

Possible candidate of the $0^+ s\bar{s}s\bar{s}$ state

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(Received 8 July 2013; published 21 November 2013)

The possibility of the $0^+ \eta\eta$ resonance $f_0(2100)$ as a candidate of the $Q^2\bar{Q}^2$ state $C^{ss}(36)$ is explored. The $\eta\eta$ channel of $f_0(2100)$ is the dominant decay mode; the $\eta\eta'$ channel has less decay rate; the decay rate of the $\eta'\eta'$ channel is very small. The $\pi\pi, K\bar{K}, 4\pi$ modes are at next leading order in N_C expansion. Other possible decay modes are discussed.

 DOI: [10.1103/PhysRevD.88.094016](https://doi.org/10.1103/PhysRevD.88.094016)

PACS numbers: 12.39.Mk, 12.39.Ba, 13.20.Gd

Among many $I^G(J^{PC}) = 0^+(0^{++})$ resonances discovered [1] around 2 GeV there are $f_0(2100)$ and $f_0(2020)$ two scalar mesons [1]. The parameters of the $f_0(2100)$ are determined to be [1]

$$M = 2103 \pm 8 \text{ MeV}, \quad \Gamma = 209 \pm 19 \text{ MeV}.$$

The $f_0(2020)$ is a broad scalar meson whose decay width is about 400 MeV and it has many decay modes: $\rho\pi\pi, \pi^0\pi^0, \rho\rho, \omega\omega, \eta\eta, \dots$ [1]. The $f_0(2100)$ is different from the $f_0(2020)$. In this paper the possible nature of the $f_0(2100)$ is investigated. In 1993 $\eta\eta$ resonances have been reported in the following processes [2]:

$$\bar{p} + p \rightarrow 3\pi^0, 2\pi^2, \pi^0 2\eta, 3\eta.$$

The $f_0(2100)$ is one of the three $\eta\eta$ resonances and its mass and decay width are determined to be

$$M = 2104 \pm 20 \text{ MeV}, \quad \Gamma = 203 \pm 10 \text{ MeV}.$$

This state has not been identified definitely in the $\pi\pi$ channel and the quantum numbers are not determined in this study. In Ref. [3], $f_0(2100)$ has been discovered in

$$\bar{p} + p \rightarrow \eta\eta, \eta\eta', \dots$$

The $f_0(2100)$ appears strongly in the $\eta\eta$ channel and

$$M = 2105 \pm 10 \text{ MeV}, \quad \Gamma = 200 \pm 25 \text{ MeV}.$$

The $f_0(2100)$ appears weakly in the $\pi^0\pi^0$ data, contributing only $(4.6 \pm 1.5)\%$ of the cross section, compared with $(38 \pm 5)\%$ in the $\eta\eta$ channel. The cross section for $\eta\eta'$ contains a weak peak at about 2150 MeV.

Recently, the BES III Collaboration has reported the discovery of the $\eta\eta$ resonance $f_0(2100)$ in $J/\psi \rightarrow \gamma\eta\eta$ [4]. Its mass and decay width are determined to be

$$m = 2081 \pm 13 \text{ MeV}, \quad \Gamma = 273_{-24}^{+27} \text{ MeV},$$

respectively, which are in agreement with the measurements [2,3]. The product branching ratio is measured to be

$$B(J/\psi \rightarrow \gamma f_0(2100) \rightarrow \gamma\eta\eta) = (1.13_{-0.1}^{+0.09}) \times 10^{-4}.$$

On the other hand, the discovery of $f_0(2100)$ has not been reported in $K\bar{K}$ channels of $J/\psi \rightarrow \gamma K\bar{K}$ [4] and $pp \rightarrow p_f(K\bar{K})p_s$ [5,6].

