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Gravitational interaction of Higgs boson and weak boson scattering

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With the LHC discovery of a 125 GeV Higgs-like boson, we study the gravitational interaction of the Higgs boson via the unique dimension-four operator involving the Higgs doublet and scalar curvature, $\xi H^\dagger H\mathcal{R}$, with nonminimal coupling ξ. This Higgs portal term can be transformed away in the Einstein frame and induces gauge-invariant effective interactions in the Higgs sector. We study the weak boson scattering in the Einstein frame and explicitly demonstrate the longitudinal-Goldstone boson equivalence theorem in the presence of ξ coupling. With these, we derive a perturbative unitarity bound on the Higgs gravitational coupling ξ in the Einstein frame, which is stronger than that inferred from the LHC Higgs measurements. We further study ξ-dependent weak boson scattering cross sections at the TeV scale and propose a new LHC probe of Higgs-gravity coupling ξ via weak boson scattering experiments.

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I. INTRODUCTION

The standard model (SM) of electromagnetic, weak, and strong interactions has been extremely successful. It has the well established $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ gauge structure and hypothesizes a single Higgs doublet to generate masses for weak gauge bosons as well as all three families of quarks, leptons, and neutrinos. The recent LHC discovery of a 125 GeV Higgs-like boson [1,2] appears to provide the last missing piece of the SM. However, the SM is definitely incomplete for many reasons—above all, the SM does not contain gravitational force. At present, the best theory of gravitation is Einstein's general relativity (GR), but is notoriously nonrenormalizable [3] despite that the gravity itself is still weakly coupled over vast energy ranges (below the Planck mass), and thus the perturbation should hold well. Given the fact that all SM particles participate in gravitation, we have to treat both the SM and GR together as a joint low energy effective theory below the Planck scale.

In the modern effective theory formulation [4], we study low energy physics by writing down the most general operators in the Lagrangian, under perturbative expansion and consistent with all given symmetries. Indeed, the SM gives such a general effective Lagrangian up to dimension-four operators under the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ gauge structure and the known particle content. On the other hand, the GR of gravitation is described by Einstein-Hilbert action, containing two leading terms of a series of operators consistent with general covariance,

$$S_{\rm GR} = M_{\rm Pl}^2 \int {\rm d}^4x\sqrt{-g}\Big(-\Lambda + \frac{1}{2}\mathcal{R}\Big), \qquad (1)$$

where Λ denotes the cosmological constant, $\mathcal{R}$ is the Ricci scalar, and $M_{\rm Pl} = (8\pi G)^{-1/2} \simeq 2.44 \times 10^{18}$ GeV gives the reduced Planck mass. To complete this series of operators up to dimension four, we include two more terms,

$$\Delta S_{\rm GR} = \int {\rm d}^4x\sqrt{-g}(c_1\mathcal{R}^2 + c_2\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}). \qquad (2)$$

The other dimension-four term $\mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}$ is not independent up to integration by parts.

Combining the SM and GR together, one usually requires that the SM particles couple to gravity through their energy-momentum tensor, so the resultant theory is consistent with the SM gauge symmetries and the equivalence principle. Practically, this amounts to the replacement $\eta_{\mu\nu} \to g_{\mu\nu}$ and $\partial_\mu \to \nabla_\mu$ in the SM Lagrangian, where ∇_μ is the covariant derivative associated with metric $g_{\mu\nu}$. (For fermions, vierbein and spin connection will be used.) This gives the so-called minimal couplings between the SM and gravity.

In light of the recent LHC Higgs discovery, we are strongly motivated to study gravitational interactions of the Higgs boson because the Higgs boson is the origin of inertial mass generation for elementary particles. For this, there is a unique dimension-four Higgs portal operator that should be included in the SM + GR system,

$$\Delta S_{\rm NMC} = \xi \int {\rm d}^4x\sqrt{-g}H^\dagger H\mathcal{R}, \qquad (3)$$

where H denotes the SM Higgs doublet, and the dimensionless parameter ξ is conventionally called the nonminimal coupling that describes the Higgs-curvature interaction. This

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dimension-four operator respects all symmetries of the SM and has to be retained in the effective theory formulation of the SM + GR. Even if we naively set the nonminimal coupling $\xi = 0$ at tree level, it will reappear at loop level [5]. Furthermore, since setting $\xi = 0$ does not enhance the symmetry, there is no reason that ξ would be small. A special choice of $\xi = -1/6$ will make the theory conformally invariant, but the conformal symmetry is not respected by the SM Lagrangian, and in most cases will be further broken by quantum effects. Hence, such a dimensionless coupling ξ could be rather large, *a priori*. This fact has already been put to use for the Higgs inflation scenario, where $\xi \gg 1$ is required, and the typical input is $\xi \sim 10^4$ [6,7]. A recent interesting study by Atkins and Calmet derived a bound on ξ from the LHC data, $|\xi| < 2.6 \times 10^{15}$ [8].

