PHYSICAL REVIEW D **88,** 091501(R) (2013)

Two-loop Wess-Zumino model with exact supersymmetry on the lattice

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(Received 31 May 2013; published 13 November 2013)

We consider a lattice formulation of the four-dimensional $N = 1$ Wess-Zumino model in terms of the Ginsparg-Wilson relation. This formulation has an exact supersymmetry on the lattice. The lattice action is invariant under a deformed supersymmetric transformation, which is nonlinear in the scalar fields, and it is determined by an iterative procedure in the coupling constant to all orders in perturbation theory. We also show that the corresponding Ward-Takahashi identity is satisfied at fixed lattice spacing. The calculation is performed in lattice perturbation theory up to order g^3 (two loops), and the Ward-Takahashi identity (containing 110 connected nontadpole Feynman diagrams) is satisfied at fixed lattice spacing thanks to this exact lattice supersymmetry.

DOI: 10.1103/PhysRevD.88.091501 PACS numbers: 11.15.Ha, 12.38.Bx, 12.60.Jv

I. INTRODUCTION

Nonperturbative dynamics play an important role in the theory of supersymmetry breaking needed in order to produce a low-energy four-dimensional effective action with a residual $N = 1$ supersymmetry. For this reason, much effort has been dedicated to formulating a lattice version of supersymmetric theories (see Refs. [1–6]).

The major obstacle in formulating a supersymmetric theory on the lattice is that the supersymmetry is a part of the super Poincaré group, which is explicitly broken by the lattice. Ordinary Poincaré invariance is also broken by the lattice, but the operators that violate Poincaré symmetry are all irrelevant (i.e., go to zero in the continuum limit, $a \to 0$). In the case of a supersymmetric theory, these operators are relevant, and a fine-tuning is needed in order to eliminate their contribution. This is the case of the Wilson fermion approach, for the $N = 1$ super Yang—Mills theory, in which the only operator that violates the $N = 1$ supersymmetry is a fermion mass term [7]. By tuning the fermion mass to the supersymmetric limit, one recovers supersymmetry in the continuum limit (see, for example, Refs. [8–11]). Alternatively, using domain-wall fermions [12,13] this fine-tuning is not required. See also Refs. [14,15]. This is in contrast with lower-dimensional models (with extended supersymmetry) in which the lattice symmetries can eliminate the need for such a fine-tuning. Basically, the strategy here is to realize part of the supercharges as an exact symmetry on the lattice. This exact supersymmetry is expected to play a key role to restore continuum supersymmetry without (or with less) fine-tuning [16–22].

In this paper, we consider the $N = 1$ four-dimensional lattice Wess-Zumino model introduced in Refs. [23–25] and studied in Refs. [26,27] (for a numerical approach, see Refs. [28–30]). Although it is a toy model, the main difficulties of lattice supersymmetry are already present (we do not consider gauge fields here). A necessary condition to have exact lattice supersymmetry is that the associated Ward-Takahashi identity (WTi) has to be exactly satisfied on the lattice. This exact symmetry is responsible for the restoration of supersymmetry in the continuum limit without the fine-tuning of the parameters of the action.

Here, we extend the formulation introduced in Ref. [26] and show that it is possible to define a deformed lattice supersymmetric transformation, which leaves the full action invariant at fixed lattice spacing, to all orders in perturbation theory. This transformation is nonlinear in the scalar fields. The action and the transformation are written in terms of the Ginsparg-Wilson operator and reduce to their continuum expression in the naive continuum limit [26]. Since in the presence of any exact symmetry all the WTi are fulfilled, we did check that the simplest nontrivial one, i.e., the one-point WTi, is exactly satisfied on the lattice for both one-(order g) and two-(order g^3) loops. Although in a one-point WTi calculation, the order g^3 is a nontrivial zero, it shows cancellations between fermion and scalar field contributions as required by the supersymmetry. This result extends to two-loop order the results already obtained in Ref. [27] for a different WTi, i.e., the one-loop (two-points) WTi (order g^2). In this case, the exact lattice supersymmetry determines the finite part of the scalar and fermion renormalization wave function, which coincide in the continuum limit, and leads to the restoration of the continuum supersymmetry.

