Branching ratios and *CP* violations of $B \to K_0^*(1430)K^*$ decays in the perturbative QCD approach

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We investigate $B \to K_0^*(1430)K^*$ decays in the perturbative QCD (pQCD) factorization approach, where B denotes B_u, B_d, and B_s mesons, respectively, and the scalar $K_0^*(1430)$ is considered as a meson based on the model of the conventional two-quark structure. With the light-cone distribution amplitude of $K_0^*(1430)$ defined in two scenarios, namely Scenario 1 and Scenario 2, we make the first estimation for the branching ratios and *CP*-violating asymmetries for those concerned decay modes in the pQCD factorization approach. For all considered $B \to K_0^*(1430)K^*$ decays in this paper, only one preliminary upper limit on the branching ratio of $B^0 \to K_0^*(1430)^0 \bar{K}^{*0}$ measured at 90% C.L. by the Belle Collaboration is available now. It is therefore of great interest to examine the predicted physical quantities at two *B* factories, the Large Hadron Collider experiments, and the forthcoming Super-*B* facility, and then to test the reliability of the pQCD approach employed to study the considered decay modes involving a *p*-wave scalar meson as one of the final state mesons. Furthermore, these pQCD predictions combined with the future precision measurements are also helpful to explore the complicated QCD dynamics involved in the light scalars.

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I. INTRODUCTION

The inner structure of the light scalars, generally below 2 GeV, has been explored by the physicists at both experimental and theoretical aspects for several decades. However, unfortunately, their underlying structure has not yet been well established and the identification of the considered light scalars is known as a long-standing puzzle [1]. But, it is lucky for us that the light scalars could be studied in the decay channels of heavy flavor *B* mesons, with the rich data provided by *B* factories, the Large Hadron Collider (LHC) experiments [2], and the forthcoming Super-*B* factory [3,4]. Ever since the $B \to f_0(980)K$ mode was first measured by the Belle Collaboration in 2002 [5] and then confirmed by the *BABAR* Collaboration in 2004 [6], more and more channels with *p*-wave light scalars in the final states of *B* meson decays have been opened and more precise data have been obtained [1]. With the gradually increasing numbers of data samples collected in the running LHC experiments and the forthcoming Super-*B* factory, it is therefore believed that as a different unique insight to the nature of the light scalars, the *B* meson decays involving *p*-wave light scalars will provide good places to investigate the physical properties of light scalars. It is expected that the old puzzles related to the nature of the scalars could receive new attention through the studies on rare *B* meson decays involving scalars, apart from those well-known primary tasks in heavy flavor physics.

Although the underlying structure of the light scalars is still controversial, the scalar $a_0(1450)$ has recently been confirmed to be a conventional $q\bar{q}$ meson in lattice calculations [7–11]. Furthermore, a good SU(3) flavor symmetry is indicated in the scalar sector through the calculations in lattice QCD [7] on the masses of $a_0(1450)$ and $K_0^*(1430)$. The evaluations on the relevant $K_0^*(1430)$ modes therefore draw more attention now. (Hereafter, unless otherwise stated, K_0^* will be adopted to describe $K_0^*(1430)$ throughout the paper for the sake of simplicity.) Recently, the authors in Ref. [12] proposed two possible scenarios, namely, Scenario 1 (S1) and Scenario 2 (S2), to describe the components of the K_0^* meson in the QCD sum rule method based on the assumption of the conventional two-quark structure:

(i) In S1, the lighter state κ near 1 GeV is treated as the lowest lying $q\bar{q}$ state, while the heavier state K_0^* above 1 GeV is considered as the corresponding first excited $q\bar{q}$ state.

(ii) In S2, K_0^* is regarded as the ground $q\bar{q}$ state, and the corresponding first excited state lies between 2.0 and 2.3 GeV. Then κ is viewed as the four-quark bound state or hybrid state.

The two body charmless hadronic *B* meson decays to the scalar K_0^* meson have been studied intensively, for example, in Refs. [12–16] by employing different factorization approaches, respectively, or in Ref. [17] even with the inclusion of the new physics contributions from a Z' boson. This year, the authors of Ref. [18] revisited the $B \to SP$, *SV* decays in the framework of QCD factorization.

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On the theoretical side, it is necessary for us to make all possible investigations on the decay modes of the B meson with the scalar K_0^* to identify the favorite one from the proposed S1 and S2 scenarios, which will also be helpful to obtain the new insights in the properties of the scalar K_0^*. On the experimental side, however, so far only a preliminary upper limit at 90% C.L. on the branching ratio of the $B^0 \to K_0^{*0}\bar{K}^{*0}$ decay has been measured by the Belle Collaboration [19],

$$\text{Br}(B^0 \to K_0^{*0}\bar{K}^{*0}) < 3.3 \times 10^{-6}. \tag{1}$$

Of course, this measurement would be improved rapidly with the LHC experiments at CERN, and other relevant channels considered in this work would also be observed in the near future.

In this work, we will study the branching ratios and CP-violating asymmetries of $B \to K_0^* K^*$ decays in the standard model (SM) by employing the low energy effective Hamiltonian [20] and the perturbative QCD (pQCD) factorization approach [21–23], where B stands for $B_{u,d}$ and B_s, respectively. Based on k_T factorization, the pQCD approach is one of the popular factorization methods for dealing with the B meson exclusive decays. In the pQCD approach, the parton transverse momentum k_T is kept in order to eliminate the end-point singularity, while the Sudakov factor plays an important role in suppressing the long-distance contribution [23]. We here not only consider the usual factorizable emission diagrams but also evaluate the nonfactorizable spectator and the annihilation type contributions simultaneously. As far as the annihilation contributions are concerned, both the soft-collinear effective theory [24] and the pQCD approach can work, but with rather different viewpoints on the relevant perturbative calculations [25,26]. However, the predictions of the pure annihilation decays based on the pQCD approach can accommodate the experimental data well, for example, for the $B_s \to \pi^+\pi^-$ and $B^0 \to K^+K^-$ decays as have been done in Refs. [27–30]. In this work, we will therefore leave the controversies aside and adopt this approach in our analysis.

The paper is organized as follows. Section II is devoted to the ingredients of the basic formalism in the pQCD approach. The analytic expressions for the decay amplitudes of $B \to K_0^* K^*$ modes in the pQCD approach are also collected in this section. The numerical results and phenomenological analysis for the branching ratios and CP-violating asymmetries of the considered decays are given in Sec. III. We summarize and conclude in Sec. IV.

II. FORMALISM

The pQCD approach is one of the popular methods used to evaluate the hadronic matrix elements in the heavy b-flavor mesons' decays. The basic idea of the pQCD approach is that it takes into account the transverse momentum k_T of the valence quarks in the calculation of

the hadronic matrix elements. The B meson transition form factors and the spectator and annihilation contributions are then all calculable in the framework of the k_T factorization, where three energy scales m_W, m_B, and $t \approx \sqrt{m_B \Lambda_{\text{QCD}}}$ are involved [21,22,31]. The running of the Wilson coefficients $C_i(t)$ with $t \gtrsim \sqrt{m_B \Lambda_{\text{QCD}}}$ are controlled by the renormalization group equation and can be calculated perturbatively. The dynamics below $\sqrt{m_B \Lambda_{\text{QCD}}}$ is soft, which is described by the meson wave functions. The soft dynamics is not perturbative but universal for all channels. In the pQCD approach, a $B \to M_2 M_3$ decay amplitude is therefore factorized into the convolution of the six-quark hard kernel (H), the jet function (J) and the Sudakov factor (S) with the bound-state wave functions (Φ) as follows:

$$\mathcal{A}(B \to M_2 M_3) = \Phi_B \otimes H \otimes J \otimes S \otimes \Phi_{M_2} \otimes \Phi_{M_3}. \tag{2}$$

The jet function J comes from the threshold resummation, which exhibits a strong suppression effect in the small x (quark momentum fraction) region [32]. The Sudakov factor S comes from the k_T resummation, which provides a strong suppression in the small k_T region [33]. Therefore, these resummation effects guarantee the removal of the end-point singularities.

A. Wave functions and distribution amplitudes

Throughout this paper, we will use light-cone coordinate $(P^+, P^-, \mathbf{P_T})$ to describe the meson's momenta with the definitions $P^\pm = (p_0 \pm p_3)/\sqrt{2}$ and $\mathbf{P_T} = (p_1, p_2)$. The heavy B meson is usually treated as a heavy-light system, and its light-cone wave function can generally be defined as [21,22,34]

$$\begin{aligned} \Phi_{B,\alpha\beta,ij} &\equiv \langle 0|\bar{b}_{\beta j}(0)q_{\alpha i}(z)|B(P)\rangle \\ &= \frac{i\delta_{ij}}{\sqrt{2N_c}}\int dx d^2 k_T e^{-i(xP^- z^+ - k_T z_T)} \\ &\quad \times \{(\not{P}+m_B)\gamma_5\phi_B(x,k_T)\}_{\alpha\beta}, \end{aligned} \tag{3}$$

where the indices i, j and α, β are the Lorentz indices and color indices, respectively, $P(m)$ is the momentum (mass) of the B meson, N_c is the color factor, and k_T is the intrinsic transverse momentum of the light quark in the B meson. Note that, in principle, there are two Lorentz structures of the wave function to be considered in the numerical calculations; however, the contribution induced by the second Lorentz structure is numerically small and approximately negligible [34].

In Eq. (3), $\phi_B(x, k_T)$ is the B meson distribution amplitude and obeys the following normalization condition:

$$\int_0^1 dx \phi_B(x, b = 0) = \frac{f_B}{2\sqrt{2N_c}}, \tag{4}$$

where b is the conjugate space coordinate of transverse momentum k_T and f_B is the decay constant of the B meson.

For the *B* meson, the distribution amplitude in the impact *b* space has been proposed,

$$\phi_B(x, b) = N_B x^2 (1-x)^2 \exp\left[-\frac{1}{2}\left(\frac{xm_B}{\omega_b}\right)^2 - \frac{\omega_b^2 b^2}{2}\right], \quad (5)$$

in Refs. [21,22], where the normalization factor N_B is related to the decay constant f_B through Eq. (4). The shape parameter ω_b has been fixed at $\omega_b = 0.40 \pm 0.04$ GeV by using the rich experimental data on the $B_{u/d}$ mesons with $f_{B_{u/d}} = 0.19$ GeV based on lots of calculations of form factors [34] and other well-known decay modes of $B_{u/d}$ mesons [21,22] in the pQCD approach in recent years. By considering the small SU(3) flavor symmetry breaking effect, the shape parameter ω_b for the B_s meson is taken as $\omega_{B_s} = 0.50 \pm 0.05$ GeV [29].

The light-cone wave function of the light vector meson K^* has been given in the QCD sum rule method up to twist-3 as [35]

$$\Phi^L_{K^*,\alpha\beta,ij} \equiv \langle K^*(P, \epsilon_L)|\bar{q}(z)_{\beta j} q(0)_{\alpha i}|0\rangle$$
$$= \frac{\delta_{ij}}{\sqrt{2N_c}} \int_0^1 dx\, e^{ixP\cdot z} \{m_{K^*} \not{\epsilon}_L \phi_{K^*}(x)$$
$$+ \not{\epsilon}_L \not{P} \phi^t_{K^*}(x) + m_{K^*} \phi^s_{K^*}(x)\}_{\alpha\beta}, \quad (6)$$

for longitudinal polarization, where ϵ_L denotes the longitudinal polarization vector of K^*, satisfying $P \cdot \epsilon_L = 0$, and *x* denotes the momentum fraction carried by the quark in the meson.