In this work, we study the Higgs gravitational coupling in the Einstein frame (Sec. II), where the original operator (3), as defined in the Jordan frame, is transformed away and induces effective interactions for the Higgs sector. These induced new Higgs interactions modify Higgs couplings with weak gauge bosons and Goldstone bosons. With this formulation, we analyze the scattering amplitudes of longitudinal weak bosons and Goldstone bosons in Sec. III. We give the *first demonstration* of the longitudinal-Goldstone boson equivalence theorem with nonzero ξ coupling. Then, we derive the perturbative unitarity bound of ξ in the Einstein frame, to the leading order of $M_{\rm Pl}^{-2}$. Furthermore, we study an intriguing scenario of SM + GR effective theory with a low ultraviolet (UV) cutoff around $\mathcal{O}(10\ \text{TeV})$, as a natural solution to the hierarchy problem. With this we make the *first proposal* to study the impacts of ξ coupling on TeV-scale weak boson scattering cross sections and discuss the LHC tests. Finally, we conclude in Sec. IV.

II. INDUCED HIGGS PORTAL INTERACTIONS FOR EINSTEIN FRAME

Even though the SM has been impressively consistent with all the experimental data so far—including the recent LHC discovery of a Higgs-like particle [1,2]—the unique dimension-four Higgs-gravity interaction (3) is an unambiguous portal to the new physics beyond SM. It is possible that the effective theory combining the SM with general relativity (1) and (2) via the Higgs portal (3) will describe the Universe all the way up to very high scales (below Planck mass $M_{\rm Pl}$), including inflation as well (with proper extensions) [6,7]. For the electroweak gauge and Higgs sectors, we can write this effective theory up to dimension-four operators, as the following:

$$S = \int d^4x\sqrt{-g^{(J)}}\Big[-M_{\rm Pl}^2\Lambda + \Big(\frac{M^2}{2} + \xi H^\dagger H\Big)\mathcal{R}^{(J)} - \frac{1}{4}F^a_{j\mu\nu}F^{a\mu\nu}_j + (D_\mu H)^\dagger(D^\mu H) - V(H)\Big], \tag{4}$$

where $\mathcal{R}^{(J)}$ is the scalar curvature associated with metric $g^{(J)}_{\mu\nu}$, and $F^{a\mu\nu}_j (= W^{a\mu\nu}, B^{\mu\nu})$ denotes field strengths of $SU(2)_L \otimes U(1)_Y$ electroweak gauge fields. The Higgs doublet H has the potential $V(H) = \lambda(H^\dagger H - \frac{1}{2}v^2)^2$. The frame including the ξ term in (4) is conventionally called the Jordan frame. Here we have used a superscript "(J)" for the metric $g^{(J)}_{\mu\nu}$ and Ricci scalar $\mathcal{R}^{(J)}$. One can always make a field-dependent Weyl transformation for the Jordan frame metric $g^{(J)}_{\mu\nu}$ such that this ξ term is transformed away, and the resultant system is called the Einstein frame in which the field equation of $g_{\mu\nu}$ is the standard Einstein equation for GR. The field redefinition in the Einstein frame reads $g_{\mu\nu} = \Omega^2 g^{(J)}_{\mu\nu}$, where $\Omega^2 = (M^2 + 2\xi H^\dagger H)/M_{\rm Pl}^2$. For simplicity, hereafter we will denote all quantities in the Einstein frame without extra superscripts. From (4) we can identify the Planck mass, $M_{\rm Pl}^2 = M^2 + \xi v^2$, where v is the Higgs vacuum expectation value. It shows that the nonminimal coupling strength ξ is bounded by $\xi \leq M_{\rm Pl}^2/v^2 \simeq 9.8 \times 10^{31}$, which is saturated in the limit $M^2 = 0$.

Given the above relation between $g^{(J)}_{\mu\nu}$ and $g_{\mu\nu}$, the Ricci scalar transforms as follows:

$$\mathcal{R}^{(J)} = \Omega^2[\mathcal{R} - 6g^{\mu\nu}\nabla_\mu\nabla_\nu \log\Omega + 6g^{\mu\nu}(\nabla_\mu \log\Omega)(\nabla_\nu \log\Omega)], \tag{5}$$

where the covariant derivative ∇_μ is associated with the Einstein frame metric $g_{\mu\nu}$. Thus, the effective action (4) can be rewritten in the Einstein frame,

$$S = \int d^4x\sqrt{-g}\Big[-\frac{M_{\rm Pl}^2}{\Omega^4}\Lambda + \frac{M_{\rm Pl}^2}{2}\mathcal{R} - \frac{1}{4}F^a_{j\mu\nu}F^{a\mu\nu}_j - \frac{3\xi}{\Omega^2}\nabla^2(H^\dagger H) + \frac{9\xi^2}{M_{\rm Pl}^2\Omega^4}(\nabla_\mu(H^\dagger H))^2 + \frac{1}{\Omega^2}(D^\mu H)^\dagger(D_\mu H) - \frac{1}{\Omega^4}V(H)\Big], \tag{6}$$

where the fourth term is a total derivative for $\Omega = 1$ and is always suppressed by $1/M_{\rm Pl}^2$ for the $\Omega \neq 1$ part. We note that the form of the gauge field kinetic term does not change under this field transformation since the Weyl factor Ω^{-4} from the determinant of the metric $\sqrt{-g}$ is compensated by additional two factors of Ω^2 from the inverse metrics in the gauge kinetic term $-\frac{1}{4}g^{\mu\nu}g^{\rho\sigma}F^a_{j\mu\rho}F^a_{j\nu\sigma}$.

