Nucleon decay via dimension-6 operators in anomalous $U(1)_A$ supersymmetric GUT

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Nucleon lifetimes for various decay modes via dimension-6 operators are calculated in the anomalous $U(1)_A$ grand unified theory (GUT) scenario, in which the unification scale Λ_u becomes smaller than the usual supersymmetric (SUSY) unification scale $\Lambda_G = 2\times 10^{16}$ GeV in general. Since the predicted lifetime $\tau(p \rightarrow \pi^0 + e^c)$ falls around the experimental lower bound, though it is strongly dependent on the explicit models, the discovery of the this nucleon decay can be expected in the near future. We explain why the two ratios $R_1 \equiv \frac{\Gamma_{n\rightarrow\pi^0+\nu^c}}{\Gamma_{p\rightarrow\pi^0+e^c}}$ and $R_2 \equiv \frac{\Gamma_{p\rightarrow K^0+\mu^c}}{\Gamma_{p\rightarrow\pi^0+e^c}}$ are important in identifying the grand unification group, and we show that three anomalous $U(1)_A$ SUSY GUT models, with $SU(5)$, $SO(10)$ and E_6 grand unification groups, can be identified by measuring the two ratios. If R_1 is larger than 0.4, the grand unification group is not $SU(5)$, and moreover, if R_2 is larger than 0.3, the grand unification group is implied to be E_6.

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I. INTRODUCTION

Grand unified theory (GUT) [1] is one of the most promising possibilities among models beyond the standard model (SM). Theoretically, it can unify not only the three gauge interactions into a single gauge interaction, but also quarks and leptons into fewer multiplets. Moreover, experimentally, not only can measured values of the three gauge couplings in the SM be explained quantitatively in the supersymmetric (SUSY) GUTs, but also the various hierarchies of masses and mixings of quarks and leptons can be understood qualitatively by the unification of quarks and leptons in one generation into the **10** and $\bar{\mathbf{5}}$ of $SU(5)$, if it is assumed that the **10** matter induces stronger hierarchies for Yukawa couplings than the $\bar{\mathbf{5}}$ matter [2].

One of the most important predictions in the GUTs is nucleon decay [1,3–5]. In general, GUTs require some new particles which are not included in the SM. Some of these new particles induce the nucleon decay. For example, the adjoint representation of the $SU(5)$ group has 24 dimensions, while the sum of the dimensions for the adjoint representations of the SM gauge groups is just 12. There are new gauge bosons in the $SU(5)$ GUT, $X(\bar{\mathbf{3}},\mathbf{2})_{\frac{5}{6}}$ and $\bar{X}(\mathbf{3},\mathbf{2})_{-\frac{5}{6}}$, where $\bar{\mathbf{3}}$ and **2** mean the antifundamental representation of $SU(3)_C$ and the fundamental representation of $SU(2)_L$, respectively, and $\frac{5}{6}$ is the hypercharge. These new gauge bosons induce dimension-6 effective operators which break both the baryon and lepton numbers and induce the nucleon decay. Usually, the main decay mode of the proton via these dimension-6 operators is $p \rightarrow \pi^0 + e^c$. Since the mass of the superheavy gauge boson can be roughly estimated by the meeting scale of the three running gauge couplings, the lifetime of the nucleon can be estimated in principle. Unfortunately, in the SM, the three gauge couplings do not meet at a scale exactly, and the lifetime is proportional to the unification scale to the fourth power, and therefore, the prediction covers quite a wide range. However, if supersymmetry is introduced, the unification scale Λ_G becomes 2×10^{16} GeV, and therefore, the lifetime can be estimated at roughly 10^{36} years, which is much larger than the experimental lower bound, 10^{34} years [6].

The partner of the SM doublet Higgs, which is called the triplet (colored) Higgs, also induces nucleon decay through Yukawa interactions. Since the Yukawa couplings for the first- and second-generation matter are much smaller than the gauge couplings, the constraint for the colored Higgs mass from the experimental limits of the nucleon decay is not so severe without SUSY. However, once SUSY is introduced, dimension-5 effective operators can break both the baryon and lepton numbers and induce nucleon decay [5]. This can compensate for the smallness of the Yukawa couplings. Actually, in the minimal $SU(5)$ SUSY GUT, the lower bound for the colored Higgs mass becomes larger than the unification scale Λ_G [7,8]. The experimental bound from the nucleon decay via dimension-5 operators gives severe constraints for SUSY GUTs.

These constraints for the colored Higgs mass lead to the most difficult problem in SUSY GUTs, i.e., the doublet-triplet splitting problem. As noted above, the colored Higgs mass must be larger than the unification scale, while the SM Higgs must be around the weak scale. Of course, a fine-tuning can realize such a large mass splitting even in the minimal SUSY $SU(5)$ GUT, but it is unnatural. In the literature, a lot of attempts have been proposed to address this problem [9]. However, in most of the solutions, some terms which are allowed by the symmetry are just neglected, or the coefficients are taken to be very small.

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Such requirements are, in a sense, fine-tuning, and therefore, some mechanism is required which can realize such a large mass splitting in a natural way.

Another famous problem in the SUSY GUTs concerns the unrealistic Yukawa relations. The unification of matter results in the unification of the Yukawa couplings, which often leads to unrealistic mass relations. In the minimal $SU(5)$ GUT, the Yukawa matrix of the down-type quarks becomes the same as that of the charged leptons, which gives unrealistic predictions between masses of these particles. In the minimal $SO(10)$ GUT, all the Yukawa matrices become equivalent due to the unification of all quarks and leptons in one generation into a single multiplet, **16**. This Yukawa unification leads to unrealistic relations between the masses of quarks and leptons.

It has been pointed out that if the anomalous $U(1)_A$ gauge symmetry is introduced, the doublet-triplet splitting problem can be solved under the natural assumption that all the interactions which are allowed by the symmetry of the theory are introduced with $O(1)$ coefficients [10–14]. Note that the introduced interactions include higher-dimensional interactions. In the scenario, the nucleon decay via dimension-5 operators can be strongly suppressed [10,11]. Moreover, with this natural assumption, realistic quark and lepton masses and mixings can be obtained [10,13]. In this paper, we denote such SUSY GUTs as the natural GUTs. One of the most interesting predictions of the natural GUTs is that the nucleon decay via dimension-6 operators is enhanced; i.e., the unification scale Λ_u becomes lower than Λ_G. In the natural GUT, the unification scale is given as

$$\Lambda_u \sim \lambda^{-a}\Lambda_G, \tag{1}$$

where $\lambda < 1$ is the ratio of the Fayet-Iliopoulos parameter to the cutoff Λ, which is taken to be the usual SUSY GUT scale Λ_G in the natural GUT in order to explain the success of the gauge coupling unification [11,12]. Since a is the anomalous $U(1)_A$ charge of the adjoint Higgs and is negative, the unification scale becomes smaller than the usual SUSY GUT scale.

In this paper, we study the nucleon decay via dimension-6 operators in the natural GUTs. The grand unification group is $SU(5)$,[1] $SO(10)$, or E_6. In $SO(10)$ and E_6 unification, we have additional gauge bosons which induce nucleon decay in addition to the X gauge bosons in $SU(5)$ GUT. We will include these new effects due to the extra gauge bosons. Moreover, we will include the effects of the matrices which make Yukawa matrices diagonal. The diagonalizing matrices are roughly fixed in the natural GUT in order to obtain the realistic quark and lepton mass matrices. In the estimation, we will use the hadron matrix elements calculated by lattice [16].

[1]Strictly, in the literature, $SU(5)$ natural GUT has not been proposed. However, we think that $SU(5)$ natural GUT is possible if the missing partner mechanism [15] is adopted.

II. DECAY WIDTHS OF THE NUCLEON

In this section, we show how to estimate the partial decay widths of nucleons from the effective Lagrangian which induces nucleon decays. The description in this section is based on Ref. [16].