The twist-2 distribution amplitude ϕ_{K^*} can be parametrized as

$$\phi_{K^*}(x) = \frac{3f_{K^*}}{\sqrt{2N_c}} x(1-x)\left[1 + 3a^{\parallel}_{1K^*}(2x-1)\right.$$
$$\left. + a^{\parallel}_{2K^*} \frac{3}{2}(5(2x-1)^2 - 1)\right]. \quad (7)$$

And the asymptotic forms of the twist-3 distribution amplitudes $\phi^t_{K^*}$ and $\phi^s_{K^*}$ are adopted [36],

$$\phi^t_{K^*}(x) = \frac{3f^T_{K^*}}{2\sqrt{2N_c}}(2x-1)^2,$$
$$\phi^s_{K^*}(x) = -\frac{3f^T_{K^*}}{2\sqrt{2N_c}}(2x-1). \quad (8)$$

Here f_{K^*} and $f^T_{K^*}$ are the decay constants of the K^* meson with longitudinal and transverse polarization, respectively, whose values are

$$f_{K^*} = 0.217 \pm 0.005 \text{ GeV}, \quad f^T_{K^*} = 0.185 \pm 0.010 \text{ GeV}. \quad (9)$$

The Gegenbauer moments are taken from the recent updates [37],

$$a^{\parallel}_{1K^*} = 0.03 \pm 0.02, \quad a^{\parallel}_{2K^*} = 0.11 \pm 0.09. \quad (10)$$

The light-cone wave function of the light scalar K^*_0 has been analyzed in the QCD sum rule method [12]

$$\Phi_{K^*_0,\alpha\beta,ij} \equiv \langle K^*_0(P)|\bar{q}(z)_{\beta j} q(0)_{\alpha i}|0\rangle$$
$$= \frac{i\delta_{ij}}{\sqrt{2N_c}} \int_0^1 dx\, e^{ixP\cdot z} \{\not{P} \phi_{K^*_0}(x) + m_{K^*_0} \phi^S_{K^*_0}(x)$$
$$+ m_{K^*_0}(\not{n}\not{\upsilon} - 1)\phi^T_{K^*_0}(x)\}_{\alpha\beta}, \quad (11)$$

where $n = (1, 0, \mathbf{0_T})$ and $\upsilon = (0, 1, \mathbf{0_T})$ are the unit vectors pointing to the plus and minus directions on the light cone, respectively, and *x* denotes the momentum fraction carried by the quark in the K^*_0 meson.

For the light scalar meson K^*_0, its leading twist (twist-2) light-cone distribution amplitude $\phi_{K^*_0}(x, \mu)$ can be generally expanded as the Gegenbauer polynomials [12,38]:

$$\phi_{K^*_0}(x, \mu) = \frac{3}{\sqrt{2N_c}} x(1-x)\left\{f_{K^*_0}(\mu)\right.$$
$$\left. + \bar{f}_{K^*_0}(\mu) \sum_{m=1}^{\infty} B_m(\mu) C^{3/2}_m(2x-1)\right\}, \quad (12)$$

where $f_{K^*_0}(\mu)$ and $\bar{f}_{K^*_0}(\mu)$, $B_m(\mu)$, and $C^{3/2}_m(t)$ are the vector and scalar decay constants, Gegenbauer moments, and Gegenbauer polynomials, respectively. There is a relation between the vector and scalar decay constants,

$$\bar{f}_{K^*_0} = \mu_{K^*_0} f_{K^*_0} \quad \text{and} \quad \mu_{K^*_0} = \frac{m_{K^*_0}}{m_2(\mu) - m_1(\mu)}, \quad (13)$$

where m_1 and m_2 are the running current quark masses in the scalar K^*_0. According to Eq. (13), one can clearly find that the vector decay constant $f_{K^*_0}$ is proportional to the mass difference between the constituent *s* and *u*(*d*) quarks, which will result in $f_{K^*_0}$ being of order $m_s - m_{u(d)}$. Therefore, contrary to the case of pseudoscalar mesons, the contribution from the factorizable diagrams with the emission of K^*_0 will be largely suppressed.

The values for scalar decay constants and Gegenbauer moments in the distribution amplitudes of K^*_0 have been estimated at scale $\mu = 1$ GeV in the scenarios S1 and S2 [12]:

S1: $\bar{f}_{K^*_0} = -0.300 \pm 0.030$ GeV, $\quad B_1 = 0.58 \pm 0.07,$

$\quad B_3 = -1.20 \pm 0.08, \quad$ S2: $\bar{f}_{K^*_0} = 0.445 \pm 0.050$ GeV,

$\quad B_1 = -0.57 \pm 0.13, \quad B_3 = -0.42 \pm 0.22. \quad (14)$

As for the twist-3 distribution amplitudes $\phi^S_{K^*_0}$ and $\phi^T_{K^*_0}$, we here adopt the asymptotic forms in our numerical calculations as in Ref. [12],

$$\phi^S_{K^*_0} = \frac{1}{2\sqrt{2N_c}} \bar{f}_{K^*_0}, \quad \phi^T_{K^*_0} = \frac{1}{2\sqrt{2N_c}} \bar{f}_{K^*_0}(1 - 2x). \quad (15)$$

Here, we should stress that the k_T dependence of the distribution amplitudes in the final states has been neglected, since its contribution is very small as indicated in Refs. [31]. The underlying reason is that the contribution from k_T correlated with a soft dynamics is strongly suppressed by the Sudakov effect through resummation for the

wave function, which is dominated by a collinear dynamics. Another reason is just that, unfortunately, up to now, the distribution amplitudes with intrinsic k_T dependence for the above-mentioned light mesons K^* and K_0^* are not available.

B. Perturbative calculations

For the considered $B \to K_0^* K^*$ decays, the related weak effective Hamiltonian H_{eff} [20] can be written as

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}^* V_{uq} [C_1(\mu) O_1^u(\mu) + C_2(\mu) O_2^u(\mu)] \right.$$

$$\left. - V_{tb}^* V_{tq} \left[\sum_{i=3}^{10} C_i(\mu) O_i(\mu) \right] \right\} + \text{H.c.}, \quad (16)$$

with $q = d$ or s, the Fermi constant $G_F = 1.16639 \times 10^{-5}$ GeV^{-2}, Cabibbo-Kobayashi-Maskawa (CKM) matrix elements V, and Wilson coefficients $C_i(\mu)$ at the renormalization scale μ. The local four-quark operators $O_i (i = 1, \ldots, 10)$ are written as

(1) current-current (tree) operators

$$O_1^u = (\bar{q}_\alpha u_\beta)_{V-A} (\bar{u}_\beta b_\alpha)_{V-A},$$
$$O_2^u = (\bar{q}_\alpha u_\alpha)_{V-A} (\bar{u}_\beta b_\beta)_{V-A}; \quad (17)$$

(2) QCD penguin operators

$$O_3 = (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{V-A},$$
$$O_4 = (\bar{q}_\alpha b_\beta)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{V-A},$$
$$O_5 = (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{V+A},$$
$$O_6 = (\bar{q}_\alpha b_\beta)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{V+A}; \quad (18)$$

(3) electroweak penguin operators

$$O_7 = \frac{3}{2} (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\beta)_{V+A},$$

$$O_8 = \frac{3}{2} (\bar{q}_\alpha b_\beta)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\alpha)_{V+A},$$

$$O_9 = \frac{3}{2} (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\beta)_{V-A}, \quad (19)$$

$$O_{10} = \frac{3}{2} (\bar{q}_\alpha b_\beta)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\alpha)_{V-A},$$

with the color indices α, β and the notations $(\bar{q}' q')_{V \pm A} = \bar{q}' \gamma_\mu (1 \pm \gamma_5) q'$. The index q' in the summation of the above operators runs through u, d, s, c, and b. The standard combinations a_i of the Wilson coefficients are defined as follows:

$$a_1 = C_2 + \frac{C_1}{3}, \qquad a_2 = C_1 + \frac{C_2}{3},$$

$$a_i = C_i + \frac{C_{i \pm 1}}{3} (i = 3\text{–}10), \quad (20)$$

where the upper (lower) sign applies, when i is odd (even).

Similar to $B \to K_0^* K$ decays [15], there are eight types of diagrams contributing to $B \to K_0^* K^*$ modes at leading order, as illustrated in Fig. 1. They involve two classes of topologies with spectator and annihilation, respectively. Each kind of topology is classified into factorizable diagrams, in which a hard gluon connects the quarks in the same meson, e.g., Figs. 1(a) and 1(b), and nonfactorizable diagrams, in which a hard gluon attaches the quarks in two different mesons, e.g., Figs. 1(c) and 1(d). By calculating these Feynman diagrams, one can get the decay amplitudes of $B \to K_0^* K^*$ decays. Because the formulas of $B \to K_0^* K^*$ are similar to those of $B \to K_0^* K_0^*$ [15], one can therefore
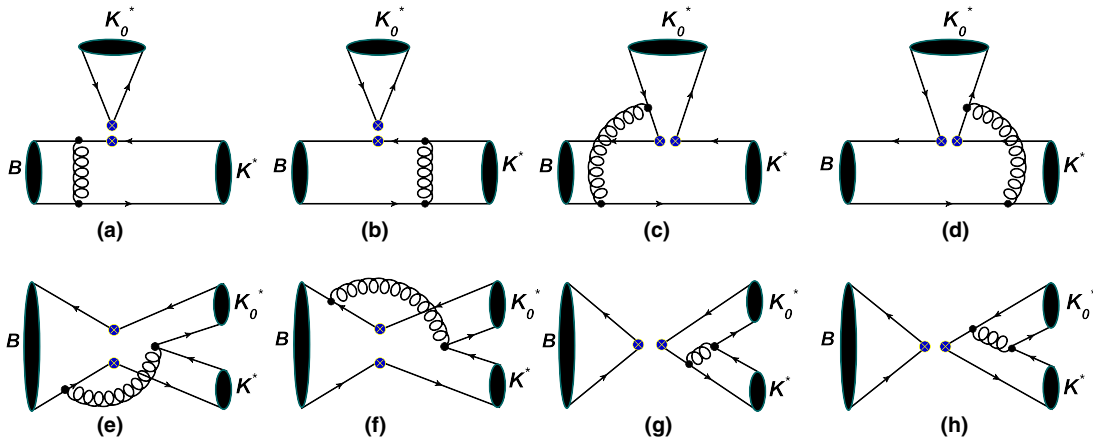

FIG. 1 (color online). Typical Feynman diagrams contributing to $B \to K_0^* K^*$ decays at leading order. In this figure, B stands for B_u, B_d, and B_s, respectively. When we exchanged the positions of K_0^* and K^*, the other eight diagrams contributing to the considered decay modes will be easily obtained.

obtain the expressions for all the diagrams just by replacing the corresponding wave functions and input parameters from $B \to K_0^* \bar{K}_0^*$. So we do not present the detailed formulas in this paper.