In the following, we briefly review the $N = 1$ four-dimensional lattice Wess-Zumino model based on the Ginsparg-Wilson fermion operator and show how to build up a lattice supersymmetry transformation, which is an exact symmetry of the lattice action, to all orders in perturbation theory. In the remaining part, we derive the WTi and explicitly check that the one-point WTi up to two loops is exactly satisfied at fixed lattice spacing.

II. THE LATTICE WESS-ZUMINO MODEL

We formulate the lattice Wess-Zumino model in four dimensions introducing a Dirac operator D that satisfies the Ginsparg-Wilson relation [31]:

$$\gamma_5 D + D\gamma_5 = aD\gamma_5 D. \tag{1}$$

This relation implies a continuum symmetry of the fermion action, which may be regarded as a lattice form of the chiral symmetry [32,33] and protects the fermion masses from additive renormalization. Although our analysis is valid for all Ds that satisfy Eq. (1), we use the solution given by Neuberger [34,35],

$$D = \frac{1}{a}\left(1 - X\frac{1}{\sqrt{X^\dagger X}}\right), \qquad X = 1 - aD_w, \tag{2}$$

$$D_w = \frac{1}{2}\gamma_\mu(\nabla^\star_\mu + \nabla_\mu) - \frac{a}{2}\nabla^\star_\mu\nabla_\mu, \tag{3}$$

where $\nabla_\mu\phi(x)$ and $\nabla^\star_\mu\phi(x)$, are the forward and backward lattice derivatives, respectively. Plugging Eq. (3) in Eq. (2), we find it convenient to isolate in D the part containing the gamma matrices [26], and write D as $D = D_1 + D_2$, where $D_1 = \frac{1}{4}\mathrm{Tr}(D)$ and $D_2 = \frac{1}{4}\gamma_\mu \mathrm{Tr}(\gamma_\mu D)$. More explicitly, we have

$$D_1 = \frac{1}{a}\left[1 - \left(1 + \frac{a^2}{2}\nabla^\star_\mu\nabla_\mu\right)\frac{1}{\sqrt{X^\dagger X}}\right], \tag{4}$$

$$D_2 = \frac{1}{2}\gamma_\mu(\nabla^\star_\mu + \nabla_\mu)\frac{1}{\sqrt{X^\dagger X}} \equiv \gamma_\mu D_{2\mu}. \tag{5}$$

In terms of D_1 and D_2, the Ginsparg-Wilson relation (1) becomes [26]

$$D_1^2 - D_2^2 = \frac{2}{a}D_1, \text{ and } \left(1 - \frac{a}{2}D_1\right)^{-1}D_2^2 = -\frac{2}{a}D_1. \tag{6}$$

The action of the four-dimensional Wess-Zumino model on the lattice was introduced in Refs. [23–25]. In our notation,

$$\begin{aligned} S_{\mathrm{WZ}} = \sum_x \Big\{ &\frac{1}{2}\bar\chi\left(1 - \frac{a}{2}D_1\right)^{-1}D_2\chi - \frac{2}{a}\phi^\dagger D_1\phi \\ &+ F^\dagger\left(1 - \frac{a}{2}D_1\right)^{-1}F + \frac{1}{2}m\bar\chi\chi \\ &+ m(F\phi + (F\phi)^\dagger) + g\bar\chi(P_+\phi P_+ + P_-\phi^\dagger P_-)\chi \\ &+ g(F\phi^2 + (F\phi^2)^\dagger)\Big\}, \end{aligned}$$

where ϕ and F are scalar fields and χ is a Majorana fermion that satisfies the Majorana condition, $\bar\chi = \chi^T C$, and C is the charge conjugation matrix that satisfies $C^T = -C$ and $CC^\dagger = 1$. Moreover, $C\gamma_\mu C^{-1} = -(\gamma_\mu)^T$, and $C\gamma_5 C^{-1} = (\gamma_5)^T$. In the continuum limit, i.e., $a \to 0$, S_{WZ} reduces to the continuum Wess-Zumino action,

$$\begin{aligned} S = \int\Big\{ &\frac{1}{2}\bar\chi(\not\partial + m)\chi + \phi^\dagger\partial^2\phi + F^\dagger F \\ &+ m(F\phi + (F\phi)^\dagger) + g\bar\chi(P_+\phi P_+ + P_-\phi^\dagger P_-)\chi \\ &+ g(F\phi^2 + (F\phi^2)^\dagger)\Big\}. \end{aligned}$$