Next, we study the longitudinal weak boson scattering and the Goldstone boson scattering. We explicitly demonstrate the longitudinal-Goldstone boson equivalence theorem in the presence of ξ coupling. For this purpose, we will derive relevant scalar and gauge couplings from the action (6). We parametrize the Higgs doublet as usual, $H = (\pi^+, \frac{1}{\sqrt{2}}(v + h^0 + i\pi^0))^T$. Thus, we can expand, $\Omega^2 = 1 + \xi M_{\rm Pl}^{-2}[2vh^0 + (h^0)^2 + 2\pi^+\pi^- + (\pi^0)^2]$. From now on, we will also set the Einstein frame metric to be flat, $g_{\mu\nu} = \eta_{\mu\nu}$, since the graviton contributions from $g_{\mu\nu}$ are suppressed by more powers of $1/M_{\rm Pl}^2$ and thus will not affect our following analysis of leading ξ terms at $\mathcal{O}(M_{\rm Pl}^{-2})$. Then, we derive kinetic terms for the Higgs field and would-be Goldstone fields,

$$\frac{1}{2}\left(1+\frac{6\xi^2 v^2}{M_{\rm Pl}^2}\right)(\partial_\mu h^0)^2 + \frac{1}{2}(\partial_\mu \pi^0)^2 + \partial_\mu \pi^- \partial^\mu \pi^+. \quad (7)$$

Hence, the Higgs field h^0 should be renormalized via $h^0 \to \zeta h^0$, with $\zeta \equiv (1 + 6\xi^2 v^2/M_{\rm Pl}^2)^{-1/2}$. In consequence [9], the Higgs boson mass is given by $m_h^2 = 2\lambda v^2 \zeta^2$, and all the SM Higgs couplings should be rescaled accordingly. Furthermore, under the Weyl transformation the original nonminimal term in (4) has led to new Higgs couplings in the Einstein frame, as shown in Eq. (6). Analyzing the second and third lines of (6), we systematically present the ξ-dependent couplings up to the first order in $M_{\rm Pl}^{-2}$. For scalar derivative interactions, we deduce the Lagrangian,

$$\begin{aligned}
&-\frac{\xi}{2M_{\rm Pl}^2}[|\partial_\mu \vec{\pi}|^2 + \zeta^2(\partial_\mu h^0)^2][|\vec{\pi}|^2 + \zeta^2 (h^0)^2 + 2v\zeta h^0] \\
&-\frac{3\xi^2}{4M_{\rm Pl}^2}[|\vec{\pi}|^2 + \zeta^2 (h^0)^2 + 2v\zeta h^0] \\
&\times \partial^2 [|\vec{\pi}|^2 + \zeta^2 (h^0)^2 + 2v\zeta h^0], \quad (8)
\end{aligned}$$

where $|\vec{\pi}|^2 = 2\pi^+\pi^- + (\pi^0)^2$ and $|\partial_\mu \vec{\pi}|^2 = 2\partial_\mu \pi^+ \partial^\mu \pi^- + (\partial_\mu \pi^0)^2$. For Higgs couplings with WW/ZZ, we derive the Lagrangian,

$$\begin{aligned}
&(2m_W^2 W_\mu^+ W^{\mu -} + m_Z^2 Z_\mu Z^\mu) \\
&\quad \times \left[\left(\frac{1}{v} - \frac{\xi v}{M_{\rm Pl}^2}\right)\zeta h^0 + \frac{1}{2}\left(\frac{1}{v^2} - \frac{5\xi}{M_{\rm Pl}^2}\right)\zeta^2 (h^0)^2\right], \quad (9)
\end{aligned}$$

which reduces to the SM couplings in the limit $\xi = 0$.

For completeness, we also discuss the fermion sector in the current formulation. The kinetic term of a Dirac fermion Ψ in the curved background can be written as

$$S_{\rm f} = \int {\rm d}^4 x \, \det(e_\nu^q) \bar{\Psi} \gamma^p e_p^\mu \left({\rm i}\partial_\mu - \frac{1}{2}\omega_\mu{}^{mn}\sigma_{mn}\right)\Psi, \quad (10)$$

where e_ν^q is vierbein, $\omega_\mu{}^{mn}$ denotes spin connection, and $\sigma_{mn} = \frac{\rm i}{2}[\gamma_m, \gamma_n]$. We define this curved background in Jordan frame, which is connected to Einstein frame with a flat metric via $g_{\mu\nu}^{(J)} = \Omega^{-2}\eta_{\mu\nu}$. Thus, we deduce the expressions, $e_\mu^m = \Omega^{-1}\delta_\mu^m$ and $\omega_\mu{}^{mn} = -\Omega^{-1}(\delta_\mu^m \partial^n \Omega - \delta_\mu^n \partial^m \Omega)$. With these, the above kinetic term becomes,

$$S_{\rm f} = \int {\rm d}^4 x \left(\frac{1}{\Omega^3}\bar{\Psi}{\rm i}\not{\partial}\Psi + \frac{3}{\Omega^4}\bar{\Psi}({\rm i}\not{\partial}\Omega)\Psi\right). \quad (11)$$