By combining various contributions from the relevant Feynman diagrams together, the total decay amplitudes for the considered $B \to K_0^* K^*$ decays can then read as follows:

(1) The total decay amplitudes for charged B_u decays:

$$\mathcal{A}(B_u \to K^{*+} \bar{K}_0^{*0}) = \lambda_u [M_{nfa} C_1] - \lambda_t \left[F_{fs}\left(a_4 - \frac{1}{2} a_{10}\right) + F_{fs}^{P2}\left(a_6 - \frac{1}{2} a_8\right) + M_{nfs}\left(C_3 - \frac{1}{2} C_9\right) + M_{nfs}\left(C_5 - \frac{1}{2} C_7\right) \right.$$

$$\left. + M_{nfa}(C_3 + C_9) + M_{nfa}^{P1}(C_5 + C_7) + f_B F_{fa}^{P2}(a_6 + a_8) \right],$$

(21)

where $\lambda_u = V_{ub}^* V_{ud}$ and $\lambda_t = V_{tb}^* V_{td}$. The decay amplitude of $B_u \to K_0^{*+} \bar{K}^{*0}$ can be obtained directly from Eq. (21) with the replacement of $K^* \leftrightarrow K_0^*$, but without the contributions from the term F_{fs}^{P2}. The reason is that the emitted vector K^* meson cannot be produced via the scalar or pseudoscalar current.

(2) The total decay amplitudes for neutral B_d decays:

$$\mathcal{A}(B_d \to K^{*+} K_0^{*-}) = \lambda_u [M_{nfa} C_2] - \lambda_t \left[M_{nfa}(C_4 + C_{10}) + M_{nfa}^{P2}(C_6 + C_8) + M_{nfa}[K^{*+} \leftrightarrow K_0^{*-}]\left(C_4 - \frac{1}{2} C_{10}\right) \right.$$

$$\left. + M_{nfa}^{P2}[K^{*+} \leftrightarrow K_0^{*-}]\left(C_6 - \frac{1}{2} C_8\right) \right],$$

(22)

$$\mathcal{A}(B_d \to K^{*0} \bar{K}_0^{*0}) = -\lambda_t \left[F_{fs}\left(a_4 - \frac{1}{2} a_{10}\right) + F_{fs}^{P2}\left(a_6 - \frac{1}{2} a_8\right) + (M_{nfs} + M_{nfa})\left(C_3 - \frac{1}{2} C_9\right) \right.$$

$$+ (M_{nfs}^{P1} + M_{nfa}^{P1})\left(C_5 - \frac{1}{2} C_7\right) + (M_{nfa} + [K^{*0} \leftrightarrow \bar{K}_0^{*0}])\left(C_4 - \frac{1}{2} C_{10}\right)$$

$$\left. + (M_{nfa}^{P2} + [K^{*0} \leftrightarrow \bar{K}_0^{*0}])\left(C_6 - \frac{1}{2} C_8\right) + f_B F_{fa}^{P2}\left(a_6 - \frac{1}{2} a_8\right) \right].$$

(23)

Similarly, the decay amplitudes of $B_d \to K_0^{*+} K^{*-}$ and $B_d \to K_0^{*0} \bar{K}^{*0}$ can also be obtained easily from Eqs. (22) and (23) with the replacements $K^* \leftrightarrow K_0^*$, respectively, and with the dropping of term F_{sf}^{P2} for the latter mode.

(3) The total decay amplitudes for B_s decays:

$$\mathcal{A}(B_s \to K_0^{*+} K^{*-}) = \lambda_u' [F_{fs} a_1 + M_{nfs} C_1 + M_{nfa} C_2] - \lambda_t' \left[F_{fs}(a_4 + a_{10}) + F_{fs}^{P2}(a_6 + a_8) + M_{nfs}(C_3 + C_9) \right.$$

$$+ M_{nfs}^{P1}(C_5 + C_7) + M_{nfa}\left(C_3 - \frac{1}{2} C_9 + C_4 - \frac{1}{2} C_{10}\right) + M_{nfa}[K_0^{*+} \leftrightarrow K^{*-}](C_4 + C_{10})$$

$$\left. + M_{nfa}^{P1}\left(C_5 - \frac{1}{2} C_7\right) + M_{nfa}^{P2}\left(C_6 - \frac{1}{2} C_8\right) + M_{nfa}^{P2}[K_0^{*+} \leftrightarrow K^{*-}](C_6 + C_8) + f_{B_s} F_{fa}^{P2}\left(a_6 - \frac{1}{2} a_8\right) \right],$$

(24)

where $\lambda_u' = V_{ub}^* V_{us}$ and $\lambda_t' = V_{tb}^* V_{ts}$, and

$$\mathcal{A}(B_s \to K_0^{*0} \bar{K}^{*0}) = -\lambda_t' \left[F_{fs}\left(a_4 - \frac{1}{2} a_{10}\right) + F_{fs}^{P2}\left(a_6 - \frac{1}{2} a_8\right) + (M_{nfs} + M_{nfa})\left(C_3 - \frac{1}{2} C_9\right) \right.$$

$$+ (M_{nfs}^{P1} + M_{nfa}^{P1})\left(C_5 - \frac{1}{2} C_7\right) + (M_{nfa} + [K_0^{*0} \leftrightarrow \bar{K}^{*0}])\left(C_4 - \frac{1}{2} C_{10}\right)$$

$$\left. + (M_{nfa}^{P2} + [K_0^{*0} \leftrightarrow \bar{K}^{*0}])\left(C_6 - \frac{1}{2} C_8\right) + f_{B_s} F_{fa}^{P2}\left(a_6 - \frac{1}{2} a_8\right) \right].$$

(25)

There are two other B_s decay channels, i.e., $B_s \to K^{*+} K_0^{*-}$ and $B_s \to K^{*0} \bar{K}_0^{*0}$, whose decay amplitudes can be derived from Eqs. (24) and (25) by the exchange of $K_0^* \leftrightarrow K^*$, respectively. Certainly, the F_{fs}^{P2} term has no contribution to them either. Note that, based on the discussions of the factorizable annihilation contributions F_{fa} in Ref. [15], we here neglected this term in the above decay amplitudes for the considered $B \to K_0^* K^*$ decays analytically.

III. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we will present the theoretical predictions for the branching ratios and CP-violating asymmetries for those considered $B \to K_0^* K^*$ decay modes in the pQCD approach. In numerical calculations, central values of the input parameters will be used implicitly unless otherwise stated. The relevant QCD scale (GeV), masses (GeV), and B meson lifetime (ps) are the following [1,21,22]:

$$\Lambda_{\overline{\mathrm{MS}}}^{(f=4)} = 0.250, \qquad m_W = 80.41, \qquad m_B = 5.28,$$

$$m_{B_s} = 5.37, \qquad m_b = 4.8; \qquad \tau_{B_u} = 1.641,$$

$$\tau_{B_d} = 1.519, \qquad \tau_{B_s} = 1.497, \qquad m_{K^*} = 0.892,$$

$$m_{K_0^*(1430)} = 1.425. \tag{26}$$

For the CKM matrix elements, we adopt the Wolfenstein parametrization and the updated parameters $A = 0.811$, $\lambda = 0.22535$, $\bar{\rho} = 0.131^{+0.026}_{-0.013}$, and $\bar{\eta} = 0.345^{+0.013}_{-0.014}$ [1].

A. Branching ratios

In this subsection, we will analyze the branching ratios of the considered $B \to K_0^* K^*$ decays in the pQCD

approach. For $B \to K_0^* K^*$ decays, the decay rate can be written as

$$\Gamma = \frac{G_F^2 m_B^3}{32\pi}(1 - 2r_{K_0^*}^2)|\mathcal{A}(B \to K_0^* K^*)|^2, \tag{27}$$

where the corresponding decay amplitudes $\mathcal{A}$ have been given explicitly in Eqs. (21)–(25). Using the decay amplitudes obtained in the last section, it is straightforward to calculate the branching ratios with uncertainties as displayed in Tables I, II, and III for the considered decay modes. The major errors are induced by the uncertainties of the shape parameters $\omega_b = 0.40 \pm 0.04$ GeV for $B_{u,d}$ decays, $\omega_{B_s} = 0.50 \pm 0.05$ GeV for B_s decays, the scalar decay constant $\bar{f}_{K_0^*}$ of the K_0^* meson, the decay constants $f_{K^*}^{(T)}$ of the vector K^* meson, the Gegenbauer moments $B_i(i = 1, 3)$ for the scalar K_0^*, the Gegenbauer moments $a_i(i = 1, 2)$ for the vector K^* meson, and the CKM matrix elements V_i $(\bar{\rho}, \bar{\eta})$, respectively.

Based on the above numerical results of the branching ratios given at leading order in the pQCD approach for the considered decay modes, some remarks are as follows:

(1) Generally speaking, the theoretical predictions for the considered decays in the pQCD approach have

TABLE I. The pQCD predictions for the branching ratios of $B_u \to K_0^{*+} \bar{K}^{*0}$ and $K^{*+} \bar{K}_0^{*0}$ decays in different scenarios: the first (second) entry corresponds to S1 (S2).

Decay modes	Branching ratios
$B_u \to K_0^{*+} \bar{K}^{*0}$	$2.1^{+1.5}_{-0.9}(\omega_b)^{+0.4}_{-0.4}(\bar{f}_{K_0^*})^{+0.3}_{-0.3}(f_{K^*}^{(T)})^{+0.1}_{-0.2}(B_i)^{+0.2}_{-0.1}(a_i)^{+0.1}_{-0.2}(V_i)^{+0.4}_{-0.2}(a_t) \times 10^{-7}$
	$1.3^{+0.5}_{-0.3}(\omega_b)^{+0.3}_{-0.3}(\bar{f}_{K_0^*})^{+0.1}_{-0.1}(f_{K^*}^{(T)})^{+0.1}_{-0.1}(B_i)^{+0.1}_{-0.1}(a_i)^{+0.0}_{-0.1}(V_i)^{+0.4}_{-0.2}(a_t) \times 10^{-6}$
$B_u \to K^{*+} \bar{K}_0^{*0}$	$6.0^{+1.4}_{-1.0}(\omega_b)^{+1.3}_{-1.2}(\bar{f}_{K_0^*})^{+0.3}_{-0.3}(f_{K^*}^{(T)})^{+0.4}_{-0.3}(B_i)^{+0.1}_{-0.1}(a_i)^{+0.5}_{-0.5}(V_i)^{+0.8}_{-0.5}(a_t) \times 10^{-7}$
	$1.5^{+0.5}_{-0.3}(\omega_b)^{+0.4}_{-0.3}(\bar{f}_{K_0^*})^{+0.1}_{-0.1}(f_{K^*}^{(T)})^{+0.3}_{-0.1}(B_i)^{+0.2}_{-0.1}(a_i)^{+0.1}_{-0.1}(V_i)^{+0.5}_{-0.3}(a_t) \times 10^{-6}$

TABLE II. Same as Table I but for neutral $B_d \to K_0^* K^*$ decays in both scenarios.

