How conventional measures overestimate electroweak fine-tuning in supersymmetric theory

Howard Baer,[1,*] Vernon Barger,[2,†] and Dan Mickelson[1,‡]

[1]*Department of Physics and Astronomy, University of Oklahoma, Norman, Oklahoma 73019, USA*
[2]*Department of Physics, University of Wisconsin, Madison, Wisconsin 53706, USA*
(Received 17 September 2013; published 18 November 2013)

The lack of evidence for superparticles at the CERN LHC, along with the rather high value of the Higgs boson mass, has sharpened the perception that what remains of supersymmetric model parameter space suffers a high degree of electroweak fine-tuning (EWFT). We compare three different measures of fine-tuning in supersymmetric models. First, Δ_{HS} measures a subset of terms containing large log contributions to m_Z (and m_h) that are inevitable in models defined at scales much higher than the electroweak scale. Second, the traditional Δ_{BG} measures fractional variation in m_Z against fractional variation of model parameters and allows for *correlations* among high scale parameters which are not included in Δ_{HS}. Third, the model-independent Δ_{EW} measures how naturally a model can generate the measured value of $m_Z = 91.2$ GeV (or m_h) in terms of *weak scale parameters* alone. We hypothesize an overarching ultimate theory (UTH) wherein the high scale soft terms are all correlated. The UTH might be contained within the more general effective supersymmetry theories which are popular in the literature. In the case of Δ_{HS}, EWFT can be grossly overestimated by neglecting additional nonindependent terms which lead to large cancellations. In the case of Δ_{BG}, EWFT can be overestimated by applying the measure to the effective theories instead of to the UTH. The measure Δ_{EW} allows for the possibility of parameter correlations which should be present in the UTH and, since it is model independent, provides the same value of EWFT for the effective theories as should occur for the UTH. We find that the well-known minimal supergravity model/constrained minimal supersymmetric model is fine-tuned under all three measures so that it is unlikely to contain the UTH. The nonuniversal Higgs model NUHM2 appears fine-tuned with $\Delta_{HS,BG} \gtrsim 10^3$. But since Δ_{EW} can be as small as 7 (corresponding to 14% fine-tuning), it may contain the UTH for parameter ranges which allow for low true EWFT.

DOI: 10.1103/PhysRevD.88.095013 PACS numbers: 12.60.Jv, 14.80.Va, 14.80.Ly

I. INTRODUCTION

The recent discovery of a Standard Model (SM)-like Higgs boson with mass $m_h = 125.5 \pm 0.5$ GeV [1,2] at the LHC seemingly provides credence to the simplest SUSY models of particle physics [3,4], which had predicted $m_h \lesssim 135$ GeV [5]. On the other hand, no sign of supersymmetric matter has yet emerged at the LHC, leading to mass limits $m_{\tilde{g}} \gtrsim 1.5$ TeV (for $m_{\tilde{g}} \simeq m_{\tilde{q}}$) and $m_{\tilde{g}} \gtrsim 1$ TeV (for $m_{\tilde{g}} \ll m_{\tilde{q}}$) [6,7]. These limits, obtained within the context of popular models such as minimal supergravity model/constrained minimal supersymmetric model (mSUGRA/CMSSM) [8] or simplified models, are qualitatively also valid in many other frameworks as long as we understand that the squark mass limit refers to first-generation squarks. The squark and gluino mass limits have caused considerable concern since for many years the storyline has been promoted that, in order to maintain naturalness in SUSY models, sparticles ought to be well below the TeV scale [9–28]. Indeed, the absence of any hint of deviations from the SM in the LHC8 data has led some to question whether supersymmetry (SUSY) could be the solution to the naturalness problem of the SM.

The fine-tuning situation in the minimal supersymmetric Standard Model (MSSM) is further exacerbated by the uncomfortably large value of the newly discovered Higgs particle: its value $m_h \simeq 125$ GeV lies well beyond its tree-level upper bound $m_h \leq m_Z$. Radiative corrections can accommodate $m_h \simeq 125$ GeV but only at the expense of either, having top squark masses beyond the TeV scale along with large mixing [29], or else, enlarging the MSSM to contain additional contributions to m_h [30–32]. The first of these possibilities again seems in violation of naturalness limits, which according to many studies require $m_{\tilde{t}_{1,2}}$, $m_{\tilde{b}_1} \lesssim 500$ GeV [33–36].

Thus, the question arises: are SUSY models now unnatural, and, if so, how unnatural are they? Or, do there exist portions of parameter space where SUSY remains natural? If so, a credible goal of collider [37,38] and dark matter [39] search experiments is to conduct a thorough search for natural SUSY.

In this paper, we compare and contrast three different measures of SUSY naturalness: 1) Δ_{HS} measures a subset of terms containing large log contributions to m_Z (and m_h) that are inevitable in models defined at scales much higher than the electroweak scale. 2) The traditional Δ_{BG} measures fractional variation in m_Z against fractional variation

*baer@nhn.ou.edu
†barger@pheno.wisc.edu
‡mickelso@nhn.ou.edu

of model parameters and allows for *correlations* among high scale parameters which are not included in Δ_{HS}. 3) The model-independent Δ_{EW} measures how naturally a model can generate the measured value of $m_Z = 91.2$ GeV (or m_h) in terms of *weak scale parameters* alone. Low values of Δ_i ($i = HS$, BG or EW) mean low fine-tuning; e.g., $\Delta_i = 100$ corresponds to $\Delta_i^{-1} = 1\%$ electroweak fine-tuning (EWFT).

For illustrative purposes, we apply these measures to two popular high scale SUSY models: the paradigm mSUGRA/CMSSM model [8] based on the parameter set

$$m_0, \quad m_{1/2}, \quad A_0, \quad \tan\beta, \quad \mathrm{sign}(\mu) \tag{1}$$

and the more general two-extra-parameter nonuniversal Higgs model NUHM2 [40] defined by the parameter set

$$m_0, \quad m_{1/2}, \quad A_0, \quad \tan\beta, \quad \mu, \quad m_A \tag{2}$$

[where we have traded the grand unified theory (GUT) scale soft SUSY breaking Higgs mass parameters $m_{H_u}^2$ and $m_{H_d}^2$ for the weak scale parameters μ and m_A for convenience]. We find the measures ordered according to

$$\Delta_{EW} < \Delta_{BG} \lesssim \Delta_{HS}. \tag{3}$$

We argue that the semi-model-independent Δ_{HS} omits non-independent terms from its measure, which leads to large cancellations giving rise to an overestimate of EWFT. The measure Δ_{BG} properly combines these correlated terms so as to avoid the pitfall contained within Δ_{HS}. To interpret Δ_{BG} properly, we hypothesize an overarching ultimate theory for which the low energy limit for $Q < \Lambda = m_{\mathrm{GUT}}$ is the MSSM wherein the high scale soft terms are all correlated (hereafter referred to as the UTH). The UTH might be contained within the more general effective SUSY theories, which are popular in the literature. Examples include the mSUGRA/CMSSM model and the NUHM2 model. In the case of Δ_{BG}, EWFT can be overestimated by applying the measure to the effective theories instead of to the UTH. The measure Δ_{EW} allows for the possibility of parameter correlations which should be present in the UTH and, since it is model-independent, provides the same value of Δ_{EW} for the effective theories as should occur for the UTH. We find that the well-known mSUGRA/CMSSM model is fine-tuned under all three measures so that it cannot contain the UTH. The nonuniversal Higgs model NUHM2 appears fine-tuned with $\Delta_{HS,BG} \gtrsim 10^3$. But since Δ_{EW} can be as small as 7 (corresponding to 14% fine-tuning), it may contain the UTH for the range of parameter choices which allow for low true EWFT.

The low Δ_{EW} models are characterized by a superpotential μ term with $|\mu| \sim m_Z \sim 100$–300 GeV. This leads to a prediction of light Higgsino states $\tilde{W}_1^{\pm}$, $\tilde{Z}_2$, and $\tilde{Z}_1$ with mass $\sim$100–300 GeV, which, due to their compressed spectra, may easily elude LHC searches but which should be accessible to an e^+e^- collider with $\sqrt{s} \gtrsim 2|\mu| \sim$ 500–600 GeV.

In Sec. II, we define and review the measures Δ_{HS}, Δ_{BG}, and Δ_{EW}, which were mentioned above. In Sec. III, we evaluate the three measures as a function of parameters in the mSUGRA/CMMSM model. We repeat our evaluation for the NUHM2 model in Sec. IV. In Sec. V, we interpret our results in terms of an overarching UTH. In Sec. VI, we present our general conclusion which is that the conventional measures of naturalness Δ_{HS} and Δ_{BG} lead to large overestimates of EWFT in supersymmetric theory. Parameter choices exist within the NUHM2 model (and of course other more general models), which lead to low Δ_{EW} and about one part in ten EWFT. Such parameter choices should be a guide to model builders seeking to find the correct UTH which predicts their values in terms of few or even no adjustable parameters.[1]

II. THREE FINE-TUNING MEASURES

A. Δ_{HS}

1. Standard Model

In the SM, with a Higgs potential given by $V = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$ where ϕ is the Higgs doublet, one may calculate the physical mass of the Higgs boson m_h as

$$m_h^2 = m_h^2|_{\mathrm{tree}} + \delta m_h^2|_{\mathrm{rad}}, \tag{4}$$

where $m_h^2|_{\mathrm{tree}} = \sqrt{2}\mu^2$ and $\delta m_h^2|_{\mathrm{rad}} = \frac{c}{16\pi^2}\Lambda^2$, and where Λ represents the cutoff of quadratically divergent loop diagrams which provides an upper limit to which the SM is considered a valid effective field theory. The coefficient c depends on the various SM couplings, and here will be taken $c \sim 1$ (e.g., the top quark loop gives $c = -6f_t^2$, where f_t is the top quark Yukawa coupling). Since $m_h^2|_{\mathrm{tree}}$ and $\delta m_h^2|_{\mathrm{rad}}$ are *independent*, we would expect naturally that $m_h^2 \sim m_h^2|_{\mathrm{tree}} > \delta m_h^2|_{\mathrm{rad}}$ since otherwise if $\delta m_h^2|_{\mathrm{rad}} \gg m_h^2$, then $m_h^2|_{\mathrm{tree}}$ will have to be fine-tuned to a high degree to obtain m_h of just $\sim$125 GeV. We may define a fine-tuning measure

$$\Delta_{\mathrm{SM}} \equiv \delta m_h^2|_{\mathrm{rad}}/(m_h^2/2), \tag{5}$$

which compares the radiative correction to the physical Higgs boson mass. Requiring $\Delta_{\mathrm{SM}} \lesssim 1$ then requires $\Lambda \sim 1$ TeV; i.e., the SM should only be valid up to at most the TeV scale.

2. MSSM

Analogous reasoning has been applied to supersymmetric models [33]. In the MSSM, then

$$m_h^2 \simeq \mu^2 + m_{H_u}^2 + \delta m_{H_u}^2|_{\mathrm{rad}}, \tag{6}$$

where now μ is the superpotential Higgs/Higgsino mass term and $m_{H_u}^2$ is the up-type soft SUSY breaking Higgs mass evaluated at $m_{\mathrm{SUSY}} \sim 1$ TeV. In gravity mediation, then m_{H_u} is expected $\sim m_{3/2} \sim 1$ TeV. The largest

[1] An example along these lines is provided in Ref. [41].

contributions to $\delta m^2_{H_u}|_{\rm rad}$ contain divergent logarithms; these can be found by integrating the renormalization group equation [42] for $m^2_{H_u}$:

$$\frac{dm^2_{H_u}}{dt} = \frac{1}{8\pi^2}\left(-\frac{3}{5}g_1^2 M_1^2 - 3g_2^2 M_2^2 + \frac{3}{10}g_1^2 S + 3f_t^2 X_t\right), \tag{7}$$

where $t = \ln(Q^2/Q_0^2)$, $S = m^2_{H_u} - m^2_{H_d} + {\rm Tr}[\mathbf{m}^2_Q - \mathbf{m}^2_L - 2\mathbf{m}^2_U + \mathbf{m}^2_D + \mathbf{m}^2_E]$, and where $X_t = m^2_{Q_3} + m^2_{U_3} + m^2_{H_u} + A_t^2$. By neglecting gauge terms and S ($S = 0$ in models with scalar soft term universality) and also neglecting the $m^2_{H_u}$ contribution to X_t and the fact that f_t and the soft terms evolve under Q^2 variation, then this expression may be readily integrated from $m_{\rm SUSY}$ to the cutoff Λ to obtain

$$\delta m^2_{H_u}|_{\rm rad} \sim -\frac{3f_t^2}{8\pi^2}(m^2_{Q_3} + m^2_{U_3} + A_t^2)\ln(\Lambda^2/m^2_{\rm SUSY}). \tag{8}$$

Inspired by gauge coupling unification, Λ may be taken as high as $m_{\rm GUT} \simeq 2\times 10^{16}$ GeV or even the reduced Planck mass $m_P \simeq 2.4\times 10^{18}$ GeV. Also, we take $m^2_{\rm SUSY} \simeq m_{\tilde t_1} m_{\tilde t_2}$. One may again create a fine-tuning measure $\Delta \equiv \delta m^2_{H_u}/(m_h^2/2)$.

Two related dangers are contained within this approach, which are different from the case of the SM:

(i) The first is that $m^2_{H_u}$ and $\delta m^2_{H_u}|_{\rm rad}$ are not independent; the value of $m^2_{H_u}$ feeds directly into evaluation of $\delta m^2_{H_u}|_{\rm rad}$ via the X_t term. It also feeds indirectly into $\delta m^2_{H_u}|_{\rm rad}$ by contributing to the evolution of the $m^2_{Q_3}$ and $m^2_{U_3}$ terms. In fact, the larger the value of $m^2_{H_u}(\Lambda)$, then the larger is the cancelling correction $\delta m^2_{H_u}|_{\rm rad}$. We return to this issue later.

