Size of the Gribov region in curved spacetime

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Recent work in the literature has argued that the joint effect of spacetime curvature and the Gribov ambiguity may introduce further modifications to the Green functions in the infrared. This paper focuses on a simple criterion for studying the effect of spacetime curvature on the size of the Gribov region, improving the accuracy of the previous analysis. It is shown that, depending on the sign of the scalar and Riemann curvature, the Gribov horizon moves inward or, instead, outward with respect to the case of flat spacetime. This is made clear by two novel inequalities, derived here for the first time.

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I. INTRODUCTION

Ever since Gribov [1] and other authors [2] discovered the limitations of quantum Yang-Mills theory in the Coulomb gauge, many efforts have been devoted to studying the issue in a variety of contexts. In general, in nonperturbative quantum gauge theory, it may happen that a gauge orbit intersects the surface defined by the gauge-fixing condition at more than one point. This leads to a ghost operator having zero modes, i.e., nonvanishing eigenfunctions belonging to the zero eigenvalue. The preservation of the gauge-fixing condition under gauge transformations is then expressed by a partial differential equation admitting a number of solutions rather than a unique solution, a property usually studied through the so-called Gribov pendulum example [1,3,4].

In our paper, we study the Gribov problem in curved spacetime, motivated by the following results:

(i) The work in Ref. [5] has shown that, in a curved background, a proper gauge fixing cannot be achieved, not even in the Abelian case.

(ii) When black holes, neutron stars, quark and hybrid stars, and cosmological setups are studied, it is important to consider the dynamics of quantum chromodynamics on a curved background [6].

(iii) The coupling to the gravitational field destroys the perturbative renormalizability of the Yang-Mills field with field strength $F^{\alpha}_{\mu\nu}$ even in the purely Yang-Mills sector [7]. In addition to the familiar term in $F_{\alpha\mu\nu}F^{\alpha\mu\nu}$ in the heat-kernel a_2 coefficient, there are now not only the usual terms in

$$R^2,\ R_{\mu\nu}R^{\mu\nu},\ R_{\mu\nu\sigma\tau}R^{\mu\nu\sigma\tau},$$

but also terms in [7]

$$\mu^{-2}F_{\alpha\rho\nu;}{}^{\nu}F^{\alpha\rho\sigma}{}_{;\sigma},\quad \mu^{-2}R_{\rho\nu}T^{\rho\nu},\quad \mu^{-4}T_{\rho\nu}T^{\rho\nu}$$

as well, where $T^{\rho\nu}$ is the Yang-Mills stress-energy density in curved spacetime, and μ is the Planck mass. The presence of the last three terms means that—although the Yang-Mills coupling constant gets renormalized—the finite part of the effective action now depends on the auxiliary mass in a way that cannot be absorbed into a running coupling constant. Each choice of auxiliary mass corresponds to a different theory [7].

(iv) The Yang-Mills field, in turn, spoils a basic property of pure gravity based on Einstein's general relativity. Indeed, although many remarkable cancellations occur in the computation, the presence of the Yang-Mills field destroys [7] the one-loop finiteness of pure gravity [8].

Section II outlines the method proposed in Ref. [4] to study the effects of curvature on the size of the Gribov region. Sections III and IV study the effects of the Ricci and Riemann tensors. The challenge of evaluating gluon and ghost propagators in curved spacetime is analyzed in Sec. V, while our results are interpreted in Sec. VI.

II. EFFECTS OF THE CURVATURE ON THE SIZE OF THE GRIBOV REGION

Consider a point p on a given spacetime (M, g), and choose Riemann normal coordinates in a neighborhood of p. Such a coordinate system is built as follows. For each $X \in U \subset T_p(M)$, consider the affinely parametrized

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geodesic γ_X starting from p with initial velocity X. By definition, the exponential map is such that

$$e^X: X \in U \to q = \gamma_X(1).$$

The points q form a neighborhood I of p. If U is sufficiently small, the exponential is invertible and one can use coordinates of the vector X in $T_p(M)$ to identify the point q. In such a coordinate system, if one takes the coordinate lines to be orthogonal at p, the metric tensor $g_{\mu\nu}$ and Christoffel symbols $\Gamma^\lambda_{\mu\nu}$ are given by the following approximate formulas:

$$g_{\mu\nu} = \delta_{\mu\nu} - \frac{1}{3} R_{\mu\alpha\nu\beta} X^\alpha X^\beta + O(\|X\|^3), \tag{2.1}$$

$$\Gamma^\lambda_{\mu\nu} = -\frac{1}{3}(R^\lambda_{\mu\nu\beta} + R^\lambda_{\nu\mu\beta}) X^\beta + O(\|X\|^2). \tag{2.2}$$

Thus, in Riemannian geodesic coordinates, spacetime displays only a tiny deviation from flatness. If the gravitational field, described by the Riemann curvature tensor, is weak, one can use perturbation theory to study the modifications introduced by a nonvanishing Riemann tensor.