In Ref. [7] in $J/\psi \rightarrow \gamma\pi^+\pi^-$ besides a 2^{++} state $\theta(1700)$ a $X(2100)$ is reported

$$M = 2027 \pm 12 \text{ MeV}, \quad \Gamma = 220 \pm 30 \text{ MeV}.$$

However, it is claimed that the angular distributions of $X(2100)$ are similar to those of the $\theta(1700)$ which has been determined to be a 2^{++} state. In Ref. [7] in the decay $J/\psi \rightarrow \gamma\pi\pi$ a wide resonance $f_0(2020)$ is seen, which is listed in Ref. [1]. It seems that the results from these two experiments do not agree each other. On the other hand, in Ref. [1] the scalar resonance $f_0(2100)$ is not listed in the $\pi\pi$ channel in $J/\psi \rightarrow \gamma\pi\pi$.

The experimental study of the scalar resonance in the 4π channel lasts a pretty long time. Now it is needed to check whether the $f_0(2100)$ has 4π decay mode. $J/\psi \rightarrow \gamma 4\pi$ and $pp \rightarrow p_f(4\pi)p_s$ are the two processes to search for $X \rightarrow 4\pi$.

- (1) MARK II has done a study of $J/\psi \rightarrow \gamma 4\pi$ [8] and it is found that the $\gamma\rho\rho$ are the components of these channels. It is not mentioned whether there is resonance of $\rho\rho$ around 2 GeV.
- (2) In Ref. [9] a 0^{-+} resonance $\eta(2100)$ which decays to both $\rho^+\rho^-$ and $\rho^0\rho^0$ has been observed by DM2 and $f_0(2100) \rightarrow \rho\rho$ has not been reported.
- (3) MARK III did a study on $J/\psi \rightarrow \gamma\pi^+\pi^-\pi^+\pi^-$ [10]. The $f_0(2104)$ has been reported in the 4π channel with $\Gamma = 203 \text{ MeV}$ and $\sigma\sigma$ is the dominant decay channel. Large branching ratios are reported:

$$\begin{aligned} B(J/\psi \rightarrow \gamma X(2104))B(X(2104) \rightarrow 4\pi) \\ = (3.0 \pm 0.8) \times 10^{-4}, \end{aligned}$$

$$\begin{aligned} B(J/\psi \rightarrow \gamma X(2104))B(X(2104) \rightarrow \rho\rho) \\ = (6.8 \pm 1.8) \times 10^{-4}. \end{aligned}$$

- (4) BES has reported a partial wave analysis of $J/\psi \rightarrow \gamma\pi^+\pi^-\pi^+\pi^-$ [11]. $f_0(2100)$ has been found in the 4π channel with

$$M = 2090_{-30}^{+30} \text{ MeV}, \quad \Gamma = 330_{-100}^{+100} \text{ MeV}.$$

$f_0(2100) \rightarrow \sigma\sigma$ decay is reported. From the values of the mass and width of f_0 , it is hard to say the resonance is $f_0(2100)$ or $f_0(2020)$.

From the current data it is difficult to draw a conclusion whether the $f_0(2100)$ has been found in the 4π channel produced in the J/ψ radiative decays.

The process $pp \rightarrow p_f(4\pi)p_s$ has been studied by WA102 [12] and broad scalar with

$$m = 2020 \pm 35 \text{ MeV}, \quad \Gamma = 410 \pm 50 \text{ MeV}$$

has been found. The resonance decays to $\rho\pi\pi$ and $\rho\rho$. It is the $f_0(2020)$ resonance. In Ref. [13] a spin analysis of the 4π channels produced in central pp interactions has been done. It is mentioned that the $J^P = 0^+ \rho\rho$ distribution shows a peak at 1.45 MeV together with a broad enhancement around 2 GeV. These experiments show that the 0^+ resonance found in the 4π channel produced in pp collision is the broad $f_0(2020)$.