III. WEAK BOSON SCATTERING FROM HIGGS-GRAVITY INTERACTIONS

It is expected that combining Higgs-curvature coupling (3) with the SM makes the theory perturbatively nonrenormalizable. This is manifest in the Einstein frame action (6) via ξ-dependent new higher dimensional operators. Hence, the high energy longitudinal weak boson scattering amplitude will exhibit noncanceled E^2 behavior from these ξ-dependent interactions, and thus lead to perturbative unitarity violation. From the derivative scalar couplings in (8), we will show that the same E^2 behavior appears in the corresponding Goldstone boson scattering amplitude. Then, we present the *first demonstration* of the longitudinal-Goldstone boson equivalence theorem (ET) [10], in the presence of nonminimal coupling ξ. This is highly nontrivial because the ξ-dependent scalar derivative interactions (8) take very different forms from the new Higgs-gauge boson couplings (9). We will further derive the perturbative unitarity bound on ξ by studying the scattering amplitudes of both longitudinal weak bosons and Goldstone bosons.

Let us analyze the scalar scattering among four electrically neutral states $|\pi^+\pi^-\rangle$, $\frac{1}{\sqrt{2}}|\pi^0\pi^0\rangle$, $\frac{1}{\sqrt{2}}|h^0h^0\rangle$, and $|\pi^0 h^0\rangle$. With systematical computation based on Lagrangian (8), we derive the leading scattering amplitudes,

$$\begin{aligned}
\mathcal{T}[\pi^+\pi^- \to \pi^+\pi^-] &= \frac{3\xi^2+\xi}{M_{\rm Pl}^2}(1+\cos\theta)E^2, \\
\mathcal{T}[\pi^+\pi^- \to \pi^0\pi^0] &= \frac{2(3\xi^2+\xi)}{M_{\rm Pl}^2}E^2, \\
\mathcal{T}[\pi^+\pi^- \to h^0h^0] &= \frac{2(3\xi^2+\xi)}{M_{\rm Pl}^2}E^2, \\
\mathcal{T}[\pi^0\pi^0 \to \pi^0\pi^0] &= \mathcal{O}(E^0), \quad (12)\\
\mathcal{T}[\pi^0\pi^0 \to h^0h^0] &= \frac{2(3\xi^2+\xi)}{M_{\rm Pl}^2}E^2, \\
\mathcal{T}[h^0h^0 \to h^0h^0] &= \mathcal{O}(E^0), \\
\mathcal{T}[\pi^0h^0 \to \pi^0h^0] &= -\frac{3\xi^2+\xi}{M_{\rm Pl}^2}(1-\cos\theta)E^2,
\end{aligned}$$

where E is the center-of-mass energy.

In parallel, we have studied the longitudinal weak boson scattering for $|W_L^+W_L^-\rangle$, $\frac{1}{\sqrt{2}}|Z_L^0Z_L^0\rangle$, $\frac{1}{\sqrt{2}}|h^0h^0\rangle$, and $|Z_L^0h^0\rangle$. For instance, taking the process $W_L^+W_L^- \to Z_L^0Z_L^0$ and using (9), we derive the leading high energy scattering amplitude in unitary gauge,

$$\begin{aligned}
\mathcal{T}[W_L^+W_L^- \to Z_L^0Z_L^0] &= \frac{8(3\xi^2+\xi)}{M_{\rm Pl}^2}\frac{(E^2-2m_W^2)^2}{4(E^2-m_h^2)} \\
&= \frac{2(3\xi^2+\xi)}{M_{\rm Pl}^2}E^2 + \mathcal{O}(E^0). \quad (13)
\end{aligned}$$

This equals the amplitude $\mathcal{T}[\pi^+\pi^- \to \pi^0\pi^0]$ in (12) at $\mathcal{O}(E^2)$. We have performed systematical analyses for all other longitudinal weak boson scattering processes and find full agreements to the leading Goldstone amplitudes (12). These explicitly justify the validity of the ET in the presence of ξ coupling, which was not studied before. This also serves as a highly nontrivial self-consistency check of our amplitudes (12).