The ghost operator for quantum Yang-Mills theory in curved spaces (omitting hereafter, for simplicity of notation, Lie-algebra indices) is

$$FP(A) = -\nabla_\mu \nabla^\mu - [A_\mu, \nabla^\mu]. \tag{2.3}$$

It acts on anticommuting scalar fields, and reads eventually, in the above coordinates,

$$FP(A)\omega = -g^{\mu\nu}(\partial_\mu \partial_\nu \omega - \Gamma^\lambda_{\mu\nu} \partial_\lambda \omega) - [A_\mu, \partial^\mu \omega]. \tag{2.4}$$

By virtue of the formula expressing Christoffel symbols in Riemann normal coordinates, one finds

$$\delta^{\mu\nu} \Gamma^\lambda_{\mu\nu} = \frac{2}{3} R^\lambda_\alpha X^\alpha, \tag{2.5}$$

and hence

$$FP(A)\omega = \left(-\Box + \frac{2}{3} R^\lambda_\alpha X^\alpha \partial_\lambda - \frac{1}{3} R^\mu{}_\alpha{}^\nu{}_\beta X^\alpha X^\beta \partial_\mu \partial_\nu \right)\omega - [A_\mu, \partial^\mu \omega]. \tag{2.6}$$

The second and third terms, involving the Ricci and Riemann tensors, respectively, are corrections which account for the presence of the gravitational field.

III. EFFECT OF THE RICCI TERM

Suppose now that ω is a real zero mode of the flat ghost operator. We can thus use perturbation theory to evaluate the shift to the zero-energy level. The Ricci contribution, denoted by ε_1, takes the form

$$\begin{aligned} \varepsilon_1 &\equiv \frac{2}{3} \frac{\int R^\lambda_\alpha X^\alpha \operatorname{Tr}(\omega \partial_\lambda \omega)}{\int \operatorname{Tr}(\omega^2)} = \frac{2}{3} R^\lambda_\alpha \frac{\int X^\alpha \frac{1}{2} \partial_\lambda \operatorname{Tr}(\omega^2)}{\int \operatorname{Tr}(\omega^2)} \\ &= -\frac{1}{3} R^\lambda_\alpha \frac{\int (\partial_\lambda X^\alpha) \operatorname{Tr}(\omega^2)}{\int \operatorname{Tr}(\omega^2)} = -\frac{1}{3} R^\lambda_\alpha \delta^\alpha_\lambda = -\frac{R}{3}, \end{aligned} \tag{3.1}$$

where, by choosing Dirichlet boundary conditions for the ghost field, we have been able to set to zero the boundary term after integration by parts. (See the work in Ref. [9] for a detailed discussion of these ghost boundary conditions in quantum Yang-Mills theory.)

IV. CONTRIBUTION OF THE RIEMANN TERM

No conclusion can be reached without a proper treatment of the Riemann term since—as will be shown below—it also involves a term linear in X. Indeed, the Riemann tensor contributes through

$$\varepsilon_2 \equiv -\frac{1}{3} R^\mu{}_\alpha{}^\nu{}_\beta \frac{\int X^\alpha X^\beta \operatorname{Tr}(\omega \partial_\mu \partial_\nu \omega)}{\int \operatorname{Tr}(\omega^2)}. \tag{4.1}$$

At this stage, we first use the identity

$$\operatorname{Tr}(\omega \partial_\mu \partial_\nu \omega) = \frac{1}{2} \partial_\mu \partial_\nu \operatorname{Tr}(\omega^2) - \operatorname{Tr}((\partial_\mu \omega)(\partial_\nu \omega)) \tag{4.2}$$

to reexpress ε_2 in the form

$$\begin{aligned} \varepsilon_2 = &-\frac{1}{6} R^\mu{}_\alpha{}^\nu{}_\beta \frac{\int X^\alpha X^\beta \partial_\mu \partial_\nu \operatorname{Tr}(\omega^2)}{\int \operatorname{Tr}(\omega^2)} \\ &+ \frac{1}{3} R^\mu{}_\alpha{}^\nu{}_\beta \frac{\int X^\alpha X^\beta \operatorname{Tr}((\partial_\mu \omega)(\partial_\nu \omega))}{\int \operatorname{Tr}(\omega^2)}. \end{aligned} \tag{4.3}$$

