Magnetic moments of bottom baryons: Effective mass and screened charge

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We calculate the magnetic moments of low-lying heavy-flavor bottom baryons using effective quark mass and shielded quark charge scheme. We obtain the magnetic moments of both $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ baryon states. We compare our predictions with other theoretical approaches.

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I. INTRODUCTION

The study of properties of heavy flavor baryons provides valuable insight into the nonperturbative aspects of QCD. Particularly, investigation of baryons containing bottom (b) quark is considered to be a necessary ingredient for understanding b-hadron phenomenology. In recent years investigations of the heavy baryon properties have become a subject of growing interest due to the experimental observation of many heavy flavor baryons [1]. All spatial-ground-state baryons carrying single charm quark have already been observed, and their masses have also been measured. Many spin-$\frac{1}{2}$ b-baryons Λ_b, Σ_b, Ξ_b, and Ω_b and spin-$\frac{3}{2}$ baryon Σ_b^* have also been discovered [1–5]. Recently, SELEX collaboration announced the doubly heavy spin-$\frac{1}{2}$ baryon state Ξ_{cc}^+ with two charm quarks [3]. Very recently, CMS collaboration at CERN observed the spin-$\frac{3}{2}$ heavy Ξ_b^* baryon state and reported a new measurement of lifetime of Λ_b baryon, i.e., $\tau_{\Lambda_b} = 1.503 \pm 0.052(\text{stat}) \pm 0.031(\text{syst})$ ps [6,7]. The hope for detection of doubly heavy and triply heavy baryons predicted by the quark model at the LHCb detector has been further raised by remarkable improvements in instrumentation and technology. Masses and magnetic moments serve as a rich source of information on the internal structure of hadrons. Experimentally, there exist measurements of the baryon magnetic moments of all the octet $J^P = \frac{1}{2}^+$ baryons (except for the Σ^0) and two of the magnetic moments of $J^P = \frac{3}{2}^+$ baryon decuplet [2,8]. Theoretically, there exist serious discrepancies between the quark model predictions and experimental results. Magnetic moments of heavy baryons have been considered in several theoretical approaches. Extensive literature based on naive quark models, nonrelativistic quark model (NRQM), logarithmic potential approach, bound state approach, relativistic quark model, effective mass scheme, power-law potential model, the skyrmion model, chiral quark model, chiral perturbation theory, QCD spectral sum rules, etc. [9–29] have been employed to analyze masses and magnetic moments of heavy baryons. Due to increasing experimental activity in bottom quark sector the theoretical focus has now been shifted on the b-baryon properties. Most recent theoretical analyses employ NRQM using AL1 potential [30,31], light cone QCD sum rules [32–34], relativistic three quark model [35–37], hypercentral model [38] and MIT bag model [39,40] to calculate the magnetic moments and radiative decays of b-baryons. Earlier, Kumar, Dhir, and Verma [27] used effective quark mass and screened quark charge formalism to predict the magnetic moments of spin-$\frac{1}{2}$ charm baryons, which was later extended to spin-$\frac{3}{2}$ charmed ($C = 1$, 2 and 3) baryons. In the present work, we further extend our analysis to bottom sector to determine the magnetic moments of baryons containing one or more b-quarks in effective quark mass and screened quark charge scheme. We compare our predictions with results from other theoretical approaches.

II. EFFECTIVE QUARK MASS SCHEME

We calculate the effective mass of the quark resulting from its interaction with the spectator quarks by single gluon exchange. Magnetic moment of baryons are obtained by using effective quark masses. The baryon mass is taken to be the sum of the quark masses plus spin-dependent hyperfine interaction [16],

$$M_B = \sum_i m_i^{\mathcal{E}} = \sum_i m_i + \sum_{i<j} b_{ij} \mathbf{s}_i \cdot \mathbf{s}_j, \tag{1}$$

where, $\mathbf{s}_i$ and $\mathbf{s}_j$ are the spin operators of the ith and jth quark, respectively; $m_i^{\mathcal{E}}$ denote the effective mass of the quark inside a baryon and b_{ij} is given by

$$b_{ij} = \frac{16\pi\alpha_s}{9m_i m_j} \langle \Psi_0 | \delta^3(\vec{r}) | \Psi_0 \rangle \tag{2}$$

for baryons $B(qqq)$ where Ψ_0 is the baryon wave function.

There may also be a spin-independent interaction term, the effect of which can be approximated by the renormalization of quark masses. Thus, the mass of the quark inside the baryon $B(123)$ may get modified due to its interaction with other quarks. For quarks 1 and 2 to be identical, we write

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$$m_1^{\mathcal{E}} = m_2^{\mathcal{E}} = m + \alpha b_{12} + \beta b_{13}, \tag{3}$$

$$m_3^{\mathcal{E}} = m_3 + 2\beta b_{13}, \tag{4}$$

where we use $m_1 = m_2 = m$ and $b_{13} = b_{23}$; α and β are the parameters to be determined as follows.