In the standard model (SM), the dimension-6 operators which induce nucleon decay are classified completely [4] and are written by one lepton l and three quarks q as $\epsilon^{\alpha\beta\gamma}(\overline{l^c_\Gamma}q_{1\Gamma\alpha})(\overline{q^c_{2\Gamma'\beta}}q_{3\Gamma'\gamma})$, where α, β, and γ are color indices. Here, l^c is a charge-conjugated field of the lepton l, and in this paper we denote l^c_Γ as $(l_\Gamma)^c$, with the chirality indices $\Gamma, \Gamma' = L, R$. In the following, the color indices α, β, and γ are omitted in the operator $\epsilon^{\alpha\beta\gamma}(\overline{l^c_\Gamma}q_{1\Gamma\alpha})\times(\overline{q^c_{2\Gamma'\beta}}q_{3\Gamma'\gamma})$; i.e., we write it as $(\overline{l^c_\Gamma}q_{1\Gamma})(\overline{q^c_{2\Gamma'}}q_{3\Gamma'})$ for simplicity. Once we calculate the effective Lagrangian which induces nucleon decays as

$$\mathcal{L}_{\rm eff} = \sum_I C^I[(\overline{l^c_\Gamma}q_{1\Gamma})(\overline{q^c_{2\Gamma'}}q_{3\Gamma'})]^I, \tag{2}$$

where C^I is a coefficient of the operator $[(\overline{l^c_\Gamma}q_{1\Gamma})(\overline{q^c_{2\Gamma'}}q_{3\Gamma'})]^I$, we can estimate the partial decay widths of the nucleon as follows:

In order to calculate the decay widths, we must know the hadron matrix elements with the initial nucleon state $|N(\boldsymbol{k}, s)\rangle$, with the momentum $\boldsymbol{k}$ and the spin s, and the final meson state $\langle{\rm meson}(\boldsymbol{p})|$, with the momentum $\boldsymbol{p}$. These can be written as

$$\begin{aligned}\langle{\rm meson}(\boldsymbol{p})|q_{1\Gamma}(\overline{q^c_{2\Gamma'}}q_{3\Gamma'})|N(\boldsymbol{k}, s)\rangle \\ = P_\Gamma[W_0^{\Gamma\Gamma'}(q^2) - i\not{q}W_q^{\Gamma\Gamma'}(q^2)]u_N(\boldsymbol{k}, s),\end{aligned} \tag{3}$$

where $W_0^{\Gamma\Gamma'}$, $W_q^{\Gamma\Gamma'}$ are form factors and $\boldsymbol{q} \equiv \boldsymbol{p} - \boldsymbol{k}$ is a momentum of the antilepton. Here, P_Γ is a chiral projection operator and $u_N(\boldsymbol{k}, s)$ is a wave function of the nucleon. Usually, the first term in Eq. (3) dominates over the second term because the antilepton is lighter than the nucleon. Therefore, the hadron matrix elements can be estimated as

$$\langle{\rm meson}(\boldsymbol{p})|q_{1\Gamma}(\overline{q^c_{2\Gamma'}}q_{3\Gamma'})|N(\boldsymbol{k}, s)\rangle \simeq P_\Gamma W_0^{\Gamma\Gamma'}(q^2)u_N(\boldsymbol{k}, s). \tag{4}$$

In our calculation, we use the form factor $W_0^{\Gamma\Gamma'}$, which has been calculated by lattice [16] as in Table I.

Then, we can estimate the partial decay widths for the process $N \to {\rm meson} + l^c$ as

$$\begin{aligned}\Gamma(N \to {\rm meson} + l^c) \\ = \frac{1}{2m_N}\int\frac{d^3p}{(2\pi)^3}\frac{1}{2E_p}\frac{d^3q}{(2\pi)^3}\frac{1}{2E_q} \\ \times|\mathcal{M}(m_N \to p + q)|^2(2\pi)^4\delta^{(4)}(k - p - q) \\ \simeq \frac{m_N}{32\pi}\left\{1 - \left(\frac{m_{\rm meson}}{m_N}\right)^2\right\}^2\left|\sum_I C^I W_0^I(N \to {\rm meson})\right|^2,\end{aligned} \tag{5}$$

TABLE I. Form factors for nucleon decays, which have been calculated by lattice [16]. The first and second error values given for W_0^{RL}, W_0^{LR} represent statistical and systematic error, respectively.

Matrix element	W_0^{RL}, W_0^{LR}
$\langle\pi^0\vert(ud)u\vert p\rangle$, $\langle\pi^0\vert(du)d\vert n\rangle$	−0.103(23)(34)
$\langle\pi^+\vert(ud)d\vert p\rangle$, $-\langle\pi^-\vert(du)u\vert n\rangle$	−0.146(33)(48)
$\langle K^0\vert(us)u\vert p\rangle$, $-\langle K^-\vert(ds)d\vert n\rangle$	0.098(15)(12)
$\langle K^+\vert(us)d\vert p\rangle$, $-\langle K^0\vert(ds)u\vert n\rangle$	−0.054(11)(9)
$\langle K^+\vert(ud)s\vert p\rangle$, $-\langle K^0\vert(du)s\vert n\rangle$	−0.093(24)(18)
$\langle K^+\vert(ds)u\vert p\rangle$, $-\langle K^0\vert(us)d\vert n\rangle$	−0.044(12)(5)
$\langle\eta\vert(ud)u\vert p\rangle$, $-\langle\eta\vert(du)d\vert n\rangle$	0.015(14)(17)

where m_N and $m_{\rm meson}$ correspond to the masses of nucleons and mesons, respectively. Partial lifetimes of the nucleon are defined as the inverse of the partial decay widths.

Therefore, once the coefficient C^I, which is dependent on the concrete models, is known, the partial decay widths can be calculated.

III. CALCULATION OF THE COEFFICIENT C^I

In this section, we explain how to obtain the coefficients of the dimension-6 operators C^I at the scale $\mu = m_N$. Firstly, we discuss the effective interactions which are induced via superheavy gauge boson exchange. Secondly, we consider the effect of the unitary matrices which transform the flavor eigenstates to the mass eigenstates of quarks and leptons. Finally, we calculate the renormalization factors by using the renormalization group.

The coefficients are strongly dependent on the explicit GUT models. Therefore, we have to fix the GUT models which we consider in this paper. First of all, we fix the grand unification group as $SU(5)$, $SO(10)$, or E_6, since the superheavy gauge bosons which induce the nucleon decay are dependent on the grand unification group. We introduce the **10** of $SO(10)$ in addition to the **16** in $SO(10)$ GUT as matter fields. This is important in obtaining realistic quark and lepton masses and mixings in a natural way [10,11]. [In E_6 GUT [17,18], the fundamental representation **27** includes a **10** of $SO(10)$ as well as a **16**.] Moreover, we adopt Cabibbo-Kobayashi-Maskawa (CKM)-like matrices and Maki-Nakagawa-Sakata (MNS)-like matrices as the unitary matrices which transform flavor eigenstates to mass eigenstates of the **10** matter of $SU(5)$ and the $\bar{\mathbf{5}}$ matter, respectively.

A. Dimension-6 effective interactions via superheavy gauge boson exchange

Before discussing the dimension-6 interactions which induce nucleon decay, let us recall how to unify the quarks and leptons in the SM into E_6 GUT multiplets, because it is important to understand the embedding in E_6 GUTs in calculating the nucleon decay and in grasping the meaning of the GUT models discussed in this paper. All quarks and leptons are embedded into three **27** multiplets of E_6. The fundamental representation **27** is divided into several multiplets of $SO(10)$ as

$$\mathbf{27} \to \mathbf{16} + \mathbf{10} + \mathbf{1}. \tag{6}$$

The spinor **16** and the vector **10** of $SO(10)$ contain the SM multiplets as

$$\begin{aligned}\mathbf{16} \to & \underbrace{q_L(\mathbf{3},\mathbf{2})_{\frac{1}{6}} + u_R^c(\bar{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}} + e_R^c(\mathbf{1},\mathbf{1})_1}_{10} \\ & + \underbrace{d_R^c(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}} + l_L(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}}_{\bar{5}} + \underbrace{\nu_R^c(\mathbf{1},\mathbf{1})_0}_{1},\end{aligned} \tag{7}$$

$$\mathbf{10} \to \underbrace{D_R^c(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}} + L_L(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}}_{\bar{5}'} + \underbrace{\overline{D_R^c}(\mathbf{3},\mathbf{1})_{-\frac{1}{3}} + \overline{L_L}(\mathbf{1},\mathbf{2})_{\frac{1}{2}}}_{\mathbf{5}}, \tag{8}$$

where the numbers denote the representations under the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. Note that we have two $\bar{\mathbf{5}}$ fields in one **27**. Therefore, if we introduce three **27**'s for three generations of quarks and leptons, we have six $\bar{\mathbf{5}}$ fields of $SU(5)$. Three of the six $\bar{\mathbf{5}}$'s become superheavy with three **5** fields after breaking E_6 into the SM gauge group. The other three $\bar{\mathbf{5}}$ fields and three **10**'s of $SU(5)$ become quarks and leptons in three generations in the SM. In this paper, $\bar{\mathbf{5}}'$ denotes the $\bar{\mathbf{5}}$ fields from the **10** of $SO(10)$ to distinguish them from the $\bar{\mathbf{5}}$ fields from the **16**. In the literature, it has been argued that the main components of matter in the SM come from the first- and second-generation $\mathbf{27}_1$ and $\mathbf{27}_2$ as $(\bar{\mathbf{5}}_1, \bar{\mathbf{5}}'_1, \bar{\mathbf{5}}_2)$ [13], which plays an important role in obtaining realistic quark and lepton masses and mixings. Here the index denotes the original flavor index for the **27** of E_6. More details will be discussed in the next subsection. Note that we are required to calculate the dimension-6 interactions not only for the usual unified fields (**10** and $\bar{\mathbf{5}}$ fields), but also for the $\bar{\mathbf{5}}'$ fields for E_6 GUT models. [Also, in $SO(10)$ GUT models, the interactions which induce $\bar{\mathbf{5}}'$ fields must be calculated, because we introduce the **10** of $SO(10)$ as a matter field.]