The experimental data mentioned above show that the scalar resonance $f_0(2100)$ discovered in $p\bar{p}$ annihilation and J/ψ radiative decay decays to $\eta\eta$ dominantly. It has weak coupling with the $\pi\pi$ channel in $p\bar{p}$ annihilation and it is not found in $J/\psi \rightarrow \gamma\pi\pi$. The $f_0(2100)$ weakly decays to $\eta\eta'$. It is not discovered in the $K\bar{K}$ channels of J/ψ radiative decay and $p\bar{p}$ collision. The data of $p\bar{p} \rightarrow p_s(4\pi)p_f$ show that there is no sign of the $f_0(2100) \rightarrow 4\pi$.

The authors of Ref. [3] claim that $f_0(2100)$ decays dominantly through a $s\bar{s}$ component and the strong production in $p\bar{p}$ strongly suggest exotic character. It is either a glueball or a hybrid and there may be mixing with $q\bar{q}$ and $s\bar{s}$.

In this paper, the possible exotic character of the $f_0(2100)$ is investigated. We explore the possibility that the $f_0(2100)$ is a four quark state of $s\bar{s}s\bar{s}$ [14]. The possible direct decay channels of $s\bar{s}s\bar{s}$ are $\eta\eta$, $\eta\eta'$, $\eta'\eta'$, $\phi\phi$, etc. The $\eta\eta$ mode has the largest phase space. Therefore, the $\eta\eta$ is the dominant decay mode of the $f_0(2100)$. The $\pi\pi$, $K\bar{K}$, 4π decays of the $f_0(2100)$ are via the meson loop diagrams, $\eta\eta \rightarrow \pi\pi$, $K\bar{K}$, 4π completed. In a meson theory [15] it shows that the tree diagrams of mesons are at the leading order in the N_C expansion and the meson loop diagrams are at next leading order. For example, in this meson theory [15], the amplitude of $\phi \rightarrow K\bar{K}$ is at $O(N_C)$. The decay $\phi \rightarrow \rho\pi$ is via one-loop meson diagrams completed. The amplitude of this decay is at $O(1)$ in the N_C expansion. Comparing with $\phi \rightarrow K\bar{K}$, the decay $\phi \rightarrow \rho\pi$ is suppressed. For the decays of the $f_0(2100)$ the $f_0(2100) \rightarrow \eta\eta$ result in a tree diagram and the $\pi\pi$, $K\bar{K}$, 4π channels result in loop diagrams of mesons; therefore, they are suppressed in the N_C expansion.

In Ref. [14] the spectrum and the properties of $Q^2\bar{Q}^2$ states have been studied in the MIT bag model. The $Q^2\bar{Q}^2$ states studied in Ref. [14] have been successfully applied to study the reactions $\gamma\gamma \rightarrow \rho^0\rho^0$ and $\gamma\gamma \rightarrow \rho^+\rho^-$ [16]. Recently, a 0^{++} resonance, $f_0(1810)$, has been discovered in $J/\psi \rightarrow \gamma\omega\phi$ [17]. If this new resonance is just one of the ordinary mesons, the process $J/\psi \rightarrow \gamma f_0(1810)$, $f_0(1810) \rightarrow \omega\phi$ would be doubled Okubo-Zweig-Iizuka (OZI) suppressed. In Ref. [18] the new $\omega\phi$ resonance has

been interpreted as the production of the $Q^2\bar{Q}^2$ state, $C^s(9)$ [14], in J/ψ radiative decay and the doubled OZI suppression is avoided.