For $J^P = \frac{1}{2}^+$ states,

$$M_B = \sum_i m_i + \sum_{i<j} b_{ij} \mathbf{s}_i \cdot \mathbf{s}_j, \tag{5}$$

simplified to

$$M_{B_{\frac{1}{2}^+}} = 2m + m_3 + \frac{b_{12}}{4} - b_{13}, \tag{6}$$

for

$$\mathbf{s}_1 \cdot \mathbf{s}_2 = \frac{1}{4}, \qquad \mathbf{s}_1 \cdot \mathbf{s}_3 = \mathbf{s}_2 \cdot \mathbf{s}_3 = -\frac{1}{2}, \tag{7}$$

thereby giving,

$$\alpha = \frac{1}{8} \quad \text{and} \quad \beta = -\frac{1}{4}, \tag{8}$$

Equation (1) can be written in generalized form for $J^P = \frac{1}{2}^+$ baryons as

$$M_{B_{\frac{1}{2}^+}} = m_1 + m_2 + m_3 + \frac{b_{12}}{4} - \frac{b_{23}}{2} - \frac{b_{13}}{2}, \tag{9}$$

where 1, 2, 3 represents u, d, s, c, and b quarks. Following the formalism described above, for $J^P = \frac{3}{2}^+$ baryons we get,

$$M_{B_{\frac{3}{2}^+}} = m_1 + m_2 + m_3 + \frac{b_{12}}{4} + \frac{b_{23}}{4} + \frac{b_{13}}{4}, \tag{10}$$

for

$$\alpha = \beta = \frac{1}{8}. \tag{11}$$

The parametrization used here seems to go beyond the leading order in quark mass splitting because the m_i term appears as $1/m_i m_j$ through the hyperfine interaction. However, higher order effects are at least partially absorbed in the nonlinear fitting of the m_i. It has been shown [14,15] that contributions from new nonlinear terms must be small because the fitted masses satisfy the Gell-Mann–Okubo mass formula, which is exact to leading order in the quark mass splitting. Therefore, the effective quark mass defined here is equivalent to first order in baryon mass splitting to the leading order parametrization of the baryon masses in chiral perturbation theory [14].

Values of quark masses and hyperfine interaction terms b_{ij} are obtained from the known isomultiplet masses. We wish to point out that as compared to our previous work the b_{ij}'s are obtained here in a more realistic manner corresponding to strange, charm, and bottom mass scales. N, Δ, Λ, Λ_c, and Λ_b gives,

$$m_u = m_d = 362 \text{ MeV}, \qquad m_s = 539 \text{ MeV},$$
$$m_c = 1710 \text{ MeV}, \qquad m_b = 5043 \text{ MeV} \quad \text{and} \tag{12}$$
$$b_{uu} = b_{ud} = b_{dd} = 196 \text{ MeV}.$$

From Σ and Ω we obtain:

$$b_{us} = b_{ds} = 118 \text{ MeV}, \qquad b_{ss} = 76 \text{ MeV}. \tag{13}$$

In charm sector, Σ_c gives

$$b_{uc} = b_{dc} = 28 \text{ MeV}, \tag{14}$$

which in turn yields

$$b_{sc} = \left(\frac{m_u}{m_s}\right) b_{uc} = 19 \text{ MeV}, \qquad b_{cc} = \left(\frac{m_u}{m_c}\right) b_{uc} = 6 \text{ MeV}. \tag{15}$$

In bottom sector, Σ_b and Σ_b^*, leads to

$$b_{ub} = b_{db} = 7 \text{ MeV}, \tag{16}$$

which gives

$$b_{sb} = \left(\frac{m_u}{m_s}\right) b_{ub} = 5 \text{ MeV}, \qquad b_{cb} = \left(\frac{m_u}{m_c}\right) b_{ub} = 1.5 \text{ MeV},$$
$$b_{bb} = \left(\frac{m_u}{m_b}\right) b_{ub} = 0.5 \text{ MeV}. \tag{17}$$

Assuming matrix element of spatial part of baryonic wave function to be flavor and spin independent, we have now extracted ratios of α_s for different quark mass scales i.e.,

$$\frac{\alpha_s(ss)}{\alpha_s(uu)} = 0.86, \qquad \frac{\alpha_s(cc)}{\alpha_s(ss)} = 0.80, \qquad \frac{\alpha_s(bb)}{\alpha_s(cc)} = 0.75,$$
$$\frac{\alpha_s(us)}{\alpha_s(uu)} = 0.90, \qquad \frac{\alpha_s(uc)}{\alpha_s(uu)} = 0.68, \qquad \frac{\alpha_s(ub)}{\alpha_s(uu)} = 0.51. \tag{18}$$

However, the spatial part of the hadron wave function may show flavor dependence, since the size of the hadron may vary with quark flavors. In the meson sector, leptonic decay width shows flavor dependence of spatial part of the wave function. Similar α_s ratios at other mass scales can be obtained from b_{ij} relations.

Using these values of quark masses and hyperfine interaction terms b_{ij}, we can obtain the effective quark masses for $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ baryons.

III. MAGNETIC MOMENTS OF ($J^P = \frac{1}{2}^+$) BARYONS IN EFFECTIVE MASS SCHEME

In the present scheme, magnetic moments of $J^P = \frac{1}{2}^+$ baryons are obtained by sandwiching the following magnetic moment operator between the appropriate baryon wave functions:

$$\boldsymbol{\mu} = \sum_i \mu_i^{\mathcal{E}} \boldsymbol{\sigma}_i, \tag{19}$$

where

$$\mu_i^{\mathcal{E}} = \frac{e_i}{2m_i^{\mathcal{E}}}, \tag{20}$$

for $i = u$, d, s,c and b; e_i represent the quark charge. Expressions for magnetic moments of $J^P = \frac{1}{2}^+$ bottom baryons are given in Table I. We also obtain magnetic transition moments $B'_{1/2^+}(123) \to B_{1/2^+}(123)$ following the general expression given by

$$\mu^{\mathcal{E}}_{B'_{1/2}\to B_{1/2}} = \mu^{\mathcal{E}}_{B'_{1/2}}(123) - \mu^{\mathcal{E}}_{B_{1/2}}(123) = [\mu^{\mathcal{E}}(2) - \mu^{\mathcal{E}}(1)]/\sqrt{3}. \tag{21}$$

