]. However, no CDM particles have been detected directly. Besides, the CDM model also encounters many well-known unresolved issues. For example, the results of numerical N-body simulations based on the CDM theory predict that the density profile of dark matter (the Navarro-Frenk-White profile) should be singular at the center [5], while observations in dwarf galaxies indicate the contrary [6–8]. This problem with CDM is known as the core-cusp problem [9]. Another problem with CDM is that computer simulations predict that there should exist thousands of small dark halos or dwarf galaxies in the Local Group while the observations only reveal less than 100 such galaxies [2,10,11]. This discrepancy is known as the missing satellite problem. Recent studies on the baryonic effects suggest that supernova feedback or the radiation pressure of massive stars may provide a solution to these problems [9,12,13]. Alternatively, the problems can also be solved if the dark matter particles are weakly interacting or warm [2,11,14].

An alternative theory to dark matter uses modified Newtonian dynamics (MOND)—a modification of Newton's second law in the weak acceleration limit [15,16]. It is suggested that a wide range of observational data, including the rotation curves of galaxies and the Tully-Fisher relation, are consistent with MOND's predictions but not those of the CDM model [16,17]. However, recent analyses claim that it would be oversimplified to falsify the CDM paradigm based on such data [18–20]. Moreover, recent data from gravitational lensing and hot gas in clusters challenge the original idea of MOND without any dark matter (classical MOND) [16,21,22]. Thus, Sanders studied 93 X-ray emitting clusters and pointed out that the missing mass still exists in cluster dark matter [23]. Later, studies of gravitational lensing and hot gas in clusters showed that the existence of 2 eV active neutrinos—the current upper limit of the active electron neutrino mass—is still not enough to explain the missing mass in clusters. Some more massive dark matter particles (e.g., sterile neutrinos) are required to account for the missing mass [22,24–26]. In addition, the observational data from the Bullet cluster and the CMB indicate that a large amount of dark matter is needed to explain the lensing result and the CMB spectral shape, respectively [4,27,28]. Another big challenge facing MOND is the observed shape of the matter power spectrum, which does not match the prediction from MOND [28]. Besides these numerous big problems, a long list of fundamental physics difficulties such as violating the conservation of momentum exist in MOND theory [29]. The relativistic version of MOND theory is also not supported by recent gravitational lensing results in clusters [30]. In summary, the observations at small scales may favor the classical MOND theory, but there are many conceptual problems and discrepancies in large-scale observations.

Previously, Kaplinghat and Turner suggested that the MOND theory may be just a misleading coincidence. They have shown that Milgrom's law—i.e., that the gravitational effect of dark matter in galaxies only becomes important where accelerations are less than about 10^{-8} cm s^{-2}—can be explained with a cosmological cold dark matter model [31]. This suggests that the MOND theory may be just another equivalent form of dark matter theory. Later, Dunkel showed that the generalized MOND equation can be derived from Newtonian dynamics for some specified dark matter contribution [32]. Besides, for a suitably chosen interaction between dark matter, baryons, and gravity, the cold dark matter model and MOND appear in different physical regimes of the same theory [33]. The above studies suggest that MOND is probably not a new

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theory, but rather only a coincidence. In fact, most of the apparent successes are related to flat rotation curves in galaxies, which can also be explained with a dark matter model. Following from this idea, we will show in another way that MOND is equivalent to a particular specified type of dark matter density profile. This profile can be derived exactly from existing physical laws. The key equation and the universal constant a_0 suggested by MOND can also be derived from this specified profile. This claim also gives an explanation of why MOND apparently only works well at small scales, and it supports the standard dark matter model in cosmology.

II. THE PREDICTIONS FROM MOND THEORY

The apparent gravitation in MOND is given by

$$g = \sqrt{g_N a_0} \tag{1}$$

when $g_\mathrm{N} \ll a_0$, where $g_\mathrm{N} = GM_\mathrm{B}/r^2$ is Newtonian gravitation without dark matter, M_B is the enclosed baryonic mass, and $a_0 \approx 10^8\ \mathrm{cm\,s^{-2}}$ is a constant [16]. Generally, this simple form gives four important predictions for galaxies and clusters without the need for dark matter. First, the rotational speeds of stars in a galaxy at large radius is given by [16]