Decay modes	Branching ratios
$B_d \to K_0^{*0} \bar{K}^{*0} + K^{*0} \bar{K}_0^{*0}$	$6.4^{+0.5}_{-0.6}(\omega_b)^{+1.3}_{-1.2}(\bar{f}_{K_0^*})^{+0.7}_{-0.7}(f_{K^*}^{(T)})^{+1.6}_{-1.5}(B_i)^{+0.8}_{-0.8}(a_i)^{+0.2}_{-0.3}(V_i)^{+0.2}_{-0.5}(a_t) \times 10^{-7}$
	$5.9^{+0.7}_{-0.6}(\omega_b)^{+1.4}_{-1.2}(\bar{f}_{K_0^*})^{+0.6}_{-0.4}(f_{K^*}^{(T)})^{+5.9}_{-3.2}(B_i)^{+0.5}_{-0.4}(a_i)^{+0.2}_{-0.3}(V_i)^{+0.7}_{-0.6}(a_t) \times 10^{-7}$
$B_d/\bar{B}_d \to K_0^{*0} \bar{K}^{*0}(K^{*0} \bar{K}_0^{*0})$	$5.0^{+2.2}_{-1.3}(\omega_b)^{+1.1}_{-1.0}(\bar{f}_{K_0^*})^{+0.6}_{-0.6}(f_{K^*}^{(T)})^{+0.4}_{-0.3}(B_i)^{+0.2}_{-0.1}(a_i)^{+0.1}_{-0.3}(V_i)^{+0.5}_{-0.5}(a_t) \times 10^{-7}$
	$2.3^{+0.8}_{-0.5}(\omega_b)^{+0.5}_{-0.5}(\bar{f}_{K_0^*})^{+0.1}_{-0.1}(f_{K^*}^{(T)})^{+0.4}_{-0.2}(B_i)^{+0.2}_{-0.1}(a_i)^{+0.0}_{-0.1}(V_i)^{+0.7}_{-0.5}(a_t) \times 10^{-6}$

Decay modes	Branching ratios (10^{-6})
$B_d \to K_0^{*+} K^{*-} + K^{*+} K_0^{*-}$	$2.8^{+0.2}_{-0.3}(\omega_b)^{+0.6}_{-0.6}(\bar{f}_{K_0^*})^{+0.1}_{-0.2}(f_{K^*}^{(T)})^{+0.4}_{-0.4}(B_i)^{+0.1}_{-0.0}(a_i)^{+0.1}_{-0.2}(V_i)^{+0.0}_{-0.1}(a_t)$
	$1.1^{+0.1}_{-0.1}(\omega_b)^{+0.2}_{-0.3}(\bar{f}_{K_0^*})^{+0.0}_{-0.1}(f_{K^*}^{(T)})^{+1.1}_{-0.6}(B_i)^{+0.1}_{-0.1}(a_i)^{+0.0}_{-0.1}(V_i)^{+0.0}_{-0.1}(a_t)$
$B_d/\bar{B}_d \to K_0^{*+} K^{*-}$	$4.5^{+0.4}_{-0.4}(\omega_b)^{+0.9}_{-0.9}(\bar{f}_{K_0^*})^{+0.2}_{-0.2}(f_{K^*}^{(T)})^{+0.6}_{-0.6}(B_i)^{+0.3}_{-0.3}(a_i)^{+0.3}_{-0.3}(V_i)^{+0.1}_{-0.1}(a_t)$
	$2.9^{+0.3}_{-0.4}(\omega_b)^{+0.6}_{-0.7}(\bar{f}_{K_0^*})^{+0.2}_{-0.1}(f_{K^*}^{(T)})^{+2.0}_{-1.4}(B_i)^{+0.1}_{-0.1}(a_i)^{+0.2}_{-0.2}(V_i)^{+0.1}_{-0.2}(a_t)$
$B_d/\bar{B}_d \to K^{*+} K_0^{*-}$	$1.7^{+0.2}_{-0.2}(\omega_b)^{+0.3}_{-0.3}(\bar{f}_{K_0^*})^{+0.1}_{-0.1}(f_{K^*}^{(T)})^{+0.2}_{-0.2}(B_i)^{+0.1}_{-0.1}(a_i)^{+0.2}_{-0.2}(V_i)^{+0.1}_{-0.1}(a_t)$
	$1.1^{+0.1}_{-0.2}(\omega_b)^{+0.2}_{-0.3}(\bar{f}_{K_0^*})^{+0.1}_{-0.0}(f_{K^*}^{(T)})^{+0.6}_{-0.5}(B_i)^{+0.0}_{-0.1}(a_i)^{+0.1}_{-0.1}(V_i)^{+0.0}_{-0.1}(a_t)$

TABLE III. Same as Table I but for strange $B_s \to K_0^* K^*$ decays in both scenarios.

Decay modes	Branching ratios (10^{-5})
$B_s \to K_0^{*0}\bar{K}^{*0} + K^{*0}\bar{K}_0^{*0}$	$1.3^{+0.2}_{-0.1}(\omega_{bs})^{+0.3}_{-0.2}(\bar{f}_{K_0^*})^{+0.2}_{-0.1}(f_{K^*}^{(T)})^{+0.4}_{-0.3}(B_i)^{+0.2}_{-0.1}(a_i)^{+0.0}_{-0.0}(V_i)^{+0.1}_{-0.1}(a_t)$
	$1.3^{+0.2}_{-0.2}(\omega_{bs})^{+0.3}_{-0.3}(\bar{f}_{K_0^*})^{+0.1}_{-0.1}(f_{K^*}^{(T)})^{+1.2}_{-0.7}(B_i)^{+0.1}_{-0.1}(a_i)^{+0.0}_{-0.0}(V_i)^{+0.1}_{-0.2}(a_t)$
$B_s/\bar{B}_s \to K_0^{*0}\bar{K}^{*0}(K^{*0}\bar{K}_0^{*0})$	$0.9^{+0.3}_{-0.2}(\omega_{bs})^{+0.2}_{-0.2}(\bar{f}_{K_0^*})^{+0.1}_{-0.1}(f_{K^*}^{(T)})^{+0.1}_{-0.1}(B_i)^{+0.1}_{-0.1}(a_i)^{+0.0}_{-0.0}(V_i)^{+0.1}_{-0.1}(a_t)$
	$5.4^{+1.5}_{-0.9}(\omega_{bs})^{+1.3}_{-1.1}(\bar{f}_{K_0^*})^{+0.2}_{-0.2}(f_{K^*}^{(T)})^{+0.9}_{-0.5}(B_i)^{+0.9}_{-0.6}(a_i)^{+0.0}_{-0.0}(V_i)^{+1.7}_{-1.1}(a_t)$
Decay modes	**Branching ratios (10^{-5})**
$B_s \to K_0^{*+}K^{*-} + K^{*+}K_0^{*-}$	$1.3^{+0.2}_{-0.1}(\omega_{bs})^{+0.3}_{-0.2}(\bar{f}_{K_0^*})^{+0.1}_{-0.1}(f_{K^*}^{(T)})^{+0.4}_{-0.2}(B_i)^{+0.1}_{-0.0}(a_i)^{+0.1}_{-0.0}(V_i)^{+0.1}_{-0.1}(a_t)$
	$1.5^{+0.4}_{-0.3}(\omega_{bs})^{+0.4}_{-0.3}(\bar{f}_{K_0^*})^{+0.1}_{-0.0}(f_{K^*}^{(T)})^{+1.0}_{-0.4}(B_i)^{+0.1}_{-0.0}(a_i)^{+0.1}_{-0.0}(V_i)^{+0.3}_{-0.2}(a_t)$
$B_s/\bar{B}_s \to K_0^{*+}K^{*-}$	$0.9^{+0.3}_{-0.1}(\omega_{bs})^{+0.2}_{-0.2}(\bar{f}_{K_0^*})^{+0.1}_{-0.1}(f_{K^*}^{(T)})^{+0.2}_{-0.1}(B_i)^{+0.0}_{-0.0}(a_i)^{+0.0}_{-0.0}(V_i)^{+0.1}_{+0.1}(a_t)$
	$4.0^{+1.3}_{-0.7}(\omega_{bs})^{+1.0}_{-0.8}(\bar{f}_{K_0^*})^{+0.2}_{-0.1}(f_{K^*}^{(T)})^{+0.8}_{-0.3}(B_i)^{+0.6}_{-0.4}(a_i)^{+0.1}_{-0.1}(V_i)^{+1.4}_{-0.8}(a_t)$
$B_s/\bar{B}_s \to K^{*+}K_0^{*-}$	$0.9^{+0.2}_{-0.1}(\omega_{bs})^{+0.2}_{-0.2}(\bar{f}_{K_0^*})^{+0.1}_{-0.1}(f_{K^*}^{(T)})^{+0.1}_{-0.1}(B_i)^{+0.2}_{-0.2}(a_i)^{+0.0}_{-0.0}(V_i)^{+0.2}_{-0.2}(a_t)$
	$6.1^{+1.5}_{-0.9}(\omega_{bs})^{+1.4}_{-1.3}(\bar{f}_{K_0^*})^{+0.3}_{-0.2}(f_{K^*}^{(T)})^{+1.0}_{-0.6}(B_i)^{+1.1}_{-0.9}(a_i)^{+0.1}_{-0.1}(V_i)^{+1.9}_{-1.2}(a_t)$

relatively large errors arising from the still large uncertainties of many input parameters. Furthermore, the numerical results for the branching ratios suffer more from the errors induced by the less constrained hadronic parameters of the light scalar K_0^*, such as the scalar decay constant $\bar{f}_{K_0^*}$ and the Gegenbauer coefficients $B_i (i = 1, 3)$. Additionally, in this work, as displayed in the tables, the higher order contributions are also simply investigated by exploring the variation of the hard scale t_{max}, i.e., from $0.8t$ to $1.2t$ (not changing $1/b_i$, $i = 1, 2, 3$), in the hard kernel, which have been counted into one of the sources of theoretical uncertainties.

(2) The pQCD predictions for the *CP*-averaged $\mathrm{Br}(B_u \to K_0^{*+}\bar{K}^{*0})$ and $\mathrm{Br}(B_u \to K^{*+}\bar{K}_0^{*0})$ are in the order of 10^{-6} in S2, which are larger than those in S1, and can be tested by the future *B* physics experiments. Moreover, one can define the ratios of the branching ratios of the same decay mode but in different scenarios as the following:

$$\frac{\mathrm{Br}(B_u \to K_0^{*+}\bar{K}^{*0})_{S2}}{\mathrm{Br}(B_u \to K_0^{*+}\bar{K}^{*0})_{S1}} = 6.2,$$
$$\frac{\mathrm{Br}(B_u \to K^{*+}\bar{K}_0^{*0})_{S2}}{\mathrm{Br}(B_u \to K^{*+}\bar{K}_0^{*0})_{S1}} = 2.5, \qquad (28)$$

where the central values are quoted for clarification. The above two patterns imply the different QCD dynamics involved in the corresponding decay channels, which can be tested with the future precision measurements.

(3) The neutral B_d decays include the pure penguin contribution modes, i.e., $B_d \to K^{*0}\bar{K}_0^{*0}$ and $B_d \to K_0^{*0}\bar{K}^{*0}$, and the pure annihilation contribution channels, i.e., $B_d \to K^{*+}K_0^{*-}$ and $B_d \to K_0^{*+}K^{*-}$, respectively. The analysis for these four decay modes are a little complicated, which is just because both B_d and $\bar{B}_d$ can decay into the same final states simultaneously; in other words, the final states in the considered B_d decays are not the *CP* eigenstates. Because of the $B_d - \bar{B}_d$ mixing, it is very difficult for us to distinguish the B_d from the $\bar{B}_d$. However, fortunately, it is easy to identify the final states in the considered decays. We therefore sum up $B_d/\bar{B}_d \to K^{*0}\bar{K}_0^{*0}$ as one channel, and $B_d/\bar{B}_d \to K_0^{*0}\bar{K}^{*0}$ as another. Similarly, we will have $B_d/\bar{B}_d \to K^{*+}K_0^{*-}$ as one mode, and $B_d/\bar{B}_d \to K_0^{*+}K^{*-}$ as another. Moreover, following the convention by the experimental measurements [1,39], we also define the averaged quantity of the two channels, i.e., $B_d \to K^{*0}\bar{K}_0^{*0} + K_0^{*0}\bar{K}^{*0}$ and $B_d \to K^{*+}K_0^{*-} + K_0^{*+}K^{*-}$. The same phenomena will also occur in the decays of the B_s meson.

