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Comment on "Bare Higgs mass at Planck scale"

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Old and new calculations of the Higgs mass quadratic divergence are compared.

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I. INTRODUCTION

The standard model (SM) Higgs-like particle of mass 125 GeV recently discovered at the LHC [1,2] has resulted in a revival of interest in the old Veltman observation [3] that it is possible to arrange the cancellation of the quadratic divergence in the Higgs mass by imposing a certain relation upon the coupling constants of the theory [4–8]. In the notation of Ref. [4], the quadratic divergence at one loop is proportional to

$$Q_1 = \lambda + \frac{1}{8}g'^2 + \frac{3}{8}g^2 - y_t^2. \tag{1}$$

Veltman, in fact, expressed the relation in terms of the particle masses,

$$Q'_1 = 2Q_1 v^2 = m_H^2 + 2m_W^2 + m_Z^2 - 4m_t^2, \tag{2}$$

whereas Ref. [9] opted to use mass ratios. In the notation of Ref. [9],

$$\Delta_1 = Q'_1/m_W^2 = H + 3 + \tan^2\theta_W - 4T \tag{3}$$

where $m_W^2 = \frac{1}{4}g^2v^2$, $m_Z^2 = \frac{1}{4}(g^2 + g'^2)v^2$, $m_H^2 = 2\lambda v^2$, $m_t^2 = \frac{1}{2}y_t^2 v^2$, $H = m_H^2/m_W^2$, $T = m_t^2/m_W^2$, and v is the Higgs vacuum expectation value. Veltman, I believe, thought of the relation as existing for the *physical* masses of the particles, and in his original paper opted to perform the calculation in the broken phase of the theory (although the symmetric-phase calculation is much simpler, as in fact I remarked to him when he showed me a draft of the paper). Requiring $Q'_1 = 0$ predicts $m_H \approx 315$ GeV, clearly at odds with the recent observations.

Now if it really was in terms of physical masses, Eq. (2) would be renormalization-group-invariant. However Eq. (1), expressed as it is in terms of renormalized couplings, is clearly renormalization-scale dependent, and recent interest in it has centered on the effect of running Q_1 up to higher energies and perhaps matching it on to an underlying supersymmetric theory at some scale [7,10]. The observation [4,10] that Q_1 changes sign at some high scale (the value of this scale being quite sensitive to the precise value of the top mass) has led to the remarkable suggestion [5] that this sign change is actually the trigger for electroweak symmetry breaking.

In fact the issue of the scale dependence of Q_1 was considered in general theories and in the particular case of the SM many years ago [9,11–14]. This work included the observation that in a Yukawa-scalar nongauge theory, there exists an intriguing relationship between the scale dependence of Q_1 and the leading quadratic divergence at the two-loop level. In fact, requiring Q_1 to be both zero and scale independent to leading order in the β functions leads to precisely the same condition as requiring the two-loop leading quadratic divergence to vanish!

In Ref. [11], the *leading* quadratic divergence at L loops was defined in the context of regularization by dimensional reduction (DRED) [15,16] as the residue of the pole at $d = 4 - 2/L$ in the IR-regulated two-particle amplitude. This definition corresponds, in fact, to associating the *leading* quadratic divergence at two loops with the (IR-regulated) integral

$$I_2 = \int \frac{d^d k d^d q}{k^2 q^2 (k+q)^2}, \tag{4}$$

which is precisely what was done in Ref. [4]. At two loops one also encounters

$$I_1 = \int \frac{d^d k d^d q}{(k^2)^2 q^2}, \tag{5}$$

which has a pole at $d = 2$ and is cancelled by the one-loop counterterm insertion contribution.