A few comments are in order concerning the ξ-dependent leading scattering amplitudes (12). We first note that the ξ coupling enters our results in two ways. One is through the Higgs field rescaling factor $\zeta = (1 + 6\xi^2 v^2/M_{\rm Pl}^2)^{-1/2}$, and the other arises from the Weyl factor $\Omega^2 = 1 + \xi(2H^\dagger H - v^2)/M_{\rm Pl}^2$. Thus, at the first nontrivial order of $M_{\rm Pl}^{-2}$, we have both ξ^2 and ξ contributions to the amplitudes (12). For typical situations with $|\xi| \gg 1$, the ξ^2 terms dominate over ξ terms and will thus control our final perturbative unitarity bound. We can classify such processes into three categories. The first one is a universal suppression factor $\zeta < 1$ for each Higgs field in the Higgs boson productions [8]. The second class of ξ^2-dependent processes is the longitudinal weak boson scattering. We find that the *anomalous* quartic scalar couplings and cubic Higgs-gauge couplings of (8) and (9) cause noncanceled ξ^2 (and ξ) dependent E^2 contributions in the longitudinal and Goldstone boson scattering amplitudes [cf. (12) and (13)], which can become significant for large E. Hence, *the $W_L W_L$ scattering can provide a sensitive probe of ξ coupling via the energy-enhanced leading contributions of $\mathcal{O}(\xi^2 E^2/M_{\rm Pl}^2)$*. The third class of ξ^2-involved processes includes those containing the cubic Higgs coupling. As shown in (8), such processes will also be enhanced at high energies by the ξ^2-dependent derivative cubic Higgs couplings. The high luminosity runs at LHC (14 TeV), the future TeV linear colliders (ILC and CLIC), and high energy circular pp colliders (TLEP and SPPC) will further probe such anomalous cubic Higgs couplings. Finally, the $\mathcal{O}(\xi/M_{\rm Pl}^2)$ couplings arise from the Weyl factor Ω, which are negligible relative to $\mathcal{O}(\xi^2/M_{\rm Pl}^2)$ for $\xi \gg 1$.

From Eq. (12), we compute the partial wave scattering amplitude,

$$a_\ell(E) = \frac{1}{32\pi}\int_{-1}^{1} d\cos\theta\, P_\ell(\cos\theta)\mathcal{T}(E,\theta). \quad (14)$$

In our case, the partial wave amplitudes form a 4×4 matrix among the four states $|\pi^+\pi^-\rangle$, $\frac{1}{\sqrt{2}}|\pi^0\pi^0\rangle$, $\frac{1}{\sqrt{2}}|h^0h^0\rangle$, and $|\pi^0h^0\rangle$. Thus, for $\ell = 0$, we derive the following matrix of leading s-wave amplitudes:

$$a_0(E) = \frac{(3\xi^2+\xi)E^2}{16\pi M_{\rm Pl}^2}\begin{pmatrix} 1 & \sqrt{2} & \sqrt{2} & 0 \\ \sqrt{2} & 0 & 1 & 0 \\ \sqrt{2} & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad (15)$$

whose eigenvalues are

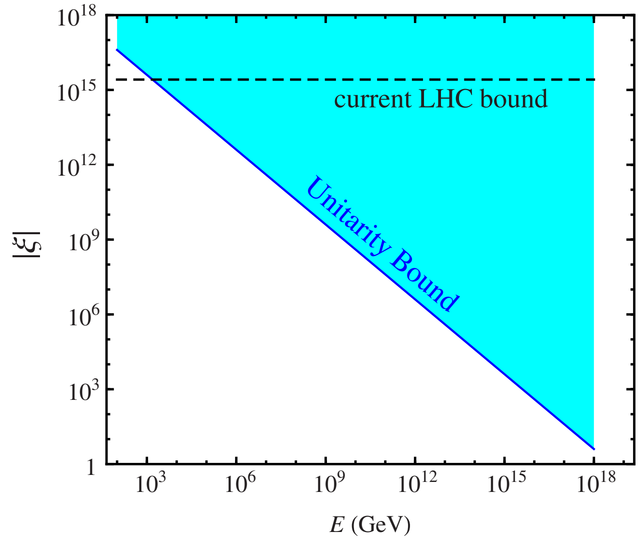

FIG. 1 (color online). Perturbative unitarity bound on Higgs-gravity coupling ξ as a function of WW scattering energy E. The shaded region violates perturbative unitarity. To compare with our bound at each given E [13], the black dashed line depicts a limit $|\xi| < 2.7\times 10^{15}$ (3σ) derived from current LHC Higgs data [2], based on [8].

$$a_0^{\rm diag}(E) = \frac{(3\xi^2+\xi)E^2}{16\pi M_{\rm Pl}^2}\,{\rm diag}(3,-1,-1,-1). \quad (16)$$

Then, we impose the partial wave unitarity condition, $|\Re e\, a_0| < 1/2$, on the maximal eigenvalue of (16). From this, we infer the perturbative unitarity bound [11] on ξ,

$$-\frac{1}{6}[(C_0^2+1)^{\frac{1}{2}}+1] < \xi < \frac{1}{6}[(C_0^2+1)^{\frac{1}{2}}-1], \quad (17)$$

where $C_0 \equiv \sqrt{32\pi}M_{\rm Pl}/E$.

In the present effective theory formulation, we have the Planck mass $M_{\rm Pl}$ serve as the natural UV cutoff on the scattering energy, $E < M_{\rm Pl}$. Thus, the inequality $C_0^2 > 32\pi \simeq 100.5 \gg 1$ holds well. Hence, to good approximation, the above unitarity bound (17) reduces to the following:

$$|\xi| < \frac{C_0}{6} = \frac{\sqrt{8\pi}}{3}\frac{M_{\rm Pl}}{E}. \quad (18)$$