(4) The theoretical predictions on the branching ratios of the B_d meson decays in the pQCD approach have been presented in Table II. For the pure penguin $B_d/\bar{B}_d \to K_0^{*0}\bar{K}^{*0}$, $B_d/\bar{B}_d \to K^{*0}\bar{K}_0^{*0}$, and $B_d \to K_0^{*0}\bar{K}^{*0} + K^{*0}\bar{K}_0^{*0}$ channels, the pQCD predictions for the former two decays show that the branching ratios (about 2×10^{-6}) in S2 are larger than that (about 5×10^{-7}) in S1, which results in the ratio $\mathrm{Br}(B_d/\bar{B}_d \to K_0^{*0}\bar{K}^{*0}(K^{*0}\bar{K}_0^{*0}))_{S2}/\mathrm{Br}(B_d/\bar{B}_d \to K_0^{*0}\bar{K}^{*0}(K^{*0}\bar{K}_0^{*0}))_{S1} \approx 4.6$; while the pQCD predictions for the latter one in both scenarios are similar, which leads to the ratio $\mathrm{Br}(B_d \to K^{*0}\bar{K}_0^{*0} + K_0^{*0}\bar{K}^{*0})_{S1}/\mathrm{Br}(B_d \to K^{*0}\bar{K}_0^{*0} + K_0^{*0}\bar{K}^{*0})_{S2} \approx 1.1$. Note that because of the charge conjugation between the pure penguin channels $B_d \to K_0^{*0}\bar{K}^{*0}$ and $\bar{B}_d \to \bar{K}_0^{*0}K^{*0}$ and the domination of the real contributions arising from the factorizable emission diagrams, e.g., Figs. 1(a) and 1(b), in the considered channels, the same branching ratios for

$B_d/\bar{B}_d \to K_0^{*0}\bar{K}^{*0}$ and $B_d/\bar{B}_d \to \bar{K}_0^{*0}K^{*0}$ modes can thus be easily obtained, as presented in Table II. Certainly, the similar phenomena will also appear in the related B_s meson decays.

As shown in Eq. (1), only a preliminary upper limit for the $B^0 \to K_0^*(1430)^0\bar{K}^{*0}$ decay is available now. By comparison, one can easily find from Table II that the pQCD predictions in both scenarios are all consistent with this upper limit. The branching ratios in the order of 10^{-6} and above are expected to be tested in the near future B meson experiments.

(5) For the pure annihilation $B_d/\bar{B}_d \to K_0^{*+}K^{*-}$, $B_d/\bar{B}_d \to K^{*+}K_0^{*-}$, and $B_d \to K_0^{*+}K^{*-} + K^{*+}K_0^{*-}$ channels, as listed in Table II, the pQCD predictions for the branching ratios in both scenarios are in order of 10^{-6}. Furthermore, one can find that the numerical pQCD results for the branching ratios in S1 are clearly larger than those in S2, which are rather different from the situation of $B_d \to K_0^{*0}\bar{K}^{*0}$ and $K^{*0}\bar{K}_0^{*0}$ decays. Although the charge conjugation also exists in the channels $B_d \to K^{*+}K_0^{*-}$ and $\bar{B}_d \to K^{*-}K_0^{*+}$, the interference between tree and penguin topologies makes the decay amplitudes for the considered modes different from those for the B_d meson decaying into two neutral final states, which therefore give different branching ratios for $B_d/\bar{B}_d \to K_0^{*+}K^{*-}$ and $B_d/\bar{B}_d \to K^{*+}K_0^{*-}$ as exhibited in Table II.

Additionally, one can define the interesting ratios among the same decay modes but in different scenarios,

$$\frac{\text{Br}(B_d \to K_0^{*+}K^{*-} + K^{*+}K_0^{*-})_{S1}}{\text{Br}(B_d \to K_0^{*+}K^{*-} + K^{*+}K_0^{*-})_{S2}} = 2.5, \quad (29)$$

$$\frac{\text{Br}(B_d/\bar{B}_d \to K_0^{*+}K^{*-})_{S1}}{\text{Br}(B_d/\bar{B}_d \to K_0^{*+}K^{*-})_{S2}} = 1.6,$$
$$\frac{\text{Br}(B_d/\bar{B}_d \to K^{*+}K_0^{*-})_{S1}}{\text{Br}(B_d/\bar{B}_d \to K^{*+}K_0^{*-})_{S3}} = 1.5, \quad (30)$$

where only the central values of the branching ratios are considered for clarification.

Very recently, LHCb [40] and CDF [41] Collaborations have measured the pure annihilation modes of charmless hadronic B meson decays, such as $B_d \to K^+K^-$ and $B_s \to \pi^+\pi^-$, respectively. It is therefore believed that such large decay rates [about $(1 \sim 5) \times 10^{-6}$] for the considered pure annihilation decays in this paper could be tested by the ongoing LHC experiments and/or the forthcoming Super-B factory in the near future. Once the numerical results of the pure annihilation decays can be confirmed by the future measurements at the predicted level, on one hand, which will provide much more evidence to support the successful pQCD

approach in calculating the annihilation diagrams; on the other hand, which will provide more important information on the sizable annihilation contributions in heavy B meson physics and further shed light on the underlying mechanism of the annihilated B meson decays.

(6) For the considered B_s meson decays, all the predicted branching ratios are in the range of $(1-6) \times 10^{-5}$, which can be seen in Table III and will be tested by the LHC experiments. In terms of the channels with the neutral final states, which are the pure penguin induced decays, the branching ratios for the averaged channel $B_s \to K_0^{*0}\bar{K}^{*0} + K^{*0}\bar{K}_0^{*0}$ are equal to each other in two scenarios. The branching ratios for the two summed channels, $B_s/\bar{B}_s \to K_0^{*0}\bar{K}^{*0}$ and $B_s/\bar{B}_s \to K^{*0}\bar{K}_0^{*0}$ in S2, however, are larger than those in S1 with a factor of about 6.

The pQCD predictions for the branching ratios for the averaged channel $B_s \to K_0^{*+}K^{*-} + K^{*+}K_0^{*-}$ are similar in size in both scenarios. Analogous to the decays with the neutral final states, the pQCD results for $B_s/\bar{B}_s \to K_0^{*+}K^{*-}$ and $B_s/\bar{B}_s \to K^{*+}K_0^{*-}$ in S2 are larger than those in S1 with a factor of about 4 and 6, respectively. These results are expected to be examined by the measurements in the future.

(7) For the considered pure penguin decays, $B_{d/s} \to K_0^{*0}\bar{K}^{*0}$ and $K^{*0}\bar{K}_0^{*0}$, we get the ratios in two scenarios between the branching ratios of B_d and B_s decays in the pQCD approach,

$$\frac{\tau_{B_d}}{\tau_{B_s}} \cdot \frac{\text{Br}(B_s \to K_0^{*0}\bar{K}^{*0} + K^{*0}\bar{K}_0^{*0})}{\text{Br}(B_d \to K_0^{*0}\bar{K}^{*0} + K^{*0}\bar{K}_0^{*0})} = 20.6,$$
$$\frac{\tau_{B_d}}{\tau_{B_s}} \cdot \frac{\text{Br}(B_s/\bar{B}_s \to K_0^{*0}\bar{K}^{*0}(K^{*0}\bar{K}_0^{*0}))}{\text{Br}(B_d/\bar{B}_d \to K_0^{*0}\bar{K}^{*0}(K^{*0}\bar{K}_0^{*0}))} = 18.3, \quad (31)$$

in S1, and

$$\frac{\tau_{B_d}}{\tau_{B_s}} \cdot \frac{\text{Br}(B_s \to K_0^{*0}\bar{K}^{*0} + K^{*0}\bar{K}_0^{*0})}{\text{Br}(B_d \to K_0^{*0}\bar{K}^{*0} + K^{*0}\bar{K}_0^{*0})} = 22.4,$$
$$\frac{\tau_{B_d}}{\tau_{B_s}} \cdot \frac{\text{Br}(B_s/\bar{B}_s \to K_0^{*0}\bar{K}^{*0}(K^{*0}\bar{K}_0^{*0}))}{\text{Br}(B_d/\bar{B}_d \to K_0^{*0}\bar{K}^{*0}(K^{*0}\bar{K}_0^{*0}))} = 23.8, \quad (32)$$

in S2, in which the central values of the branching ratios are quoted. From the analytical expressions for the decay amplitudes of these B_d and B_s modes, e.g., Eqs. (23) and (25), one can easily find that the main difference is just from the involved CKM factors λ_t and λ_t' with $|\lambda_t'/\lambda_t|^2 = 22.5$.

(8) Frankly speaking, the measurements at the experimental aspect are not yet available up to now. We therefore cannot make any judgments on whether scenario 1 or scenario 2 of the scalar K_0^* is favored by the considered decays. The pQCD predictions for

the branching ratios of the considered $B \to K_0^* K^*$ decays will be tested by the LHC experiments and/or forthcoming Super-*B* facility.

Here, based on the numerical calculations of the branching ratios, we also examine the effects coming from the annihilation diagrams. In those considered $B \to K_0^* K^*$ decays, when the annihilation contributions are not taken into account, the relevant predictions on the branching ratios in the pQCD approach are as follows:

$$\mathrm{Br}(B_u \to K_0^{*+} \bar{K}^{*0}) = 7.8 \times 10^{-7},$$
$$\mathrm{Br}(B_u \to K^{*+} \bar{K}_0^{*0}) = 6.1 \times 10^{-7}, \tag{33}$$

$$\mathrm{Br}(B_d \to K_0^{*0} \bar{K}^{*0} + K^{*0} \bar{K}_0^{*0}) = 1.7 \times 10^{-8},$$
$$\mathrm{Br}(B_d/\bar{B}_d \to K_0^{*0} \bar{K}^{*0}(K^{*0} \bar{K}_0^{*0})) = 1.3 \times 10^{-6}, \tag{34}$$

$$\mathrm{Br}(B_s \to K_0^{*0} \bar{K}^{*0} + K^{*0} \bar{K}_0^{*0}) = 3.2 \times 10^{-7},$$
$$\mathrm{Br}(B_s/\bar{B}_s \to K_0^{*0} \bar{K}^{*0}(K^{*0} \bar{K}_0^{*0})) = 2.4 \times 10^{-5}, \tag{35}$$

$$\mathrm{Br}(B_s \to K_0^{*+} K^{*-} + K^{*+} K_0^{*-}) = 0.3 \times 10^{-5}, \tag{36}$$

$$\mathrm{Br}(B_s/\bar{B}_s \to K_0^{*+} K^{*-}) = 2.2 \times 10^{-5},$$
$$\mathrm{Br}(B_s/\bar{B}_s \to K^{*+} K_0^{*-}) = 2.1 \times 10^{-5}, \tag{37}$$