In Ref. [4], a calculation of the two-loop quadratic divergence in the Higgs mass was presented, and the coefficient of I_2 was found to be proportional to Q_2 where

$$Q_2 = -\left(9y_t^4 + y_t^2\left(-\frac{7}{12}g'^2 + \frac{9}{4}g^2 - 16g_3^2\right) + \frac{77}{16}g'^4 + \frac{243}{16}g^4 + \lambda(-18y_t^2 + 3g'^2 + 9g^2) - 10\lambda^2\right). \tag{6}$$

It appears the authors were unaware of the previous calculation of the same quantity [17] (using DRED) of Ref. [9], where the result found was proportional to Δ_2, given by

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$$\Delta_2 = \frac{9}{2}H^2 + 27HT - 54T^2 - 9H(3 + \tan^2\theta_W) - T(27 - 7\tan^2\theta_W - s) + \frac{189}{2} + 45\tan^2\theta_W + \frac{261}{2}\tan^4\theta_W, \tag{7}$$

where $s = 192g_3^2/g^2$.

Reducing Δ_2 to the same notation as Q_2 we obtain

$$m_W^4\Delta_2 = -\frac{3}{2}\Big(9y_t^4 + y_t^2\Big(-\frac{7}{12}g'^2 + \frac{9}{4}g^2 - 16g_3^2\Big) - \frac{87}{16}g'^4 - \frac{63}{16}g^4 - \frac{15}{8}g^2g'^2 + \lambda(-18y_t^2 + 3g'^2 + 9g^2) - 12\lambda^2\Big), \tag{8}$$

and we see that most terms agree. (The overall factor is not significant; in Refs. [9,11–13] we were concerned with seeking theories *without* quadratic divergences). However the λ^2, g^4, g'^4 and $g^2g'^2$ terms do not agree, in both magnitude and sign in the case of the g^4, g'^4 terms. The disagreement was noted in Ref. [5], the author of which opted to believe the result of Ref. [4].

Note that the result of Ref. [4] has no $g^2g'^2$ term. On this particular point we can easily see, I believe, that Ref. [4] is incorrect as follows.

The calculations of Ref. [4] were done in the Landau gauge, in which gauge, as they remark, it is easy to see that graphs of the general form of Fig. 1 do not contribute. In the Landau gauge there is, however, one graph that *does* give rise to a $g^2g'^2$ term, shown in Fig. 2.

I have calculated the graph shown in Fig. 2 in the Landau gauge, and obtained a result in agreement with Eq. (7).

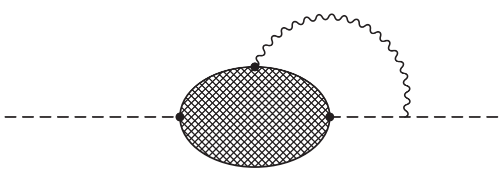

FIG. 1. A class of graphs free of quadratic divergences.

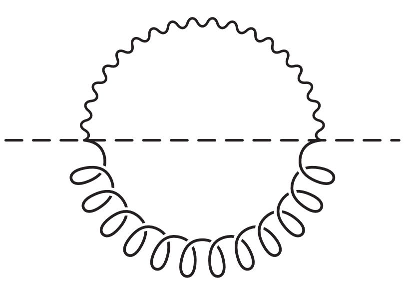

FIG. 2. Nonzero contribution proportional to $g^2g'^2$.

It seems to me likely that the authors of Ref. [4] have inadvertently omitted this graph.

With regard to the remaining discrepancies, the difference in the λ^2 terms presumably results from an error by one group or the other. For the g^4, g'^4 terms, two issues arise. The first is gauge invariance; I am not aware of a proof that the whole result is gauge invariant, but I believe it is. The fact that I have obtained the same result for the $g^2g'^2$ term using the Landau gauge as that of Ref. [9] (where the calculations were performed in a background Feynman gauge, using configuration-space methods) is some evidence for this. The second issue arises from the fact that using DRED, the ϵ-scalars peculiar to that scheme *themselves* develop a one-loop self-energy quadratic divergence. As described in Ref. [9], this leads to a breakdown in the relationship between the leading two-loop divergence Q_2 and the quantity

$$A_{11} = \beta_{\lambda_i}^{(1)}\cdot\frac{\partial}{\partial\lambda_i}Q_1 - Q_1\cdot\frac{\partial}{\partial\lambda_i}\beta_{\lambda_i}^{(1)} \tag{9}$$

that, as mentioned above, had been observed in nongauge theories. It would thus have been very interesting had the result of Ref. [4] for Q_2 agreed with A_{11} but it does not. In any event, I believe that using DRED and identifying the $d = 3$ pole is equivalent to the procedure of Ref. [4].