$$v^4 = GM_\mathrm{B}a_0. \tag{2}$$

If baryons are mainly concentrated at the central part of a galaxy, then M_B is nearly a constant which gives flat rotation curves [34]. Also, this equation represents the baryonic Tully-Fisher relation $M_\mathrm{B} \propto v^4$. The power-law dependence and the proportionality constant $(Ga_0)^{-1} \approx 75M_\odot\ \mathrm{km^{-4}\,s^4}$ generally agree with the fits from observations, $M_\mathrm{B} = [(47 \pm 6)M_\odot\ \mathrm{km^{-4}\,s^4}]v^4$ [17]. Secondly, there exists a critical surface density $\Sigma_m \approx a_0/\pi G$ such that there should be a large discrepancy between the visible and dynamical masses when the surface density $\Sigma \le \Sigma_m$ [16]. This means that the apparent dark matter content is larger in the low surface brightness galaxies. Moreover, MOND predicts that the rotation curves in low surface brightness galaxies would continuously rise to the final asymptotic value [16]. These predictions have been verified by observations [16,35]. Thirdly, since dark matter does not exist in MOND, the features of rotation curves can be traced back to the features in the baryon mass distribution [36]. This prediction is generally supported by the rotation curves observed in galaxies [16]. Lastly, the dynamical mass in a cluster at large radius predicted by MOND is given by [37]

$$M_\mathrm{dyn} = (Ga_0)^{-1}\left(\frac{kT}{m}\right)^2\left(\frac{d\ln\rho}{d\ln r}\right)^2, \tag{3}$$

where T, m are the mean temperature and mass of a hot gas particle, respectively, and ρ is the density profile of the hot gas. As no dark matter is present in clusters, MOND predicts $M_\mathrm{B} = M_\mathrm{dyn}$. However, the observed hot gas mass does not match the predicted dynamical mass, even when active neutrinos are taken into account [22,23,25]. Moreover, since $d\ln\rho/d\ln r$ is nearly a constant at large r [38], we have $M_\mathrm{dyn} \propto T^2$. Recent observations from 118 clusters indicate that $M_\mathrm{dyn} \propto T^{1.57\pm0.06}$ [39], which shows a large discrepancy with MOND's prediction.

III. EQUIVALENT DARK MATTER DENSITY PROFILE OF MOND

All the above predictions can be deduced from Eq. (1)—the only key equation in MOND. In the following, we will show that the above key equation in MOND and the universal constant a_0 can indeed be deduced by existing physical laws and some special properties of the dark matter distribution.

We assume that the baryonic component in a galaxy is spherically symmetric and isotropic. Since the mean free path of the baryonic matter is small ($\lambda \sim 0.001$ pc), the collisions among baryonic matter are vigorous. If the interaction among baryons is larger than the gravitational interaction between the baryons and dark matter, the baryonic distribution would become isothermal and provide a feedback to the dark matter distribution. This can be justified by observations in many galaxies [40]. The effect of gravity by the baryonic component can be analyzed by using the steady-state Jeans equation [40],

$$\frac{d(\rho_\mathrm{B}\sigma^2)}{dr} = -\rho_\mathrm{B}\frac{d\psi}{dr}, \tag{4}$$

where ρ_B is the baryonic mass density, σ is the velocity dispersion of baryonic matter, and ψ is the total gravitational potential (which includes the baryonic matter and dark matter). Since the isothermal distribution of baryons corresponds to the constant-velocity dispersion σ, by Eq. (4) we get [40]

$$\sigma^2\frac{d\rho_\mathrm{B}}{d\psi} + \rho_\mathrm{B} = 0. \tag{5}$$

The solution to the above equation is $\psi = \psi_0 - \sigma^2\ln\rho_\mathrm{B}$, where ψ_0 is a constant. Substituting the function ψ into the Poisson equation and assuming the total mass is dominated by dark matter, the dark matter density is given by

$$\rho_\mathrm{D} = -\frac{\sigma^2}{4\pi G}\left[\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\ln\rho_\mathrm{B}}{dr}\right)\right]. \tag{6}$$

Letting $\gamma = -d\ln\rho_\mathrm{B}/d\ln r$, we get

$$\frac{\gamma}{r^2} + \frac{1}{r}\frac{d\gamma}{dr} = \frac{4\pi G\rho_\mathrm{D}}{\sigma^2}. \tag{7}$$

Since the isothermal baryonic component gives $\gamma = 2$, we get

$$\rho_\mathrm{D} = \frac{\sigma^2}{2\pi G r^2}. \tag{8}$$

Nevertheless, observational data in galaxies strongly support the existence of a core in the dark matter density profile [9]. The existence of a small core (size $\sim$ kpc) may be due to the self-interaction between dark matter particles [14] or the baryonic processes such as supernova feedback [9]. Therefore, we may slightly modify Eq. (8) without destroying the isothermal distribution at large radius by introducing a cored isothermal profile,

$$\rho_{\rm D} = \frac{\sigma^2}{2\pi G(r^2 + r_c^2)} = \frac{\rho_c}{1 + (r/r_c)^2}, \tag{9}$$

where ρ_c and r_c are the central density and core radius of the dark matter profile, respectively. When $r \gg r_c$, Eq. (9) will reduce to Eq. (8).