in scenario 1, and

$$\mathrm{Br}(B_u \to K_0^{*+} \bar{K}^{*0}) = 1.8 \times 10^{-6},$$
$$\mathrm{Br}(B_u \to K^{*+} \bar{K}_0^{*0}) = 1.5 \times 10^{-6}, \tag{38}$$

$$\mathrm{Br}(B_d \to K_0^{*0} \bar{K}^{*0} + K^{*0} \bar{K}_0^{*0}) = 3.4 \times 10^{-8},$$
$$\mathrm{Br}(B_d/\bar{B}_d \to K_0^{*0} \bar{K}^{*0}(K^{*0} \bar{K}_0^{*0})) = 3.0 \times 10^{-6}, \tag{39}$$

$$\mathrm{Br}(B_s \to K_0^{*0} \bar{K}^{*0} + K^{*0} \bar{K}_0^{*0}) = 3.0 \times 10^{-7},$$
$$\mathrm{Br}(B_s/\bar{B}_s \to K_0^{*0} \bar{K}^{*0}(K^{*0} \bar{K}_0^{*0})) = 5.4 \times 10^{-5}, \tag{40}$$

$$\mathrm{Br}(B_s \to K_0^{*+} K^{*-} + K^{*+} K_0^{*-}) = 0.7 \times 10^{-5}, \tag{41}$$

$$\mathrm{Br}(B_s/\bar{B}_s \to K_0^{*+} K^{*-}) = 5.0 \times 10^{-5},$$
$$\mathrm{Br}(B_s/\bar{B}_s \to K^{*+} K_0^{*-}) = 4.9 \times 10^{-5}, \tag{42}$$

in scenario 2, in which only the central values are considered for estimating the contributions arising from the annihilation diagrams in various decay channels. By comparison, one can easily find the following points:

(1) For the charged B_u decays, the weak annihilation contributions play more important roles in the $B_u \to K_0^{*+} \bar{K}^{*0}$ than that in the $B_u \to K^{*+} \bar{K}_0^{*0}$.

(2) For the pure penguin B_d and B_s decays, apart from the $B_s/\bar{B}_s \to K_0^{*0} \bar{K}^{*0}(K^{*0} \bar{K}_0^{*0})$ in S2, other channels are basically dominated by the weak annihilation contributions, particularly, for the $B_{d/s} \to K_0^{*0} \bar{K}^{*0} + K^{*0} \bar{K}_0^{*0}$ modes.

(3) For other B_s decays, the significant contributions given by the weak annihilation diagrams can also be clearly observed. Of course, the reliability of the contributions from the annihilation diagrams to these considered decays calculated in the pQCD approach will be examined by the relevant experiments in the future.

B. *CP*-violating asymmetries

Now we turn to the evaluations of the *CP*-violating asymmetries of $B \to K_0^* K^*$ decays in the pQCD approach. For the charged B_u meson decays, the direct *CP* violation A_{CP}^{dir} can be defined as

$$A_{CP}^{\mathrm{dir}} = \frac{|\bar{\mathcal{A}}_f|^2 - |\mathcal{A}_f|^2}{|\bar{\mathcal{A}}_f|^2 + |\mathcal{A}_f|^2}, \tag{43}$$

where $\mathcal{A}_f$ stands for the decay amplitude of $B_u \to K_0^{*+} \bar{K}^{*0}$ and $B_u \to K^{*+} \bar{K}_0^{*0}$, respectively, while $\bar{\mathcal{A}}_f$ denotes the charge conjugation one correspondingly. Using Eq. (43), we find the following pQCD predictions (in units of 10^{-2}):

$$A_{CP}^{\mathrm{dir}}(B_u \to K_0^{*+} \bar{K}^{*0}) = \begin{cases} -32.6^{+10.5}_{-11.3}(\omega_b)^{+0.7}_{-0.8}(\bar{f}_{K_0^*})^{+3.1}_{-4.0}(f_{K^*})^{+3.6}_{-4.8}(B_i)^{+5.4}_{-1.7}(a_i)^{+1.1}_{-2.4}(V_i)^{+2.1}_{-3.9}(a_t) & (\text{S1}) \\ -34.9^{+5.0}_{-4.5}(\omega_b)^{+0.5}_{-0.4}(\bar{f}_{K_0^*})^{+1.6}_{-1.5}(f_{K^*})^{+6.9}_{-9.0}(B_i)^{+1.5}_{-1.6}(a_i)^{+1.4}_{-2.2}(V_i)^{+1.6}_{-0.2}(a_t) & (\text{S2}) \end{cases}, \tag{44}$$

$$A_{CP}^{\mathrm{dir}}(B_u \to K^{*+} \bar{K}_0^{*0}) = \begin{cases} 43.6^{+3.7}_{-2.2}(\omega_b)^{+1.1}_{-1.3}(\bar{f}_{K_0^*})^{+2.0}_{-1.7}(f_{K^*})^{+13.0}_{-13.7}(B_i)^{+2.8}_{-2.6}(a_i)^{+1.7}_{-1.6}(V_i)^{+2.0}_{-2.7}(a_t) & (\text{S1}) \\ -67.9^{+4.9}_{-5.2}(\omega_b)^{+0.8}_{-1.0}(\bar{f}_{K_0^*})^{+1.7}_{-1.6}(f_{K^*})^{+16.0}_{-14.9}(B_i)^{+1.2}_{-0.6}(a_i)^{+1.9}_{-3.8}(V_i)^{+3.3}_{-1.3}(a_t) & (\text{S2}) \end{cases}. \tag{45}$$

Note that these two channels exhibit large direct *CP*-violating asymmetries in both scenarios in the pQCD approach, which indicates that the contribution of the penguin diagrams is sizable. Combining the large *CP*-averaged branching ratios [$\mathcal{O}(10^{-6})$] in S2 with the large *CP* violations, which could be clearly detected in *B*

factories and LHC experiments and will provide important information for further understanding the QCD dynamics involved in the considered scalar K_0^*.

As for the *CP*-violating asymmetries for the neutral $B_{d/s} \to K_0^* K^*$ decays, the effects of $B_{d/s} - \bar{B}_{d/s}$ mixing should be considered. First, for $B_{d/s}/\bar{B}_{d/s} \to K_0^{*0} \bar{K}^{*0}$ and

$K^{*0}\bar{K}_0^{*0}$ decays, they will not exhibit *CP* violation in both scenarios 1 and 2, since they involve the pure penguin contributions at the leading order in the SM, which can be seen from the decay amplitudes as given in Eqs. (23) and (25). If the measurements from experiments for the direct *CP* asymmetries $A_{CP}^{\rm dir}$ in $B_{d/s} \to K_0^{*0}\bar{K}^{*0}$ and $K^{*0}\bar{K}_0^{*0}$ decays exhibit obviously nonzero, then this will indicate the existence of new physics beyond the SM and will provide a very promising place to look for this exotic effect.

However, the study of *CP* violation for $B_{d/s} \to K_0^{*+}K^{*-}$ and $K^{*+}K_0^{*-}$ becomes more complicated as $K_0^{*+}\bar{K}^{*-}$ and $K^{*+}\bar{K}_0^{*-}$ are not *CP* eigenstates. The time-dependent *CP* asymmetries for $B_{d/s} \to K_0^{*\pm}K^{*\mp}$ decays are thus given by

$$
\begin{aligned}
a_{CP} &\equiv \frac{\Gamma(\bar{B}_{d/s}(\Delta t) \to K_0^{*\pm}K^{*\mp}) - \Gamma(B_{d/s}(\Delta t) \to K_0^{*\pm}K^{*\mp})}{\Gamma(\bar{B}_{d/s}(\Delta t) \to K_0^{*\pm}K^{*\mp}) + \Gamma(B_{d/s}(\Delta t) \to K_0^{*\pm}K^{*\mp})} \\
&= (A_{CP}^{\rm dir} \pm \Delta A_{CP}^{\rm dir}) \cos(\Delta m_{(d/s)}\Delta t) \\
&\quad + (A_{CP}^{\rm mix} \pm \Delta A_{CP}^{\rm mix}) \sin(\Delta m_{(d/s)}\Delta t),
\end{aligned} \tag{46}
$$

where $\Delta m_{(d/s)}$ is the mass difference between the two neutral $B_{d/s}$ mass eigenstates, and $\Delta t = t_{CP} - t_{\rm tag}$ is the time difference between the tagged $B_{d/s}$ $(\bar{B}_{d/s})$ and the accompanying $\bar{B}_{d/s}$ $(B_{d/s})$ with opposite b flavor decaying to the final *CP* eigenstate $K_0^{*\pm}K^{*\mp}$ at the time t_{CP}. The quantities $A_{CP}^{\rm dir}(\mathcal{C}_f)$ and $A_{CP}^{\rm mix}(\mathcal{S}_f)$ parametrize flavor-dependent direct *CP* violation and mixing-induced *CP* violation, respectively, and the parameters $\Delta A_{CP}^{\rm dir}$ and $\Delta A_{CP}^{\rm mix}$ are related *CP*-conserving quantities: $\Delta A_{CP}^{\rm dir}$ describes the asymmetry between the rates $\Gamma(B_{d/s} \to K_0^{*+}K^{*-}) + \Gamma(\bar{B}_{d/s} \to K_0^{*-}K^{*+})$ and $\Gamma(B_{d/s} \to K_0^{*-}K^{*+}) + \Gamma(\bar{B}_{d/s} \to K_0^{*+}K^{*-})$, while $\Delta A_{CP}^{\rm mix}$ measures the strong phase difference between the amplitudes contributing to $B_{d/s} \to K_0^{*\pm}K^{*\mp}$ decays. Here, we should stress that in the definition of the above equation, i.e., Eq. (46), the effects arising from the width difference of B_s mesons have been neglected for simplicity.