In Fig. 1, we present the perturbative unitarity bound of ξ as a function of scattering energy E, up to $E = 10^{18}$ GeV, which is still significantly below the Planck scale $M_{\rm Pl} \simeq 2.44\times 10^{18}$ GeV. For the effective theory of SM + GR with Planck mass $M_{\rm Pl}$ as the natural UV cutoff, the weak boson scattering energy can reach up to $E = 10^{17-18}$ GeV, and thus our perturbative unitarity bound requires $\xi \lesssim \mathcal{O}(10-1)$. We also note that Atkins and Calmet [8] recently derived an interesting bound on ξ from the 2012 LHC data [1]. Combining ATLAS and CMS new data in this spring [2] gives the Higgs signal strength $\hat{\mu} = 1.00\pm 0.10$ [12]. Thus, we have a refined 3σ upper limit, $|\xi| < 2.7\times 10^{15}$. For comparison, we

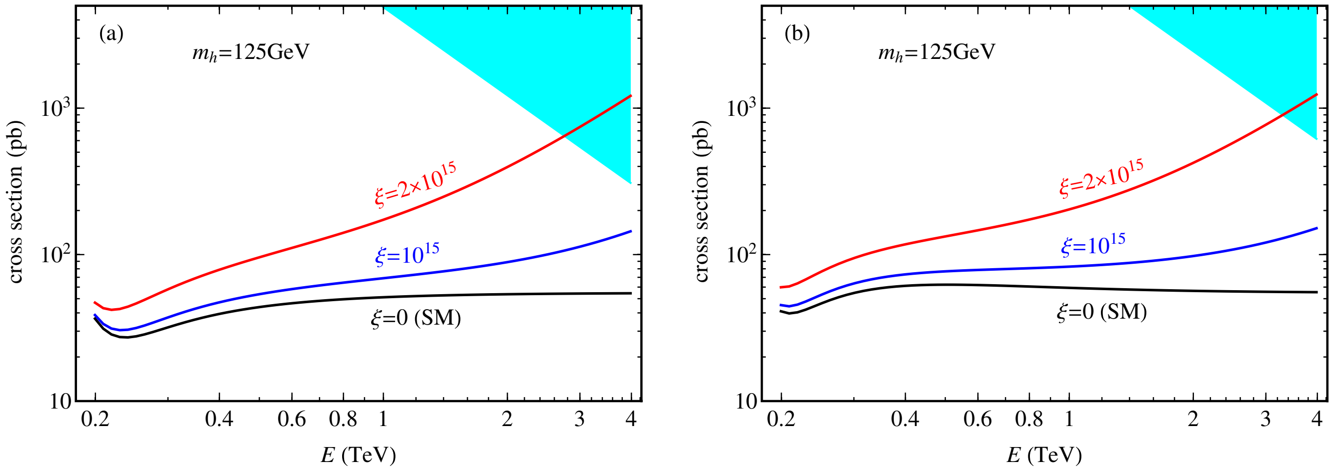

FIG. 2 (color online). Weak boson scattering cross section as a function of center-of-mass energy E. We present $W_L^+W_L^- \to Z_L^0Z_L^0$ in (a) and $W_L^\pm W_L^\pm \to W_L^\pm W_L^\pm$ in (b). In each plot, the (red, blue, black) curves depict our predictions of $\xi = (2\times 10^{15}, 10^{15}, 0)$, respectively. The black curve of $\xi = 0$ is the pure SM expectation. We have input Higgs boson mass $m_h = 125$ GeV. The shaded light-blue area shows the perturbative unitarity violation region.

depict this bound in Fig. 1 by the dashed line [13]. It shows that once the scattering energy E exceeds $\mathcal{O}$ (TeV), the perturbative unitarity bound becomes much more stringent.

If nature has chosen a much lower UV cutoff for the effective theory of SM + GR, then a much larger ξ will be allowed. A very intriguing case is that UV cutoff lies at the TeV scale, say, $\Lambda_{\rm UV} = \mathcal{O}(10\ \text{TeV})$. Thus, the coupling ξ can reach $\xi = \mathcal{O}(10^{14-15})$. As a nice theory motivation for this case, it gives a conceptually simple solution to the hierarchy problem and makes the SM Higgs sector natural up to the UV cutoff [14], $\Lambda_{\rm UV} = \mathcal{O}(10\ \text{TeV})$. This further opens up an exciting possibility that the LHC (14 TeV) and its future upgrades can effectively probe such Higgs-gravity interactions via weak boson scattering experiments.

Let us study this intriguing effective theory of TeV scale quantum gravity with $\Lambda_{\rm UV} = \mathcal{O}(10\ \text{TeV})$. It is well known that weak boson scattering experiments serve as a key task of LHC for probing new physics of electroweak symmetry breaking and the Higgs mechanism [15]. Hence, we analyze weak boson scattering cross sections and consider two typical processes, $W_L^+W_L^- \to Z_L^0Z_L^0$ and $W_L^\pm W_L^\pm \to W_L^\pm W_L^\pm$, over the energy regime ($<\Lambda_{\rm UV}$) to be probed by the LHC (14 TeV). In this case, Fig. 1 shows that ξ can be fairly large and reach $\xi = O(10^{15})$. In Fig. 2, we demonstrate WW scattering cross sections for two sample inputs, $\xi = (2\times 10^{15}, 10^{15})$, as compared to the pure SM result of $\xi = 0$ [16]. In addition, we impose perturbative unitarity bound on the scattering cross section, $\sigma < 4\pi\rho_e/E^2$ [17], as denoted by the shaded light-blue region in each plot. (Here ρ_e is the identical particle factor for the final state as defined in [17].)