My confidence in the result of Ref. [9] relies on the general results Eqs. (3.5) and (3.8) given there and the renormalization-group check on the reduction to the SM case described in the Appendix of that reference. In this context, however, I should remark that there is a typo in Eq. (4.3) of the published version of that reference, which should read

$$A_{11} = \frac{9}{2}H^2 + 27HT - 54T^2 - 9H(3 + \tan^2\theta_W) - T(27 - 7\tan^2\theta_W - s) + \frac{21}{2} + 45\tan^2\theta_W + \frac{109}{2}\tan^4\theta_W. \tag{10}$$

Note that Eq. (4.5) of Ref. [9], which is obtained by substituting $\Delta_1 = 0$ from Eq. (4.1) into Eq. (4.3), is in fact correct. From Eq. (10) we obtain

$$m_W^4A_{11} = -\frac{3}{2}\Big(9y_t^4 + y_t^2\Big(-\frac{7}{12}g'^2 + \frac{9}{4}g^2 - 16g_3^2\Big) - \frac{109}{48}g'^4 - \frac{7}{16}g^4 - \frac{15}{8}g^2g'^2 + \lambda(-18y_t^2 + 3g'^2 + 9g^2) - 12\lambda^2\Big). \tag{11}$$

The difference between A_{11} and Δ_2 was, as we indicated above, associated by Ref. [9] with the ϵ-scalar self-energy component of the diagrams shown in Fig. 3 [in fact only Fig. 3(b) contributes]. It is easy to check that the difference

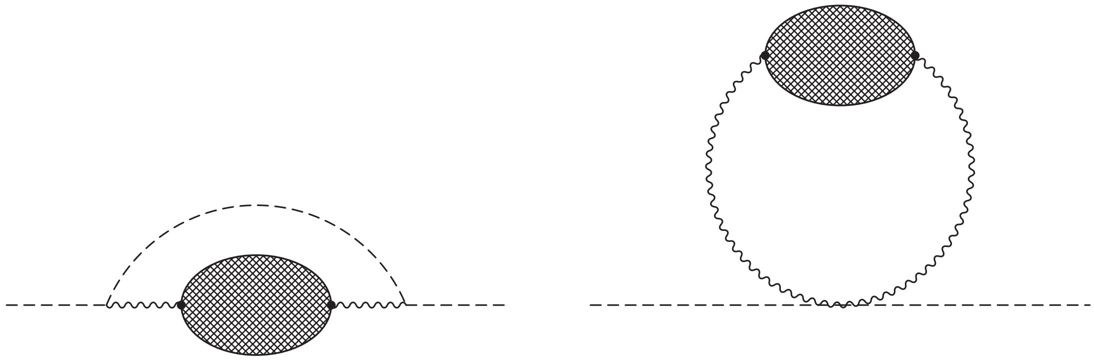

FIG. 3. A class of graphs with the ϵ-scalar self energy.

between Eq. (8) and (11) above is consistent with Eq. (3.9) of Ref. [9].

With a *physical* cutoff for the quadratic divergence, it is reasonable to argue [4] that, away from $Q_1 = 0$, the effect of the two-loop quadratic divergence Q_2 is small compared to that of Q_1. Therefore the disagreements I have indicated above will not have much impact on the thrust of the arguments presented in Refs. [4–8], although it may well change the scale at which the *total* quadratic divergence reaches zero by an appreciable amount. For possible future applications it is as well to clarify which of the two calculations discussed here is correct.

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