In Ref. [19] the approach of effective Lagrangian has been applied to study the possible mixing of quark antiquark states with other states made of two quarks and two antiquarks in the low-lying scalar and pseudoscalar mesons, such as $f_0(500)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$, $f_0(1710)$, etc. The study shows the $q\bar{q}q\bar{q}$ is the dominant component of the $f_0(500)$ and some of the states have significant $q\bar{q}q\bar{q}$ components [19]. The mixing between the scalar mesons and scalar glueball has been investigated in Ref. [19] too. As mentioned above, the $s\bar{s}s\bar{s}$ state at about 2 GeV has been predicted by the MIT bag model [14]. The work done in this paper is a phenomenological study of this $s\bar{s}s\bar{s}$ state. Based on the study in Ref. [19], in principle the mixing between scalar $q\bar{q}$, $q\bar{q}q\bar{q}$, and glueball state and the $s\bar{s}s\bar{s}$ state should exist. As mentioned above, the experimental data show that the decay rates of the $f_0(2100)$ to $\pi\pi$, 4π , and $K\bar{K}$ are smaller (suppressed). It is shown in Ref. [1] that the $\pi\pi$ is the dominant decay channel of the $f_0(500)$ and $f_0(980)$. The decays of the $f_0(1370) \rightarrow \pi\pi$, 4π , $\rho\rho$ are found [1] and the $\rho\rho$ is the dominant channels of the $f_0(1370)$. The decay channels of $\pi\pi$ and 4π of the $f_0(1500)$ have large branching ratios, respectively. The decay modes of $K\bar{K}$, $\eta\eta$, $\pi\pi$, $\omega\omega$ of the $f_0(1710)$ are found. As mentioned above, the experimental data show the decay rates of the $f_0(2100) \rightarrow \pi\pi$, $K\bar{K}$, etc. are smaller. It is known that scalar glueball decays to $\pi\pi$, $K\bar{K}$, ... too. Therefore, phenomenologically the mixing between the $s\bar{s}s\bar{s}$ state and $f_0(500)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$, $f_0(1710)$, scalar glueballs should be smaller. In this paper these smaller mixings are not studied.

Now we take the 0^{++} $C^{ss}(36)$ state studied in Ref. [14] as the $s\bar{s}s\bar{s}$ state,

$$C^{ss}(36) = \eta_s \eta_s, \quad \eta_s = s\bar{s}, \quad (1)$$

whose mass has been predicted to be

$$m = 1950 \text{ MeV}. \quad (2)$$

This value is very close to the mass of the $f_0(2100)$. The color wave function of the $C^{ss}(36)$ state consists of color octet–color octet and color singlet–color singlet two parts [14]. The recoupling coefficients of this state are shown in Table I [14] (the color octet is indicated by an underline).

In this study the $C^{ss}(36)$ with mass 2100 MeV is taken as the $f_0(2100)$ and the decays and productions of the $C^{ss}(36)$ state are investigated. We study the decays first, then the productions. Through a “fall apart” [14] mechanism the

TABLE I. Recoupling coefficients.

	PP	VV	$\underline{P} \cdot \underline{P}$	$\underline{V} \cdot \underline{V}$
$C^{ss}(36)$	-0.644	0.269	-0.322	-0.639

$C^{ss}(36)$ decays to $\eta\eta$, $\eta\eta'$, $\eta'\eta'$, $\phi\phi$, etc. For the decay $s\bar{s}s\bar{s} \rightarrow PP$ the $s\bar{s}s\bar{s}$ is expressed as

$$\begin{aligned} s\bar{s} &= \frac{1}{\sqrt{3}}\eta' - \sqrt{\frac{2}{3}}\eta, \\ s\bar{s}s\bar{s} &= \frac{2}{3}\eta\eta + \frac{1}{3}\eta'\eta' - \frac{2\sqrt{2}}{3}\eta\eta', \end{aligned} \quad (3)$$

where $\eta = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$ and $\eta' = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$ are taken. The three decay amplitudes are via the mechanism of fall apart obtained:

$$\begin{aligned} \langle \eta\eta | T | f_0(2100) \rangle &= -0.644 a m \frac{4}{3}, \\ \langle \eta\eta' | T | f_0(2100) \rangle &= 0.644 a m \frac{2\sqrt{2}}{3}, \\ \langle \eta'\eta' | T | f_0(2100) \rangle &= -0.644 a m \frac{2}{3}, \end{aligned} \quad (4)$$