(ii) The second is that, whereas $SU(2)_L\times U(1)_Y$ gauge symmetry can be broken at tree level in the SM, in the SUSY case, $m^2_{H_u} \sim m_{3/2} > 0$, and electroweak (EW) symmetry is not even broken until one includes radiative corrections. For high scale SUSY models, EW symmetry is broken radiatively by $m^2_{H_u}$ being driven to large negative values. This suggests a regrouping of terms [43,44]:

$$m_h^2|_{\rm phys} = \mu^2 + (m^2_{H_u}(\Lambda) + \delta m^2_{H_u}), \tag{9}$$

where instead both μ^2 and $(m^2_{H_u} + \delta m^2_{H_u})$ should be comparable to $m_h^2|_{\rm phys}$.

Nonetheless, using the measure Δ, Eq. (8) may be rearranged to provide a bound on third-generation squarks [33,34,36]:

$$\sqrt{m^2_{\tilde t_1} + m^2_{\tilde t_2}} \lesssim 600\ {\rm GeV}\frac{\sin\beta}{\sqrt{1+R_t^2}}\left(\frac{\log\frac{\Lambda}{\rm TeV}}{3}\right)^{-1/2}\left(\frac{\Delta}{5}\right)^{1/2}, \tag{10}$$

where $R_t = A_t/\sqrt{m^2_{\tilde t_1} + m^2_{\tilde t_2}}$. Taking $\Delta = 10$ (i.e., $\Delta^{-1} =$ 10% EWFT) and Λ as low as 20 TeV corresponds to

(i) $m_{\tilde t_i}, m_{\tilde b_1} \lesssim 600$ GeV,

(ii) $m_{\tilde g} \lesssim 1.5$–2 TeV.

The last of these conditions arises because the squark radiative corrections $\delta m^2_{\tilde t_i} \sim (2g_s^2/3\pi^2)m^2_{\tilde g}\times\log\Lambda$. Setting the log to unity and requiring $\delta m^2_{\tilde t_i} < m^2_{\tilde t_i}$ then implies $m_{\tilde g} \lesssim 3m_{\tilde t_i}$, or $m_{\tilde g} \lesssim 1.5$–2 GeV for $\Delta \lesssim 10$. Taking Λ as high as $m_{\rm GUT}$ leads to even tighter constraints: $m_{\tilde t_{1,2}}, m_{\tilde b_1} \lesssim$ 200 GeV and $m_{\tilde g} \lesssim 600$ GeV, almost certainly in violation of LHC sparticle search constraints. Since (degenerate) first-/second-generation squarks and sleptons enter the Higgs potential only at the two loop level, these can be much heavier: beyond LHC reach and also possibly heavy enough to provide a (partial) decoupling solution to the SUSY flavor and *CP* problems [45].

To bring the fine-tuning measure Δ into closer accord with the measures described below, we write it in terms of $m_Z^2/2$ instead of in terms of $m_h^2/2$. The minimization condition for the Higgs potential $V_{\rm tree} + \Delta V$ in the MSSM reads

$$\frac{m_Z^2}{2} = \frac{m^2_{H_d} + \Sigma_d^d - (m^2_{H_u} + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2, \tag{11}$$

where Σ_u^u and Σ_d^d are radiative corrections that arise from the derivatives of ΔV evaluated at the minimum. The radiative corrections Σ_u^u and Σ_d^d include contributions from various particles and sparticles with sizeable Yukawa and/or gauge couplings to the Higgs sector. Expressions for the Σ_u^u and Σ_d^d are given in the Appendix of Ref. [43]. We may include explicit dependence on the high scale Λ at which the SUSY theory may be defined, by writing the *weak scale* parameters $m^2_{H_{u,d}}$ as

$$m^2_{H_{u,d}} = m^2_{H_{u,d}}(\Lambda) + \delta m^2_{H_{u,d}}; \qquad \mu^2 = \mu^2(\Lambda) + \delta\mu^2, \tag{12}$$

where $m^2_{H_{u,d}}(\Lambda)$ and $\mu^2(\Lambda)$ are the corresponding parameters renormalized at the high scale Λ. The $\delta m^2_{H_{u,d}}$ terms contain the $\log\Lambda$ dependence emphasized in constructs of natural SUSY models [33,34,36]. Thus, one obtains

$$\frac{m_Z^2}{2} = \frac{(m^2_{H_d}(\Lambda) + \delta m^2_{H_d} + \Sigma_d^d) - (m^2_{H_u}(\Lambda) + \delta m^2_{H_u} + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - (\mu^2(\Lambda) + \delta\mu^2). \tag{13}$$

We can now define a fine-tuning measure that encodes the information about the high scale origin of the parameters by requiring that each of the terms on the right-hand side of Eq. (13) (normalized to $m_Z^2/2$) be smaller than a value Δ_{HS}. The high scale fine-tuning measure Δ_{HS} is thus defined to be

$$\Delta_{HS} \equiv \max{}_i |B_i|/(m_Z^2/2), \tag{14}$$

with $B_{H_d} \equiv m_{H_d}^2(\Lambda)/(\tan^2\beta - 1)$, etc. In models such as mSUGRA, for which the domain of validity extends to very high scales, because of the large logarithms, one would expect that the $B_{\delta H_u}$ contributions to Δ_{HS} would be the dominant term.

An advantage of Δ_{HS} over Δ is that the dominant term $B_{\delta H_u}$ is extracted now from the renormalization group equation (RGE) solution and thus includes large logs arising from gauge terms as well as the effect of running parameters which are not contained in Eq. (8). However, it still maintains the split among the *dependent* terms $m_{H_u}^2(\Lambda)$ and $\delta m_{H_u}^2$.[2]

B. Δ_{BG}

The fine-tuning measure Δ_{BG} can be regarded as the traditional measure, in use now for over 25 years [9,10,13]. We start again with the scalar potential minimization condition (this time at tree level)

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1} - \mu^2 \simeq -m_{H_u}^2 - \mu^2, \tag{15}$$

where the latter partial equality obtains for moderate-to-large $\tan\beta$ values. The traditional measure is then defined as

$$\Delta_{BG} \equiv \max{}_i[c_i] \quad \text{where } c_i = \left|\frac{\partial \ln m_Z^2}{\partial \ln a_i}\right| = \left|\frac{a_i}{m_Z^2}\frac{\partial m_Z^2}{\partial a_i}\right|, \tag{16}$$

where the a_i constitute the fundamental parameters of the model. Thus, Δ_{BG} measures the fractional change in m_Z^2 due to fractional variation in high scale parameters a_i. The c_i are known as *sensitivity coefficients* [46].

An advantage of Δ_{BG} over Δ_{HS} or Δ is that it maintains the correlation between $m_{H_u}^2(\Lambda)$ and $\delta m_{H_u}^2$ by replacing $m_{H_u}^2(m_{\mathrm{SUSY}}) = m_{H_u}^2(\Lambda) + \delta m_{H_u}^2$ by its expression in terms of high scale parameters. To evaluate Δ_{BG}, one needs to know the explicit dependence of $m_{H_u}^2$ and μ^2 on the fundamental parameters. Expressions can be gained by semianalytic solutions to the one-loop RGEs, as found for instance in Ref. [47]. In the case where $\tan\beta = 10$, it is found in Refs. [46,48,49] that

$$-2\mu^2(m_{\mathrm{SUSY}}) = -2.18\mu^2 \tag{17}$$

$$\begin{aligned} -2m_{H_u}^2(m_{\mathrm{SUSY}}) = {} & 3.84M_3^2 + 0.32M_3M_2 + 0.047M_1M_3 - 0.42M_2^2 \\ & + 0.011M_2M_1 - 0.012M_1^2 - 0.65M_3A_t - 0.15M_2A_t \\ & - 0.025M_1A_t + 0.22A_t^2 + 0.004M_3A_b \\ & - 1.27m_{H_u}^2 - 0.053m_{H_d}^2 \\ & + 0.73m_{Q_3}^2 + 0.57m_{U_3}^2 + 0.049m_{D_3}^2 - 0.052m_{L_3}^2 + 0.053m_{E_3}^2 \\ & + 0.051m_{Q_2}^2 - 0.11m_{U_2}^2 + 0.051m_{D_2}^2 - 0.052m_{L_2}^2 + 0.053m_{E_2}^2 \\ & + 0.051m_{Q_1}^2 - 0.11m_{U_1}^2 + 0.051m_{D_1}^2 - 0.052m_{L_1}^2 + 0.053m_{E_1}^2, \end{aligned} \tag{18}$$

where the parameters on the right-hand side are evaluated at the GUT scale. For different values of $\tan\beta$, then somewhat different relations are obtained. At this point, the derivatives in Eq. (16) can be explicitly evaluated so that Δ_{BG} can be easily computed.

1. *Importance of high scale correlations*

An important difference between Δ_{HS} and Δ_{BG} is that the latter combines the dependent terms $m_{H_u}^2(\Lambda)$ and $\delta m_{H_u}^2$ which were separated in Δ_{HS}. Including these allows for *cancellations* between various terms which occur if certain correlations between HS parameters arise in the model under consideration. For instance, in lines 6 and 7 of Eq. (18), if we impose

$$m_{Q_{1,2}} = m_{U_{1,2}} = m_{D_{1,2}} = m_{L_{1,2}} = m_{E_{1,2}} \equiv m_{16}(1,2) \tag{19}$$

as might be expected in an $SO(10)$ GUT theory, then each line collapses to $\sim 0.007m_{16}^2(1,2)$; the various terms now conspire via cancellations to yield much less fine-tuning than otherwise might be expected.

More importantly, if

$$m_{H_u}^2 = m_{H_d}^2 = m_{16}^2(3) \equiv m_0^2 \tag{20}$$

as is imposed in models with scalar mass universality, then lines 4 and 5 of Eq. (18) conspire to yield a term $\sim -0.017m_0^2$, which again yields far less fine-tuning in the third-generation sector than one might otherwise expect due to cancellations of terms, many of which contain the large logs which are measured by Δ_{HS}. This latter

[2]The possibility of models with low Δ_{HS} is explored within the context of GUT models with nonuniversal gaugino masses [19] and the 19-parameter SUGRA model [20]. Low Δ_{HS} requires small m_{H_u} ($\Lambda = m_{\mathrm{GUT}}$) and then minimal evolution of $m_{H_u}^2$ between m_{GUT} and m_{SUSY}. The low Δ_{HS} models tend to have sub-TeV top squarks, which lead typically to large deviations in $BF(b \to s\gamma)$.

case is usually referred to as "focus point SUSY" [14,50]; it provides a concrete example that in the case of very heavy top squarks, the fine-tuning which follows from Δ_{HS} may be a large overestimate.[3] Further cancellations among terms in Eq. (18) can occur when the A_t parameters obey certain relations to $m_{1/2}$. Thus, the allowance for cancellations in the log terms of Δ_{BG} gives rise to the expectation that

$$\Delta_{BG} \lesssim \Delta_{HS}. \tag{21}$$

2. Model dependence of Δ_{BG}

At this point it is important to note that, while Eq. (18) provides a good example of how large log and other cancellations can occur due to HS parameter correlations, it is not at all clear that usage of Eq. (18) is the correct way to proceed. There is often dispute in the literature as to which parameters should be included in the set a_i which enters into the evaluation of Δ_{BG}. Surely the high scale soft SUSY breaking parameters would be included, but should also, e.g., the top quark Yukawa coupling f_t or other Yukawa or even gauge couplings be included?[4] Furthermore, shall one use the Lagrangian trilinear soft SUSY breaking parameter a_t as occurs in $\mathcal{L} \ni a_t \epsilon_{ab} \tilde{Q}_3^a H_u^b \tilde{u}_{R3}^\dagger$ or the more common A_t where $a_t = f_t A_t$? Different prescriptions as to what one includes in the "fundamental parameters" a_i will lead to different expressions for m_Z^2 in terms of the a_i.

A further concern with Δ_{BG} is that different models with exactly the same weak scale spectra can give rise to wildly different values of Δ_{BG}. We will see that in the HB/FP region [50] of the mSUGRA model, Δ_{BG} can be greatly reduced due to $m_{H_u} = m_{H_d} \equiv m_0$ at the GUT scale. Yet, using the exact same input parameters within the NUHM2 model (or any other model with greater parameter freedom which contains mSUGRA as a subset), then the value of Δ_{BG} will be quite a bit larger. An example is given in Table I, which lists the various sensitivity coefficients of the Δ_{BG} measure for mSUGRA and for NUHM2 but where the mSUGRA output values of μ and m_A are used as inputs to NUHM2. In this case, the two models have exactly the same weak scale spectra. But due to the greater correlations amongst HS parameters present in mSUGRA, the value of Δ_{BG} has dropped by an order of magnitude compared to NUHM2.

Here, it must be remembered that models like mSUGRA, NUHM2, etc., are to be regarded as *effective theories* valid up to $\Lambda = M_{\rm GUT}$ and where the parameters parametrize our ignorance of high scale physics such as the mechanism for SUSY breaking. It is usually regarded that such SUSY GUT models are the low energy effective field theories of some more encompassing theory (ultimate theory, or perhaps string theory) where further parameter correlations are to be expected, or perhaps there are no free parameters. In such a case, the effective theory may look fine-tuned while the high scale correlations present in the UTH lead to little or no fine-tuning.