We wish to point out that we use a simplistic approach based on the nonrelativistic magnetic moment. It has been shown by Morpurgo [14] that for the static properties of baryons the nonrelativistic constituent quark model approach is completely equivalent to a parametrization of relativistic field theory of strong interactions in a spin-flavor basis. A similar argument connecting the constituent quark model and HBChPT is floated by Durand *et al.* [15]. Remarkable success of the usual additive quark model is thus a consequence of the relative smallness of the non-additive two- and three-body operators arising from spin-dependent interactions. So, the use of the one-body operator is justified in light of the decoupling of spatial and spins of the ground state baryon wave functions [14,15].

We use (9) to obtain more general expressions for effective masses of the quarks inside the baryon as follows:

(1) For (aab)-type baryons with quarks 1 and 2 being identical,

$$m_1^{\mathcal{E}} = m_2^{\mathcal{E}} = m + \frac{b_{12}}{8} - \frac{b_{13}}{4}, \tag{22}$$

and

$$m_3^{\mathcal{E}} = m_3 - \frac{b_{13}}{2} \quad \text{for } 1 = 2 \neq 3. \tag{23}$$

(2) The baryonic states with three quarks of different flavor (abc) can have antisymmetric $\Lambda_{[12]3}$-type and symmetric $\Sigma_{\{12\}3}$-type flavor configurations under the exchange of quarks 1 and 2.

(a) For (abc) Λ-type baryons,

$$m_1^{\mathcal{E}} = m_1 - \frac{3b_{12}}{8}, \tag{24}$$

$$m_2^{\mathcal{E}} = m_2 - \frac{3b_{21}}{8}, \tag{25}$$

and

$$m_3^{\mathcal{E}} = m_3 \quad \text{for } 1 \neq 2 \neq 3. \tag{26}$$

(b) For (abc) Σ-type baryons,

$$m_1^{\mathcal{E}} = m_1 + \frac{b_{12}}{8} - \frac{b_{13}}{4}, \tag{27}$$

$$m_2^{\mathcal{E}} = m_2 + \frac{b_{12}}{8} - \frac{b_{23}}{4}, \tag{28}$$

and

$$m_3^{\mathcal{E}} = m_3 - \frac{b_{23}}{4} - \frac{b_{13}}{4} \quad \text{for } 1 \neq 2 \neq 3. \tag{29}$$

Using these relations we obtain the effective quark masses for $J^P = \frac{1}{2}^+$ baryons as follows:

(1) For singly heavy baryons,

$$\begin{aligned} &m_u^{\Lambda_b} = m_d^{\Lambda_b} = 288\text{ MeV}, \quad m_b^{\Lambda_b} = 5043\text{ MeV};\\ &m_u^{\Xi_b} = m_d^{\Xi_b} = 318\text{ MeV}, \quad m_s^{\Xi_b} = 495\text{ MeV},\\ &m_b^{\Xi_b} = 5043\text{ MeV}; \quad m_s^{\Omega_b} = 547\text{ MeV},\\ &m_b^{\Omega_b} = 5041\text{ MeV}; \quad m_u^{\Sigma_b} = m_d^{\Sigma_b} = 385\text{ MeV},\\ &m_b^{\Sigma_b} = 5039\text{ MeV}; \quad m_u^{\Xi'_b} = m_u^{\Xi'_b} = 375\text{ MeV},\\ &m_s^{\Xi'_b} = 553\text{ MeV}, \quad m_b^{\Xi'_b} = 5040\text{ MeV}. \end{aligned} \tag{30}$$

TABLE I. Expressions for magnetic moments of $(J^P = \frac{1}{2}^+)$ baryons using effective quark masses (in nuclear magneton).

Particles	Magnetic Moment
Singly heavy	
Σ_b^+	$(-\mu_b + 4\mu_u)/3$
Σ_b^0	$(-\mu_b + 2\mu_d + 2\mu_u)/3$
Σ_b^-	$(-\mu_b + 4\mu_d)/3$
$\Xi_b'^0$	$(-\mu_b + 2\mu_s + 2\mu_u)/3$
$\Xi_b'^-$	$(-\mu_b + 2\mu_d + 2\mu_s)/3$
Λ_b^0	μ_b
Ξ_b^0	μ_b
Ξ_b^-	μ_b
Ω_b^-	$(-\mu_b + 4\mu_s)/3$
Doubly heavy	
$\Xi_{cb}'^+$	μ_u
$\Xi_{cb}'^0$	μ_d
$\Omega_{cb}'^0$	μ_s
Ξ_{cb}^+	$(2\mu_b + 2\mu_c - \mu_u)/3$
Ξ_{cb}^0	$(2\mu_b + 2\mu_c - \mu_d)/3$
Ω_{cb}^0	$(2\mu_b + 2\mu_c - \mu_s)/3$
Ξ_{bb}^0	$(4\mu_b - \mu_u)/3$
Ξ_{bb}^-	$(4\mu_b - \mu_d)/3$
Ω_{bb}^-	$(4\mu_b - \mu_s)/3$
Triply Heavy	
Ω_{ccb}^+	$(-\mu_b + 4\mu_c)/3$
Ω_{cbb}^0	$(4\mu_b - \mu_c)/3$

TABLE II. Masses of ($J^P = \frac{1}{2}^+$) bottom baryons using effective quark masses (in GeV).