where a is an unknown constant from the mechanism of the fall apart, -0.644 is the recoupling coefficient from Table I, $m = 2100$ MeV. The decay widths are derived as

$$\begin{aligned} \Gamma(f_0 \rightarrow \eta\eta) &= \frac{0.644^2}{18\pi} a^2 m \left(1 - \frac{4m_\eta^2}{m^2}\right)^{\frac{1}{2}} \\ &= 1.34 \times 10^{-2} a^2 \text{ GeV}, \\ \Gamma(f_0 \rightarrow \eta\eta') &= \frac{0.644^2}{9\pi} a^2 \left\{ \frac{1}{4m^2} (m^2 + m_\eta^2 - m_{\eta'}^2)^2 - m_\eta^2 \right\}^{\frac{1}{2}} \\ &= 1.05 \times 10^{-2} a^2 \text{ GeV}, \\ \Gamma(f_0 \rightarrow \eta'\eta') &= \frac{0.644^2}{72\pi} a^2 m \left(1 - \frac{4m_{\eta'}^2}{m^2}\right)^{\frac{1}{2}} \\ &= 1.58 \times 10^{-3} a^2 \text{ GeV}. \end{aligned} \quad (5)$$

The ratios of the three decay channels of the $f_0(2100)$ are

$$\Gamma(\eta\eta) : \Gamma(\eta\eta') : \Gamma(\eta'\eta') \sim 1 : 0.78 : 0.12. \quad (6)$$

This $s\bar{s}s\bar{s}$ scheme [14] predicts a very small decay rate for the channel $\eta'\eta'$, which is caused by two factors: small phase space and small coefficient of the decomposition (3). This mechanism also predicts a smaller decay rate for the channel $\eta\eta'$. In Ref. [3] a weaker peak in the channel $\eta\eta'$ at 2150 MeV has been reported.

It is known that there is mixing between η , η' and the 0^{-+} glueball which is believed to be $\eta(1405)$ [20]. The mixing makes the 0^{-+} glueball $\eta(1405)$ have an $s\bar{s}$ component [20]. Therefore, the decay $f_0(2100) \rightarrow \eta\eta(1405)$ exists and has a small decay rate. Using the results presented in the first paper of Ref. [20], it is estimated $\Gamma(f_0(2100) \rightarrow \eta\eta(1405)) \sim 0.09\Gamma(f_0(2100) \rightarrow \eta\eta)$. However, this decay channel will provide useful information for accurate determination of the $\eta - \eta' - \eta(1405)$ mixing. Because the coefficients of the mixing determined by different authors [20] are different, the mixing effects on Eqs. (5) and (6) will not be studied in this paper.

In the same way the decay $f_0(2100) \rightarrow \phi\phi$ can be studied

$$\begin{aligned} \langle \phi_{\lambda_1}(k_1) \phi_{\lambda_2}(k_2) | T | f_0(2100) \rangle &= 2 b m 0.269 \epsilon^{\lambda_1}(k_1) \cdot \epsilon^{\lambda_2}(k_2), \\ \Gamma(f_0 \rightarrow \phi\phi) &= \frac{1}{8\pi} (0.269)^2 m \left(1 - \frac{4m_\phi^2}{m^2}\right)^{\frac{1}{2}} \\ &\quad \times \left\{ 2 + \frac{m^4}{4m_\phi^4} \left(1 - \frac{2m_\phi^2}{m^2}\right) \right\} b^2 \\ &= 0.633 \times 10^{-2} b^2 \text{ GeV}, \end{aligned} \quad (7)$$

where b is a parameter and the relationship between a and b is unknown.