In $SU(5)$ GUTs, the superheavy gauge bosons for the nucleon decay are X and $\bar{X}$, which are included in the adjoint gauge multiplet **24** of $SU(5)$. Since $SU(5)$ is a subgroup of $SO(10)$ and E_6, the X field contributes to nucleon decay even in $SO(10)$ and E_6 GUTs. The X field induces the effective dimension-6 interactions, which can be written in $SU(5)$ notation as $(\mathbf{10_i^\dagger 10_i} + \bar{\mathbf{5}}_\mathbf{i}^\dagger \bar{\mathbf{5}}_\mathbf{i} + \bar{\mathbf{5}}_\mathbf{i}'^\dagger \bar{\mathbf{5}}_\mathbf{i}') \times (\mathbf{10_j^\dagger 10_j} + \bar{\mathbf{5}}_\mathbf{j}^\dagger \bar{\mathbf{5}}_\mathbf{j} + \bar{\mathbf{5}}_\mathbf{j}'^\dagger \bar{\mathbf{5}}_\mathbf{j}')$, where $\mathbf{10_i}$ and $\bar{\mathbf{5}}_\mathbf{i}$ of $SU(5)$ are matter fields with flavor indices i, j. Here, the terms including $\bar{5}'$ must be taken into account in $SO(10)$ or E_6 GUT. In $SO(10)$ GUT models, additional fields X' and $\overline{X'}$ also induce nucleon decay. They are included in the adjoint gauge field **45**, divided as

$$\mathbf{45} \to \underbrace{G(\mathbf{8},\mathbf{1})_0 + W(\mathbf{1},\mathbf{3})_0 + \overline{X}(\mathbf{3},\mathbf{2})_{-\frac{5}{6}} + X(\bar{\mathbf{3}},\mathbf{2})_{\frac{5}{6}} + N^c(\mathbf{1},\mathbf{1})_0}_{24} + \underbrace{X'(\mathbf{3},\mathbf{2})_{\frac{1}{6}} + U_R'^c(\bar{3},\mathbf{1})_{-\frac{2}{3}} + E_R'^c(\mathbf{1},\mathbf{1})_1}_{10}$$
$$+ \underbrace{\overline{X'}(\bar{\mathbf{3}},\mathbf{2})_{-\frac{1}{6}} + \overline{U_R'^c}(\mathbf{3},\mathbf{1})_{\frac{2}{3}} + \overline{E_R'^c}(\mathbf{1},\mathbf{1})_{-1}}_{\overline{10}} + \underbrace{N'^c(\mathbf{1},\mathbf{1})_0}_{1}. \qquad (9)$$

The effective interactions induced by the X' field are included in the effective interaction $(\mathbf{10}_\mathbf{i}^\dagger \bar{\mathbf{5}}_\mathbf{i}) \cdot (\bar{\mathbf{5}}_\mathbf{j}^\dagger \mathbf{10}_\mathbf{j})$. Note that it does not include $\bar{\mathbf{5}}'$ fields, because the superfield $\mathbf{5}$'s will inevitably appear in the effective interactions with $\bar{\mathbf{5}}'$ fields. In E_6 GUTs, the additional superheavy gauge bosons X'' and $\overline{X''}$ can produce the nucleon decay. The new superheavy gauge bosons are included in $\mathbf{16}$ and $\overline{\mathbf{16}}$ of $SO(10)$ in the adjoint $\mathbf{78}$ of E_6, which is divided as

$$\mathbf{78} \to \mathbf{45} + \mathbf{16} + \overline{\mathbf{16}} + \mathbf{1}. \qquad (10)$$

The X'' field is included in $\mathbf{10}$ of $SU(5)$ in $\mathbf{16}$ and has the same quantum numbers as X' under the SM gauge group. This X'' field induces the effective interactions included in $(\mathbf{10}_i^\dagger \bar{\mathbf{5}}_\mathbf{i}') \cdot (\bar{\mathbf{5}}_\mathbf{j}'^\dagger 10_\mathbf{j})$.

By using the technique of decomposition of E_6 into the subgroup $SU(3)_C \times SU(3)_L \times SU(3)_R$ [18,19], the dimension-6 effective interactions for quark and lepton flavor eigenstates can be calculated as

$$\begin{aligned}\mathcal{L}_{\rm eff} = &\frac{g_{\rm GUT}^2}{M_X^2}\{(\overline{e_{Ri}^c}u_{Rj})(\overline{u_{Lj}^c}d_{Li}) + (\overline{e_{Ri}^c}u_{Rj})(\overline{u_{Li}^c}d_{Lj}) \\ &+ (\overline{e_{Li}^c}u_{Lj})(\overline{u_{Rj}^c}d_{Ri}) + (\overline{E_{Li}^c}u_{Lj})(\overline{u_{Rj}^c}D_{Ri}) \\ &- (\overline{\nu_{Li}^c}d_{Lj})(\overline{u_{Rj}^c}d_{Ri}) - (\overline{N_{Li}^c}d_{Lj})(\overline{u_{Rj}^c}D_{Ri})\} \\ &+ \frac{g_{GUT}^2}{M_{X'}^2}\{(\overline{e_{Li}^c}u_{Lj})(\overline{u_{Ri}^c}d_{Rj}) - (\overline{\nu_{Li}^c}d_{Lj})(\overline{u_{Ri}^c}d_{Rj})\} \\ &+ \frac{g_{\rm GUT}^2}{M_{X''}^2}\{(\overline{E_{Li}^c}u_{Lj})(\overline{u_{Ri}^c}D_{Rj}) - (\overline{N_{Li}^c}d_{Lj})(\overline{u_{Ri}^c}D_{Rj})\},\end{aligned} \qquad (11)$$

where $g_{\rm GUT}$ is the unified gauge coupling, and the superheavy gauge boson masses M_X, $M_{X'}$, and $M_{X''}$ are dependent on the vacuum expectation values (VEVs) of the GUT Higgs which break E_6 into the SM gauge group. In this paper, we assume that the adjoint Higgs has a Dimopoulos-Wilczek (DW)-type VEV [20],

$$\langle \mathbf{45}_A \rangle = i\sigma_2 \begin{pmatrix} x & & & & \\ & x & & & \\ & & x & & \\ & & & 0 & \\ & & & & 0 \end{pmatrix}, \qquad (12)$$

to solve the doublet-triplet splitting problem. Here $\mathbf{45}_A$ is the $\mathbf{45}$ component field of the E_6 adjoint Higgs A in $SO(10)$ decomposition, and $\sigma_i (i = 1, 2, 3)$ is the Pauli matrix. This is because in the anomalous $U(1)_A$ GUTs, the DW-type VEV can be obtained in a natural way, and it is easier to obtain the realistic quark and lepton masses and mixings than with the other mechanism for solving the doublet-triplet splitting problem. This DW-type VEV breaks $SO(10)$ into $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. The superheavy gauge boson masses are given by[2]

$$M_X^2 = g_{\rm GUT}^2 x^2, \qquad M_{X'}^2 = g_{\rm GUT}^2 (x^2 + v_c^2),$$
$$M_{X''}^2 = g_{\rm GUT}^2 \left(\frac{1}{4}x^2 + v_\phi^2\right). \qquad (13)$$

Here, v_ϕ and v_c are the VEV of the E_6 Higgs $\Phi(\mathbf{27})$, which breaks E_6 into $SO(10)$, and the VEV of the $SO(10)$ Higgs $C(\mathbf{27})$, which breaks $SO(10)$ into $SU(5)$, respectively. [And $\bar{\Phi}(\overline{\mathbf{27}})$ and $\bar{C}(\overline{\mathbf{27}})$ are also needed to satisfy the D flatness conditions of E_6.] Note that the mass of the X' gauge boson is almost the same as that of X in anomalous $U(1)_A$ GUT, because $v_c \ll x$ in order to obtain the DW-type VEV in a natural way [10,11]. In some of the typical E_6 GUTs with anomalous $U(1)_A$[14], v_ϕ is smaller than x. And therefore the X'' as well as the X' can play an important role in nucleon decay.

Note that the interactions induced by the X'' gauge boson are only between the $\mathbf{10}$ and $\bar{\mathbf{5}}'$ fields, while the interactions induced by X' are only between the $\mathbf{10}$ and $\bar{\mathbf{5}}$ fields, and those induced by X include various interactions among $\mathbf{10}$, $\bar{\mathbf{5}}$, and $\bar{\mathbf{5}}'$ fields. Therefore, the X'' gauge boson contributes to the nucleon decay only for the restricted models in which some of the first and second generations of quarks and leptons include the $\bar{\mathbf{5}}'$ fields as the components.