As a second step we exploit the Leibniz rule, which provides

$$\begin{aligned} \partial_\mu (X^\alpha X^\beta \partial_\nu \operatorname{Tr}(\omega^2)) = &\partial_\mu (X^\alpha X^\beta) \partial_\nu \operatorname{Tr}(\omega^2) \\ &+ X^\alpha X^\beta \partial_\mu \partial_\nu \operatorname{Tr}(\omega^2), \end{aligned} \tag{4.4}$$

in order to integrate by parts. Hence we find

$$\begin{aligned} \int X^\alpha X^\beta \partial_\mu \partial_\nu \operatorname{Tr}(\omega^2) &= \int \partial_\mu (X^\alpha X^\beta \partial_\nu \operatorname{Tr}(\omega^2)) \\ &\quad - \int (\delta^\alpha_\mu X^\beta + \delta^\beta_\mu X^\alpha) \partial_\nu \operatorname{Tr}(\omega^2) \\ &= -\delta^\alpha_\mu \int X^\beta \partial_\nu \operatorname{Tr}(\omega^2) \\ &\quad - \delta^\beta_\mu \int X^\alpha \partial_\nu \operatorname{Tr}(\omega^2), \end{aligned} \tag{4.5}$$

where we have again exploited Dirichlet boundary conditions for the ghost field, bearing in mind that $\partial_\nu \operatorname{Tr}(\omega^2) = 2 \operatorname{Tr}(\omega \partial_\nu \omega)$.

As a third step, we again use the Leibniz rule, i.e.,

$$\partial_\nu (X^\alpha \operatorname{Tr}(\omega^2)) = \delta^\alpha_\nu \operatorname{Tr}(\omega^2) + X^\alpha \partial_\nu \operatorname{Tr}(\omega^2), \tag{4.6}$$

and the same with X^α replaced by X^β, to find, by virtue of Dirichlet boundary conditions for ω,

$$\int X^\alpha X^\beta \partial_\mu \partial_\nu \mathrm{Tr}(\omega^2) = (\delta^\alpha_\mu \delta^\beta_\nu + \delta^\alpha_\nu \delta^\beta_\mu) \int \mathrm{Tr}(\omega^2). \tag{4.7}$$

The first term on the right-hand side in our formula for ε_2 is therefore

$$\tilde{\varepsilon}_2 = -\frac{1}{6}(R^\mu{}_\mu{}^\beta{}_\beta + R^\mu{}_\nu{}^\nu{}_\mu) = \frac{R}{6}, \tag{4.8}$$

which implies that

$$\varepsilon_2 = \frac{R}{6} + \frac{1}{3} \frac{\int \gamma^{\mu\nu} \mathrm{Tr}(\omega_{,\mu}\omega_{,\nu})}{\int \mathrm{Tr}(\omega^2)}, \tag{4.9}$$

having defined

$$\gamma^{\mu\nu} \equiv R^\mu{}_\alpha{}^\nu{}_\beta X^\alpha X^\beta. \tag{4.10}$$

For example, in the so-called Euclidean de Sitter space, upon defining a constant K, $\gamma^{\mu\nu}$ is

$$\begin{aligned}\gamma^{\mu\nu} &= K(g^{\mu\nu} g_{\alpha\beta} - \delta^\mu_\beta \delta^\nu_\alpha) X^\alpha X^\beta \\ &= K(g^{\mu\nu} \|X\|^2 - X^\mu X^\nu),\end{aligned} \tag{4.11}$$

where $\|X\|^2 = g(X, X) = X_\alpha X^\alpha > 0$. Our $\gamma^{\mu\nu}$ acts as $g(X, X) g^{\mu\nu}$ on the hyperplane orthogonal to X, while it vanishes on the subspace generated by X. Hence the sign of $\gamma^{\mu\nu}$ is ruled by the constant K, which is positive.

V. THE CHALLENGE OF GLUON AND GHOST PROPAGATORS

When Yang-Mills theory is studied in flat space, it is rather important to evaluate the gluon and ghost propagators, since their behavior in the infrared depends crucially on the Gribov mass parameter, which is determined through the so-called gap equation. More precisely, investigations of lattice gauge theory on very large volumes [10] have found an infrared finite gluon propagator and a ghost propagator which is no longer enhanced in the infrared. The work in Ref. [11] has exploited a refinement of the Gribov-Zwanziger method to obtain analytical results in agreement with these lattice data. Moreover, the authors of Ref. [12] obtained—in various truncations of Dyson-Schwinger equations and functional renormalization group equations—a one-parameter family of solutions for the ghost and gluon dressing functions of Landau gauge Yang-Mills theory, each member of the one-parameter family being confining. In a general curved spacetime, however, no momentum-space representation is available in the first place, since the homogeneity required for its existence is lacking, and the local momentum-space formalism built in Ref. [13] is only appropriate for studying ultraviolet divergences. Thus, one needs a radical departure from the calculational techniques available in flat space.