Because the parameter b is unknown, it is not able to do a reliable prediction for the decay rate of the $\phi\phi$ channel. It is worth searching for the resonance $f_0(2100)$ in the $\phi\phi$ channel. Comparing with the $\eta\eta$ channel, the small phase space and the recoupling coefficient of the $\phi\phi$ channel make the decay rate of $f_0(2100) \rightarrow \phi\phi$ too small. However, there are another two factors which enhance the decay rate of $f_0(2100) \rightarrow \phi\phi$. The amplitude of this decay contains a factor $\epsilon^{\lambda_1}(k_1) \cdot \epsilon^{\lambda_2}(k_2)$, where $\lambda_i (i = 1, 2)$ and $k_i (i = 1, 2)$ are the polarization and momentum of the two ϕ mesons, respectively. This polarization factor contributes a factor 4.38 to the decay rate. The second factor is that for the channel $\phi\phi$ the factor for the $s\bar{s}s\bar{s}$ component is one and for the $\eta\eta$ channel this factor is $\frac{4}{9}$ (3). All the factors together make the decay rate of $f_0(2100) \rightarrow \phi\phi$ not too small. If $a \sim b$ is assumed we obtain

$$\Gamma(f_0(2100) \rightarrow \eta\eta) : \Gamma(f_0(2100) \rightarrow \phi\phi) \sim 1 : 0.47.$$

There are experimental studies on $J/\psi \rightarrow \gamma\phi\phi$ [21], in which the $0^{++}\phi\phi$ at 2100 MeV is not studied. It is worth searching for the $f_0(2100)$ in the $J/\psi \rightarrow \gamma\phi\phi$.

In the picture of four quark states the mechanism of the productions of the $\eta\eta$ resonance $f_0(2100)$ ($C^{ss}(36)$) in $p\bar{p}$ collisions and J/ψ radiative decay can be understood qualitatively. A proton is made of uud quarks and the $f_0(2100)$ is a $s\bar{s}s\bar{s}$ state. How is this $s\bar{s}s\bar{s}$ state produced in $p\bar{p}$ collision? It is known that half of the energy of a proton is carried by gluons. Therefore, $p\bar{p} \rightarrow gg + \dots$, $gg \rightarrow f_0(2100)$ is the process for the production of the $f_0(2100)$ in $p\bar{p}$ collisions. In QCD the J/ψ radiative decay is described as $J/\psi \rightarrow \gamma gg$. Therefore, the same $gg \rightarrow f_0(2100) \rightarrow \dots$ is responsible for the production of the $f_0(2100)$ in J/ψ radiative decay. In this study the $f_0(2100)$ is taken as the $C^{ss}(36)$ in which there is a component $-0.639 \underline{\phi} \cdot \underline{\phi}$ (Table I), where $\underline{\phi}$ is the color octet ϕ . The vector meson dominance works well in particle physics, in which the photon is coupled to the vector mesons (ρ , ω , ϕ). A similar mechanism is proposed in Refs. [22,23], in which the gluon is coupled to the color octet vector, \underline{V} ,

$$\frac{1}{\sqrt{2}} g_s g_\phi g^a \phi^a, \quad (8)$$

where the a is the color index, g_s is the coupling constant of QCD, $g_\phi^2 = \frac{2}{3} g^2$, $g = 0.395$ is determined in Ref. [15], and g^a is the gluon field. The process $g + g \rightarrow \phi \phi \rightarrow f_0(2100) \rightarrow \eta\eta$ is responsible for the productions of the $f_0(2100)$ in both $p\bar{p}$ annihilation and J/ψ radiative decay. Using this mechanism, the amplitude of the production of $f_0(2100)$ is at $O(g_s^2)$. Comparing with glueball production in J/ψ radiative decay, the amplitude of the production of the $f_0(2100)$ is suppressed by $O(\frac{1}{N_C})$ and is at the same order of magnitude of the production of the hadrons made of quarks. Roughly speaking, the production rate of the glueball in J/ψ radiative decay is at about $O(10^{-3})$ and the $f_0(2100)$ is at $O(10^{-4})$. This is consistent with the analysis above. This mechanism has been applied to study $J/\psi \rightarrow \gamma X(1810)$, $X(1810) \rightarrow \omega\phi$ [18]. Usually, the $J/\psi \rightarrow \gamma\omega\phi$ is a double OZI suppressed process. However, if $X(1810)$ is a four quark state the double OZI suppression no longer exists.