These VEVs can be fixed by their anomalous $U(1)_A$ charges as

$$x \sim \lambda^{-a}\Lambda, \qquad v_c \sim \lambda^{-\frac{1}{2}(c+\bar{c})}\Lambda, \qquad v_\phi \sim \lambda^{-\frac{1}{2}(\phi+\bar{\phi})}\Lambda, \qquad (14)$$

where a, ϕ, $\bar{\phi}$, c, and $\bar{c}$ are the anomalous $U(1)_A$ charges for A, Φ, $\bar{\Phi}$, C, and $\bar{C}$, respectively [13]. Each VEV has an

[2]Under the $SU(3)_C \times SU(3)_L \times SU(3)_R$ decomposition of E_6, the gauge fields $\bar{X}$, X', and X'' are included in the $(\mathbf{3},\mathbf{3},\mathbf{3})$ representation. Since $\bar{X}$ and X' are $SU(2)_R$ doublets, the same contribution to the masses comes from the adjoint Higgs VEV x, which breaks $SO(10)$ into $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. From the fact that the $B-L$ charge is proportional to $\lambda_8^L + \lambda_8^R$, which is one of the generators of $SU(3)_L$ and $SU(3)_R$, we can calculate the contributions to the masses of X, X', and X'' as in Eq. (13). Here λ_A $(A = 1, 2, \ldots, 8)$ denotes the Gell-Mann matrices.

$O(1)$ uncertainty that comes from $O(1)$ ambiguities in each term in the Lagrangian. As mentioned in the Introduction, the unification scale $\Lambda_u \equiv \langle A \rangle \sim x$ becomes lower than the usual GUT scale, $\Lambda_G \sim 2 \times 10^{16}$ GeV, because the cutoff $\Lambda = \Lambda_G$, $\lambda < 1$, and the $U(1)_A$ charges for the Higgs fields like A are negative in general. Therefore, the nucleon decay via dimension-6 operators is enhanced in the anomalous $U(1)_A$ GUT scenario [11]. Here we consider two typical $U(1)_A$ charge assignments: ($a = -1$, $\phi + \bar{\phi} = -1$, $c + \bar{c} = -4$) and ($a = -1/2$, $\phi + \bar{\phi} = -2$, $c + \bar{c} = -5$) [14]. In this paper we take $\lambda \sim 0.22$. Note that the relation $x \gg v_c$ is always satisfied in the anomalous $U(1)_A$ GUT with the DW-type VEV, because the term which destabilizes the DW-type VEV is allowed if $c + \bar{c}$ becomes larger. It means that the X' gauge boson has a sizable contribution to the nucleon decay in the anomalous $U(1)_A$ GUTs. On the other hand, the relation $v_\phi > x$ is obtained in the former model, but $v_\phi < x$ in the latter model. In this paper, we study the latter model because its X'' gauge boson has a larger contribution to the nucleon decay. The prediction of the former model is similar to that of the $SO(10)$ model, because the contribution of the E_6 gauge boson X'' becomes smaller.

The results in Eq. (11) in E_6 GUT models can be applied to the $SO(10)$ GUTs in the limit $M_{X''} \to \infty$, and to the $SU(5)$ GUTs in the limit $M_{X''}$, $M_{X'} \to \infty$. If the **10** of $SO(10)$ is not introduced in $SO(10)$ models, just neglect the terms which include the $\bar{\mathbf{5}}'$ fields in Eq. (11).

B. Realistic flavor mixings in anomalous $U(1)_A$ GUT models

One of the most important features in the anomalous $U(1)_A$ models is that the interactions can be determined by the anomalous $U(1)_A$ charges of the fields except for the $O(1)$ coefficients. For example, the Yukawa interactions and the right-handed neutrino masses are

$$Y_u^{ij} q_{Li} u_{Rj}^c h_u + Y_d^{ij} q_{Li} d_{Rj}^c h_d + Y_e^{ij} l_{Li} e_{Rj}^c h_d + Y_{\nu_D}^{ij} l_{Li} \nu_{Rj}^c h_u + M_{\nu_R}^{ij} \nu_{Ri}^c \nu_{Rj}^c, \quad (15)$$

where the Yukawa matrices and the right-handed neutrino masses can be written as [21]

$$\begin{aligned} Y_u^{ij} &= \lambda^{q_{Li}+u_{Rj}^c+h_u}, \qquad Y_d^{ij} = \lambda^{q_{Li}+d_{Rj}^c+h_d}, \\ Y_e^{ij} &= \lambda^{l_{Li}+e_{Rj}^c+h_d}, \qquad Y_{\nu_D}^{ij} = \lambda^{l_{Li}+\nu_{Rj}^c+h_u}, \\ M_{\nu_R}^{ij} &= \lambda^{\nu_{Ri}^c+\nu_{Rj}^c}\Lambda. \end{aligned} \quad (16)$$

Here, h_u and h_d are the Higgs doublets for up quarks and for down quarks, respectively. We have used the notation in which the matter and Higgs fields and the minimal SUSY SM Higgs h_u and h_d and the $U(1)_A$ charges are represented by the same characters as the corresponding fields. By unitary transformation,

$$\psi'_{Li} = (L_\psi^\dagger)_{ij} \psi_{Lj}, \qquad \psi'^c_{Ri} = (R_\psi^\dagger)_{ij} \psi_{Rj}^c, \quad (17)$$

where $\psi = u$, d, e, ν, these Yukawa matrices can be diagonalized. Since $u_L(\nu_L)$ and $d_L(e_L)$ are included in $q_L(l_L)$, we use $q_L = u_L = d_L$ ($l_L = \nu_L = e_L$) as their $U(1)_A$ charges. Here, ψ' is a mass eigenstate, and ψ is a flavor eigenstate. What is important in the anomalous $U(1)_A$ theory is that not only quark and lepton masses, but also the CKM matrix [22] and the MNS matrix [23], which are defined as

$$U_{\rm CKM} = L_u^\dagger L_d, \qquad U_{\rm MNS} = L_\nu^\dagger L_e, \quad (18)$$

can be determined by their anomalous $U(1)_A$ charges as

$$\begin{aligned} m_{ui} &= \lambda^{q_{Li}+u_{Ri}^c+h_u}\langle h_u \rangle, \qquad m_{di} = \lambda^{q_{Li}+d_{Ri}^c+h_d}\langle h_d \rangle, \\ m_{ei} &= \lambda^{l_{Li}+e_{Ri}^c+h_d}\langle h_d \rangle, \qquad m_{\nu_i} = \lambda^{2l_{Li}+2h_u}\frac{\langle h_u \rangle^2}{\Lambda}, \\ (U_{\rm CKM})_{ij} &= \lambda^{|q_{Li}-q_{Lj}|}, \qquad (U_{\rm MNS})_{ij} = \lambda^{|l_{Li}-l_{Lj}|}, \end{aligned} \quad (19)$$

except for $O(1)$ coefficients. Any mass hierarchies can be obtained by choosing the appropriate $U(1)_A$ charges, but we have several simple predictions for mixings: $(U_{\rm CKM})_{13} \sim (U_{\rm CKM})_{12}(U_{\rm CKM})_{23}$, $(U_{\rm MNS})_{13} \sim (U_{\rm MNS})_{12} \times (U_{\rm MNS})_{23}$, $(U_{\rm MNS})_{23}^4 \sim (m_{\nu 3}^2 - m_{\nu 2}^2)/(m_{\nu 2}^2 - m_{\nu 1}^2)$. Note that a normal hierarchy for neutrino masses is also predicted. Not only are these predictions consistent with the observations, but also realistic quark and lepton masses and mixings can be obtained by choosing the $U(1)_A$ charges. For example, if we take $q_{L1} - q_{L2} = 1$ and $q_{L2} - q_{L3} = 2$, we can obtain the realistic CKM matrix when $\lambda \sim 0.22$. Taking $l_{L1} \sim l_{L2} \sim l_{L3}$, the neutrino mixings become large.