TABLE I. Sensitivity coefficients and Δ_{BG} for the mSUGRA and NUHM2 models with $m_0 = 9993.4$ GeV, $m_{1/2} = 691.7$ GeV, $A_0 = -4788.6$ GeV, and $\tan\beta = 10$. The mSUGRA output values of $\mu = 309.7$ GeV and $m_A = 9859.9$ GeV serve as NUHM2 inputs so that the two models have exactly the same weak scale spectra.

Model	c_{m_0}	$c_{m_{1/2}}$	c_{A_0}	c_μ	c_{H_u}	c_{H_d}	Δ_{BG}
mSUGRA	156	762	1540	−25.1	⋯	⋯	1540
NUHM2	16041	762	1540	−25.1	−15208	−643.6	16041

The fundamental lesson here is that examples exist where correlations among model parameters which are present in more restrictive theories, but not in the effective theory within which they are contained, lead to cancellations in contributions to EWFT. In such cases, one may gain a false impression as to the amount of EWFT needed in a theory. Is one then to give up on EWFT as a guide to a supersymmetric theory?

C. Δ_{EW}

A less ambitious, more conservative and model-independent, fine-tuning measure has been advocated in Refs. [43,44,51].[5] Starting again with the scalar potential minimization condition, this time including radiative corrections, we have

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2. \tag{22}$$

Noting that all entries in Eq. (22) are defined at the weak scale, the *electroweak fine-tuning measure*

$$\Delta_{EW} \equiv \max_i |C_i|/(m_Z^2/2) \tag{23}$$

may be constructed, where $C_{H_d} = m_{H_d}^2/(\tan^2\beta - 1)$, $C_{H_u} = -m_{H_u}^2 \tan^2\beta/(\tan^2\beta - 1)$, and $C_\mu = -\mu^2$. Also, $C_{\Sigma_u^u(k)} = -\Sigma_u^u(k)\tan^2\beta/(\tan^2\beta - 1)$ and $C_{\Sigma_d^d(k)} = \Sigma_d^d(k)/(\tan^2\beta - 1)$, where k labels the various loop contributions included in Eq. (22).

[3] Note that if $m_{H_u}^2(\Lambda)$ is subtracted out of Eq. (18) [as is done in Eq. (8)], then the nearly complete cancellation of Higgs and third-generation soft terms will no longer occur.

[4] Also, different papers will use varying powers of parameters as fundamental inputs. For instance, in mSUGRA, does one use m_0 or m_0^2? These differences lead to just factors of 2 in the evaluation of Δ_{BG}.

[5] The importance of low $|\mu| \sim m_Z$ was emphasized in Ref. [14]. Reference [50] also remarks that there be no large cancellation between $m_{H_u}^2$ and μ^2. References [21,52–54] essentially adopt weak scale fine-tuning. Reference [44] creates Δ_{EW} including radiative corrections and notes that large A_t suppresses radiative corrections while lifting the value of m_h.

Constructed in this way, it is clear that

$$\lim_{\Lambda\to m_{\rm SUSY}} \Delta_{HS} = \Delta_{EW}. \tag{24}$$

It can also be checked that

$$\lim_{\Lambda\to m_{\rm SUSY}} \Delta_{BG} \sim \Delta_{EW}, \tag{25}$$

since the most important terms in Eq. (22) appear linearly in $m_{H_u}^2$ and μ^2. Thus, we expect that

$$\Delta_{EW} < \Delta_{BG} \lesssim \Delta_{HS} \tag{26}$$

for any particular point in a given model parameter space.

The measure Δ_{EW} is created from weak scale SUSY parameters and so contains no information about any possible high scale origin, hence its model-independence. Since it evaluates the fine-tuning which remains upon taking the limit $\Lambda \to m_{\rm SUSY}$, it makes an allowance for cancellations of large logs which may enter into $m_{H_u}^2(m_{\rm SUSY})$. In this sense, Δ_{EW} captures the *minimal* amount of EWFT required of any SUSY model, including those defined at some high scale $\Lambda \gg m_{\rm SUSY}$. Δ_{EW} can be thought of as providing a *lower bound* on electroweak fine-tuning [55]. Any model with a large value of Δ_{EW} is always fine-tuned. However, if Δ_{EW} is low, it need not mean the model is not fine-tuned; rather, it allows for the possibility that some model might exist with low fine-tuning, which might be hidden by the naive application of either Δ_{HS} or Δ_{BG}. As such, low Δ_{EW} is a *necessary, albeit not sufficient*, measure of electroweak fine-tuning.

The quantity Δ_{EW} measures the largest *weak scale* contribution to the Z mass. Model parameter choices which lead to low values of Δ_{EW} are those which would naturally generate a value of $m_Z \sim 91.2$ GeV. In order to achieve low Δ_{EW}, it is necessary that $-m_{H_u}^2$, μ^2 and $-\Sigma_u^u$ all be nearby to $m_Z^2/2$ to within a factor of a few [43,44]; the low Δ_{EW} models are typified by the presence of light Higgsinos $\tilde{W}_1^\pm$, $\tilde{Z}_{1,2}$ with mass $\sim|\mu| \sim 100$–300 GeV.

1. Utility of Δ_{EW}

We have emphasized that Δ_{EW} is a measure of the *minimal fine-tuning* that is present in a given weak scale SUSY spectrum. While a model with a small value of Δ_{EW} is not necessarily free of fine-tuning, any model with a large value of Δ_{EW} is always fine-tuned.

The utility of Δ_{EW} arises from the fact that it is determined by just the weak scale spectrum [43]; i.e., different high scale theories that lead to the same sparticle spectrum will yield the same value of Δ_{EW}, even though these may have vastly different values of Δ_{HS} or Δ_{BG}. A small value of Δ_{EW} in some region of parameter space of a SUSY effective theory offers the possibility that there may exist an overarching UTH with essentially the same spectrum but for which the parameter correlations lead to small values of Δ_{BG}. This UTH would then be the underlying theory with low true EWFT. Since the broad features of the phenomenology are determined by the spectrum, we expect that the phenomenological consequences of the (unknown) UTH will be the same as for the more general effective theory, which includes the UTH as a special case.

III. Δ_i IN THE MSUGRA/CMSSM MODEL

To calculate superparticle mass spectra in SUSY models, we employ the Isajet 7.83 [56] SUSY spectrum generator Isasugra [57]. We begin with a scan over mSUGRA/CMSSM parameter space for a fixed value of $\tan\beta = 10$. Results for other $\tan\beta$ values are qualitatively similar. Then we scan over

$$\begin{gathered} m_0\colon 0\text{–}15\ \text{TeV}, \qquad m_{1/2}\colon 0\text{–}2\ \text{TeV}, \\ -2.5 < A_0/m_0 < 2.5. \end{gathered} \tag{27}$$

We will show results for both $\mu > 0$ and $\mu < 0$. For each solution generated, we require:

(1) electroweak symmetry to be radiatively broken (REWSB);
(2) that the neutralino $\tilde{Z}_1$ is the lightest MSSM particle;
(3) that the light chargino mass obeys the LEP2 limit that $m_{\tilde{W}_1} > 103.5$ GeV [58];
(4) $m_h = 125 \pm 2$ GeV (assuming ± 2 GeV theory error in the m_h calculation) in accord with the recent Higgs-like resonance discovery at LHC [1,2];
(5) that LHC search constraints on $m_{\tilde{q}}$ and $m_{\tilde{g}}$ are obeyed, where $m_{\tilde{g}} \gtrsim 1$ TeV for $m_{\tilde{g}} \ll m_{\tilde{q}}$ and $m_{\tilde{g}} \gtrsim 1.5$ TeV for $m_{\tilde{g}} \sim m_{\tilde{q}}$.[6]

For mSUGRA, all GUT scale soft SUSY breaking scalar masses are equal to m_0 while all gaugino masses equal $m_{1/2}$. In this case, from Eq. (18) we can calculate the Δ_{BG} sensitivity coefficients:

$$\begin{aligned} c_{m_{1/2}} &= (7.57 m_{1/2} - 0.821 A_0)(m_{1/2}/m_Z^2), \\ c_{m_0} &= 0.013(m_0^2/m_Z^2), \\ c_{A_0} &= (0.44 A_0 - 0.821 m_{1/2})(A_0/m_Z^2), \\ c_\mu &= -2.18(\mu^2/m_Z^2). \end{aligned} \tag{28}$$

Notice that in this model, since $m_{H_u} = m_0(3) \equiv m_0$, there are large cancellations in Eq. (18), which suppress the contribution to c_{m_0}.

In Fig. 1, we show Δ_{HS}, Δ_{BG}, and Δ_{EW} vs m_0 from our mSUGRA parameter space scan. In frame *a*), we see that Δ_{HS} is highly correlated with m_0. This is to be expected since the larger m_0 becomes, the larger the top squark contributions are to $\delta m_{H_u}^2$. Thus, Δ_{HS} prefers the lowest m_0 values possible. We also see that the minimal value of $\Delta_{HS} \sim 10^3$, corresponding to $\Delta^{-1} \sim 0.1\%$ fine-tuning at best. In frame *b*), we see that Δ_{BG} has a similar minimal value of $\Delta_{BG} \sim 10^3$, but the shape vs m_0 is very different.

[6] Explicit contours are shown in Ref. [51].

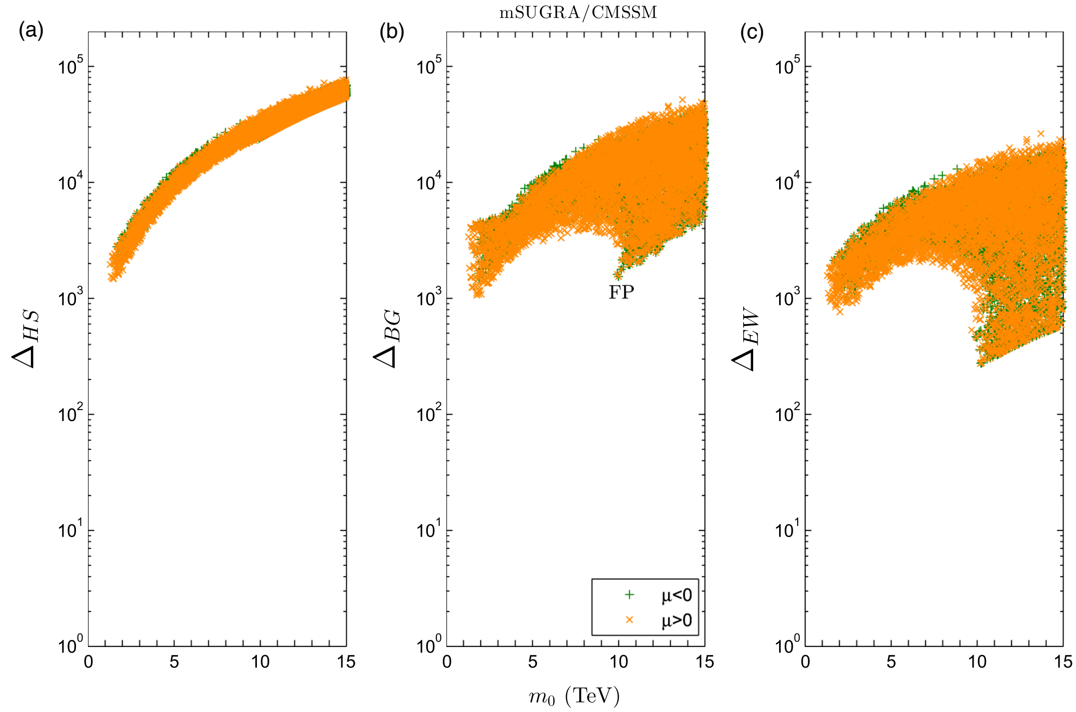

FIG. 1 (color online). Plot of (a) Δ_{HS}, (b) Δ_{BG}, and (c) Δ_{EW} vs m_0 from a scan over mSUGRA/CMSSM model parameter space for $\tan\beta = 10$. The location of the HB/FP regions is denoted FP in frame b).

One minimum occurs around $m_0 \sim 2$ TeV, while another minimum occurs at $m_0 \sim 9$ TeV. For $A_0 \neq 0$, the contours of μ increase with m_0, and c_μ dominates Δ_{BG}. At very high m_0, one begins approaching what is known as the hyperbolic branch/focus point region [14,50], where μ decreases with increasing m_0; this causes the dip around $m_0 \sim 9$ TeV and corresponds to reduced fine-tuning even when scalar masses are very heavy [50]. Note that even though the minimum of Δ_{BG} occurs around $m_0 \sim 9$ TeV, the minimal value is still $\Delta_{BG} \sim 10^3$, or at best $\sim 0.1\%$ EWFT. In frame c), we plot Δ_{EW} vs m_0. For mSUGRA, $\mu^2 \sim -m_{H_u}^2$ at the weak scale, and since μ^2 drops as one increases m_0 (for not too large A_0), then the HB/FP region has the lowest Δ_{EW}. Once m_0 exceeds ~ 10 TeV, then the Σ_u^u terms dominate, and Δ_{EW} again increases with increasing top squark masses. The minimum of Δ_{EW} is ~ 250, or $\sim 0.4\%$ fine-tuning in constructing m_Z. Thus, mSUGRA seems rather highly electroweak fine-tuned under all three measures.

In Fig. 2, we show the three Δ measures vs $m_{1/2}$. The minimum of Δ_{HS} is soft but occurs around ~ 1 TeV, as does the minimum of Δ_{BG}. However, the distributions are really quite diffuse, and for any $m_{1/2}$ value, a wide range of Δ_i values can occur. For Δ_{EW}, there seems no preference for any $m_{1/2}$ values, which is just a reflection that μ increases with m_0 and not $m_{1/2}$.