Based on the MIT model [14], the investigation of the $f_0(2100)$ presented in this paper is a phenomenological study only. On the other hand, there are established four-quark scalar nonets below 1 GeV [19]. Based on the MIT model these four quark states below 1 GeV have been studied too [19]. For example, $C^0(\underline{9}, 0^+) = \frac{1}{\sqrt{2}} u\bar{u}d\bar{d}$ ($m = 650$ MeV), $C^s(\underline{9}, 0^+) = \frac{1}{\sqrt{2}} s\bar{s}(u\bar{u} + d\bar{d})$ ($m = 1100$ MeV), etc. The $C^{ss}(36)$ studied in this paper is the lowest state of $s\bar{s}s\bar{s}$ [14]. The $C^{ss}(36)$ and $C^0(\underline{9}, 0^+)$, $C^s(\underline{9}, 0^+)$ have very different structure of the flavors (see Table I [14]).

On the other hand, besides the $C^{ss}(36)$ studied in this paper, another $C^{ss}(36^*)$ whose mass is 2350 MeV has been predicted in Ref. [14] too. According to Ref. [14], these two states have different flavor structure. The PP and the

$\underline{V} \cdot \underline{V}$ components of the $C^{ss}(36^*)$ are 0.041 and -0.089 , respectively [14]. They are much smaller than corresponding components of the $C^{ss}(36)$ (see Table I). Therefore, very small production rates of $p\bar{p} \rightarrow C^{ss}(36^*) \rightarrow \eta\eta$ and $J/\psi \rightarrow \gamma C^{ss}(36^*) \rightarrow \gamma\eta\eta$ should be expected. It is reasonable to assume that the $f_0(2100)$ is the $C^{ss}(36)$.

It is worth mentioning that in Ref. [14] in the range of 2 GeV there is another scalar,

$$C^0(36^*) = -\frac{1}{2} \pi\pi + \frac{\sqrt{3}}{2} \eta_0\eta_0, \quad (9)$$

where $\eta_0 = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$. The mass of this state is predicted to be $m = 2100$ MeV [14]. Equation (9) shows that this state is very different from the state $C^{ss}(36)$ studied above. At the leading order this state decays into mesons made of u and d quarks. It is worth investigating the possibility that this state is related to the broad resonance $f_0(2020)$ [1] mentioned above. The width of the $f_0(2020)$ is 442 ± 60 MeV and the decay modes $\rho\pi\pi$, $\pi^0\pi^0$, $\rho\rho$, $\omega\omega$, and $\eta\eta$ are seen [1]. A detailed phenomenological study of this state is beyond the scope of this paper.

In summary, the study shows that all the decay properties of the $\eta\eta$ resonance $f_0(2100)$ can be understood by the four quark state $s\bar{s}s\bar{s}$, $C^{ss}(36)$. The $\eta\eta$ channel is the dominant decay mode. The study predicts that the $f_0(2100) \rightarrow \eta\eta'$ has less decay rate. The decay channels, $\pi\pi$, $K\bar{K}$, 4π are at next leading order in N_C expansion and suppressed. The existence of the decay mode of $f_0(2100) \rightarrow \phi\phi$ is predicted and the measurement of this channel is significant for the $q^2\bar{q}^2$ scheme of the $f_0(2100)$. Because of $\eta - \eta' - \eta(1405)$ mixing the $\eta(1405)$ should have a small $s\bar{s}$ component. Therefore, $f_0(2100) \rightarrow \eta\eta(1405)$ exists. The production of $f_0(2100)$ in J/ψ radiative decay and $p\bar{p}$ collisions result in $gg \rightarrow f_0(2100)$.

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