In the $SU(5)$ unification, because of the unification of matter, we have some constraints among their $U(1)_A$ charges, as $a_i \equiv q_{Li} = u_{Ri}^c = e_{Ri}^c$ and $\bar{f}_i \equiv d_{Ri}^c = l_{Li}$. Then, basically, these charges are fixed in order to obtain realistic quark and lepton mixings. It is quite impressive that even with these charges, realistic hierarchical structures of quark and lepton masses are also obtained. Actually, the requirement results in up-type quarks having the largest mass hierarchy, neutrinos having the weakest, and down-type quarks and charged leptons having middle mass hierarchies. These are nothing but the observed mass hierarchies for quarks and leptons, though the first-generation neutrino mass has not been observed yet. The unrealistic GUT relation $Y_d = Y_e^t$ can easily be avoided in anomalous $U(1)_A$ GUT, because the higher-dimensional interaction $\lambda^{a_i+\bar{f}_j+a+h_d} A_i A \bar{F}_j H_d$, which breaks the unrealistic GUT relation $Y_d = Y_e^t$ after developing the VEV of the adjoint Higgs A as $\langle A \rangle \sim \lambda^{-a}$, gives the same order contribution to the Yukawa couplings as the original Yukawa interactions $\lambda^{a_i+\bar{f}_j+h_d} A_i \bar{F}_j H_d$. Here A_i is **10** matter of $SU(5)$, and $\bar{F}_i$ is $\bar{\mathbf{5}}$ matter.

However, in the minimal $SO(10)$ GUT, all quarks and leptons in one generation can be unified into a single multiplet, and therefore, the $U(1)_A$ charges for q_L become the same as l_L. That leads to the same mixings of quarks and leptons. This is an unrealistic prediction in the minimal $SO(10)$ GUT with the anomalous $U(1)_A$ gauge symmetry.

There are several solutions that realize realistic flavor mixings in $SO(10)$ GUT models. One of them is to introduce one or a few additional 10's of $SO(10)$ fields as matter fields. When one of the $\bar{\mathbf{5}}$ fields of $SU(5)$ from a **10** of $SO(10)$ becomes quarks and leptons, we have a different $U(1)_A$ charge hierarchy for $\bar{\mathbf{5}}$ fields from that for a **10** of $SU(5)$. As a result, the realistic quark and lepton masses and mixings can be obtained [10,11].

One of the most important features in E_6 unification is that the additional **10**'s of $SO(10)$ fields in $SO(10)$ unification are automatically introduced, because the fundamental representation **27** includes a **10** in addition to a **16** of $SO(10)$. Moreover, the assumption in $SU(5)$ unification that the **10** fields induce stronger hierarchical Yukawa couplings than $\bar{\mathbf{5}}$ fields can be derived in E_6 unification. Since three **27** matters are introduced for the quarks and leptons, we have six $\bar{\mathbf{5}}$ fields. Three of the six $\bar{\mathbf{5}}$ fields become superheavy with three **5** fields, as noted in the previous subsection. Since the third-generation field $\mathbf{27_3}$ has larger Yukawa couplings due to smaller $U(1)_A$ charge, it is natural that two $\bar{\mathbf{5}}$ fields from $\mathbf{27_3}$ become superheavy, and therefore, three massless $\bar{\mathbf{5}}$ fields come from the first- and second-generation fields $\mathbf{27_1}$ and $\mathbf{27_2}$ [13]. As a result, we can obtain a milder hierarchy for $\bar{\mathbf{5}}$ fields than the original hierarchy for **10** fields, which is nothing but what we would like to explain. The main modes of three typical massless $\bar{\mathbf{5}}$ fields are $\bar{\mathbf{5}}_1$, $\bar{\mathbf{5}}'_1$, and $\bar{\mathbf{5}}_2$. It is important that the Yukawa couplings of $\bar{\mathbf{5}}'$ can also be controlled by $U(1)_A$ charges of matters and Higgs. Therefore, we can choose which $\bar{\mathbf{5}}$ field becomes a $\bar{\mathbf{5}}'_1$ by fixing the $U(1)_A$ charges. In order to obtain the larger neutrino mixings, it is the best that the main component of the second-generation $\bar{\mathbf{5}}$ be $\bar{\mathbf{5}}'_1$. Though we do not discuss here the details for the realistic models and explicit charge assignments, we can obtain realistic mixing matrices as

$$U_{\rm CKM} = \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \qquad U_{\rm MNS} = \begin{pmatrix} 1 & \lambda^{\frac{1}{2}} & \lambda \\ \lambda^{\frac{1}{2}} & 1 & \lambda^{\frac{1}{2}} \\ \lambda & \lambda^{\frac{1}{2}} & 1 \end{pmatrix}. \tag{20}$$

These matrices have $O(1)$ uncertainties which come from $O(1)$ ambiguities of Yukawa interactions.

For the calculation of the nucleon decay widths, the explicit flavor structure is quite important. Strictly speaking, these massless modes have mixings with the superheavy fields $\bar{\mathbf{5}}'_2$, $\bar{\mathbf{5}}'_3$, and $\bar{\mathbf{5}}_3$, but in our calculation, we neglect these mixings because their contribution is quite small. We just consider the mixings between $\bar{\mathbf{5}}_1$ and $\bar{\mathbf{5}}'_1$, and $\bar{\mathbf{5}}_2$ in E_6 unification.

C. Renormalization factor

To calculate coefficients for the dimension-6 effective interactions at the nucleon mass scale, we have to consider the renormalization factors. For the calculation, we have to divide the scale region into two parts. The first region is from the GeV scale to the SUSY-breaking scale. We call the effect from this region the "long-distance effect," and the renormalization group factor is written as A_{Rl}[24]. The other region is from the SUSY scale to the GUT scale. We call the effect from this region the "short-distance effect," and the renormalization group factor is written as A_{Rs} [25,26]. The total renormalization factor A_R is defined as follows:

$$A_R = A_{Rl} \times A_{Rs}. \tag{21}$$

To calculate coefficients of dimension-6 effective interactions at the GeV scale, we multiply the renormalization factor by the dimension-6 effective interactions at the GUT scale.

One-loop calculation gives the renormalization factor for each region and for each gauge interaction as

$$A_{Ri} = \left(\frac{\alpha_i(M_{\rm end})}{\alpha_i(M_{\rm start})}\right)^{\frac{A_i}{b_i}}, \tag{22}$$

$$\gamma_i = -2A_i \frac{g_i^2}{(4\pi)^2}, \qquad \beta_i = b_i \frac{g_i^3}{(4\pi)^2}, \tag{23}$$

where γ_i is the anomalous dimension for dimension-6 operators for each SM gauge interaction, and β_i is the β function for each gauge coupling. $M_{\rm start}$ and $M_{\rm end}$ are the energy scales of the boundaries of each region ($M_{\rm end} > M_{\rm start}$).

The value is dependent on the explicit GUT model. In this paper, we use the renormalization factor of the minimal SUSY $SU(5)$ GUT $A_R = 3.6$ for the dimension-6 operators which include a right-handed charged lepton e^c_R, and we use $A_R = 3.4$ for the operators which include the doublet leptons l as the reference values [26]. In order to apply our results to an explicit GUT model, the correction for the renormalization factor is needed. For example, in an anomalous $U(1)_A$ SUSY $SO(10)$ GUT [explicit $U(1)_A$ charges are given in the caption of Fig. 1 in Ref. [11]], the renormalization factor can be estimated as $A_R = 3.2$ ($A_{Rl} = 1.5, A_{Rs} = 2.1$) for the operators which include the singlet charged lepton e^c_R. In this model, the gauge couplings become larger because there are a lot of superheavy particles, which increases the renormalization factor. However, the unification scale is lower, which decreases the renormalization factor. The latter effect is larger in this model. Therefore, the nucleon lifetime in this anomalous $U(1)_A$ SUSY $SO(10)$ GUT model[3] is $(3.2/3.6)^{-2} = 1.3$ times longer than the calculated values when using the renormalization group factor in the minimal $SU(5)$ SUSY GUT model.

[3]Strictly, the absolute value of the adjoint Higgs VEV $\langle A \rangle$ in this model is different from the VEVs adopted in this paper. The correction about $(2)^{-4}$ is needed for the lifetime of the nucleon.

IV. GUT MODELS

In order to obtain realistic quark and lepton masses and mixings in an anomalous $U(1)_A$ GUT scenario, the diagonalizing matrices for $\bar{\mathbf{5}}$ fields have large mixings as MNS matrices, while those for **10** fields have small mixings as CKM matrices. Namely,

$$L_u \sim L_d \sim R_u \sim R_e \sim U_{\rm CKM}, \tag{24}$$

$$R_d \sim L_e \sim L_\nu \sim U_{\rm MNS}. \tag{25}$$

Since the quark and lepton mixings are determined by the charges of left-handed quarks and leptons, respectively, the above result for diagonalizing matrices is inevitable in the anomalous $U(1)_A$ GUT.

We calculate various nucleon decay modes in the following anomalous $U(1)_A$ GUT models:

(1) *SU*(5) *Model*—In *SU*(5) unification, without loss of generality, we can take one of the diagonalizing matrices for **10** fields and one of the diagonalizing matrices for $\bar{\mathbf{5}}$ fields as unit matrices by field redefinitions. In this paper, we take $R_u = 1$ and $R_d = 1$. Because of the relations $U_{CKM} = L_u^\dagger L_d$ and $U_{MNS} = L_\nu^\dagger L_e$, we have three independent diagonalizing matrices in $SU(5)$ unification.