Particles	Masses	[30]	[33,34]	[36]	[38][b]	[41]	Expt.[2]
Singly heavy							
Σ_b	5.808	5.808	⋯	5.82	5.858	5.805	5.813[a]
Ξ'_b	5.967	5.946	5.945	5.94	⋯	5.937	⋯
Λ_b	5.620	5.643	⋯	5.624	5.693	5.622	5.619[a]
Ξ_b	5.855	5.850	5.824	5.89	5.922	5.812	5.790
Ω_b	6.135	6.034	⋯	6.04	6.052	6.065	6.071 ± 0.40
Doubly heavy							
Ξ'_{cb}	7.114	6.948	6.97 ± 0.20	6.85	⋯	6.963	
Ω'_{cb}	7.291	7.009	6.80 ± 0.30	6.93	⋯	7.116	
Ξ_{cb}	7.097	6.919	6.72 ± 0.20	6.82	6.887	6.933	
Ω_{cb}	7.281	6.986	6.75 ± 0.30	6.91	6,952	7.088	
Ξ_{bb}	10.440	10.197	9.96 ± 0.90	10.10	10.162	10.202	
Ω_{bb}	10.620	10.260	9.97 ± 0.90	10.18	10.220	10.359	
Triply Heavy							
Ω_{ccb}	8.463	⋯	8.50 ± 0.12	8.00	8.172		
Ω_{cbb}	11.795	⋯	11.72 ± 0.16	11.50	11.447		

[a]used as input
[b]averaged

(2) For doubly heavy baryons,

$$m_u^{\Xi_{cb}} = m_d^{\Xi_{cb}} = 353\text{ MeV}, \qquad m_c^{\Xi_{cb}} = 1703\text{ MeV},$$
$$m_b^{\Xi_{cb}} = 5041\text{ MeV}; \qquad m_s^{\Omega_{cb}} = 533\text{ MeV},$$
$$m_c^{\Omega_{cb}} = 1705\text{ MeV}, \qquad m_b^{\Omega_{cb}} = 5042\text{ MeV};$$
$$m_u^{\Xi'_{cb}} = m_u^{\Xi'_{cb}} = 362\text{ MeV}, \qquad m_c^{\Xi'_{cb}} = 1709\text{ MeV},$$
$$m_b^{\Xi'_{cb}} = 50421\text{ MeV}; \qquad m_s^{\Omega'_{cb}} = 539\text{ MeV},$$
$$m_c^{\Omega'_{cb}} = 1709\text{ MeV}, \qquad m_b^{\Omega'_{cb}} = 5042\text{ MeV};$$
$$m_u^{\Xi_{bb}} = m_d^{\Xi_{bb}} = 358\text{ MeV}, \qquad m_b^{\Xi_{bb}} = 5041\text{ MeV};$$
$$m_s^{\Omega_{bb}} = 537\text{ MeV}, \qquad m_b^{\Omega_{bb}} = 5042\text{ MeV}. \tag{31}$$

(3) For triply heavy baryons,

$$m_c^{\Omega_{ccb}} = 1710\text{ MeV}, \qquad m_b^{\Omega_{ccb}} = 5042\text{ MeV};$$
$$m_c^{\Omega_{cbb}} = 1709\text{ MeV}; \qquad m_b^{\Omega_{cbb}} = 5043\text{ MeV}. \tag{32}$$

Using these effective quark masses, we obtain masses of baryon isomultiplets as shown column 2 of Table II. Further, we calculate the magnetic moments of $J^P = \frac{1}{2}^+$ baryons as given in column 2 of Table III. To determine magnetic $B'_{1/2} \to B_{1/2}$ transition moments, we take the geometric average of effective quark masses of constituent quarks of initial and final state baryons, i.e., $m_i^{B' \to B} = \sqrt{m_i^{B'} m_i^{B}}$. For the sake of comparison we also give the results of the different models namely MIT bag model [39], hypercentral potential [38], relativistic three quark [36], light cone QCD sum rules [32], NRQM using AL1 potential [30], power-law potential [17], etc.

It may be noted that notations of primed and unprimed states for the single heavy (qqQ) baryons Ξ_Q and Ξ'_Q are used as per convention that the physical Ξ_Q state contains a pair of light quarks $[q_1 q_2]$ mostly in a spin $S = 0$ (antisymmetric) state where q_i denotes the light and Q the heavy quarks [2]. The other state in which the light quark pair $[q_1 q_2]$ is mostly in spin triplet $S = 1$ (symmetric) state is denoted as Ξ'_Q. On the other hand, some complications arise in the case of doubly heavy (qQQ) baryons Ξ_{bc}, Ξ'_{bc} and Ω_{bc}, Ω'_{bc}. These states are identified by the set of quantum numbers (J^P, S_d) where S_d is the spin of a heavy diquark. The spins of the two heavy quarks are coupled to form ($S_d = 0$) antisymmetric spin configuration of diquark $[Q_1 Q_2]$ and ($S_d = 1$) symmetric spin configuration of diquark $\{Q_1 Q_2\}$. In literature [30,32–34,37,38,40,41], the standard convention is to denote the symmetric heavy diquark state as unprimed $|B\rangle$ state and antisymmetric one as $|B'\rangle$. In addition, the wave function mixing between $|B\rangle$ and $|B'\rangle$ states have also been considered in [17,37,39], which we have ignored in the present analysis.