In Fig. 3, we show the Δ measures vs A_0/m_0. The first aspect of note is that no solutions occur for $A_0 \sim 0$, which is because no solutions with $m_h \sim 123$–127 GeV can be found in the minimal stop mixing region. The lowest Δ_{HS} values occur at largest $|A_0|$ values. This is because low Δ_{HS} prefers low m_0, and low m_0 can only give $m_h \sim 123$–127 GeV for highly mixed stops. For Δ_{BG}, one also gets a min at $A_0 \sim 0.5m_0$. This is again the HB/FP region, where low Δ_{BG} is found at high m_0, but at high m_0, not so much stop mixing is needed to obtain $m_h \sim 123$–127 GeV. As in Fig. 1c, the min of Δ_{EW} is found for $|A_0| \sim 0.5m_0$, again in the HB/FP region.

In the mSUGRA model, the value of $m_{H_u}^2(m_{\rm SUSY})$ is generated from its initial value m_0 at $m_{\rm GUT}$ followed by renormalization group (RG) evolution. The value of μ is then chosen by using Eq. (15) to determine what $\mu^2(m_{\rm SUSY})$ should have been in order to obtain the measured value of m_Z. In Fig. 4, we plot the Δ_i vs $|\mu(m_{\rm SUSY})|$. The lowest value of Δ_{HS} occurs around $\mu \sim 2$ TeV, which corresponds to values of μ where m_0 is minimal. The lowest values of μ in the 100–200 GeV range come from the HB/FP region, but in this region Δ_{HS} is very large, owing to the heavy top squarks. In frame b), we see Δ_{BG} is also split at low μ, but this time the Higgsino region (HB/FP) with $\mu \sim 100$–200 GeV is only slightly more fine-tuned than the lowest Δ_{BG} values. In frame c), the minimal Δ_{EW} occurs around $\mu \sim 1$ TeV, and again the deep Higgsino region (the region where the Higgsino components of $\tilde{Z}_1$ are dominant) has slightly larger values of Δ_{EW} owing to the large top squark masses in the HB/FP region; these lead to large Σ_u^u.

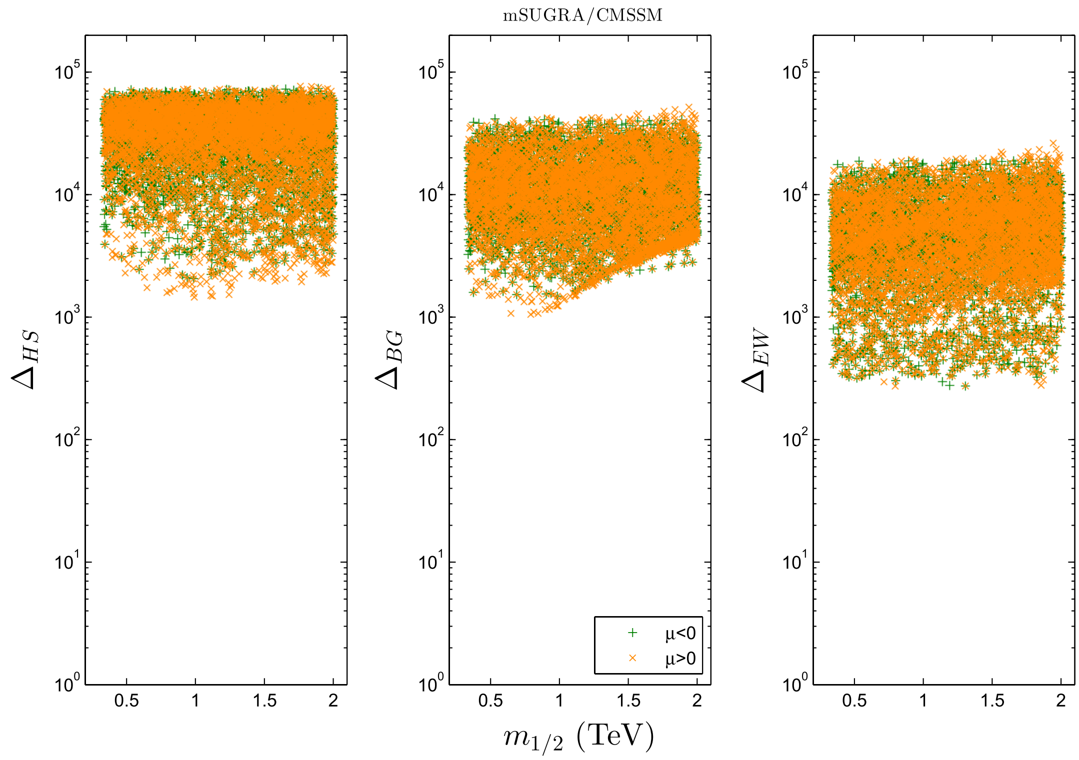

FIG. 2 (color online). Plot of Δ_{HS}, Δ_{BG}, and Δ_{EW} vs $m_{1/2}$ from a scan over mSUGRA/CMSSM model parameter space for $\tan\beta = 10$.

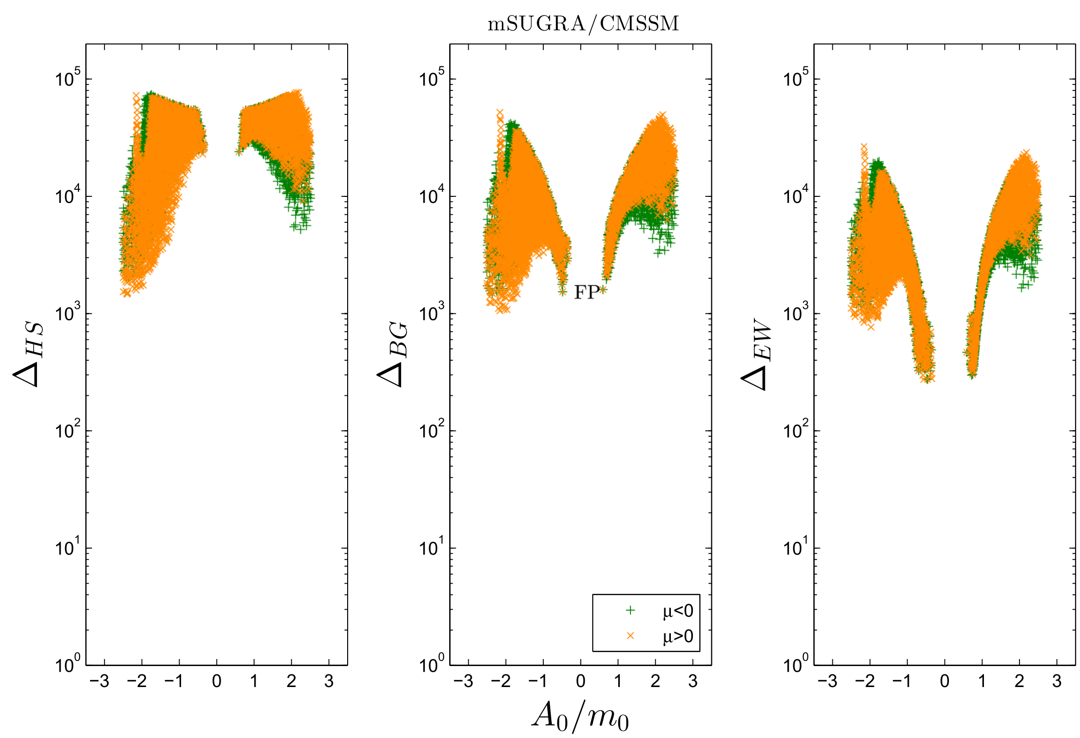

FIG. 3 (color online). Plot of Δ_{HS}, Δ_{BG}, and Δ_{EW} vs A_0/m_0 from a scan over mSUGRA/CMSSM model parameter space for $\tan\beta = 10$. The location of the HB/FP regions is denoted FP in frame *b*).

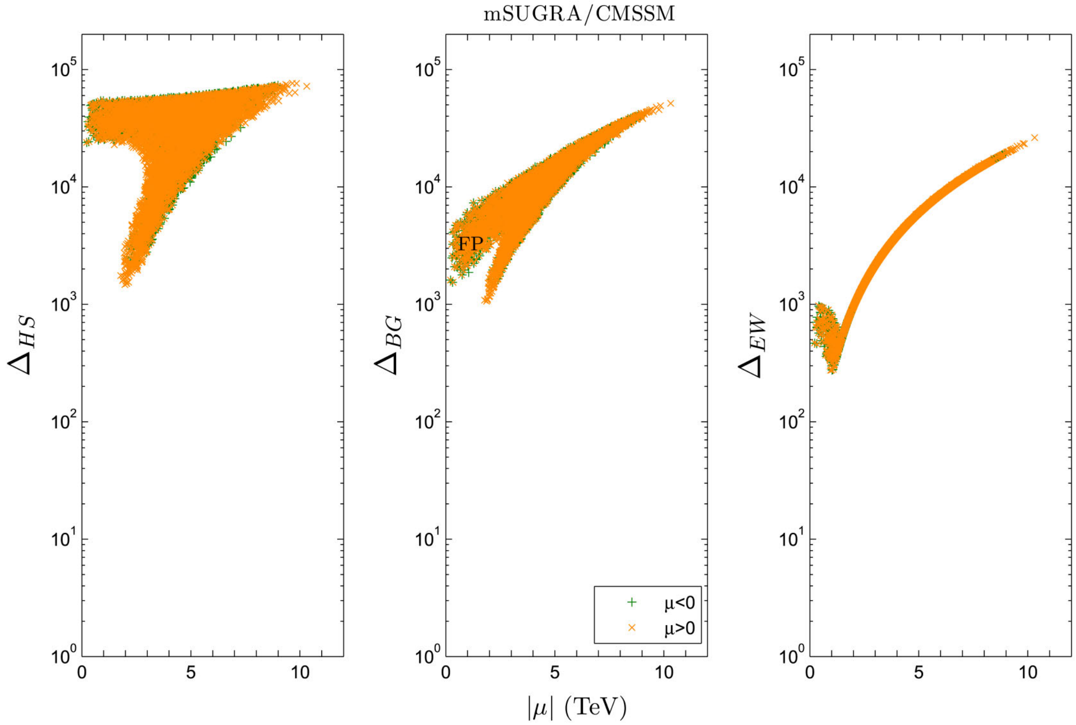

FIG. 4 (color online). Plot of Δ_{HS}, Δ_{BG}, and Δ_{EW} vs μ from a scan over mSUGRA/CMSSM model parameter space for $\tan\beta = 10$. The location of the HB/FP regions is denoted FP in frame *b*).

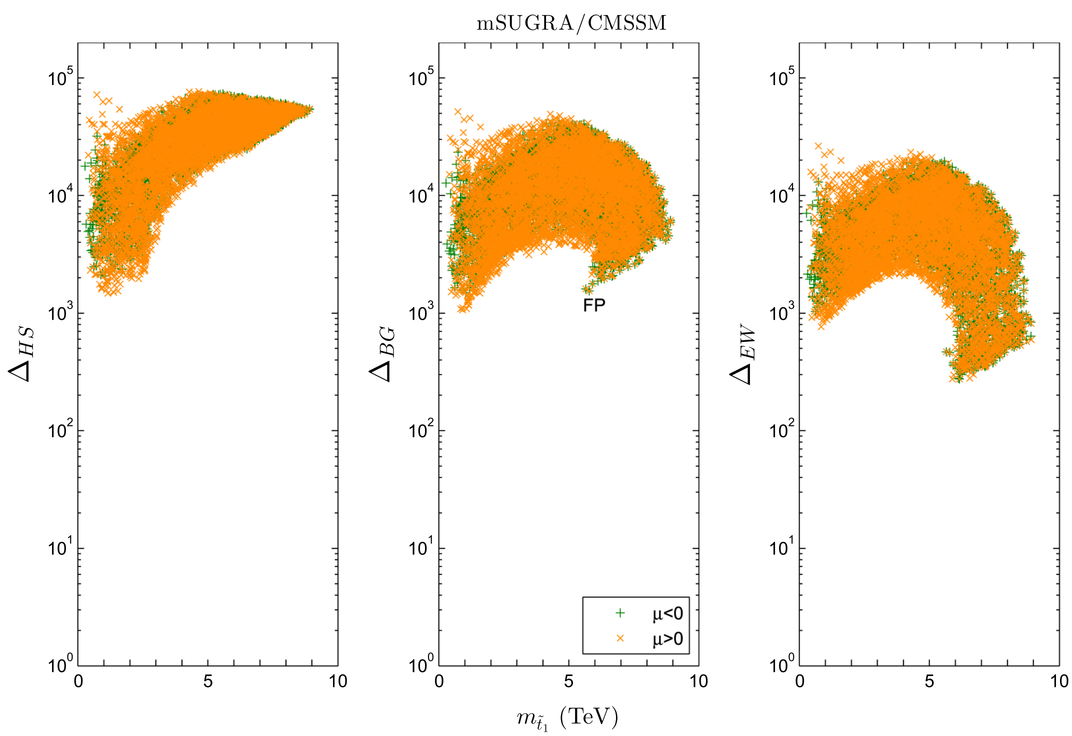

FIG. 5 (color online). Plot of Δ_{HS}, Δ_{BG}, and Δ_{EW} vs $m_{\tilde{t}_1}$ from a scan over mSUGRA/CMSSM model parameter space for $\tan\beta = 10$.