Recent observations in galaxies indicate that the product of the dark matter central density and the core radius is a constant: $\rho_c r_c = 141^{+82}_{-52} M_\odot\ {\rm pc}^{-2} = C$ [41]. By using the profile in Eq. (9), we get

$$\frac{\sigma^2}{2\pi G r_c} \approx C. \tag{10}$$

The total enclosed dark matter mass within r_c is $M_c \approx \sigma^2 r_c/G$. Therefore we have

$$\frac{\sigma^4}{2\pi G^2 M_c} \approx C. \tag{11}$$

Furthermore, since $M_c \sim 0.1 M_{\rm DM}$ [42] and $M_{\rm B} \approx 0.2 M_{\rm DM}$ [4], we have

$$\sigma^4 \sim \pi G^2 C M_{\rm B}. \tag{12}$$

As the MOND effect is important at large radius only, by substituting Eq. (12) into Eq. (8), the resulting dark matter density profile is given by

$$\rho_{\rm D} \approx \frac{1}{4\pi r^2}\sqrt{4\pi C M_{\rm B}} = \frac{1}{4\pi r^2}\sqrt{\frac{M_{\rm B} a_0}{G}}, \tag{13}$$

where we assume $a_0 = 4\pi C G \sim 10^{-8}\ {\rm cm\,s^{-2}}$. Since the baryonic mass $M_{\rm B}$ is nearly a constant at large radius, the total mass for dark matter is

$$M_{\rm DM} = 4\pi \int_0^r \rho_{\rm D} r^2 dr \approx \sqrt{\frac{M_{\rm B} a_0}{G}} r. \tag{14}$$

If the apparent gravity g in MOND is indeed a real gravitational effect from dark matter, by using the above equation we get

$$g = \frac{G M_{\rm DM}}{r^2} = \sqrt{\frac{G M_{\rm B} a_0}{r^2}} = \sqrt{g_{\rm N} a_0}, \tag{15}$$

which is the same key equation in MOND theory. The corresponding constant a_0 surprisingly matches the universal constant suggested in MOND: $a_0 \approx (1.3 \pm 0.3) \times 10^{-8}\ {\rm cm\,s^{-2}}$ [17]. In other words, our result suggests that the basic assumption in MOND theory [Eq. (1)] is equivalent to the specified dark matter profile in Eq. (13). Therefore, most of the predictions of MOND theory can also be obtained by our specified dark matter profile. For example, the baryonic Tully-Fisher relation can be obtained in Eq. (12). Moreover, since $M_{\rm DM}$ varies with $M_{\rm B}$ [see Eq. (14)], the rotational speed v also varies with $M_{\rm B}$. Therefore, a tiny variation in the baryonic mass distribution can be directly reflected in the rotation curve. This result generally matches MOND's third prediction. Therefore, the apparent success of MOND is telling us that the dark matter density distribution is related to the baryonic matter content $M_{\rm B}$ and that the velocity dispersion of dark matter particles is nearly uniform. These properties can be derived from existing physical laws.

IV. DISCUSSION

Traditionally, the dark matter problem has been mainly addressed by the existence of cold dark matter. However, the successful predictions from MOND on the galactic scale may indicate that MOND is correct to a certain extent. Generally, these two theories are highly incompatible. In this article, we have shown that the basic assumption in MOND is equivalent to a particular form of the dark matter density profile. This form can be naturally obtained if the distribution of the baryonic matter is spherically symmetric, isotropic, and isothermal. Also, empirical studies show that $\rho_c r_c$ is a constant for most galaxies. This relation can be derived in some particular models of self-interacting dark matter [43]. By using these two properties, the derived dark matter density profile is equivalent to that in MOND theory, and all predictions from MOND can be obtained. The universal constant a_0 suggested in MOND can also be derived in this model, $a_0 = 4\pi C G \sim 10^{-8}\ {\rm cm\,s^{-2}}$, which gives excellent agreement with MOND's prediction. In fact, the isothermal dark matter density profile in galaxies is well supported by many recent observations [44–46]. Moreover, since the characteristics of cores and the effect of baryonic matter are mainly found in galaxies, this result also gives an explanation of why MOND apparently works well in galaxies only.

In fact, several severe challenges facing MOND—such as the missing mass in clusters, the shapes of the matter power spectrum, and the CMB spectrum—indicate that MOND probably is not a universal law in physics. If MOND indeed represents an isothermal distribution of dark matter that only works in galaxies but not in clusters, it would not be necessary for us to make any changes to Newtonian dynamics or general relativity. Therefore, the apparent success of MOND may be just a coincidence. However, apart from the context of dark matter, MOND also predicts that some strange effects may be observable in the Solar System [47]. For example, around each equinox date, two spots emerge on the Earth where static bodies experience spontaneous acceleration due to the possible violation of Newton's second law [48]. Since these effects are independent of dark matter, MOND will still survive if these effects can really be detected in the future.

To conclude, the reconciliation of MOND and the dark matter model suggests that MOND theory is equivalent to the isothermal dark matter density profile in the dark matter model. Also, the dark matter density profile on the galactic scale is related to the baryonic mass content.

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