Following Ref. [42], we define the transition amplitudes, for B_d decays, for example, as follows:

$$
\begin{aligned}
\mathcal{A}_{+-} &\equiv \mathcal{A}(B_d \to K_0^{*+}K^{*-}), \\
\mathcal{A}_{-+} &\equiv \mathcal{A}(B_d \to K_0^{*-}K^{*+}), \\
\bar{\mathcal{A}}_{+-} &\equiv \mathcal{A}(\bar{B}_d \to K_0^{*+}K^{*-}), \\
\bar{\mathcal{A}}_{-+} &\equiv \mathcal{A}(\bar{B}_d \to K_0^{*-}K^{*+}),
\end{aligned} \tag{47}
$$

and

$$
\lambda_{+-} = \frac{q_{B_d}}{p_{B_d}}\frac{\bar{\mathcal{A}}_{+-}}{\mathcal{A}_{+-}}, \qquad \lambda_{-+} = \frac{q_{B_d}}{p_{B_d}}\frac{\bar{\mathcal{A}}_{-+}}{\mathcal{A}_{-+}}, \tag{48}
$$

where the vector K^* meson is emitted by the W boson in the case of $\mathcal{A}_{+-}$ and $\bar{\mathcal{A}}_{-+}$, while it contains the spectator quark in the case of $\mathcal{A}_{-+}$ and $\bar{\mathcal{A}}_{+-}$. Then one can get

$$
\begin{aligned}
A_{CP}^{\rm dir} + \Delta A_{CP}^{\rm dir} &\equiv \frac{|\lambda_{+-}|^2 - 1}{|\lambda_{+-}|^2 + 1} = \frac{|\bar{\mathcal{A}}_{+-}|^2 - |\mathcal{A}_{+-}|^2}{|\bar{\mathcal{A}}_{+-}|^2 + |\mathcal{A}_{+-}|^2}, \\
A_{CP}^{\rm dir} - \Delta A_{CP}^{\rm dir} &\equiv \frac{|\lambda_{-+}|^2 - 1}{|\lambda_{-+}|^2 + 1} = \frac{|\bar{\mathcal{A}}_{-+}|^2 - |\mathcal{A}_{-+}|^2}{|\bar{\mathcal{A}}_{-+}|^2 + |\mathcal{A}_{-+}|^2},
\end{aligned} \tag{49}
$$

and

$$
\begin{aligned}
A_{CP}^{\rm mix} + \Delta A_{CP}^{\rm mix} &\equiv \frac{2\,{\rm Im}\lambda_{+-}}{|\lambda_{+-}|^2 + 1} = \frac{2\,{\rm Im}(e^{-2i\beta_d}\bar{\mathcal{A}}_{+-}\mathcal{A}_{+-}^*)}{|\mathcal{A}_{+-}|^2 + |\bar{\mathcal{A}}_{+-}|^2}, \\
A_{CP}^{\rm mix} - \Delta A_{CP}^{\rm mix} &\equiv \frac{2\,{\rm Im}\lambda_{-+}}{|\lambda_{-+}|^2 + 1} = \frac{2\,{\rm Im}(e^{-2i\beta_d}\bar{\mathcal{A}}_{-+}\mathcal{A}_{-+}^*)}{|\mathcal{A}_{-+}|^2 + |\bar{\mathcal{A}}_{-+}|^2}.
\end{aligned} \tag{50}
$$

Owing to the fact that $B_d \to K_0^{*\pm}K^{*\mp}$ is not a *CP* eigenstate, one must also consider the time- and flavor-integrated charge asymmetry,

$$
\mathcal{A}_{K_0^*K^*} \equiv \frac{|\mathcal{A}_{+-}|^2 + |\bar{\mathcal{A}}_{+-}|^2 - |\mathcal{A}_{-+}|^2 - |\bar{\mathcal{A}}_{-+}|^2}{|\mathcal{A}_{+-}|^2 + |\bar{\mathcal{A}}_{+-}|^2 + |\mathcal{A}_{-+}|^2 + |\bar{\mathcal{A}}_{-+}|^2}, \tag{51}
$$

as another source of possible direct *CP*-violating asymmetry. Then, by transforming the experimentally motivated direct *CP* parameters $\mathcal{A}_{K_0^*K^*}$ and $A_{CP}^{\rm dir}$ into the physically motivated choices, one can obtain the direct *CP* asymmetries for $B_d \to K_0^{*+}K^{*-}$ and $K_0^{*-}K^{*+}$ modes as the following:

$$
\begin{aligned}
A_{K_0^{*+}K^{*-}} &\equiv \frac{\Gamma(\bar{B}_d \to K_0^{*-}K^{*+}) - \Gamma(B_d \to K_0^{*+}K^{*-})}{\Gamma(\bar{B}_d \to K_0^{*-}K^{*+}) + \Gamma(B_d \to K_0^{*+}K^{*-})} \\
&= \frac{|\kappa^{+-}|^2 - 1}{|\kappa^{+-}|^2 + 1} = -\frac{\mathcal{A}_{K_0^*K^*} - A_{CP}^{\rm dir} - \mathcal{A}_{K_0^*K^*}\Delta A_{CP}^{\rm dir}}{1 - \Delta A_{CP}^{\rm dir} - \mathcal{A}_{K_0^*K^*}A_{CP}^{\rm dir}},
\end{aligned} \tag{52}
$$

$$
\begin{aligned}
A_{K_0^{*-}K^{*+}} &\equiv \frac{\Gamma(\bar{B}_d \to K_0^{*+}K^{*-}) - \Gamma(B_d \to K_0^{*-}K^{*+})}{\Gamma(\bar{B}_d \to K_0^{*+}K^{*-}) - \Gamma(B_d \to K_0^{*-}K^{*+})} \\
&= \frac{|\kappa^{-+}|^2 - 1}{|\kappa^{-+}|^2 + 1} = \frac{\mathcal{A}_{K_0^*K^*} + A_{CP}^{\rm dir} + \mathcal{A}_{K_0^*K^*}\Delta A_{CP}^{\rm dir}}{1 + \Delta A_{CP}^{\rm dir} + \mathcal{A}_{K_0^*K^*}A_{CP}^{\rm dir}},
\end{aligned} \tag{53}
$$

where

$$
\kappa^{+-} = \frac{q_{B_d}}{p_{B_d}}\frac{\bar{\mathcal{A}}_{-+}}{\mathcal{A}_{+-}}, \qquad \kappa^{-+} = \frac{q_{B_d}}{p_{B_d}}\frac{\bar{\mathcal{A}}_{+-}}{\mathcal{A}_{-+}}. \tag{54}
$$

Note that the difference among Eq. (52) in this paper, Eq. (161) in Ref. [42], and Eq. (5.20) in Ref. [43] is just induced from the minus sign in the definition of direct *CP*-violating asymmetry, i.e., $A_{CP}^{\rm dir}$, in Eq. (46). The *CP*-violating parameters for the $B_s \to K_0^{*+}K^{*-}$ and $K^{*+}K_0^{*-}$ decays can be similarly defined.

Based on the discussions on the *CP* violations in the above sector, we can present the numerical results of the considered channels in both scenarios for the *CP*-violating asymmetries in the pQCD approach as follows:

$$
\mathcal{A}_{K_0^* K^*} =
\begin{cases}
45.4^{+1.3}_{-1.7}(\omega_b)^{+0.1}_{-0.2}(\bar{f}_{K_0^*})^{+0.0}_{-0.1}(f_{K^*}^{(T)})^{+0.7}_{-1.1}(B_i)^{+0.8}_{-1.3}(a_i)^{+1.3}_{-2.2}(V_i)^{+1.7}_{-2.5}(a_t)\% & (\text{S1}) \\
46.1^{+1.1}_{-0.9}(\omega_b)^{+0.4}_{-0.4}(\bar{f}_{K_0^*})^{+0.5}_{-0.2}(f_{K^*}^{(T)})^{+2.6}_{-3.9}(B_i)^{+0.7}_{-1.0}(a_i)^{+1.4}_{-2.1}(V_i)^{+4.2}_{-3.8}(a_t)\% & (\text{S2})
\end{cases}
\tag{55}
$$

$$
A_{CP}^{\text{dir}} =
\begin{cases}
-5.7^{+0.3}_{-0.4}(\omega_b)^{+0.1}_{-0.0}(\bar{f}_{K_0^*})^{+0.7}_{-0.2}(f_{K^*}^{(T)})^{+0.5}_{-0.7}(B_i)^{+2.9}_{-1.4}(a_i)^{+0.3}_{-0.3}(V_i)^{+0.3}_{-0.3}(a_t)\% & (\text{S1}) \\
-16.0^{+0.1}_{-0.1}(\omega_b)^{+0.3}_{-0.3}(\bar{f}_{K_0^*})^{+0.7}_{-0.8}(f_{K^*}^{(T)})^{+4.2}_{-4.9}(B_i)^{+3.4}_{-2.9}(a_i)^{+1.0}_{-0.5}(V_i)^{+1.3}_{-1.8}(a_t)\% & (\text{S2})
\end{cases}
\tag{56}
$$

$$
\Delta A_{CP}^{\text{dir}} =
\begin{cases}
86.4^{+0.8}_{-0.7}(\omega_b)^{+0.1}_{-0.2}(\bar{f}_{K_0^*})^{+0.1}_{-0.1}(f_{K^*}^{(T)})^{+0.2}_{-0.3}(B_i)^{+1.4}_{-1.9}(a_i)^{+1.4}_{-1.1}(V_i)^{+1.7}_{-1.7}(a_t)\% & (\text{S1}) \\
74.0^{+1.1}_{-1.1}(\omega_b)^{+0.5}_{-0.8}(\bar{f}_{K_0^*})^{+0.0}_{-0.1}(f_{K^*}^{(T)})^{+1.9}_{-4.8}(B_i)^{+0.4}_{-1.0}(a_i)^{+0.8}_{-1.2}(V_i)^{+0.0}_{-0.1}(a_t)\% & (\text{S2})
\end{cases}
\tag{57}
$$

$$
A_{CP}^{\text{mix}} =
\begin{cases}
-14.2^{+0.3}_{-1.2}(\omega_b)^{+0.1}_{-0.0}(\bar{f}_{K_0^*})^{+0.2}_{-0.4}(f_{K^*}^{(T)})^{+1.4}_{-1.7}(B_i)^{+0.4}_{-0.4}(a_i)^{+2.1}_{-0.9}(V_i)^{+2.7}_{-4.0}(a_t)\% & (\text{S1}) \\
10.0^{+0.8}_{-0.8}(\omega_b)^{+0.6}_{-0.3}(\bar{f}_{K_0^*})^{+0.9}_{-0.6}(f_{K^*}^{(T)})^{+2.2}_{-4.9}(B_i)^{+2.8}_{-3.0}(a_i)^{+6.2}_{-4.1}(V_i)^{+0.1}_{-1.1}(a_t)\% & (\text{S2})
\end{cases}
\tag{58}
$$

$$
\Delta A_{CP}^{\text{mix}} =
\begin{cases}
-1.8^{+0.7}_{-0.5}(\omega_b)^{+0.2}_{-0.4}(\bar{f}_{K_0^*})^{+0.1}_{-0.0}(f_{K^*}^{(T)})^{+1.6}_{-1.7}(B_i)^{+2.0}_{-1.2}(a_i)^{+2.1}_{-1.6}(V_i)^{+1.0}_{-0.3}(a_t)\% & (\text{S1}) \\
-28.1^{+1.8}_{-2.0}(\omega_b)^{+0.3}_{-0.5}(\bar{f}_{K_0^*})^{+1.5}_{-1.5}(f_{K^*}^{(T)})^{+18.9}_{-7.8}(B_i)^{+8.2}_{-7.7}(a_i)^{+0.8}_{-0.4}(V_i)^{+0.5}_{-0.0}(a_t)\% & (\text{S2})
\end{cases}
\tag{59}
$$

$$
A_{K_0^{*+} K^{*-}} =
\begin{cases}
-73.2^{+1.3}_{-2.5}(\omega_b)^{+0.5}_{-0.0}(\bar{f}_{K_0^*})^{+2.8}_{-1.1}(f_{K^*}^{(T)})^{+1.9}_{-2.7}(B_i)^{+14.6}_{-8.3}(a_i)^{+2.1}_{-3.0}(V_i)^{+2.6}_{-3.2}(a_t)\% & (\text{S1}) \\
-83.9^{+0.7}_{-0.7}(\omega_b)^{+0.6}_{-0.2}(\bar{f}_{K_0^*})^{+1.5}_{-1.4}(f_{K^*}^{(T)})^{+5.7}_{-2.9}(B_i)^{+7.9}_{-5.5}(a_i)^{+3.5}_{-1.6}(V_i)^{+4.0}_{-4.4}(a_t)\% & (\text{S2})
\end{cases}
\tag{60}
$$