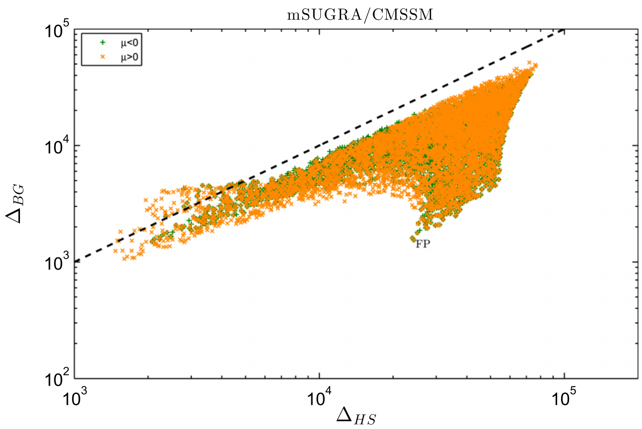

FIG. 6 (color online). Plot of Δ_{BG} vs Δ_{HS} from a scan over mSUGRA/CMSSM model parameter space for $\tan\beta = 10$. The dashed line denotes equal measures.

Thus, for the mSUGRA model, while all measures seem to favor low values of μ, the lowest EWFT is not found in the deep Higgsino (HB/FP) region, where a Higgsino-like lightest SUSY particle is expected.

In Fig. 5, we show the Δ measures vs $m_{\tilde{t}_1}$. Here, we see that low Δ_{HS} does indeed occur for relatively light top squarks, but with masses significantly above the values advocated in Refs. [35,36]. Here, $m_{\tilde{t}_1}$ as low as about 1 TeV can be found; for lower $m_{\tilde{t}_1}$ values, very large $|A_t|$ is required to satisfy the m_h constraint by having very heavy $\tilde{t}_2$; this, however, increases Δ_{HS}. There are minima of Δ_{BG} for $m_{\tilde{t}_1} \sim 1$ TeV and also in the HB/FP region where $m_{\tilde{t}_1} \sim 6$ TeV. The measure Δ_{EW} also shows two minima, with the lowest values being obtained in the HB/FP region where stops are around 6 TeV.

In Fig. 6, for each parameter set generated, we plot Δ_{HS} vs Δ_{BG} to show the correlation between the two measures. The dashed line shows where $\Delta_{BG} = \Delta_{HS}$. We see that the measures satisfy the inequality in Eq. (21)[7] and further that the two measures are highly correlated: in general, larger Δ_{HS} values also imply larger Δ_{BG}. The exception occurs in the HB/FP region where, because of correlations among the HS parameters, Δ_{BG} dips to very low values even at large Δ_{HS}.

In Fig. 7, we plot Δ_{EW} vs Δ_{BG}. We see that $\Delta_{EW} < \Delta_{BG}$, in accord with the expectation in Eq. (26). Generally speaking, the two measures are again well correlated; as before, the exception is the HB/FP region where Δ_{EW} decreases much more than Δ_{BG}.

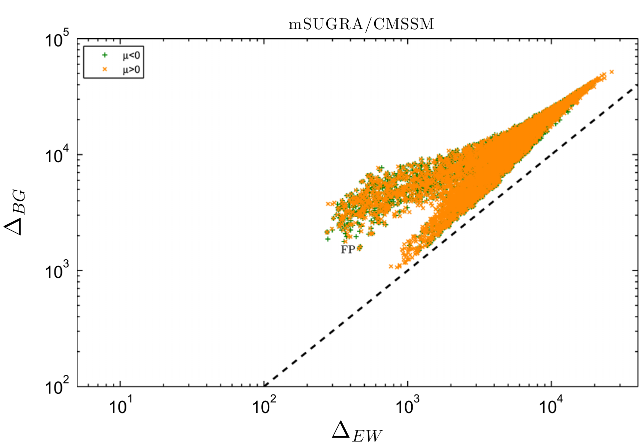

FIG. 7 (color online). Plot of Δ_{BG} vs Δ_{EW} from a scan over mSUGRA/CMSSM model parameter space for $\tan\beta = 10$. The dashed line denotes equal measures.

IV. Δ_i IN THE NUHM2 MODEL

Next we turn to a scan over the two-extra-parameter nonuniversal Higgs model NUHM2 defined by the parameter set,

$$m_0, \quad m_{1/2}, \quad A_0, \quad \tan\beta, \quad \mu, \quad m_A, \tag{29}$$

and where again we fix $\tan\beta = 10$. Then we scan over

$$\begin{gathered} m_0\text{: 0–15 TeV}, \qquad m_{1/2}\text{: 0–2 TeV}, \\ -3.0 < A_0/m_0 < 3.0, \qquad \mu\text{: 0.1–1.5 TeV}, \\ m_A\text{: 0.15–1.5 TeV}. \end{gathered} \tag{30}$$

The points from this scan are shown by red pluses in the figures that follow. We also performed a narrow scan with μ: 0.1–0.35 TeV denoted by blue crosses. The constraints on the sparticle masses are the same as in the mSUGRA scan.

For Δ_{BG} in the NUHM2 model, the strong cancellation between $m_{H_u}^2$ and the matter scalar mass terms in Eq. (18) that was operative for mSUGRA no longer occurs. Instead, the sensitivity coefficients are given by

$$\begin{aligned} c_{m_{1/2}} &= \text{same as mSUGRA}, & c_{m_0} &= 1.336(m_0^2/m_Z^2), \\ c_{m_{H_u}} &= -1.27(m_{H_u}^2/m_Z^2), & c_{m_{H_d}} &= -0.053(m_{H_d}^2/m_Z^2), \\ c_{A_0} &= \text{same as mSUGRA}, & c_\mu &= \text{same as mSUGRA}. \end{aligned} \tag{31}$$

The noncancelling terms in NUHM2 now mean that Δ_{BG} will largely be driven by c_{m_0} and $c_{m_{H_u}}$; an example is shown in Table I. The sensitivity coefficient from H_d is quite suppressed compared to the H_u term as expected for moderate-to-large $\tan\beta$.

In Fig. 8, we plot the various Δs vs m_0 for our scan of NUHM2 models. From the plot, we see very different behaviors compared to Fig. 1. Both Δ_{HS} and Δ_{BG} are highly correlated with m_0, as may be expected in the BG case if the sensitivity coefficients are dominated by c_{m_0}. Minimal EWFT occurs at the lowest m_0 points available. The minimal values of these two measures lie near 10^3,

[7]The small number of points where this inequality is violated is where $c_{m_{1/2}}$ determines Δ_{BG}; in this case, the factor 2 arising from the fact we take the derivative with respect to $m_{1/2}$ rather than $m_{1/2}^2$ plays an important role. Indeed, it is easy to see that $\Delta_{HS} \geq 2\Delta_{BG}$ is always satisfied.

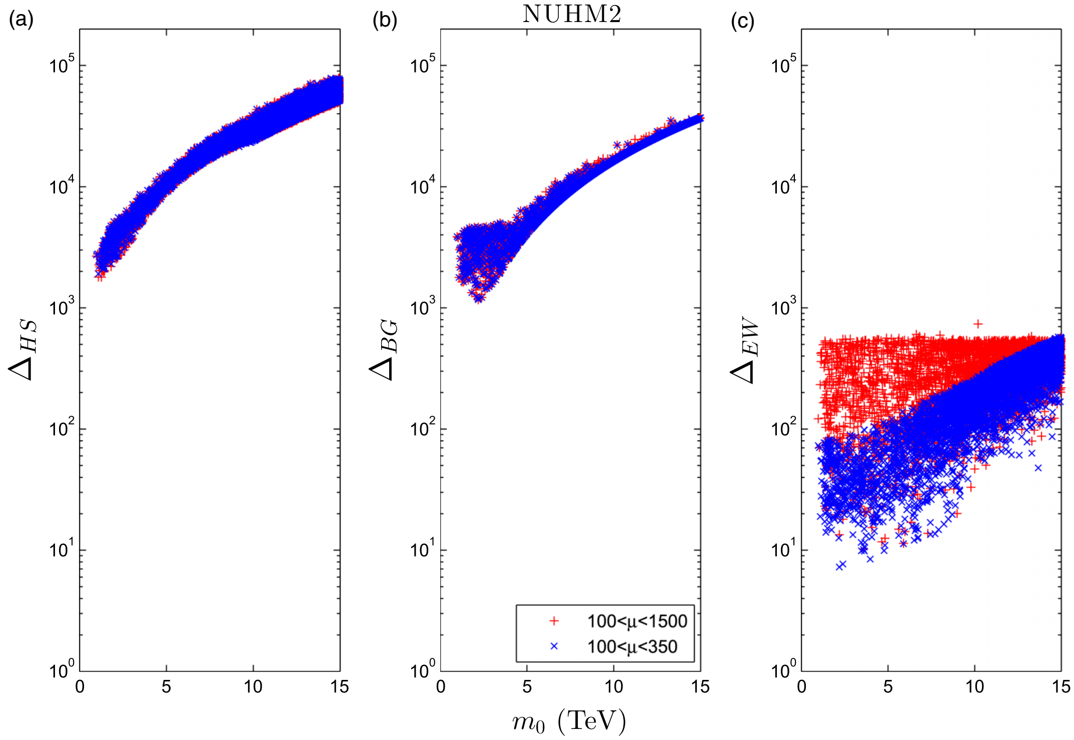

FIG. 8 (color online). Plot of (a) Δ_{HS}, (b) Δ_{BG}, and (c) Δ_{EW} vs m_0 for the two scans over NUHM2 model parameter space described in the text with $\tan\beta = 10$. The broad scan is denoted by red (lighter shading) whilst the narrow scan with low μ is denoted by blue (darker shading).

similar to the mSUGRA case. The behavior of Δ_{EW} is very different. First, the minimal value for Δ_{EW} from the NUHM2 scan is around 7 ($\sim$14% fine-tuning), which is about 2 orders of magnitude lower than the minimum from Δ_{HS} and Δ_{BG}. These very low Δ_{EW} values occur in the radiatively driven natural SUSY (RNS) scenario of Refs. [43,44].[8] At tree level, low Δ_{EW} is obtained for 1) low values of $\mu \sim m_Z$ and 2) low values of $m_{H_u}^2 \sim m_Z^2$; both these features can be realized, along with not-too-heavy stops, due to the extra parameter freedom enjoyed by NUHM2 models. Furthermore, the distribution of Δ_{EW}, while increasing with m_0, is only softly dependent on m_0, with minimal values of Δ_{EW} occurring in the $m_0 \sim 2$–5 TeV range. This is because m_0 influences the top squark masses, which enter Δ_{EW} only at one-loop level.

In Fig. 9, we show the Δ_i vs $m_{1/2}$. Here, as in the mSUGRA case discussed before, all three Δ_i exhibit only a weak dependence on $m_{1/2}$, with the minimum of Δ_i occurring around $m_{1/2} \sim 1$ TeV. The results are independent of the range of μ that is scanned.

In Fig. 10, the values of Δ_i are shown vs A_0/m_0. Here, in contrast to the mSUGRA case, we see no gap at small values of A_0/m_0 as also noted in Ref. [29]. The added freedom to choose the Higgs mass parameters allows solutions with the observed value of m_h. Both Δ_{HS} and Δ_{BG} distributions exhibit minima occurring at large $|A_0|$ values. The lowest value for $\Delta_{HS,BG}$ occurs at $A_0 \sim -(2$–$3)m_0$. For this sign of A_0, it is much easier to generate $m_h \sim 125$ GeV at low m_0 values where the $\Delta_{HS,BG}$ are lowest. The distribution in Δ_{EW} also has minima at large A_0, although the minima tend to occur around $A_0 \sim \pm 1.6 m_0$.

In Fig. 11, we show the three Δ_i measures vs μ. In this case, we see that neither Δ_{HS} nor Δ_{BG} has any preferred μ value. This is because μ does not enter the evolution of $m_{H_u}^2$ (or any other soft SUSY breaking parameter) in the case of Δ_{HS}, and c_μ is never the maximal sensitivity coefficient in Δ_{BG}. The situation is quite different for Δ_{EW}. In this case, we see a tight correlation with the low Δ_{EW} values preferring the lowest values of μ that are phenomenologically allowed, i.e., those closest to m_Z. In this case, low Δ_{EW} has a strong preference for a set of light Higgsinos $\tilde{W}_1^\pm$, $\tilde{Z}_{1,2}$ of which the $\tilde{Z}_1$ would be a higgsino-like weakly interacting massive particle candidate. Note, however, that the gaugino components of $\tilde{Z}_1$ cannot get too small since then large gaugino masses would increase $m_{\tilde{t}_{1,2}}$, thus increasing the radiative corrections Σ_u^u. We thus conclude that $\tilde{Z}_1$ has substantial Higgsino *and* gaugino components, giving it an observable spin-independent direct detection cross section $\sigma^{\rm SI}(\tilde{Z}_1 p)$ at ton-sized detectors [39]. Also, the various Higgsinos would likely be visible at a linear e^+e^- collider

[8] In the RNS model of Refs. [43,44], $|\mu|$ is required $\sim$100–200 GeV, $m_{H_u}^2$ is driven radiatively to values $m_{H_u}^2 \sim -m_Z^2$, and large mixing in the stop sector diminishes the radiative corrections $\Sigma_u^u(\tilde{t}_{1,2})$ while lifting the value of m_h to $\sim$125 GeV.