If one defines the real components by $\phi \to \frac{1}{\sqrt{2}}(A + iB)$ and $F \to \frac{1}{\sqrt{2}}(F - iG)$ (where now F is real), the Wess-Zumino action can be written as a free part or kinetic term, S_0, plus the superpotential term, S_{pot}, $S_{\mathrm{WZ}} = S_0 + S_{\mathrm{pot}}$, as

$$\begin{aligned} S_0 &= \sum_x \Big\{\frac{1}{2}\bar\chi\left(1 - \frac{a}{2}D_1\right)^{-1}D_2\chi - \frac{1}{a}(AD_1A + BD_1B) \\ &\quad + \frac{1}{2}F\left(1 - \frac{a}{2}D_1\right)^{-1}F + \frac{1}{2}G\left(1 - \frac{a}{2}D_1\right)^{-1}G\Big\}, \\ S_{\mathrm{pot}} &= \sum_x \Big\{\frac{1}{2}m\bar\chi\chi + m(FA + GB) + \frac{1}{\sqrt{2}}g\bar\chi(A + i\gamma_5 B)\chi \\ &\quad + \frac{1}{\sqrt{2}}g[F(A^2 - B^2) + 2G(AB)]\Big\}. \end{aligned}$$

III. THE SUPERSYMMETRIC TRANSFORMATION

As was discussed in Ref. [23] and then shown in Ref. [26], S_0 is invariant under a lattice supersymmetric transformation,

$$\begin{aligned} &\delta A = \bar\varepsilon\chi = \bar\chi\varepsilon, \qquad \delta B = -i\bar\varepsilon\gamma_5\chi = -i\bar\chi\gamma_5\varepsilon \\ &\delta\chi = -D_2(A - i\gamma_5 B)\varepsilon - (F - i\gamma_5 G)\varepsilon \\ &\delta F = \bar\varepsilon D_2\chi, \qquad \delta G = i\bar\varepsilon D_2\gamma_5\chi, \end{aligned} \tag{7}$$

which is similar to the continuum one except for replacing the continuum derivative with the lattice one, $D_{2\mu}$. Indeed [26], the variation of S_0 under this transformation is

$$\begin{aligned} \delta S_0 = \sum_x \Big\{ &\bar\chi\left(1 - \frac{a}{2}D_1\right)^{-1}D_2[-D_2(A - i\gamma_5 B)\varepsilon \\ &- (F - i\gamma_5 G)\varepsilon] - \frac{2}{a}\bar\chi\varepsilon D_1 A + \frac{2i}{a}\bar\chi\gamma_5\varepsilon D_1 B \\ &+ (\bar\varepsilon D_2\chi)\left(1 - \frac{a}{2}D_1\right)^{-1}F \\ &+ i(\bar\varepsilon D_2\gamma_5\chi)\left(1 - \frac{a}{2}D_1\right)^{-1}G\Big\}. \end{aligned}$$

Using the Ginsparg-Wilson relation (6) and integrating by parts (details can be found in Ref. [26]), we get

$$\sum_x\Big\{-\bar\chi\varepsilon\Big[\Big(1-\frac{a}{2}D_1\Big)^{-1}D_2^2+\frac{2}{a}D_1\Big]A + i\bar\chi\gamma_5\varepsilon\Big[\Big(1-\frac{a}{2}D_1\Big)^{-1}D_2^2+\frac{2}{a}D_1\Big]B - \bar\chi\Big(1-\frac{a}{2}D_1\Big)^{-1}D_2(F-i\gamma_5G)\varepsilon + \bar\chi D_2\varepsilon\Big(1-\frac{a}{2}D_1\Big)^{-1}F+i\bar\chi D_2\gamma_5\varepsilon\Big(1-\frac{a}{2}D_1\Big)^{-1}G\Big\}=0.$$