(2) *SO*(10) *Model 1*—In $SO(10)$ unification, one **10** of $SO(10)$ is introduced as an additional matter field in order to obtain realistic quark and lepton masses and mixings. It is essential that since $\bar{\mathbf{5}}_\mathbf{3}$ becomes superheavy with **5** and is replaced with the $\bar{\mathbf{5}}'$ from the additional fields, the diagonalizing matrices for $\bar{\mathbf{5}}$ fields can be much different from a **10** of $SU(5)$ fields. Note that the main modes of $\bar{\mathbf{5}}$ fields become $(\bar{\mathbf{5}}_1, \bar{\mathbf{5}}', \bar{\mathbf{5}}_2)$. It is reasonable that the $\bar{\mathbf{5}}'$ becomes the second-generation $\bar{\mathbf{5}}$ field to obtain the large neutrino mixings. Without loss of generality, we can take one of the diagonalizing matrices as a unit matrix, and in this paper, we take $R_u = 1$. Because of the relations $U_{CKM} = L_u^\dagger L_d$ and $U_{MNS} = L_\nu^\dagger L_e$, we have four independent diagonalizing matrices in $SO(10)$ unification.

(3) E_6 *Model 1*—In E_6 unification, the additional **10** of $SO(10)$ matter is included in the fundamental representation **27** of E_6 in addition to **16**. It is reasonable that $\bar{\mathbf{5}}$ fields from $\mathbf{27_3}$ become superheavy because they have larger couplings than $\mathbf{27_1}$ and $\mathbf{27_2}$. Therefore, $\bar{\mathbf{5}}$ fields in the standard model come from $\mathbf{27_1}$ and $\mathbf{27_2}$. The main modes become $(\bar{\mathbf{5}}_\mathbf{1}, \bar{\mathbf{5}}'_\mathbf{1}, \bar{\mathbf{5}}_\mathbf{2})$. If the $\bar{\mathbf{5}}'_\mathbf{1}$ becomes the second-generation $\bar{\mathbf{5}}$ field, the large neutrino mixings can be obtained as noted in the previous section. We have four independent diagonalizing matrices as in $SO(10)$ unification.

The values of the GUT Higgs VEVs are also important in calculating the partial decay widths of nucleons. In the anomalous $U(1)_A$ GUT, these are fixed by their $U(1)_A$ charges. In these models, we take these VEVs as

$$x = 1 \times 10^{16}\ \text{GeV}, \qquad v_c = 5 \times 10^{14}\ \text{GeV}, \qquad v_\phi = 5 \times 10^{15}\ \text{GeV}. \tag{26}$$

We have two typical $U(1)_A$ charge assignments in E_6 unification, which yield ($x \sim \lambda^{0.5}\Lambda_G$, $v_c \sim \lambda^{2.5}\Lambda_G$, $v_\phi \sim \lambda\Lambda_G$) and ($x \sim \lambda\Lambda_G$, $v_c \sim \lambda^2\Lambda_G$, $v_\phi \sim \lambda^{0.5}\Lambda_G$). We adopted the former assignment in these models because the contribution from the E_6 gauge boson X'' becomes larger. The latter assignment gives similar results to the $SO(10)$ model.

V. NUMERICAL CALCULATION

In our calculation, the ambiguities in the diagonalizing matrices are considered by randomly generating ten unitary matrices for each independent L_f and R_f ($f = u, d, e, \nu$). The unitary matrices must satisfy the following requirements:

(1) We take real unitary matrices for simplicity.

(2) $L_u = L_d U_{\rm CKM}^{(\rm exp)\dagger}$ and $L_\nu = L_e U_{\rm MNS}^{(\rm exp)\dagger}$, where [27–29]

$$U_{\rm CKM}^{(\rm exp)} = \begin{pmatrix} 0.97 & 0.23 & 0.0035 \\ -0.23 & 0.97 & 0.041 \\ 0.0086 & -0.040 & 1.0 \end{pmatrix}, \quad U_{\rm MNS}^{(\rm exp)} = \begin{pmatrix} 0.83 & 0.54 & 0.15 \\ -0.48 & 0.53 & 0.70 \\ 0.30 & -0.65 & 0.70 \end{pmatrix}. \tag{27}$$

(3) $L_u \sim L_d \sim R_e \sim U_{\rm CKM}$ and $L_\nu \sim L_e (\sim R_d) \sim U_{\rm MNS}$, where

$$U_{\rm CKM} = \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad U_{\rm MNS} = \begin{pmatrix} 1 & \lambda^{0.5} & \lambda \\ \lambda^{0.5} & 1 & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix}. \tag{28}$$

Each component has the $O(1)$ coefficient C_{ij}, and we take $0.5 \le C_{ij} \le 2$.

Since we have three independent diagonalizing matrices in $SU(5)$ unification, we examine 10^3 model points. In $SO(10)$ and E_6 unification, four independent diagonalizing matrices lead to 10^4 model points.

A. Various decay modes for the proton

We calculate the lifetime of the proton for various decay modes. The results are shown in Figs. 1–3. We plot the lifetime of the most important decay mode, $p \to \pi^0 + e^c$, on the horizontal axis and the lifetime of the other decay modes on the vertical axis. In Fig. 1, the large gray circles show the predictions of the minimal $SU(5)$ GUT model in

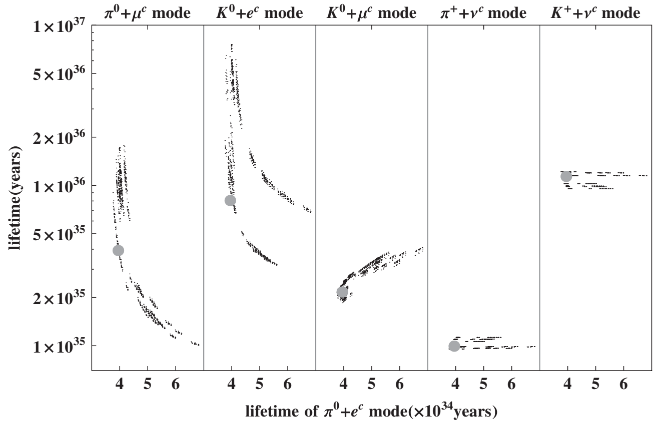

FIG. 1. Various lifetimes for the proton in the $SU(5)$ model with $M_X = g_{\rm GUT} x$ and $x = 1 \times 10^{16}$ GeV. The large gray circles show the predictions of the minimal $SU(5)$ GUT model [8].

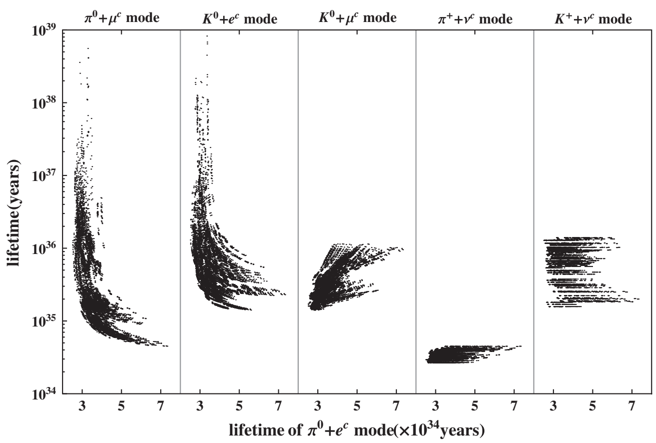

FIG. 2. Various lifetimes for the proton in $SO(10)$ model 1 with $M_X = g_{\rm GUT} x$, $M_{X'} = g_{\rm GUT}\sqrt{x^2 + v_c^2}$, $x = 1 \times 10^{16}$ GeV, and $v_c = 5 \times 10^{14}$ GeV.

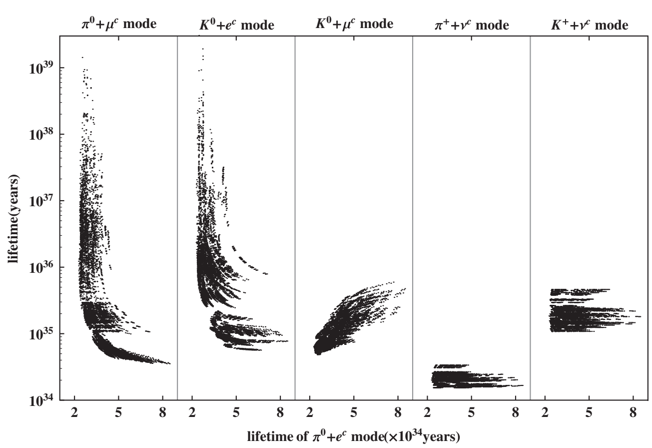

FIG. 3. Various lifetimes for the proton in E_6 model 1 with $M_X = g_{\rm GUT} x$, $M_{X'} = g_{\rm GUT}\sqrt{x^2 + v_c^2}$, $M_{X''} = g_{\rm GUT}\sqrt{\frac{x^2}{4} + v_\phi^2}$, $x = 1 \times 10^{16}$ GeV, $v_c = 5 \times 10^{14}$ GeV, and $v_c = 5 \times 10^{15}$ GeV.

which all the diagonalizing matrices can be fixed [8], although it has unrealistic GUT relations for the Yukawa couplings between the charged leptons and the down-type quarks. Here, we have used the same value for the VEV x as the value we adopted in this paper.