IV. MAGNETIC MOMENTS OF ($J^P = \frac{3}{2}^+$) BARYONS IN EFFECTIVE MASS SCHEME

Proceeding in a way similar to $J^P = \frac{1}{2}^+$, the magnetic moments of baryons are obtained by sandwiching the magnetic moment operator (14), i.e., $\boldsymbol{\mu} = \sum_i \mu_i^{\mathcal{E}} \boldsymbol{\sigma}_i$, between the appropriate baryon wave functions, where for $i = u, d, s, c$, and b. Expressions for magnetic moments of bottom baryons are given in Table IV.

In order to calculate the magnetic moments, we determine the effective quark masses of ($J^P = \frac{3}{2}^+$) baryons from the following relations derived from (10):

TABLE III. Magnetic moments of ($J^P = \frac{1}{2}^+$) bottom baryons using effective quark masses (in nuclear magneton).

	This Work							
Baryons	Effective Quark Mass	Screened Quark Charge	[39]	[12,39]	[38]	[36]	[30]	[17]
Singly heavy								
Σ_b^+	2.190	2.177	1.622	2.50	2.229	2.07	⋯	2.575
Σ_b^0	0.563	0.533	0.422	0.64	0.591	0.53	⋯	0.659
Σ_b^-	−1.064	−1.110	−0.778	−1.22	−1.047	−1.01	⋯	−1.256
$\Xi_b^{\prime 0}$	0.756	0.676	0.556	0.90	0.766	0.66	⋯	0.930
$\Xi_b^{\prime -}$	−0.913	−0.996	−0.660	−1.02	−0.902	−0.91	⋯	−0.985
Ω_b^-	−0.741	−0.863	−0.545	−0.79	−0.960	−0.82	⋯	−0.714
Λ_b^0	−0.062	−0.060	−0.066	−0.06	−0.064	−0.06	⋯	⋯
Ξ_b^0	−0.062	−0.060	−0.100	−0.110	⋯	−0.06	⋯	⋯
Ξ_b^-	−0.062	−0.066	−0.063	−0.050	⋯	−0.06	⋯	⋯
Doubly heavy								
$\Xi_{cb}^{\prime +}$	1.729	1.718	1.093	1.71	⋯	1.520	1.990	1.525
$\Xi_{cb}^{\prime 0}$	−0.864	−0.817	−0.236	−0.53	⋯	−0.76	−0.993	−0.390
$\Omega_{cb}^{\prime 0}$	−0.580	−0.621	−0.106	−0.27	⋯	−0.61	−0.542	−0.119
Ξ_{cb}^+	−0.387	−0.369	−0.157	−0.25	−0.400	−0.12	−0.475	⋯
Ξ_{cb}^0	0.499	0.480	−0.068	−0.13	0.477	0.42	0.518	⋯
Ω_{cb}^0	0.399	0.407	0.034	0.08	0.397	0.45	0.368	⋯
Ξ_{bb}^0	−0.665	−0.630	−0.432	−0.70	−0.657	−0.53	−0.742	−0.722
Ξ_{bb}^-	0.208	0.215	0.086	0.23	0.190	0.18	0.251	0.236
Ω_{bb}^-	0.111	0.138	0.043	0.12	0.109	0.04	0.101	0.100
Triply heavy								
Ω_{ccb}^+	0.508	0.522	0.505	0.54	⋯	0.53	⋯	0.476
Ω_{cbb}^0	−0.205	−0.200	−0.205	−0.21	⋯	−0.20	⋯	−0.197
$(\frac{1}{2}^{\prime +} \to \frac{1}{2}^+)$ transition moments								
$\lvert\Sigma_b^0 \to \Lambda_b^0\rvert$	1.627	1.535	1.052	1.61	⋯	⋯	⋯	⋯
$\lvert\Xi_b^{\prime 0} \to \Xi_b^0\rvert$	1.392	1.354	0.917	1.41	⋯	⋯	⋯	⋯
$\lvert\Xi_b^{\prime -} \to \Xi_b^-\rvert$	0.178	0.142	0.082	0.16	⋯	⋯	⋯	⋯
$\lvert\Xi_{bc}^{\prime +} \to \Xi_{bc}^+\rvert$	0.247	0.250	0.277	0.62	⋯	⋯	⋯	⋯
$\lvert\Xi_{bc}^{\prime 0} \to \Xi_{bc}^0\rvert$	0.247	0.242	0.508	0.70	⋯	⋯	⋯	⋯
$\lvert\Omega_{bc}^{\prime 0} \to \Omega_{bc}^0\rvert$	0.247	0.243	0.443	0.56	⋯	⋯	⋯	⋯

(1) For (aab)-type baryons,

$$m_1^{\mathcal{E}} = m_2^{\mathcal{E}} = m + \frac{b_{12}}{8} + \frac{b_{13}}{8}, \tag{33}$$

and

$$m_3^{\mathcal{E}} = m_3 + \frac{b_{13}}{4} \quad \text{for } 1 = 2 \neq 3. \tag{34}$$