As discussed in Refs. [26,27], the variation of $S_{\rm pot}$ under the supersymmetric transformation (7) does not vanish due to the failure of the Leibniz rule at finite lattice spacing [36]. To discuss the symmetry properties of the lattice Wess-Zumino model, one possibility would be to modify the action by adding irrelevant terms that make the full action invariant [37]. Another possibility is to modify the supersymmetric transformation (7) so that $S_{\rm pot}$ has an exact symmetry for $a \neq 0$. We shall see that this procedure is only possible if we use fermions that satisfy Eq. (1). Since the transformation (7) leaves S_0 invariant, the modification should vanish for $g = 0$. The supersymmetric transformation that leaves invariant $S_{\rm WZ}$ is similar to Eq. (7), where the only difference is in the variation of the fermion field [26],

$$\delta\chi = -D_2(A-i\gamma_5B)\varepsilon-(F-i\gamma_5G)\varepsilon+gR\varepsilon, \quad (8)$$

where R is a function to be determined order by order in perturbation theory, imposing the condition $\delta S_{\rm WZ}=0$. Expanding R in power of g [26],

$$R = R^{(1)}+gR^{(2)}+g^2R^{(3)}+\cdots, \quad (9)$$

and imposing the symmetry condition order by order in perturbation theory, we find [26]

$$R^{(1)} = \Big(\Big(1-\frac{a}{2}D_1\Big)^{-1}D_2+m\Big)^{-1}\Delta L, \quad (10)$$

where

$$\Delta L \equiv 1/\sqrt{2}\{2(AD_2A-BD_2B)-D_2(A^2-B^2) + 2i\gamma_5[(AD_2B+BD_2A)-D_2(AB)]\}. \quad (11)$$

For $n\geq 2$,

$$R^{(n)} = -\sqrt{2}\Big(\Big(1-\frac{a}{2}D_1\Big)^{-1}D_2+m\Big)^{-1}(A+i\gamma_5B)R^{(n-1)}.$$

Notice that the operator $((1-\frac{a}{2}D_1)^{-1}D_2+m)^{-1}$ is the free fermion propagator, and Eq. (8), as a function of R, is nonlinear in the scalar fields. Inserting these results in Eq. (9), the function R to be used in Eq. (8), resummed to all order in pertubation theory, is

$$R=\Big[\Big(1-\frac{a}{2}D_1\Big)^{-1}D_2+m+\sqrt{2}g(A+i\gamma_5B)\Big]^{-1}\Delta L. \quad (12)$$

Thanks to the Ginsparg-Wilson relation [encoded in Eq. (6)], we are able to resum Eq. (9) to obtain Eq. (12), which contains all orders in perturbation theory and is a closed form that can possibly be used for numerical simulations. In the limit $a\to 0$, Eq. (8) becomes Eq. (7) since ΔL vanishes in this limit. ΔL is different from zero due to the breaking of the Leibniz rule at finite lattice spacing. This resummation would not be possible using Wilson fermions [38].

Now we want to show that the full Wess-Zumino action, $S_{\rm WZ}$, is invariant under the supersymmetric tranformation [with Eq. (8)] and include all orders in perturbation theory. Indeed, its variation is

$$\delta S_{\rm WZ} = \sum_x\Big\{g\bar\chi\Big[\Big(1-\frac{a}{2}D_1\Big)^{-1}D_2R+mR\Big]\varepsilon - \frac{g}{\sqrt{2}}[2\bar\chi(A+i\gamma_5B)D_2(A-i\gamma_5B)\varepsilon - \bar\chi D_2(A-i\gamma_5B)^2\varepsilon]+\sqrt{2}g^2\bar\chi(A+i\gamma_5B)R\varepsilon\Big\}.$$

Using Eq. (12), after some algebra, we get indeed zero:

$$\delta S_{\rm WZ} = \sum_x\Big\{g\bar\chi\Delta L\varepsilon-\frac{g}{\sqrt{2}}[2\bar\chi(A+i\gamma_5B)D_2(A-i\gamma_5B) - \bar\chi D_2(A-i\gamma_5B)^2\varepsilon]\Big\}=0.$$

IV. ONE-POINT WARD-TAKAHASHI IDENTITY TO TWO LOOPS

Before going to two loops (order g^3), we first want to show how to obtain a WTi. This will be useful for the more involved calculation to two loops. The WTi is derived from the generating functional, which is given by $Z[\Phi,J]=\int\mathcal{D}\Phi\exp-(S_{\rm WZ}+S_J)$, where S_J is the source term, $S_J=\sum_xJ_\Phi\cdot\Phi\equiv\sum_x\{J_AA+J_BB+J_FF+J_GG+\bar\eta\chi\}$. Using the invariance of both the Wess-Zumino action and the measure with respect to the lattice supersymmetric transformation (8), the WTi reads $\langle J_\Phi\cdot\delta\Phi\rangle_J=0$, where $\delta\Phi$ is given in Eq. (8).