We have several comments on these results. First, the predicted lifetime of the $p \to \pi^0 + e^c$ decay mode is not far from the experimental lower bound, $\tau(p \to \pi^0 + e^c) > 1.29 \times 10^{34}$ years [6]. Note that these results are obtained for models with a unification scale $\Lambda_u \sim 1 \times 10^{16}$ GeV. Therefore, for the models with $a = -1$ (typically $\Lambda_u \sim 5 \times 10^{15}$ GeV), the predicted value becomes more than 1 order shorter. Of course, since we have the $O(1)$ ambiguity for the unification scale, which easily leads to a predicted lifetime more than 1 order longer, and the hadron matrix elements have still large uncertainties, these models ($a = -1$) cannot be excluded by this observation. What is important here is that we should not be surprised if nucleon decay via dimension-6 operators is observed in the very near future. Second, the lifetimes of the decay modes which include an antineutrino are calculated by summing up the partial decay widths for different antineutrino flavors, because the flavor of the neutrino cannot be distinguished by the present experiments for nucleon decay. As a result, the lifetime of the decay modes which include an antineutrino have less dependence on the parameters, because the dependence can be canceled due to unitarity of the diagonalizing matrix L_ν [30]. Third, the flavor-changing decay modes—for example, the $p \to \pi^0 + \mu^c$ and $p \to K^0 + e^c$ decay modes—have a stronger dependence on the explicit $O(1)$ parameters in the diagonalizing matrices than the flavor-unchanging decay modes, $p \to \pi^0 + e^c$ and $p \to K^0 + \mu^c$. This is mainly because off-diagonal elements have stronger ambiguities than the diagonal elements in diagonalizing matrices. Fourth, we comment on the shape for the $p \to \pi^0 + \mu^c$, $p \to K^0 + e^c$, and $p \to K^0 + \mu^c$ modes. Because of the unitarity of L_e and R_e, the longer lifetime of $p \to \pi^0 + e^c$ leads to a shorter lifetime for the $p \to \pi^0 + \mu^c$ and $p \to K^0 + e^c$ modes and a longer lifetime for the $p \to K^0 + \mu^c$ mode. These tendencies can be seen in the figures.

Finally, we comment on the shape of the figure for the decay modes which include an antineutrino. In the figures, a lot of lines which parallel the horizontal axis can be seen. This is because the $O(1)$ parameters in the diagonalizing matrices, L_e and R_e, change the lifetime of the $p \to \pi^0 + e^c$ decay mode, but they do not change the lifetime of decay modes which have an antineutrino in the final state. L_e would change the lifetime of decay modes with antineutrinos through the relation $L_\nu = L_e U_{\rm MNS}^{({\rm exp})\dagger}$. However, as noted above, the different L_ν's have the same contribution to the decay modes with antineutrinos in which all different flavors are summed up, because of the unitarity of L_ν.

In the next subsection, we would like to discuss how to identify the GUT models by the nucleon decay modes.

For identification, we use the $p \to \pi^0 + e^c$, $n \to \pi^0 + \nu^c$, and $p \to K^0 + \mu^c$ decay modes because these are less dependent on the $O(1)$ parameters, where the $n \to \pi^0 + \nu^c$ mode has also only small dependence on the $O(1)$ parameters as the $p \to \pi^+ + \nu^c$ has.

B. Identification of GUT models

In this subsection, we discuss how to distinguish GUT models by the nucleon decay. We emphasize that the ratios of the partial decay widths for $p \to \pi^0 + e^c$, $n \to \pi^0 + \nu^c$, and $p \to K^0 + \mu^c$ are important for the identification of GUT models. The partial decay width is strongly dependent on the explicit values of the VEVs. However, by taking the ratio, part of the dependence can be canceled. The results become independent of the absolute magnitudes of these VEVs and are dependent only on the ratios of the VEVs. Therefore, the results can be applied to other GUT models with different VEVs, but with the same ratios of VEVs.

First, we would like to explain that the ratio of the decay width for the $n \to \pi^0 + \nu^c$ mode to the decay width for the $p \to \pi^0 + e^c$ mode is useful for distinguishing GUT models [31], especially the grand unification group. In $SU(5)$ GUT models as in Eq. (11), there are four effective interactions which are important for nucleon decay. Three of them induce the decay modes which include e^c in the final state, while just one of them causes the decay modes which include ν^c. Therefore, in $SU(5)$ unification, the ratio becomes quite smaller than 1. In $SO(10)$ unification, two effective interactions are added which contribute to the decay modes with e^c and to those with ν^c equivalently. In E_6 unification, two effective interactions with E^c and with N^c are added, and the contribution to $n \to \pi^0 + \nu^c$ through the flavor mixings becomes larger than the contribution to $p \to \pi^0 + e^c$. Here the essential point is that the $SO(10)$ superheavy gauge boson X' and the E_6 superheavy gauge boson X'' induce only the effective interactions which include $\bar{\mathbf{5}}$ fields of $SU(5)$, while the $SU(5)$ superheavy gauge boson X can also induce the effective interactions which include only a $\mathbf{10}$ of $SU(5)$. Therefore, basically, the models with the larger grand unification group lead to the larger ratio if the contributions from X' and X'' are not negligible. This feature is useful for identifying the grand unification group, especially when the X' and X'' bosons are as light as the X. In the anomalous $U(1)_A$ GUT models, the masses of X' and X'' can be comparable to the X mass, or even smaller than the mass of X. Therefore, this identification is quite useful.

We calculate the ratio of the decay width for the $p \to \pi^0 + e^c$ mode to the decay width for the $n \to \pi^0 + \nu^c$ mode for the anomalous $U(1)_A$ GUT models as

$$R_1 \equiv \frac{\Gamma_{n\to\pi^0+\nu^c}}{\Gamma_{p\to\pi^0+e^c}} = \begin{cases} 0.18\text{–}0.34 & SU(5)\text{ model} \\ 0.35\text{–}0.90 & SO(10)\text{ model 1}\,. \\ 0.38\text{–}2.5 & E_6\text{ model 1} \end{cases} \quad (29)$$

It is obvious that the ratio $\frac{\Gamma_{n\to\pi^0+\nu^c}}{\Gamma_{p\to\pi^0+e^c}}$ becomes larger for the larger grand unification group. However, we cannot distinguish these GUT models by this ratio perfectly, because we have the $O(1)$ ambiguities in the diagonalizing matrices. There is a region in which both $SO(10)$ and E_6 GUTs are allowed.

In order to distinguish the $SO(10)$ and E_6 models, we propose an additional ratio of partial decay widths, $R_2 \equiv \frac{\Gamma_{p\to K^0+\mu^c}}{\Gamma_{p\to\pi^0+e^c}}$. One important fact is that the $SO(10)$ superheavy gauge boson X' cannot induce the effective interactions which include the second-generation fields which come from the $\mathbf{10}$ of $SO(10)$. On the other hand, the E_6 superheavy gauge boson X'' induces only the effective interactions which include the second-generation fields from the $\mathbf{10}$ of $SO(10)$. Therefore, the ratio $\frac{\Gamma_{p\to K^0+\mu^c}}{\Gamma_{p\to\pi^0+e^c}}$ can play an important role in identifying the grand unification group. See Fig. 4, in which we plot R_1 on the horizontal axis and R_2 on the vertical axis. The figure shows that various model points can be classified into three regions corresponding to the three grand unification groups, $SU(5)$, $SO(10)$, and E_6. These three GUT classes can be distinguished by these observations.