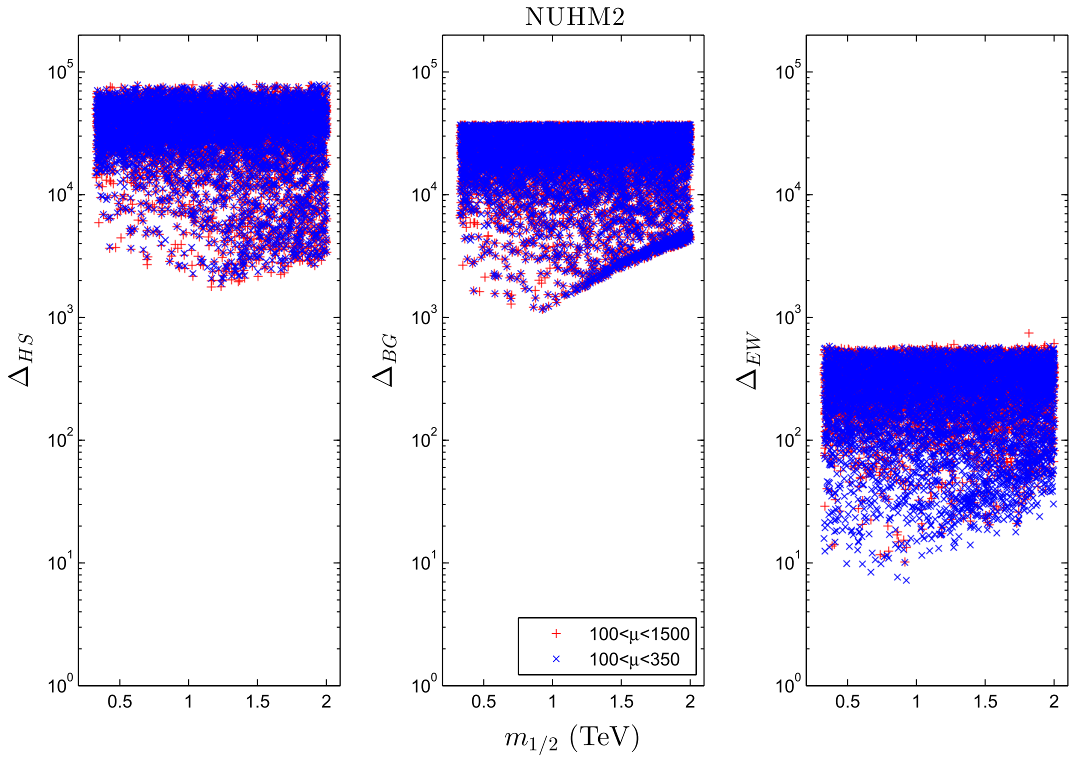

FIG. 9 (color online). Plot of Δ_{HS}, Δ_{BG}, and Δ_{EW} vs $m_{1/2}$ from the two scans over NUHM2 model parameter space described in the text with $\tan\beta = 10$.

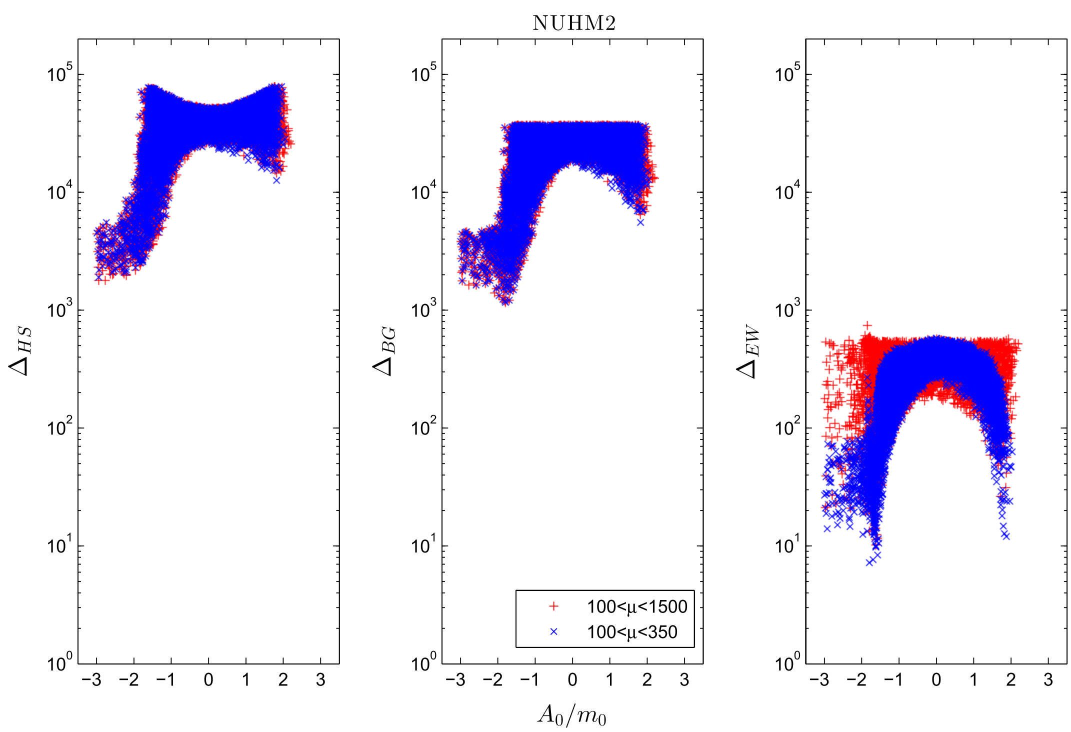

FIG. 10 (color online). Plot of Δ_{HS}, Δ_{BG}, and Δ_{EW} vs A_0/m_0 from the two scans over NUHM2 model parameter space described in the text for $\tan\beta = 10$.

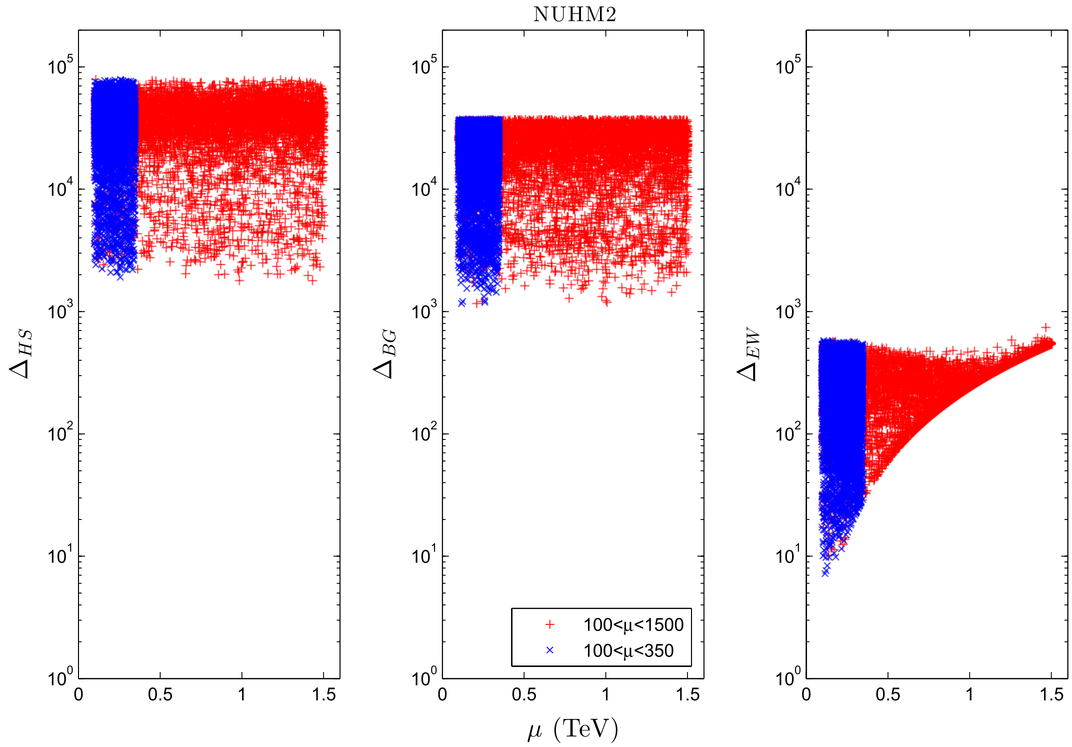

FIG. 11 (color online). Plot of Δ_{HS}, Δ_{BG}, and Δ_{EW} vs μ from the two scans over NUHM2 model parameter space described in the text for $\tan\beta = 10$.

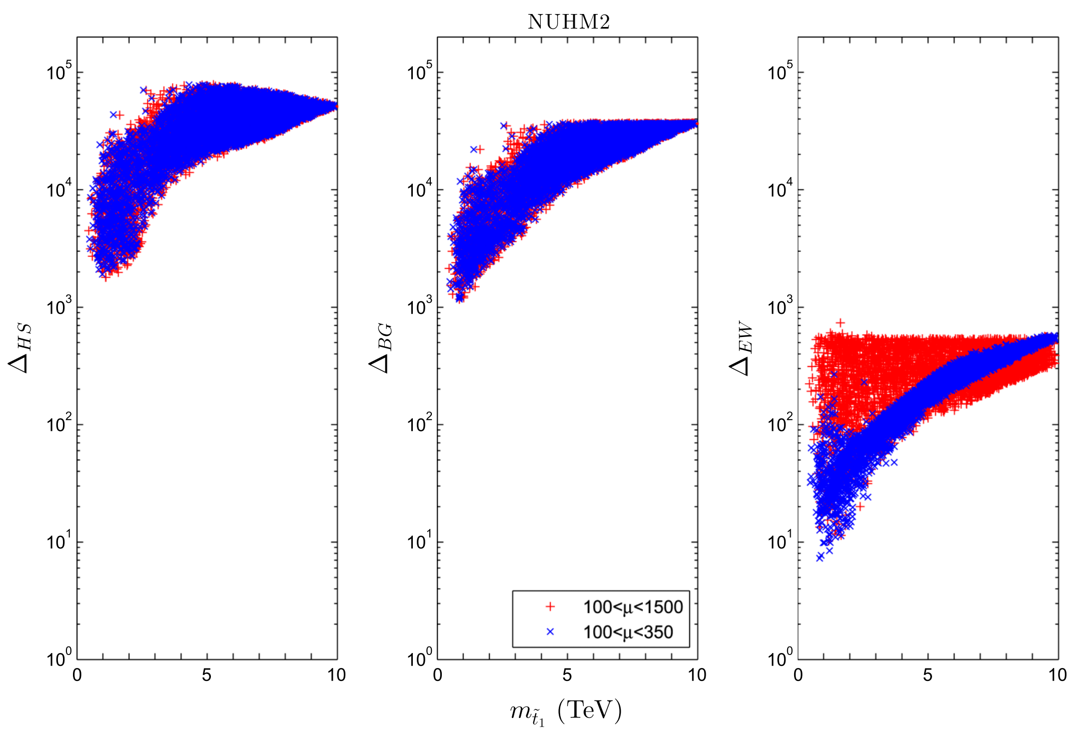

FIG. 12 (color online). Plot of Δ_{HS}, Δ_{BG}, and Δ_{EW} vs $m_{\tilde{t}_1}$ for the two scans over NUHM2 model parameter space described in the text for $\tan\beta = 10$.

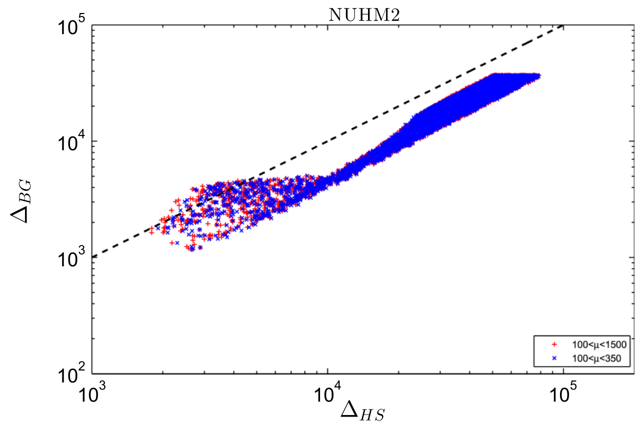

FIG. 13 (color online). Plot of Δ_{BG} vs Δ_{HS} for the two scans over NUHM2 model parameter space described in the text for $\tan\beta = 10$.

operating with $\sqrt{s} \sim 0.25$–1 TeV, although these would be difficult to directly observe at the LHC if gluinos are heavier than 1.5–2 TeV [59] because then the $\tilde{W}_1/\tilde{Z}_2 - \tilde{Z}_1$ mass gap becomes too small. For the low $|\mu|$ models, the $pp \to \tilde{W}_2\tilde{Z}_4 \to W^\pm W^\pm + \not{E}_T$ signal may be observable at the (high luminosity) LHC if the heavy winolike $\tilde{W}_2$ and $\tilde{Z}_4$ have masses up to about 800 GeV [37].

In Fig. 12 we show the distribution of Δ_i values vs $m_{\tilde{t}_1}$. Here, we find all three measures concur that the lowest fine-tuning is found for the lowest values of $m_{\tilde{t}_1}$ which lead to $m_h \sim 125$ GeV. These tend to lie in the vicinity of $m_{\tilde{t}_1} \sim 1$ TeV, well beyond current bounds from the LHC. Although we have not shown it here, we have checked that the corresponding value of $m_{\tilde{t}_2} \sim 2$ TeV.

Figure 13 shows the correlation between Δ_{HS} and Δ_{BG} from the scan over NUHM2 parameter space. For NUHM2, these two measures are highly correlated, and once again we see that the inequality (21) is broadly satisfied and further that all points satisfy $\Delta_{BG} \le 2\Delta_{HS}$ as mentioned earlier.