We start with the simplest (one-point) WTi, which is generated by taking the derivative with respect to $\bar\eta$ and setting to zero all the sources, that is

$$\langle D_2(A-i\gamma_5B)\rangle+\langle F\rangle-i\gamma_5\langle G\rangle-g\langle R\rangle=0. \quad (13)$$

The order $O(g)$ of this WTi is given by

$$\langle D_2(A-i\gamma_5B)\rangle^{(1)}+\langle F\rangle^{(1)} - i\gamma_5\langle G\rangle^{(1)}-g\langle R^{(1)}\rangle^{(0)}=0, \quad (14)$$

where the notation $\langle\mathcal{O}\rangle^{(n)}$ indicates the n-order (in g) contribution to the expectation value of $\mathcal{O}$. From the action, $S_{\rm WZ}$, the free propagators are

$$\langle AA\rangle = \langle BB\rangle = -\mathcal{M}^{-1}\mathbb{D}_1^{-1}$$
$$\langle FF\rangle = \langle GG\rangle = \frac{2}{a}\mathcal{M}^{-1}D_1 = -\mathcal{M}^{-1}\mathbb{D}_1^{-1}D_2^2$$
$$\langle AF\rangle = \langle BG\rangle = m\mathcal{M}^{-1} \tag{15}$$
$$\langle \chi\bar{\chi}\rangle = (\mathbb{D}_1^{-1}D_2 + m)^{-1} = -\mathcal{M}^{-1}(\mathbb{D}_1^{-1}D_2 - m),$$

where $\mathbb{D}_1^{-1} \equiv (1-\frac{a}{2}D_1)^{-1}$ and $\mathcal{M}^{-1} \equiv [\frac{2}{a}D_1(1-\frac{a}{2}D_1)^{-1} + m^2]^{-1}$ and the Ginsparg-Wilson relation (6) has been used to rewrite the auxiliary fields propagators. Despite the appearance of the operator $(1-\frac{a}{2}D_1)^{-1}$, there are no would-be doublers, and the propagators are regular (see Appendix A of Ref. [27] for details). For a nonperturbative approach that shows the localization of this operator, see Refs. [28,29].

Using Eq. (15), it is easy to see that this WTi is satisfied, which means that when we insert all the terms into the WTi (14), the result is zero (notice that $\langle AA\rangle = \langle BB\rangle$ and $\langle AF\rangle = \langle BG\rangle$). For instance,

$$\langle D_2A_x\rangle^{(1)} = \frac{g}{\sqrt{2}}D_{2xy}\Big[\langle A_yF_u\rangle(\langle A_uA_u\rangle - \langle B_uB_u\rangle) + 2\langle A_yA_u\rangle\Big(\langle AF\rangle_u + \langle BG\rangle_u - \frac{1}{2}\,\mathrm{Tr}\langle\bar{\chi}\chi\rangle_u\Big)\Big] = 0.$$

Similarly, $\langle G\rangle^{(1)} = 0$ because of $\mathrm{Tr}(\gamma_\mu) = 0$ and

$$\langle F_x\rangle^{(1)} = \frac{g}{\sqrt{2}}\Big[\langle F_xF_u\rangle(\langle AA\rangle_u - \langle BB\rangle_u) + 2\langle F_xA_u\rangle \times\Big(\langle AF\rangle_u + \langle BG\rangle_u - \frac{1}{2}\,\mathrm{Tr}\langle\chi_u\bar{\chi}_u\rangle\Big)\Big] = 0.$$

Finally, the term including R is given by,

$$g\langle R_x^{(1)}\rangle^{(0)} = g\langle\chi_u\bar{\chi}_u\rangle(2D_{2yl}\langle A_yA_l\rangle - 2D_{2yl}\langle B_yB_l\rangle - D_{2yl}\langle AA\rangle_l + D_{2yl}\langle BB\rangle_l) = 0.$$

Now we are ready to verify that the two-loop order, g^3, in a one-point WTi is satisfied at fixed lattice spacing. The calculation is not trivial and contains 110 connected nontadpole Feynman diagrams (from F and the operator R). The tadpole contributions cancel out separately. We start from Eq. (13):

$$\langle D_2(A - i\gamma_5B)\rangle^{(3)} + \langle F\rangle^{(3)} - i\gamma_5\langle G\rangle^{(3)} - g\langle R^{(1)}\rangle^{(2)} - g^2\langle R^{(2)}\rangle^{(1)} - g^3\langle R^{(3)}\rangle^{(0)} = 0. \tag{16}$$