(2) For (abc)-type baryons,

$$m_1^{\mathcal{E}} = m_1 + \frac{b_{12}}{8} + \frac{b_{13}}{8}, \tag{35}$$

$$m_2^{\mathcal{E}} = m_2 + \frac{b_{23}}{8} + \frac{b_{12}}{8}, \tag{36}$$

and

$$m_3^{\mathcal{E}} = m_3 + \frac{b_{13}}{8} + \frac{b_{23}}{8} \quad \text{for } 1 \neq 2 \neq 3. \tag{37}$$

(3) For (aaa)-type baryons,

$$m_1^{\mathcal{E}} = m_2^{\mathcal{E}} = m_3^{\mathcal{E}} = m + \frac{b_{12}}{4}, \tag{38}$$

and

$$b_{12} = b_{23} = b_{13} \quad \text{for } 1 = 2 = 3. \tag{39}$$

Values of quark masses and hyperfine interaction terms b_{ij} are taken from (12) and (13), which in turn yield the following effective quark masses for $J^P = \frac{3}{2}^+$ baryons:

(1) For singly heavy baryons,

$$m_u^{\Sigma_b^*} = m_d^{\Sigma_b^*} = 387\ \text{MeV}, \qquad m_b^{\Sigma_b^*} = 5046\ \text{MeV};$$
$$m_u^{\Xi_b^*} = m_d^{\Xi_b^*} = 377\ \text{MeV}, \qquad m_s^{\Xi_b^*} = 555\ \text{MeV};$$
$$m_b^{\Xi_b^*} = 5044\ \text{MeV}; \qquad m_s^{\Omega_b^*} = 549\ \text{MeV},$$
$$m_b^{\Omega_b^*} = 5044\ \text{MeV}. \tag{40}$$

TABLE IV. Expressions for magnetic moments ($J^P = \frac{3}{2}^+$) bottom baryons using effective quark masses (in nuclear magneton).

Particles	Magnetic Moment
Singly heavy	
Σ_b^{*+}	$(\mu_b + 2\mu_u)$
Σ_b^{*0}	$(\mu_b + \mu_d + \mu_u)$
Σ_b^{*-}	$(\mu_b + 2\mu_d)$
Ξ_b^{*0}	$(\mu_b + \mu_s + \mu_u)$
Ξ_b^{*-}	$(\mu_b + \mu_d + \mu_s)$
Ω_b^{*-}	$(\mu_b + 2\mu_s)$
Doubly heavy	
Ξ_{cb}^{*+}	$(\mu_b + \mu_c + \mu_u)$
Ξ_{cb}^{*0}	$(\mu_b + \mu_c + \mu_d)$
Ω_{cb}^{*0}	$(\mu_b + \mu_c + \mu_s)$
Ξ_{bb}^{*0}	$(2\mu_b + \mu_u)$
Ξ_{bb}^{*-}	$(2\mu_b + \mu_d)$
Ω_{bb}^{*-}	$(2\mu_b + \mu_s)$
Triply heavy	
Ω_{ccb}^{*+}	$(\mu_b + 2\mu_c)$
Ω_{cbb}^{*0}	$(2\mu_b + \mu_c)$
Ω_{bbb}^{*-}	$(3\mu_b)$

(2) For doubly heavy baryons,

$$m_u^{\Xi_{cb}^*} = 366\text{ MeV}, \qquad m_c^{\Xi_{cb}^*} = 1714\text{ MeV},$$
$$m_b^{\Xi_{cb}^*} = 5044\text{ MeV}; \qquad m_s^{\Omega_{cb}^*} = 542\text{ MeV},$$
$$m_c^{\Omega_{cb}^*} = 1713\text{ MeV}, \qquad m_b^{\Omega_{cb}^*} = 5044\text{ MeV};$$
$$m_u^{\Xi_{bb}^*} = m_d^{\Xi_{bb}^*} = 364\text{ MeV}, \qquad m_b^{\Xi_{bb}^*} = 5044\text{ MeV};$$
$$m_s^{\Omega_{bb}^*} = 540\text{ MeV}, \qquad m_b^{\Omega_{bb}^*} = 5044\text{ MeV}; \qquad (41)$$

(3) For triply heavy baryons,

$$m_c^{\Omega_{ccb}^*} = 1711\text{ MeV}, \qquad m_b^{\Omega_{ccb}^*} = 5043\text{ MeV};$$
$$m_c^{\Omega_{cbb}^*} = 1710\text{ MeV}; \qquad m_b^{\Omega_{cbb}^*} = 5043\text{ MeV};$$
$$m_b^{\Omega_{bbb}^*} = 5043\text{ MeV}. \qquad (42)$$

We sum these effective quark masses to obtain masses of baryon isomultiplets as shown in column 2 of Table V. These masses are also compared with results of various approaches. We calculate the magnetic moments of $J^P = \frac{3}{2}^+$ baryons as given in column 2 of Table VI. We compare our results with different works based on the bag model [39], NRQM [12,39] hypercentral potential model [38], light cone QCD sum rules [34], and NRQM with AL1 potential model [30]. The numerical results are discussed Sec. VI.

TABLE V. Masses of ($J^P = \frac{3}{2}^+$) bottom baryons using effective quark masses (in GeV).