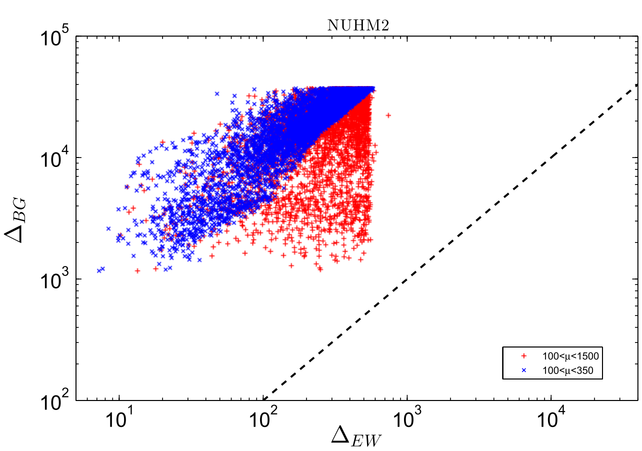

FIG. 14 (color online). Plot of Δ_{BG} vs Δ_{EW} for the two scans over NUHM2 model parameter space described in the text for $\tan\beta = 10$.

TABLE II. Values of Δ_{HS}, Δ_{BG}, and Δ_{EW} for the mSUGRA/CMSSM, NUHM2, and pMSSM models. For mSUGRA, we take $m_0 = 9993.4$ GeV, $m_{1/2} = 691.7$ GeV, $A_0 = -4788.6$ GeV, and $\tan\beta = 10$. The mSUGRA output values of $\mu = 309.7$ GeV and $m_A = 9859.9$ GeV serve as NUHM2 inputs. The weak scale outputs of mSUGRA and NUHM2 serve as pMSSM inputs so that all three models have exactly the same weak scale spectra.

Model	Δ_{HS}	Δ_{BG}	Δ_{EW}
mSUGRA	24302	1540	462
NUHM2	24302	16041	462
pMSSM	462	462	462

Finally, in Fig. 14, we plot Δ_{BG} vs Δ_{EW}. Once again, we see that Eq. (26) is satisfied. Whereas these two Δs were highly correlated in the mSUGRA case (except for the points in the HB/FP region), for NUHM2 they are much less so in that points with lowest Δ_{BG} may have Δ_{EW} ranging from its minimum at ~ 7 all the way up to near its maximum. This is because points with very large μ values can have very low values of Δ_{BG} because, as we have already noted, c_μ is never maximal in the various sensitivity coefficients.

A comparison of the three Δs for each of three models—mSUGRA, NUHM2 and pMSSM[9]—is shown in Table II. For mSUGRA, we take $m_0 = 9993.4$ GeV, $m_{1/2} = 691.7$ GeV, $A_0 = -4788.6$ GeV, and $\tan\beta = 10$. The mSUGRA output values of $\mu = 309.7$ GeV and $m_A = 9859.9$ GeV serve as NUHM2 inputs. The weak scale outputs of mSUGRA and NUHM2 serve as pMSSM inputs so that all three models have exactly the same weak scale spectra. From the table, we see under Δ_{HS} that the mSUGRA and NUHM2 models are both highly fine-tuned since Δ_{HS} mainly depends on the change $\delta m^2_{H_u}$ in running from $m_{\rm GUT}$ to $m_{\rm SUSY}$. For the pMSSM, since $\Lambda = m_{\rm SUSY}$, then Δ_{HS} collapses to Δ_{EW}. For the measure Δ_{BG}, we obtain maximal EWFT within the NUHM2 model since here we have a large set of uncorrelated parameters at the scale $\Lambda = m_{\rm GUT}$. In mSUGRA, with fewer parameters due to $m_{H_u} = m_{H_d} \equiv m_0$, the additional correlations allow for a collapse in EWFT by an order of magnitude. If additional parameter correlations are present, e.g., relating m_0 with A_0 and $m_{1/2}$, then it is possible that Δ_{BG} collapses even further to near its lower limit given by Δ_{EW}. Under Δ_{EW}, which is model independent (within the MSSM), then all three models have identical values of EWFT: $\Delta_{EW} = 462$.

V. INTERPRETATION IN TERMS OF AN ULTIMATE THEORY

In this paper, we have computed three measures of electroweak naturalness and applied them to two popular

[9]The pMSSM, or phenomenological MSSM, is the MSSM defined with 19 free weak scale parameters.

models: mSUGRA and NUHM2. We have argued that Δ_{HS} produces an overestimate of EWFT due to a separation of dependent terms $m_{H_u}^2(\Lambda)$ and $\delta m_{H_u}^2$. These terms contain large correlated cancellations since the larger $m_{H_u}^2(\Lambda)$ becomes, the larger is the radiative correction $\delta m_{H_u}^2$. In fact, the large negative correction contained in $\delta m_{H_u}^2$ is exactly what is required to cause a radiatively generated breakdown in electroweak symmetry. The measure Δ_{BG} avoids this problem by evaluating the combination $m_{H_u}^2(\Lambda) + \delta m_{H_u}^2 = m_{H_u}^2(m_{\rm SUSY})$ in terms of fundamental model parameters. By invoking HS models with increasingly constrained parameter sets, the EWFT in Δ_{BG} can be seen to collapse. An explicit demonstration occurs in moving from the six-parameter NUHM2 model to the four-parameter mSUGRA model in the HB/FP region: in this case, much lower values of Δ_{BG} are generated in the region of heavy stop masses than might otherwise be expected under Δ_{HS}.

At this point, we should note that few authors would be willing to consider either mSUGRA or NUHM2 as fundamental theories. Instead, they are to be viewed as effective field theories for which the range of validity may extend up to $\Lambda = m_{\rm GUT}$. An often unstated assumption is that most authors hypothesize the existence of an overarching ultimate theory—perhaps string theory—for which the low energy limit for $Q < \Lambda = m_{\rm GUT}$ is the MSSM but wherein the high scale soft terms are all correlated (here referred to as the UTH). Such an UTH would have fewer free parameters and perhaps even no parameters at all; in the latter case, the soft terms might all be determined in terms of the fundamental Planck scale M_P. The UTH might be contained within the more general effective SUSY theories popular in the literature. In the case of Δ_{BG}, the measure of EWFT can be overestimated by evaluating Δ_{BG} within the effective theories instead of within the UTH. Indeed, it is not even clear if Δ_{BG} has any meaning for an UTH with no free parameters.

The measure Δ_{EW} allows for the possibility of parameter correlations which should be present in the UTH and, since it is model-independent, leads to the same value of Δ_{EW} for the effective theories as should occur for the UTH. In the course of this work, we have found that the well-known mSUGRA/CMSSM model is fine-tuned under all three measures. As such, it is unlikely to contain the UTH. The nonuniversal Higgs model NUHM2 appears fine-tuned with $\Delta_{HS,BG} \gtrsim 10^3$. But since Δ_{EW} can be as small as 7 (corresponding to 14% fine-tuning or one part in 10), it may contain the UTH for selected parameter choices which allow for low Δ_{EW}. In other words, a model with derived parameters leading to low Δ_{EW} would also have low true EWFT. In the case of NUHM2, which preserves the elegant SUSY and GUT features, the UTH should lead to typical mass spectra shown in Fig. 15. For even more general frameworks (e.g., those with nonuniversal gaugino masses) other spectra with low Δ_{EW} are also possible [19,20].

The measure Δ_{EW} is model independent in the sense that any high scale model giving rise to look-alike spectra at the weak scale will have the same value of Δ_{EW}. It is also most intimately connected with data, in that it requires natural generation of $m_Z = 91.2$ GeV while maintaining LHC Higgs mass and sparticle mass constraints. In this sense, models with low Δ_{EW} solve what is known as the little hierarchy problem: how can it be that m_Z and $m_h \sim 100$ GeV while sparticle masses are beyond the TeV scale? The answer is that the SUSY models must have low Higgsino mass $\mu \sim 100$–200 GeV, they must generate $m_{H_u}^2 \sim -(100$–$200)$ GeV at the weak scale (always possible in NUHM models), and there must be large mixing in the top-squark sector with TeV-scale top squarks. An example may be seen in Fig. 16. Here we show the various scalar potential contributions to m_Z scaled to $m_Z^2/2$ for $m_0 = 7025$ GeV, $m_{1/2} = 568.3$ GeV, $A_0 = -11426.6$ GeV, and $\tan\beta = 8.55$ (benchmark point RNS2 from Ref. [44]). Red bars denote negative contributions, while blue bars denote positive contributions.

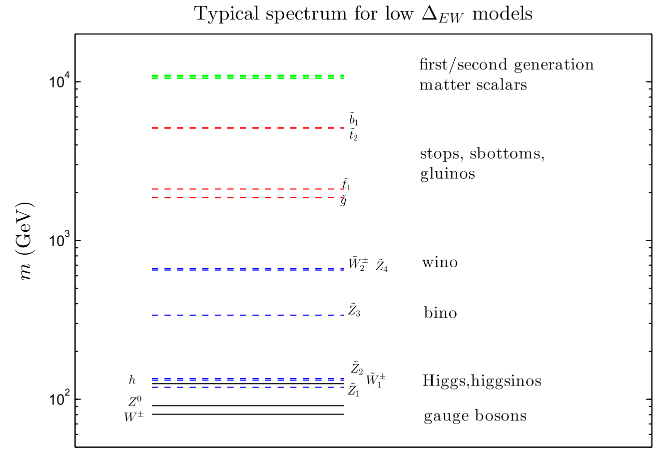

FIG. 15 (color online). Typical sparticle mass spectrum from SUSY models with low Δ_{EW}. Such a spectrum might be expected to result from an UTH with low true EWFT.

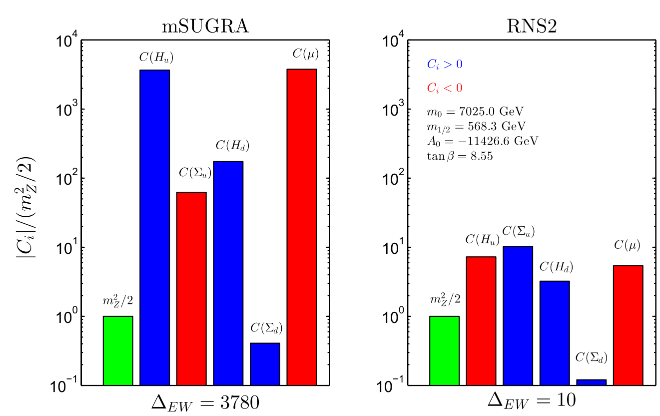

FIG. 16 (color online). Plot of contributions to $m_Z^2/2$ from the mSUGRA/CMSSM model with parameters as listed, and also for the RNS2 benchmark point with the same m_0, $m_{1/2}$, A_0, and $\tan\beta$ values, but with $\mu = 150$ GeV. Red (lighter) bars denote negative contributions, while blue (darker) bars denote positive contributions.

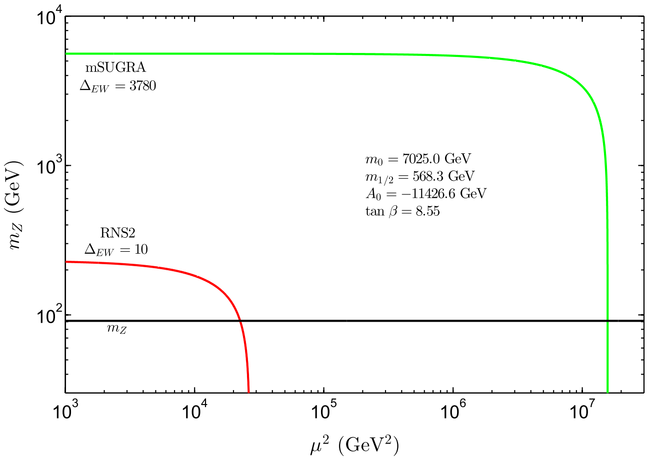

FIG. 17 (color online). Plot of m_Z vs μ^2 for mSUGRA/CMSSM with parameters as listed. We also show m_Z vs μ^2 in the NUHM2 model.

positive contributions. In frame *a*), the situation is shown for the mSUGRA model (parameters as above with $m_{H_u} = m_{H_d} = m_0$) where $m_{H_u}^2$ is driven to large negative values at the weak scale. The value of μ^2 must be dialed in (fine-tuned) so that a large, unnatural cancellation between $m_{H_u}^2$ and μ^2 is needed to gain a Z mass of just 91.2 GeV. In frame *b*), we show the case for radiatively driven natural SUSY with the same parameters as mSUGRA but with $\mu = 150$ GeV and where now $m_{H_u}(\Lambda) \neq m_{H_d}(\Lambda) \neq m_0$. All contributions are now roughly comparable to $m_Z^2/2$ so that in this case, it is easy to understand why m_Z and m_h both naturally occur around $\sim$100 GeV.

This can be further illustrated in Fig. 17, where we adopt all weak scale parameters from the two benchmark models except μ^2 but then plot the value of m_Z as is generated by varying μ^2. We see in the mSUGRA case that one would naturally expect $m_Z \sim 6$ TeV instead of 91.2 GeV. One must finely tune μ^2 to very high precision to generate $m_Z = 91.2$ GeV. In the RNS2 case, m_Z is expected to lie around 200 GeV, and it is not so far fetched that it turns out to be 91.2 GeV, which still requires $\sim$10% fine-tuning of μ^2.

VI. CONCLUSIONS

Our conclusions are summarized as follows:

(i) The measure Δ_{HS}, which essentially measures $\delta m_{H_u}^2/(m_h^2/2)$ or alternatively $\delta m_{H_u}^2/(m_Z^2/2)$, overestimates EWFT by omitting the nonindependent value of $m_{H_u}^2(\Lambda)$. This can be rectified by instead using the combined term $(m_{H_u}^2(\Lambda) + \delta m_{H_u}^2)/(m_Z^2/2)$ as occurs in Δ_{BG} and Δ_{EW}.