The first term of this WTi is zero because of the δ-momentum conservation and $D_2(k=0) = 0$. Also, $i\gamma_5\langle G\rangle^{(3)}$ is trivially zero. Then, one is left with

$$\langle F\rangle^{(3)} - g\langle R^{(1)}\rangle^{(2)} - g^2\langle R^{(2)}\rangle^{(1)} - g^3\langle R^{(3)}\rangle^{(0)} = 0. \tag{17}$$

To calculate the expectation value of F, one has to insert the interaction term until order g^3, as

$$\langle F_x\rangle^{(3)} = g^3/(2\sqrt{2})\langle F_x\{[(\chi(A + i\gamma_5B)\bar{\chi})_u + (F(A^2 - B^2) + 2GAB)_u][(\chi(A + i\gamma_5B)\bar{\chi})_v + (F(A^2 - B^2) + 2GAB)_v][(\chi(A + i\gamma_5B)\bar{\chi})_w + (F(A^2 - B^2) + 2GAB)_w]\}\rangle^{(0)},$$

where u, v, w are dummy indices. For the remaining terms of the WTi (17) involving the operator R, we have

$$g\langle R_x^{(1)}\rangle^{(2)} = g^3/2\langle\chi\bar{\chi}\rangle_{xy}\langle\Delta L_y\{[(\chi(A + i\gamma_5B)\bar{\chi})_u + (F(A^2 - B^2))_u + 2(GAB)_u][(\chi(A + i\gamma_5B)\bar{\chi})_v + (F(A^2 - B^2))_v + 2(GAB)_v]\}\rangle^{(0)},$$

where ΔL is given in Eq. (11) and $R^{(1)}$ in Eq. (10). The second term $R^{(2)}$ is given by

$$g^2\langle R_x^{(2)}\rangle^{(1)} = -g^3\langle(\mathbb{D}_1^{-1}D_2 + m)^{-1}_{xz}(A_z + i\gamma_5B_z) \times(\mathbb{D}_1^{-1}D_2 + m)^{-1}_{zu}\Delta L_w[(\chi(A + i\gamma_5B)\bar{\chi})_w + (F(A^2 - B^2))_w + 2(GAB)_w]\rangle^{(0)}.$$

The last term, $R^{(3)}$, is given by

$$g^3\langle R_x^{(3)}\rangle^{(0)} = 2g^3\langle(\mathbb{D}_1^{-1}D_2 + m)^{-1}_{xy}(A_y + i\gamma_5B_y) \times(\mathbb{D}_1^{-1}D_2 + m)^{-1}_{yz}(A_z + i\gamma_5B_z) \times(\mathbb{D}_1^{-1}D_2 + m)^{-1}_{zw}\Delta L_w\rangle^{(0)}.$$

We now write the contribution of $\langle R_x^{(2)}\rangle^{(1)}_{\mathrm{NT}}$ and $\langle R_x^{(3)}\rangle^{(0)}_{\mathrm{NT}}$, which contains 19 and 12 connected nontadpole diagrams, respectively:

$$\begin{aligned} g^2\langle R_x^{(2)}\rangle^{(1)}_{\mathrm{NT}} ={}& -g^3/\sqrt{2}\{\langle\chi\bar{\chi}\rangle_{xz}\langle\chi\bar{\chi}\rangle_{zu}[\langle A_zF_w\rangle(4\langle A_uA_w\rangle D_{2uv}\langle A_vA_w\rangle + 4\langle B_uB_w\rangle D_{2uv}\langle B_vB_w\rangle - 2D_{2uv}\langle A_vA_w\rangle\langle A_wA_v\rangle \\ &- 2D_{2uv}\langle B_vB_w\rangle\langle B_wB_v\rangle) + \langle A_zA_w\rangle(4\langle A_uF_w\rangle D_{2uv}\langle A_vA_w\rangle + 4\langle A_uA_w\rangle D_{2uv}\langle A_vF_w\rangle - 4\langle B_uG_w\rangle D_{2uv}\langle B_vB_w\rangle \\ &- 4\langle B_uB_w\rangle D_{2uv}\langle B_vG_w\rangle - 2D_{2uv}\langle A_vF_w\rangle\langle A_wA_v\rangle + 2D_{2uv}\langle B_vG_w\rangle\langle B_wB_v\rangle)] + \langle\chi\bar{\chi}\rangle_{xz}\gamma_5\langle\chi\bar{\chi}\rangle_{zu}\gamma_5[\langle B_zB_w\rangle \\ &\times(4\langle A_uF_w\rangle D_{2uv}\langle B_vB_w\rangle - 4\langle A_uA_w\rangle D_{2uv}\langle B_vG_w\rangle + 4\langle B_uB_w\rangle D_{2uv}\langle A_vF_w\rangle - 4\langle B_uG_w\rangle D_{2uv}\langle A_vA_w\rangle \\ &- 4D_{2uv}\langle A_vF_w\rangle\langle B_vB_w\rangle + 4D_{2uv}\langle A_vA_w\rangle\langle B_vG_w\rangle) + \langle B_zG_w\rangle(-4\langle A_uA_w\rangle D_{2uv}\langle B_vB_w\rangle - 4\langle B_uB_w\rangle D_{2uv}\langle A_vA_w\rangle \\ &+ 4D_{2uv}\langle A_uA_w\rangle\langle B_vB_w\rangle)]\} \\ ={}& 0, \end{aligned} \tag{18}$$

and

$$\begin{aligned} g^3\langle R_x^{(3)}\rangle_{\rm NT}^{(0)} &= 2\sqrt{2}g^3\langle\chi\bar{\chi}\rangle_{xy}\{\langle\chi\bar{\chi}\rangle_{yz}\langle\chi\bar{\chi}\rangle_{zw}[\langle A_yA_w\rangle D_{2wl}\langle A_zA_l\rangle + \langle A_zA_w\rangle D_{2wl}\langle A_yA_l\rangle - D_{2wl}\langle A_yA_l\rangle\langle A_zA_l\rangle] \\ &\quad - \langle\chi\bar{\chi}\rangle_{yz}\gamma_5\langle\chi\bar{\chi}\rangle_{zw}\gamma_5[\langle A_yA_w\rangle D_{2wl}\langle B_zB_l\rangle + \langle B_zB_w\rangle D_{2wl}\langle A_yA_l\rangle - D_{2wl}\langle A_yA_l\rangle\langle B_zB_l\rangle] \\ &\quad - \gamma_5\langle\chi\bar{\chi}\rangle_{yz}\langle\chi\bar{\chi}\rangle_{zw}\gamma_5[\langle A_zA_w\rangle D_{2wl}\langle B_yB_l\rangle + \langle B_yB_w\rangle D_{2wl}\langle A_zA_l\rangle - D_{2wl}\langle B_yB_l\rangle\langle A_zA_l\rangle] \\ &\quad + \gamma_5\langle\chi\bar{\chi}\rangle_{yz}\gamma_5\langle\chi\bar{\chi}\rangle_{zw}[\langle B_yB_w\rangle D_{2wl}\langle B_zB_l\rangle + \langle B_zB_w\rangle D_{2wl}\langle B_yB_l\rangle - D_{2wl}\langle B_yB_l\rangle\langle B_zB_l\rangle]\} \\ &= 0. \end{aligned} \quad (19)$$

A similar procedure is used to determined, $\langle R_x^{(1)}\rangle_{\rm NT}^{(2)} = 0$ and $\langle F_x\rangle_{\rm NT}^{(3)} = 0$. They contain 32 and 47 diagrams, respectively.

V. CONCLUDING REMARKS

We showed that the lattice Wess-Zumino model in four dimensions is invariant under a deformed supersymmetric transformation to all orders in perturbation theory. As a nontrivial check, we performed a two-loop calculation of a one-point WTi, associated with this lattice supersymmetry, and we showed that it is exactly satisfied at fixed lattice spacing. This guarantees the restoration of supersymmetry in the continuum limit without the fine-tuning of the parameters of the action. Although each term in Eq. (16) vanishes separately (due to the fact that we are investigating a one-point WTi), the cancellation involves bosons and fermion fields in each term of Eq. (16), as it is required in supersymmetry. Moreover, the expectation value of R is zero. This result is not in contradiction with the one in Ref. [27], in which a one-loop (two-point) WTi was investigated and a finite value of $\langle R\rangle$ was found. The reason is that one-point Ward-Takahashi identities do not contribute to the renormalization of the wave function of scalar and fermion fields.

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