Particles	Masses	[30]	[34]	[38][b]	[41]	Expt.[2]
Singly heavy						
Σ_b^*	5.820	5.882	5.83 ± 0.35	5.878	5.834	5.833[a]
Ξ_b^*	5.976	5.975	5.97 ± 0.40	5.985	5.963	5.9455
Ω_b^*	6.142	6.063	6.08 ± 0.40	6.116	6.088	...
Doubly heavy						
Ξ_{cb}^*	7.124	6.986	7.25 ± 0.20	6.921	6.980	...
Ω_{cb}^*	7.298	7.130	7.30 ± 0.20	6.997	7.130	...
Ξ_{bb}^*	10.451	10.236	10.40 ± 0.10	10.219	10.237	...
Ω_{bb}^*	10.628	10.297	10.50 ± 0.20	10.298	10.389	...
Triply heavy						
Ω_{ccb}^*	8.465	...	...	8.181	...	...
Ω_{cbb}^*	11.797	...	...	11.488	...	...
Ω_{bbb}^*	15.129	...	...	14.566	...	...

[a]used as input
[b]averaged

V. MAGNETIC MOMENTS WITH EFFECTIVE MASS AND SHIELDED QUARK CHARGE

Similar to the variation of the quark mass resulting from its environment, the charge of a quark inside a baryon may also be affected. For example, when a quark inside a baryon is probed by a soft photon, its charge may be screened due to the presence of the neighboring quarks [16]. This effect is in some sense similar to the shielding of the nuclear charge of the helium atom due to the surrounding electron cloud. We take the effective charge to be linearly dependent on the charge of the shielding quarks. Thus the effective charge of quark a in the baryon $B(a, b, c)$ is taken as [16]:

$$e_a^B = e_a + \alpha_{ab} e_b + \alpha_{ac} e_c, \qquad (43)$$

where e_a is the bare charge of quark a. Taking $\alpha_{ab} = \alpha_{ba}$ and invoking the isospin symmetry, we obtain the following constraints:

$$\alpha_{uu} = \alpha_{ud} = \alpha_{dd} = \beta, \qquad \alpha_{us} = \alpha_{ds} = \alpha, \qquad \alpha_{ss} = \gamma;$$

in charm sector:

$$\alpha_{uc} = \alpha_{dc} = \beta', \qquad \alpha_{sc} = \delta, \qquad \alpha_{cc} = \gamma';$$

in bottom sector:

$$\alpha_{ub} = \alpha_{db} = \beta'', \qquad \alpha_{sb} = \delta', \qquad \alpha_{cb} = \gamma'', \qquad \alpha_{bb} = \zeta.$$

Using the SU(3) we get,

$$\alpha = \beta = \gamma; \qquad (44)$$

$$\beta' = \delta, \quad \text{and} \quad \beta'' = \delta'. \qquad (45)$$

We can further reduce these parameters to

$$\gamma = \gamma' = \delta \quad \text{and} \quad \gamma'' = \delta'; \qquad (46)$$

$$\delta = \delta' = \zeta \tag{47}$$

using SU(4) and SU(5) flavor symmetry which are badly broken. Redefining the magnetic moment operator,

$$\boldsymbol{\mu} = \sum_i \frac{e_i^B}{2m_i^{\mathcal{E}}} \boldsymbol{\sigma}_i, \tag{48}$$

we determine the baryon magnetic moments using the p, n, and Λ moments as input, and fix the quark masses for numerical calculations: $m_u = m_d = 370$ MeV, $m_s = 494$ MeV, and $\alpha = 0.033$. Here we keep $m_c = 1680$ MeV and $m_b = 5.043$ GeV. The obtained numerical values are given in column 3 of Tables III and IV, and are correspondingly compared with various approaches.

VI. NUMERICAL RESULTS AND DISCUSSIONS

In this paper, we have used effective quark mass and screened quark charge scheme to predict the magnetic moments of all the $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ baryons up to $b = 3$. We have used isomultiplet masses N, Δ, Λ, Λ_c, Λ_b, etc. as input to obtain effective quark masses inside a baryon for both spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ baryons. Using these effective quark masses we then predict the magnetic moments of the bottom baryons. Later, we also include the effect of screened quark charge to calculate the magnetic moments. The summary of results is presented as follows.

A. Magnetic moments of $J^P = \frac{1}{2}^+$ baryons

Presently, no experimental values of magnetic moments are available for the heavy baryon (charm and bottom) sector. One may expect them to be measured experimentally in the near future, as many interesting experimental results have been put forward recently [1–7]. Theoretically, bottom baryon magnetic moments have been calculated using various approaches listed in columns 4–9 of Table III. We wish to point out that in order to compare results of various models, care must be taken of the notation of primed and unprimed states of singly and doubly heavy baryons. We observe the following:

(1) Our results for bottom baryons with one heavy quark are consistent with the predictions of the hypercentral model [38] and relativistic three quark model [36]. However, the results obtained in NRQM [12] and quark model based on power-law potential [17] are roughly 10%–15% larger than our predictions with few exceptions.

(2) Comparison with the improved bag model [39] reveals that numerical values calculated in this approach are in general smaller than all other approaches. The reason being that for heavy baryons the bag radii and center-of-mass motion corrections are smaller which in turn decrease the magnetic moment values. Following NRQM [12], they have also considered the effects of mixing in Ξ_b^0 and $\Xi_b'^0$ states resulting in different values in these cases.

(3) In light cone QCD sum rules [32,33], available predictions, i.e., $\mu_{\Xi_b^-} = -(0.08 \pm 0.02)$ n.m., $\mu_{\Xi_b^0} = -(0.045 \pm 0.005)$ n.m. are consistent with our results, however, it predicts a larger magnetic moment value for $\mu_{\Lambda_b^0} = -(0.18 \pm 0.05)$ n.m Comparing our results with a recent calculation based on hypercentral approach by [42], we find that their results are on the larger side even when compared to a similar approach [38].