From Figs. 2(a) and 2(b), we see that the predicted WW scattering cross sections with sample inputs of $\xi = 2\times 10^{15}$, 10^{15} exhibit different behaviors and give sizable excesses above the pure SM expectations $\xi = 0$). *Such nonresonance behaviors are universal and are expected to show up in all WW scattering channels* [18,19], unlike the conventional new physics models of electroweak symmetry breaking [15,20]. These distinctive features can be discriminated by the upcoming LHC runs at 14 TeV with higher luminosity. As a remark, we clarify that there is no preferred natural values of ξ from theory side. Although the dimensionless coupling ξ can have a large value, it appears in the interaction vertices always in association with the *suppression factor* $v^2/M_{\rm Pl}^2$ or $E^2/M_{\rm Pl}^2$, as shown in Eqs. (8) and (9). Hence, it is fine to have a large ξ as long as it holds the perturbative expansion (shown in Fig. 1).

IV. CONCLUSION AND DISCUSSION

The world is shaped by gravitation at the macroscopic and cosmological scales, while the gravitational force will also play a key role at the smallest Planck scale. *Then, what happens in between?* With the LHC discovery of a 125 GeV Higgs-like boson [1,2], we are strongly motivated to explore gravitational interactions of the Higgs boson in connection to the mechanism of electroweak symmetry breaking and the origin of inertial mass generation for elementary particles.

In this work, we studied Higgs-gravity interaction via the unique dimension-four operator (3) with nonminimal coupling ξ, which serves as an unambiguous portal to the new physics beyond SM. In Sec. II, we focused on its formulation in the Einstein frame where this Higgs-curvature operator is transformed into a set of ξ-dependent new Higgs interactions (6). We derived the ξ-induced Higgs/Goldstone self-interactions in (8) and the Higgs-gauge interactions in (9). In Sec. III we systematically analyzed longitudinal and Goldstone boson scatterings. We demonstrated, *for the first time*, the longitudinal-Goldstone boson equivalence theorem in the presence of ξ coupling. This is important for understanding the Higgs mechanism with a nonzero Higgs-curvature coupling ξ. We performed a coupled channel analysis of WW scattering in the Einstein frame and derived new perturbative unitarity bound on ξ as in Fig. 1. We revealed that for the SM + GR system with Planck mass $M_{\rm Pl}$ as its natural UV cutoff, the weak boson scattering energy can reach $E = 10^{17-18}$ GeV($<M_{\rm Pl}$). In this case, the validity of perturbative unitarity requires $\xi \lesssim \mathcal{O}(10-1)$. We further studied the intriguing scenario for the SM + GR effective theory with the UV cutoff around $\Lambda_{\rm UV} = \mathcal{O}(10\ \text{TeV})$. Thus, the ξ coupling can reach $\xi = \mathcal{O}(10^{15})$. In Fig. 2, we predicted, *for the first time*, the WW scattering cross sections with such ξ couplings over the TeV scale. These exhibit *different behaviors* from the pure SM result ($\xi = 0$), so they will be discriminated at the LHC (14 TeV) with higher integrated luminosity. The future TeV linear colliders (ILC and CLIC) and the future high energy circular pp colliders (TLEP and SPPC) will further probe the ξ coupling via WW scattering experiments.

As a final remark, we note that Ref. [21] studied linearized gravity in Jordan frame, and calculated scattering amplitudes of graviton exchange for external particles being spin-($0, 1/2, 1$). It considered a nonminimal coupling term $\xi R\phi^2$, with scalar ϕ having no vacuum expectation value. It obtained a unitarity bound in the form [21] $|\xi| < \mathcal{O}(M_{\rm Pl}/E)$, which has a similar structure to our (17) and (18). But our independent analysis for the realistic SM Higgs doublet in the Einstein frame is highly nontrivial, where the nonminimal ξ term (3) is transformed away and the resultant ξ-dependent new interactions (8) and (9) do not explicitly invoke gravitons. Thus, we can compute the longitudinal/Goldstone scattering amplitudes more easily and extract ξ-dependent terms in a straightforward way. Another advantage is that the Einstein frame has a canonically normalized graviton field, and the tree-level Lagrangian manifestly preserves the equivalence principle. The Einstein frame has also been widely used, including various models of Higgs inflation [6,7]. We note that the unitarity issue of nonminimal coupling was discussed in the Einstein frame for the purpose of Higgs inflation models before [7], but those discussions are mainly power counting arguments or qualitative estimates with rather different focus and context. Our current work presents systematical and quantitative unitarity analysis of Higgs-curvature interaction in the Einstein frame, with physical applications to the weak boson scattering at the TeV scale and in light of the exciting LHC Higgs discovery [1,2]. Especially, *for the first time*, we newly demonstrated that the LHC can probe ξ coupling via weak boson scattering experiments.