Of course, these results are strongly dependent on the explicit models and their parameters, especially the VEVs, which we have taken as $x = 1\times10^{16}$ GeV, $v_c = 5\times10^{14}$ GeV, and $v_\phi = 5\times10^{15}$ GeV. However, we should note that the effect of the $SO(10)$ superheavy gauge boson X' is almost maximal in these VEVs because $v_c \ll x$. On the other hand, the contribution from the E_6 superheavy gauge boson X'' can be larger, because the contributions to the X'' mass from the VEV v_ϕ and from the VEV x are comparable in these parameters. Therefore, if the ratio R_1

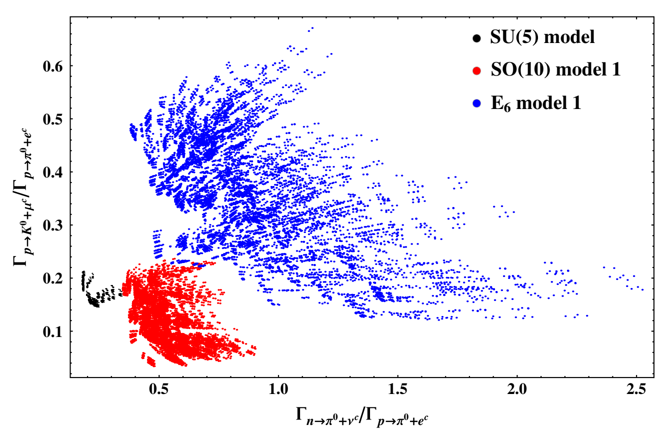

FIG. 4 (color online). In $SU(5)$, we have 10^3 model points, because we have three independent diagonalizing matrices, and we generate 10 unitary matrices for each independent matrix. In $SO(10)$ and E_6, we have 10^4 model points, because we have four independent diagonalizing matrices. VEVs are taken as $x = 1\times10^{16}$ GeV, $v_c = 5\times10^{14}$ GeV, and $v_\phi = 5\times10^{15}$ GeV.

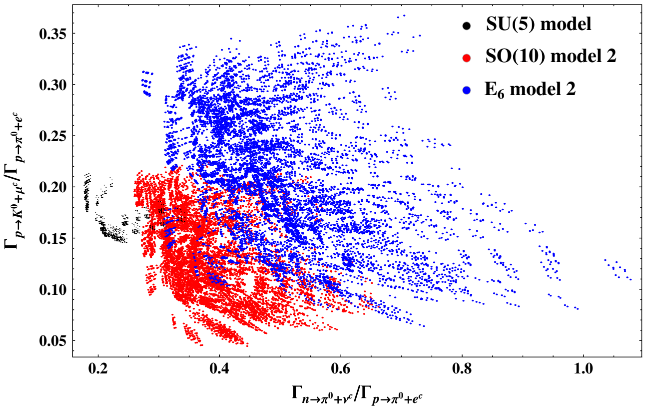

FIG. 5 (color online). In $SU(5)$, we have 10^3 model points; in $SO(10)$ and E_6, we have 10^4 model points, as noted in the caption of Fig. 4. VEVs are taken as $x = v_c = v_\phi$.

is observed to be much larger than 1, the observation suggests the E_6 gauge group strongly.

If anomalous $U(1)_A$ symmetry is not adopted, usually the VEV relations v_c, $v_\phi \geq x$ are required in order to explain the gauge coupling unification. Of course, if v_c, $v_\phi \gg x$, then the predictions of $SO(10)$ models and E_6 models become the same as those of $SU(5)$ models. Here, we show another plot by taking $x = v_c = v_\phi$, which makes the X' and X'' contributions maximal in these models without anomalous $U(1)_A$ symmetry, keeping the success of the gauge coupling unification. The results are shown in Fig. 5. It is understood that the $SO(10)$ model points come closer to the $SU(5)$ model points, and the E_6 model points come closer to the $SO(10)$ model points.

To conclude this subsection, we shall explain why we adopt the $n \to \pi^0 + \nu^c$ mode instead of the $p \to \pi^+ + \nu^c$ mode. We have two reasons: First, the former mode is easier to detect experimentally. Since the decay of π^+ includes an invisible neutrino, the latter decay mode is more difficult to observe. Second, the hadron matrix element of the former mode is the same as that of the $p \to \pi^0 + e^c$ mode, and therefore in the ratio R_1 these hadron matrix elements are canceled.

VI. DISCUSSION AND SUMMARY

We have calculated the lifetime of the nucleon for various decay modes via dimension-6 operators in the anomalous $U(1)_A$ GUT models. Since the anomalous $U(1)_A$ GUT models predict a lower unification scale in general, it is important to predict the nucleon lifetime via dimension-6 operators. The lifetime $\tau(p \to \pi^0 + e^c)$ has been calculated as $O(10^{34})$ years for the unification scale $\Lambda_u = 1 \times 10^{16}$ GeV, which is a typical value for the unification scale in an anomalous $U(1)_A$ GUT scenario with the $U(1)_A$ charge of the adjoint Higgs $a = -1/2$. Although we have several ambiguities in the calculation from $O(1)$ coefficients or the hadron matrix elements, the discovery of the nucleon decay in near-future experiments [32] can be expected because the present experimental lower limit is 1.29×10^{34} years. The predicted value can become $O(10^{33})$ years for the anomalous $U(1)_A$ GUT models with $a = -1$. In the calculation, we have taken into account the ambiguities from the quark and lepton mixings by generating the various diagonalizing unitary matrices randomly. One of the largest ambiguities for the predictions comes from the $O(1)$ coefficient of the unification scale. Since the lifetime is proportional to Λ_u^4, the factor 2 in the unification scale can make the prediction of the lifetime 16 times larger. Moreover, the ambiguities from the hadron matrix elements can easily change the prediction by a factor of 2. Therefore, we cannot reject the anomalous $U(1)_A$ GUT with $a = -1$ by these predictions. We can expect the observation of the nucleon decay in near-future experiments.

We have proposed that the two ratios $R_1 \equiv \frac{\Gamma_{n\to\pi^0+\nu^c}}{\Gamma_{p\to\pi^0+e^c}}$ and $R_2 \equiv \frac{\Gamma_{p\to K^0+\mu^c}}{\Gamma_{p\to\pi^0+e^c}}$ are important to identify the anomalous $U(1)_A$ GUT models. The ratio R_1 becomes larger for the larger rank of the grand unification group if the masses of the $SO(10)$ and E_6 superheavy gauge bosons X' and X'' are comparable to or even smaller than the $SU(5)$ superheavy gauge boson mass. This is because the superheavy gauge bosons X' and X'' induce only the effective interactions which include the doublet lepton l, while the $SU(5)$ superheavy gauge boson X induces both the effective interactions with l and the effective interactions with e_R^c. What is important is that in the anomalous $U(1)_A$ GUT models, the X' mass is always comparable with the X mass. The X'' mass can be smaller than the X mass, which is dependent on the explicit models. Therefore, at least in the anomalous $U(1)_A$ GUT scenario, measuring this ratio is critical in distinguishing the $SU(5)$ models from the other models. The ratio R_2 is important for distinguishing E_6 models from $SO(10)$ models. In most of the anomalous $U(1)_A$ GUT models with $SO(10)$ and the E_6 unification group, the $\bar{\mathbf{5}}'$ field from the **10** of $SO(10)$ becomes the main component of the second-generation $\bar{\mathbf{5}}$ field to obtain large neutrino mixings. What is important here is that the X' boson does not induce the effective interactions which include $\bar{\mathbf{5}}'$ fields, while the X'' boson induces only the effective interactions which include $\bar{\mathbf{5}}'$'s. Therefore, in E_6 unification, the nucleon decay widths for the second-generation quark and lepton must be larger than in $SO(10)$ unification. We have plotted various model points in several figures in which the horizontal axis is R_1 and the vertical axis is R_2. And we have concluded that we can identify the grand unification group by measuring these ratios if $x = 1 \times 10^{16}$ GeV, $v_c = 5 \times 10^{14}$ GeV, and $v_\phi = 5 \times 10^{15}$ GeV, which are typical values in the models with $a = -1/2$, $c + \bar{c} = -5$, and $\phi + \bar{\phi} = -2$. Of course, this conclusion is dependent on the parameters. For example, when $v_\phi \gg x$, it becomes difficult to distinguish the E_6 models from the $SO(10)$ models because the mass of

X'' becomes much larger than the other superheavy gauge bosons. However, since it is difficult to realize $R_1 > 0.4$ in $SU(5)$ unification, if R_1 is observed to be larger than 0.4, then the grand unification group is not $SU(5)$. Moreover, if R_2 is larger than 0.3, E_6 unification is implied. An important point is that $\Gamma(n \rightarrow \pi^0 + \nu^c)$ and $\Gamma(p \rightarrow K^0 + \mu^c)$ can be comparable with $\Gamma(p \rightarrow \pi^0 + e^c)$ in E_6 unification.

Note that our calculations can apply to the usual SUSY GUT models, in which the unification scale is around $\Lambda_G = 2 \times 10^{16}$ GeV, although the predicted lifetime becomes much longer. And taking account of the gauge coupling unification, the VEVs v_c and v_ϕ must be larger than Λ_G usually. Therefore, the effects of superheavy gauge bosons X' and X'' are not so large. However, the ratios R_1 and R_2 must be important in identifying GUT models even without anomalous $U(1)_A$ gauge symmetry.

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