(4) In the doubly heavy baryon sector, numerical values of magnetic moments of Ξ_{bb}^0, Ξ_{bb}^-, Ω_{bb}^-, and Λ_b^0 are consistent with all the other approaches except for the bag model [39] and relativistic three quark model [36] predictions which are smaller than our results.

(5) Comparison of magnetic moments of doubly heavy Ξ_{cb}^+, $\Xi_{cb}'^+$, Ξ_{cb}^0, $\Xi_{cb}'^0$, and Ω_{cb}^0, $\Omega_{cb}'^0$ states in different approaches show disagreements. This may be attributed to the choice of wave functions in different models. Our results are in nice agreement with the NRQM with AL1 potential [30] and hypercentral approach [38], however, they are marginally higher than predictions of the relativistic three quark model [36]. Also, it has been argued that mixing induced by color-hyperfine splitting may affect the magnetic moment values which has been included in improved bag model [39] predictions.

(6) Our results involving singly heavy magnetic transition $B'_{1/2} \to B_{1/2}$ moments are in good agreement with the NRQM approach [12]. In fact, analysis based on light cone QCD sum rules [32] predicts a similar value for transition magnetic moment $\mu_{\Sigma_b\Lambda_b} = (1.6 \pm 0.4)$ n.m. However, for doubly heavy baryon state transition magnetic moments our results are small in comparison to other approaches.

(7) Considering the fact that magnetic moments of doubly and triply heavy baryons are virtually governed by magnetic moments of heavy quarks, all the approaches give almost similar values of magnetic moments of triply heavy (Ω_{ccb}^+ and Ω_{cbb}^0) baryons.

B. Magnetic moments of $J^P = \frac{3}{2}^+$ baryons

Likewise spin-$\frac{1}{2}$ baryon sector, in the absence of any experimental information, several theoretical approaches have been used to estimate the relatively simpler case of $J^P = \frac{3}{2}^+$ b-baryon magnetic moments as listed in columns 4–8 of Table VI. We observe the following:

(1) For the case of singly heavy bottom baryons, our results are consistent with the predictions of the hypercentral model [38], though smaller than estimates given by NRQM [12].

TABLE VI. Magnetic moments of ($J^P = \frac{3}{2}^+$) bottom baryons using effective quark masses (in nuclear magneton).

	This Work						
Baryons	Effective Quark Mass	Screened Quark Charge	[39]	[12,39]	[38]	[34]	[30]
Singly heavy							
Σ_b^{*+}	3.167	3.162	2.346	3.56	3.234	2.52 ± 0.50	⋯
Σ_b^{*0}	0.746	0.705	0.537	0.87	0.791	0.50 ± 0.15	⋯
Σ_b^{*-}	−1.677	−1.752	−1.271	−1.92	−1.657	-1.50 ± 0.36	⋯
Ξ_b^{*0}	1.031	0.915	0.690	1.19	0.042	0.50 ± 0.15	⋯
Ξ_b^{*-}	−1.454	−1.585	−1.088	−1.60	−1.098	-1.42 ± 0.35	⋯
Ω_b^{*-}	−1.201	−1.389	−0.919	−1.28	−1.201	-1.40 ± 0.35	⋯
Doubly heavy							
Ξ_{cb}^{*+}	2.011	2.022	1.414	2.19	2.052	⋯	2.270
Ξ_{cb}^{*0}	−0.551	−0.508	−0.257	−0.60	−0.568	⋯	−0.712
Ω_{cb}^{*0}	−0.274	−0.309	−0.111	−0.28	−0.317	⋯	−0.261
Ξ_{bb}^{*0}	1.596	1.507	0.916	1.74	1.577	⋯	1.870
Ξ_{bb}^{*-}	−0.984	−1.029	−0.652	−1.05	−0.952	⋯	−1.110
Ω_{bb}^{*-}	−0.703	−0.805	−0.522	−0.73	−0.711	⋯	−0.662
Triply heavy							
Ω_{ccb}^{*+}	0.670	0.703	0.659	0.72	0.651	⋯	⋯
Ω_{cbb}^{*0}	0.242	0.225	0.225	0.27	0.216	⋯	⋯
Ω_{bbb}^{*-}	−0.186	−0.198	−0.194	−0.18	−0.195	⋯	−0.180

(2) As observed in the $J^P = \frac{1}{2}^+$ case, numerical results obtained by the improved bag model [39] are smaller than all approaches.

(3) The magnetic moments calculated in light cone QCD sum rules [34] are in nice agreement with our predictions, except for the Ξ_b^{*0} magnetic moment value which is smaller than our prediction.

(4) In the doubly heavy baryon sector, as expected, our results are consistent with the hypercentral potential model [38], but larger than bag model [39] predictions. The numerical values of Ξ_{cb}^{*+}, Ξ_{cb}^{*0}, and Ξ_{bb}^{*0} magnetic moments in this work are smaller in comparison to the NRQM [12] and NRQM with AL1 potential [30] predictions, while the rest of the predictions seem consistent with these approaches.

(5) Here also, magnetic moments of triply heavy baryons namely Ω_{ccb}^{*+}, Ω_{cbb}^{0}, and Ω_{bbb}^{*-} acquire roughly similar values in all theoretical works.

We hope these results will motivate experimental and theoretical analyses in this direction in the near future.

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