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[9] We note that in the Jordan frame, the nonminimal coupling (3) will induce kinematical mixing between graviton and Higgs fields. Its diagonalization leads to exactly the same Higgs rescaling $h^0 \to \zeta h^0$, and thus the Higgs mass formula, $m_h^2 = 2\lambda v^2 \zeta^2$.

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[11] Here, we clarify the interpretation and importance of perturbative unitarity bound *in general*, although it is

nothing special to our study. In our paper, we take the tree-level unitarity violation as a signal of requiring higher order nonperturbative effects and the inclusion of new resonance(s) in the effective field theory (EFT). Our analysis follows the well established standard method of B. W. Lee, C. Quigg, and H. B. Thacker, Phys. Rev. D **16**, 1519 (1977). This is well justified, because the Lee-Quigg-Thacker bound puts an upper limit on the SM Higgs mass, $m_h < \sqrt{8\pi/3}v \simeq 712$ GeV, which has been well obeyed and supported by the new LHC discovery [1,2] of a Higgs boson with mass $m_h \simeq 125$ GeV (<712 GeV). In another example, the low energy nonlinear chiral Lagrangian of QCD suffers a tree-level unitarity bound from $\pi\pi$ scattering. It gives $E < \sqrt{8\pi}f_\pi \simeq 466$ MeV for the $(I, J) = (0, 0)$ channel and $E < \sqrt{24\pi}f_\pi \simeq 808$ MeV for the $(I, J) = (1, 1)$ channel, where the second bound is close to the 1 GeV scale. Indeed, both bounds are nicely obeyed and supported by experimental data. Note that the current Particle Data Book [Phys. Rev. D **86**, 010001 (2012)] displays a new broad scalar resonance around 400–550 MeV in the (0, 0) channel, named σ meson or $f_0(500)$, while the (1, 1) channel has the well-known $\rho(770)$ meson with mass 775.5 MeV. The states $\sigma[f_0(500)]$ and $\rho(770)$ are *the new physics* for the low energy EFT of the chiral Lagrangian with pions. We want to clarify that although an exact S-matrix would be unitary, it is practically impossible to compute the exact S-matrix in any realistic theory. Within any nonrenormalizable EFT (including low energy chiral perturbation theory and Einstein gravity), there is no guarantee that adding 1-loop corrections would relax the tree-level unitarity bound; there is still no guarantee by adding 2-loop and beyond, *because every loop-order contains additional new counter-terms with fully unknown coefficients*. Furthermore, there is no unique way to sum up higher loop corrections because of extra new counter-terms (with unknown coefficients) at every loop-order, as is well-known. Without extra assumptions, there is no reliable way to sum up higher loops by ignoring all these new counter-terms in the EFT. In conclusion, the perturbative unitarity bounds à la Lee-Quigg-Thacker method are *fully justified and important*.

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[13] We clarify that the dashed line shown in Fig. 1 only means to compare the current LHC bound (derived from the fully different process such as $gg \to h$) with our unitarity bound at *each given energy E of WW scattering*. Clearly, this LHC bound has nothing to do with the WW scattering energy E shown on the horizontal axis of Fig. 1. When future measurements of Higgs production at the LHC (14 TeV) give a refined ξ bound, we can place another dashed line in Fig. 1 for comparison. Furthermore, we want to clarify that this LHC bound was derived by using the rescaling factor $\zeta = (1 + 6\xi^2 v^2/M_{\rm Pl}^2)^{-1/2}$ for the Higgs kinetic term (7), where its deviation from the SM value, $\zeta - 1$, has no energy dependence. The numerical bound (dashed line) shown in Fig. 1 is derived from the current LHC(7 + 8 TeV) data by simply rescaling the Higgs production cross section (such as $gg \to h$) by the *constant factor* of ζ^2 [cf. Eq. (10) of Ref. [8]], which has its leading term $\zeta^2 = 1$ from the SM contribution. The future LHC runs at 14 TeV will improve the measurements of the Higgs cross sections and thus may put a tighter bound on ζ^2, but it only affects ξ coupling via subleading terms in ζ^2 and thus will not be very sensitive to ξ, as expected. On the contrary, our unitarity bound in Fig. 1 is derived from the WW scattering amplitudes (15) and (16) with the net ξ^2E^2 (ξE^2) enhancements, where *the SM contribution vanishes identically*, and thus our unitarity bound is truly sensitive to ξ with the increase of WW scattering energy E.

[14] For this effective theory study, we do not concern with any detail of the UV dynamics above $\Lambda_{\rm UV} = \mathcal{O}(10~{\rm TeV})$. There exist well-defined TeV scale quantum gravity theories on the market. An explicit example is an extra-dimensional scenario with a compactification scale of $\mathcal{O}(10~{\rm TeV})$, and the Kaluza-Klein modes will reveal themselves at energies above $\mathcal{O}(10~{\rm TeV})$.

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[16] We note that the renormalization of ξ coupling will cause its running via the renormalization group (RG) equation [5,7]. Since the ξ running is logarithmic and our Fig. 2 has the relevant scattering energies ($0.2 < E < 4$ TeV) vary within only 1 order of magnitude for the LHC, we find that the RG running effects are generally small. As we have explicitly verified, the logarithmic running effect of ξ is negligible in Fig. 2.

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