$$
A_{K_0^{*-} K^{*+}} =
\begin{cases}
43.0^{+1.4}_{-2.0}(\omega_b)^{+0.1}_{-0.2}(\bar{f}_{K_0^*})^{+0.1}_{-0.4}(f_{K^*}^{(T)})^{+0.8}_{-1.4}(B_i)^{+1.4}_{-1.9}(a_i)^{+1.5}_{-2.5}(V_i)^{+1.8}_{-2.7}(a_t)\% & (\text{S1}) \\
38.5^{+1.1}_{-0.8}(\omega_b)^{+0.6}_{-0.6}(\bar{f}_{K_0^*})^{+0.2}_{-0.0}(f_{K^*}^{(T)})^{+4.6}_{-6.7}(B_i)^{+1.1}_{-1.1}(a_i)^{+1.5}_{-1.9}(V_i)^{+3.7}_{-3.4}(a_t)\% & (\text{S2})
\end{cases}
\tag{61}
$$

for $B_d \to K_0^{*+} K^{*-}$ and $K_0^{*-} K^{*+}$ decays, and

$$
\mathcal{A}_{K_0^* K^*} =
\begin{cases}
0.5^{+5.2}_{-5.0}(\omega_{bs})^{+0.6}_{-0.5}(\bar{f}_{K_0^*})^{+2.5}_{-2.3}(f_{K^*}^{(T)})^{+1.8}_{-2.2}(B_i)^{+8.7}_{-7.5}(a_i)^{+0.0}_{-0.0}(V_i)^{+0.8}_{-2.2}(a_t)\% & (\text{S1}) \\
-20.3^{+2.5}_{-1.7}(\omega_{bs})^{+0.5}_{-0.4}(\bar{f}_{K_0^*})^{+0.1}_{-0.1}(f_{K^*}^{(T)})^{+1.7}_{-0.4}(B_i)^{+3.0}_{-2.0}(a_i)^{+0.6}_{-0.8}(V_i)^{+0.6}_{-0.4}(a_t)\% & (\text{S2})
\end{cases}
\tag{62}
$$

$$
A_{CP}^{\text{dir}} =
\begin{cases}
3.2^{+4.2}_{-7.4}(\omega_{bs})^{+0.5}_{-0.5}(\bar{f}_{K_0^*})^{+2.8}_{-2.2}(f_{K^*}^{(T)})^{+2.9}_{-3.6}(B_i)^{+12.7}_{-14.2}(a_i)^{+0.1}_{-0.1}(V_i)^{+0.6}_{-2.5}(a_t)\% & (\text{S1}) \\
-27.2^{+4.1}_{-2.9}(\omega_{bs})^{+0.2}_{-0.1}(\bar{f}_{K_0^*})^{+1.3}_{-1.1}(f_{K^*}^{(T)})^{+1.8}_{-0.2}(B_i)^{+4.6}_{-2.7}(a_i)^{+0.9}_{-1.1}(V_i)^{+0.5}_{-0.0}(a_t)\% & (\text{S2})
\end{cases}
\tag{63}
$$

$$
\Delta A_{CP}^{\text{dir}} =
\begin{cases}
-11.5^{+21.7}_{-26.8}(\omega_{bs})^{+0.6}_{-0.3}(\bar{f}_{K_0^*})^{+6.1}_{-6.1}(f_{K^*}^{(T)})^{+14.8}_{-13.4}(B_i)^{+18.1}_{-15.7}(a_i)^{+2.4}_{-4.1}(V_i)^{+12.9}_{-13.6}(a_t)\% & (\text{S1}) \\
-45.4^{+5.0}_{-4.0}(\omega_{bs})^{+0.1}_{-0.0}(\bar{f}_{K_0^*})^{+3.8}_{-3.5}(f_{K^*}^{(T)})^{+2.8}_{-5.5}(B_i)^{+4.5}_{-2.2}(a_i)^{+1.4}_{-2.4}(V_i)^{+4.2}_{-3.4}(a_t)\% & (\text{S2})
\end{cases}
\tag{64}
$$

$$
A_{CP}^{\text{mix}} =
\begin{cases}
36.5^{+11.5}_{-17.5}(\omega_{bs})^{+0.5}_{-0.6}(\bar{f}_{K_0^*})^{+3.5}_{-3.6}(f_{K^*}^{(T)})^{+6.9}_{-10.5}(B_i)^{+7.1}_{-9.1}(a_i)^{+1.4}_{-1.2}(V_i)^{+1.6}_{-4.3}(a_t)\% & (\text{S1}) \\
-27.4^{+7.9}_{-5.3}(\omega_{bs})^{+0.2}_{-0.2}(\bar{f}_{K_0^*})^{+0.5}_{-0.3}(f_{K^*}^{(T)})^{+12.6}_{-5.1}(B_i)^{+3.9}_{-3.2}(a_i)^{+1.0}_{-0.8}(V_i)^{+2.9}_{-2.5}(a_t)\% & (\text{S2})
\end{cases}
\tag{65}
$$

$$
\Delta A_{CP}^{\text{mix}} =
\begin{cases}
-62.3^{+16.6}_{-14.8}(\omega_{bs})^{+0.6}_{-0.8}(\bar{f}_{K_0^*})^{+5.2}_{-4.6}(f_{K^*}^{(T)})^{+10.3}_{-9.6}(B_i)^{+9.0}_{-5.4}(a_i)^{+2.0}_{-1.2}(V_i)^{+0.6}_{-0.2}(a_t)\% & (\text{S1}) \\
-35.3^{+2.7}_{-1.2}(\omega_{bs})^{+0.5}_{-0.4}(\bar{f}_{K_0^*})^{+5.6}_{-5.4}(f_{K^*}^{(T)})^{+23.4}_{-14.7}(B_i)^{+8.8}_{-9.2}(a_i)^{+3.1}_{-1.5}(V_i)^{+7.4}_{-5.3}(a_t)\% & (\text{S2})
\end{cases}
\tag{66}
$$

$$A_{K_0^{*+}K^{*-}} = \begin{cases} 2.4^{+0.2}_{-0.9}(\omega_{bs})^{+0.9}_{-1.1}(\bar{f}_{K_0^*})^{+0.3}_{-0.2}(f_{K^*}^{(T)})^{+0.9}_{-1.4}(B_i)^{+5.6}_{-4.5}(a_i)^{+0.1}_{-0.1}(V_i)^{+0.1}_{-0.1}(a_t)\% & \text{(S1)} \\ 1.7^{+0.3}_{-0.4}(\omega_{bs})^{+0.6}_{-0.6}(\bar{f}_{K_0^*})^{+0.3}_{-0.2}(f_{K^*}^{(T)})^{+1.2}_{-1.1}(B_i)^{+0.4}_{-0.4}(a_i)^{+0.1}_{-0.0}(V_i)^{+0.1}_{-0.1}(a_t)\% & \text{(S2)} \end{cases}, \tag{67}$$

$$A_{K_0^{*-}K^{*+}} = \begin{cases} 4.1^{+8.2}_{-15.4}(\omega_{bs})^{+0.1}_{-0.0}(\bar{f}_{K_0^*})^{+6.1}_{-4.8}(f_{K^*}^{(T)})^{+5.9}_{-6.0}(B_i)^{+19.6}_{-25.5}(a_i)^{+0.3}_{-0.1}(V_i)^{+1.0}_{-4.8}(a_t)\% & \text{(S1)} \\ -63.6^{+10.7}_{-8.4}(\omega_{bs})^{+0.1}_{-0.0}(\bar{f}_{K_0^*})^{+4.4}_{-4.5}(f_{K^*}^{(T)})^{+4.0}_{-2.8}(B_i)^{+11.4}_{-6.7}(a_i)^{+1.9}_{-3.2}(V_i)^{+3.2}_{-2.1}(a_t)\% & \text{(S2)} \end{cases}, \tag{68}$$

for $B_s \to K_0^{*+}K^{*-}$ and $K_0^{*-}K^{*+}$ decays.

For the direct CP asymmetries in the pure annihilation $B_d \to K_0^{*+}K^{*-}$ and $B_d \to K^{*+}K_0^{*-}$ decays as defined in Eqs. (52) and (53), for example, one can find from the numerical results shown in Eqs. (60) and (61) that their signs and magnitudes are rather different in these two modes within theoretical errors. In the former mode, the direct CP violation is about -73% in S1 and -84% in S2; while in the latter one, the direct CP asymmetry is 43% in S1 and 39% in S2. It is clear that the magnitudes of direct CP-violating asymmetries predicted in the pQCD approach for these two modes in both scenarios are large, which can be tested at the ongoing LHC and forthcoming Super-B experiments, by combining the large branching ratios $[\sim \mathcal{O}(10^{-6})]$. Furthermore, once the predictions on the physical quantities in the pQCD approach can be confirmed at the predicted level by the precision experimental measurements in the future, which can also provide indirect evidence for the important but controversial issue (See Refs. [25,26] for detail) on the evaluation of annihilation contributions at leading power, one can ask whether it is almost real with a tiny strong phase in a soft collinear effective theory or almost imaginary with a large strong phase in the pQCD approach.

IV. SUMMARY

In this work, we studied the charmless hadronic $B_{u/d/s} \to K_0^* K^*$ decays by employing the pQCD approach based on the framework of the k_T factorization theorem. By regarding the scalar K_0^* as the conventional $q\bar{q}$ meson, then with the help of the light-cone distribution amplitude of K_0^* up to twist-3 in two scenarios, we explored the physical observables such as branching ratios and CP-violating asymmetries of the considered channels. It is worth mentioning that, in this paper, as the first estimates to the physical observables of $B \to K_0^* K^*$ decays, only the perturbatively short distance contributions at leading order are investigated. We do not consider the possible long-distance contributions, such as the rescattering effects, although they should be present, and they may be large and affect the theoretical predictions. It is beyond the scope of this work and expected to be studied in the future.

From the numerical evaluations and phenomenological analysis in the pQCD approach, we found the following results:

(i) The considered $B_u \to K_0^{*+}\bar{K}^{*0}$ and $K^{*+}\bar{K}_0^{*0}$ decays exhibit large branching ratios $[\sim \mathcal{O}(10^{-6})]$ and large direct CP-violating asymmetries in S2, which are clearly measurable in B factories and LHC experiments and are helpful to better understand the QCD behavior of the scalar K_0^* in turn.

(ii) In the considered modes, only the preliminary upper limit on $\mathrm{Br}(B_d \to K_0^{*0}\bar{K}_0^{*0})$ has been reported by the Belle Collaboration. The predicted results agree basically with this upper limit and will be tested by more precision measurements in the future.

(iii) Most of the considered decays are affected significantly by the involved weak annihilation contributions. The predictions on large branching ratios and large direct CP violations of the pure annihilation processes $B_d \to K_0^{*+}K^{*-}$ and $K^{*+}K_0^{*-}$ can be measured in the ongoing LHC experiments and forthcoming Super-B factory, which will provide more evidence to help understand the annihilation contributions in B physics.

(iv) The $B_{d/s} \to K_0^{*0}\bar{K}^{*0}$ and $K^{*0}\bar{K}_0^{*0}$ decays can be viewed as a good platform to test the exotic new physics beyond the SM if the obviously nonzero direct CP violations could be observed.

(v) Generally speaking, the pQCD predictions for the considered decays still suffer from large theoretical errors induced by the uncertainties of the input parameters, e.g., mesonic decay constants and Gegenbauer moments in the universal distribution amplitudes, which are expected to be constrained by more precision data.

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