(ii) Δ_{BG} measures fractional change in m_Z^2 against fractional change in model parameters. As such, it is by definition model dependent. To interpret Δ_{BG}, we introduce the concept of an overarching UTH with few or even no free parameters. By applying Δ_{BG} to the more general effective theories which contain the UTH, then large cancellations due to correlated high scale parameters are missed, leading to an overestimate in EWFT. An example is shown where mSUGRA functions as a four-parameter UTH contained within the six-parameter NUHM2. The correlated parameters $m_{H_u}^2 = m_{H_d}^2 \equiv m_0^2$ lead to large cancellations in the scalar sector in the well-known HB/FP region.

(iii) The model-independent measure Δ_{EW} is obtained as the limit as $\Lambda \to m_{\rm SUSY}$ of Δ_{HS} and Δ_{BG}. It measures how likely the weak scale μ parameter, soft terms, and radiative corrections can conspire to yield m_Z, $m_h \sim 100$ GeV without large uncorrelated cancellations (fine-tuning). (Typically, in models with large Δ_{EW}, a value of μ^2 must be dialed in (fine-tuned) so that a large, unnatural cancellation between $m_{H_u}^2$ and μ^2 is required to obtain a Z mass of just 91.2 GeV.)

Since Δ_{EW} is model independent and depends only on the weak scale spectra which is generated, it will produce the same value for the UTH as it would for various effective theories which contain the UTH. While mSUGRA is fine-tuned under all three measures, implying it is unlikely to contain the UTH, values of Δ_{EW} below 10 can be found for the NUHM2 model, indicating fine-tuning to one part in ten. The weak scale spectra from NUHM2 which yield $\Delta_{EW} \sim 10$ would be a good candidate for what may be expected from an UTH including SUSY/GUT relations with low true EWFT. Such models are characterized by light Higgsinos $m_{\tilde{W}_1}$, $m_{\tilde{Z}_{1,2}} \sim$ 100–300 GeV, which can elude searches at LHC14 [60] but which could easily be discovered at an e^+e^- collider with $\sqrt{s} \sim 500$–600 GeV [61].

Our overall lesson is that the conventional measures Δ_{HS} and Δ_{BG} tend to overestimate—often by orders of magnitude—the EWFT needed for supersymmetry theory. In contrast, as discussed in Ref. [43], the measure Δ_{EW} has the properties of being model-independent, conservative, measureable, unambiguous, predictive, falsifiable, and simple to calculate. In virtue of these qualitiesand in light of our current lack of knowledge of the UTHthe quantity Δ_{EW} appears to be the correct measure of EWFT to apply to the effective theories which might contain the UTH. In models such as NUHM2, which allow for Δ_{EW} as low as $\sim$10, then [62] "the SUSY (GUT) picture... remains the standard way beyond the Standard Model." Target spectra for model builders intent on constructing the UTH are provided in Fig. 15.

ACKNOWLEDGMENTS

We thank A. Mustafayev and X. Tata for discussions. This work was supported in part by the US Department of Energy, Office of High Energy Physics.

[1] G. Aad *et al.* (ATLAS Collaboration), Phys. Lett. B **716**, 1 (2012).

[2] S. Chatrchyan *et al.* (CMS Collaboration), Phys. Lett. B **716**, 30 (2012).

[3] H. Baer and X. Tata, *Weak Scale Supersymmetry: From Superfields to Scattering Events* (Cambridge University Press, Cambridge, England, 2006).

[4] S. P. Martin, arXiv:hep-ph/9709356; M. Drees, R. Godbole, and P. Roy, *Theory and Phenomenology of Sparticles* (World Scientific, Singapore, 2004).

[5] M. S. Carena and H. E. Haber, Prog. Part. Nucl. Phys. **50**, 63 (2003).

[6] G. Aad *et al.* (ATLAS Collaboration), Phys. Rev. D **87**, 012008 (2013).

[7] S. Chatrchyan *et al.* (CMS Collaboration), J. High Energy Phys. 10 (2012) 018.

[8] A. Chamseddine, R. Arnowitt, and P. Nath, Phys. Rev. Lett. **49**, 970 (1982); R. Barbieri, S. Ferrara, and C. Savoy, Phys. Lett. **119B**, 343 (1982); N. Ohta, Prog. Theor. Phys. **70**, 542 (1983); L. Hall, J. Lykken, and S. Weinberg, Phys. Rev. D **27**, 2359 (1983).

[9] J. R. Ellis, K. Enqvist, D. V. Nanopoulos, and F. Zwirner, Mod. Phys. Lett. A **01**, 57 (1986).

[10] R. Barbieri and G. Giudice, Nucl. Phys. **B306**, 63 (1988).

[11] G. Kane, C. Kolda, L. Roszkowski, and J. Wells, Phys. Rev. D **49**, 6173 (1994).

[12] G. W. Anderson and D. J. Castano, Phys. Lett. B **347**, 300 (1995); Phys. Rev. D **52**, 1693 (1995).

[13] S. Dimopoulos and G. F. Giudice, Phys. Lett. B **357**, 573 (1995).

[14] K. L. Chan, U. Chattopadhyay, and P. Nath, Phys. Rev. D **58**, 096004 (1998); S. Akula, M. Liu, P. Nath, and G. Peim, Phys. Lett. B **709**, 192 (2012); M. Liu and P. Nath, Phys. Rev. D **87**, 095012 (2013).

[15] P. H. Chankowski, J. R. Ellis, and S. Pokorski, Phys. Lett. B **423**, 327 (1998); P. H. Chankowski, J. R. Ellis, M. Olechowski, and S. Pokorski, Nucl. Phys. **B544**, 39 (1999).

[16] G. L. Kane and S. F. King, Phys. Lett. B **451**, 113 (1999); M. Bastero-Gil, G. L. Kane, and S. F. King, Phys. Lett. B **474**, 103 (2000).

[17] J. A. Casas, J. R. Espinosa, and I. Hidalgo, J. High Energy Phys. 01 (2004) 008.

[18] S. Cassel, D. M. Ghilencea, and G. G. Ross, Nucl. Phys. **B825**, 203 (2010); **B835**, 110 (2010); S. Cassel, D. M. Ghilencea, S. Kraml, A. Lessa, and G. G. Ross, J. High Energy Phys. 05 (2011) 120; G. G. Ross and K. Schmidt-Hoberg, Nucl. Phys. **B862**, 710 (2012); G. G. Ross, K. Schmidt-Hoberg, and F. Staub, J. High Energy Phys. 08 (2012) 074; D. M. Ghilencea and G. G. Ross, Nucl. Phys. **B868**, 65 (2013); A. Kaminska, G. G. Ross, and K. Schmidt-Hoberg, arXiv:1308.4168.

[19] I. Gogoladze, F. Nasir, and Q. Shafi, Int. J. Mod. Phys. A **28**, 1350046 (2013); arXiv:1306.5699.

[20] H. Baer, V. Barger, and M. Padeffke-Kirkland, Phys. Rev. D **88**, 055026 (2013).

[21] M. Perelstein and B. Shakya, J. High Energy Phys. 10 (2011) 142; Phys. Rev. D **88**, 085003 (2013).

[22] S. Antusch, L. Calibbi, V. Maurer, M. Monaco, and M. Spinrath, Phys. Rev. D **85**, 035025 (2012); J. High Energy Phys. 01 (2013) 187.

[23] E. Hardy, arXiv:1306.1534.

[24] S. Fichet, Phys. Rev. D **86**, 125029 (2012).

[25] K. Kowalska and E. M. Sessolo, Phys. Rev. D **88**, 075001 (2013).

[26] C. Han, K.-i. Hikasa, L. Wu, J. M. Yang, and Y. Zhang, arXiv:1308.5307.

[27] E. Dudas, G. von Gersdorff, S. Pokorski, and R. Ziegler, arXiv:1308.1090.

[28] A. Arvanitaki, M. Baryakhtar, X. Huang, K. Van Tilburg, and G. Villadoro, arXiv:1309.3568.

[29] H. Baer, V. Barger, and A. Mustafayev, Phys. Rev. D **85**, 075010 (2012).

[30] L. J. Hall, D. Pinner, and J. T. Ruderman, J. High Energy Phys. 04 (2012) 131.

[31] K. J. Bae, K. Choi, E. J. Chun, S. H. Im, C. B. Park, and C. S. Shin, J. High Energy Phys. 11 (2012) 118; K. J. Bae, T. H. Jung, and H. D. Kim, Phys. Rev. D **87**, 015014 (2013).

[32] P. Athron, M. Binjonaid, and S. F. King, Phys. Rev. D **87**, 115023 (2013).

[33] R. Kitano and Y. Nomura, Phys. Lett. B **631**, 58 (2005); Phys. Rev. D **73**, 095004 (2006).

[34] N. Arkani-Hamed, in WG2 Meeting, CERN, Geneva, 2012 (unpublished).

[35] C. Brust, A. Katz, S. Lawrence, and R. Sundrum, J. High Energy Phys. 03 (2012) 103.

[36] M. Papucci, J. T. Ruderman, and A. Weiler, J. High Energy Phys. 09 (2012) 035; C. Wymant, Phys. Rev. D **86**, 115023 (2012); H. Baer, V. Barger, P. Huang, and X. Tata, J. High Energy Phys. 05 (2012) 109.

[37] H. Baer, V. Barger, P. Huang, D. Mickelson, A. Mustafayev, W. Sreethawong, and X. Tata, Phys. Rev. Lett. **110**, 151801 (2013).

[38] A. G. Delannoy *et al.*, Phys. Rev. Lett. **111**, 061801 (2013).

[39] H. Baer, V. Barger, and D. Mickelson, Phys. Lett. B **726**, 330 (2013).

[40] J. Ellis, K. Olive, and Y. Santoso, Phys. Lett. B **539**, 107 (2002); J. Ellis, T. Falk, K. Olive, and Y. Santoso, Nucl. Phys. **B652**, 259 (2003); H. Baer, A. Mustafayev, S. Profumo, A. Belyaev, and X. Tata, J. High Energy Phys. 07 (2005) 065.

[41] C. Han, F. Wang, and J. M. Yang, arXiv:1304.5724.

[42] V. D. Barger, M. S. Berger, and P. Ohmann, Phys. Rev. D **49**, 4908 (1994).

[43] H. Baer, V. Barger, P. Huang, D. Mickelson, A. Mustafayev, and X. Tata, Phys. Rev. D **87**, 115028 (2013).

[44] H. Baer, V. Barger, P. Huang, A. Mustafayev, and X. Tata, Phys. Rev. Lett. **109**, 161802 (2012).

[45] M. Dine, A. Kagan, and S. Samuel, Phys. Lett. B **243**, 250 (1990); A. Cohen, D. B. Kaplan, and A. Nelson, Phys. Lett. B **388**, 588 (1996); N. Arkani-Hamed and H. Murayama, Phys. Rev. D **56**, R6733 (1997).

[46] For a recent review, see, e.g., J. L. Feng, arXiv:1302.6587.

[47] L. E. Ibanez, C. Lopez, and C. Munoz, Nucl. Phys. **B256**, 218 (1985); A. Lleyda and C. Munoz, Phys. Lett. B **317**, 82 (1993).

[48] H. Abe, T. Kobayashi, and Y. Omura, Phys. Rev. D **76**, 015002 (2007).

[49] S. P. Martin, Phys. Rev. D **75**, 115005 (2007).

[50] J. L. Feng, K. T. Matchev, and T. Moroi, Phys. Rev. D **61**, 075005 (2000); J. L. Feng and K. T. Matchev, Phys. Rev. D

63, 095003 (2001); J. L. Feng, K. T. Matchev, and D. Sanford, Phys. Rev. D **85**, 075007 (2012); J. L. Feng and D. Sanford, Phys. Rev. D **86**, 055015 (2012).
[51] H. Baer, V. Barger, P. Huang, D. Mickelson, A. Mustafayev, and X. Tata, Phys. Rev. D **87**, 035017 (2013).
[52] M. Perelstein and C. Spethmann, J. High Energy Phys. 04 (2007) 070.
[53] S. F. King, M. Muhlleitner, and R. Nevzorov, Nucl. Phys. **B860**, 207 (2012).
[54] G. Altarelli, arXiv:1308.0545.
[55] H. Baer, V. Barger, P. Huang, D. Mickelson, A. Mustafayev, and X. Tata, arXiv:1306.2926.
[56] H. Baer, F. Paige, S. Protopopescu, and X. Tata, arXiv:hep-ph/0312045.
[57] H. Baer, C. H. Chen, R. Munroe, F. Paige, and X. Tata, Phys. Rev. D **51**, 1046 (1995); H. Baer, J. Ferrandis, S. Kraml, and W. Porod, Phys. Rev. D **73**, 015010 (2006).
[58] Joint LEP 2 Supersymmetry Working Group, Combined LEP Chargino Results up to 208 GeV, http://lepsusy.web.cern.ch/lepsusy/www/inos_moriond01/charginos_pub.html.
[59] H. Baer, V. Barger, and P. Huang, J. High Energy Phys. 11 (2011) 031.
[60] H. Baer, V. Barger, P. Huang, D. Mickelson, A. Mustafayev, W. Sreethawong, and X. Tata, arXiv:1306.3148.
[61] M. Berggren, F. Brümmer, J. List, G. Moortgat-Pick, T. Robens, K. Rolbiecki, and H. Sert, arXiv:1307.3566.
[62] G. Altarelli and F. Feruglio, arXiv:hep-ph/0306265.