For this purpose, we have carefully considered the Gusynin [14] technique, which relies in turn on the Widom [15] formalism. Following Gusynin, one can express the matrix elements of the resolvent of a positive elliptic operator H by means of the formula

$$\begin{aligned}G(x, x', \lambda) &\equiv \langle x | \frac{1}{(H - \lambda \mathrm{I})} | x' \rangle \\ &= \int \frac{d^n k}{(2\pi)^n \sqrt{g(x')}} e^{il(x,x',k)} \sigma(x, x', k; \lambda).\end{aligned} \tag{5.1}$$

Here $l(x, x', k)$ is a biscalar under general coordinate transformations and constitutes a generalization of the phase $k_\mu (x - x')^\mu$ used in the flat case. This expression for the resolvent is manifestly covariant. The generalization of the linearity property of $l(x, x', k)$ that is valid in the flat case is obtained by requiring that symmetric combinations of covariant derivatives should vanish in the coincidence limit,

$$\nabla_{(\mu_1} \nabla_{\mu_2} \ldots \nabla_{\mu_m)} l|_{x=x'} \equiv [\nabla_{(\mu_1} \nabla_{\mu_2} \ldots \nabla_{\mu_m)} l] = 0, \quad \mathrm{m} \neq 1, \tag{5.2}$$

along with

$$[\nabla_\mu l] = k_\mu. \tag{5.3}$$

The square bracket denotes the coincidence limit, and symmetrization is understood as running over the indices enclosed by the round brackets. These conditions are sufficient to determine $l(x, x', k)$ in a neighborhood of the point x'. Indeed, the commutator of covariant derivatives acts on tensors as follows:

$$\begin{aligned}[\nabla_\mu, \nabla_\nu] f^{\nu_1 \ldots \nu_k}_{\mu_1 \ldots \mu_n} &= R_{\mu\nu\lambda}{}^{\nu_i} f^{\nu_1 \ldots \nu_{i-1} \lambda \nu_{i+1} \ldots \nu_k}_{\mu_1 \ldots \mu_n} \\ &\quad - R_{\mu\nu\mu_i}{}^\lambda f^{\nu_1 \ldots \nu_k}_{\mu_1 \ldots \mu_{i-1} \lambda \mu_{i+1} \ldots \mu_n}\end{aligned} \tag{5.4}$$

$$+ T^\lambda_{\mu\nu} \nabla_\lambda f^{\nu_1 \ldots \nu_k}_{\mu_1 \ldots \mu_n} + W_{\mu\nu} f^{\nu_1 \ldots \nu_k}_{\mu_1 \ldots \mu_n}. \tag{5.5}$$

Using this formula and Eqs. (5.2) and (5.3), one can find the covariant derivatives of l in the coincidence limit. The resolvent kernel $G(x, x', \lambda)$ is a solution of the equation

$$(H(x, \nabla_\mu) - \lambda) G(x, x', \lambda) = \frac{1}{\sqrt{g}} \delta(x - x'), \tag{5.6}$$

subject to the boundary conditions which define the domain of the operator H. By inserting into this equation the integral formula for the resolvent kernel one gets the equation

$$(H(x, \nabla_\mu + i\nabla_\mu l) - \lambda) \sigma(x, x', k; \lambda) = I(x, x'). \tag{5.7}$$

The function $I(x, x')$ is a biscalar and is defined by conditions analogous to those satisfied by $l(x, x', k)$,

$$[I] = E, \tag{5.8}$$

$$[\nabla_{(\mu_1} \nabla_{\mu_2} \ldots \nabla_{\mu_m)} I] = 0, \tag{5.9}$$

where E is the unit matrix.

One then introduces an auxiliary parameter ε, which will be set to 1 at the end of the calculations, and

$\sigma(x, x', k; \lambda)$ and $H(x, \nabla_\mu + i\nabla_\mu l)$ are expanded by following the rules $l \to l/\varepsilon$, $\lambda \to \lambda/\varepsilon^{2r}$, i.e.,

$$\sigma_\varepsilon(x, x', k; \lambda) = \sum_{m=0}^{\infty} \varepsilon^{2r+m} \sigma_m(x, x', k; \lambda), \qquad (5.10)$$

$$H(x, \nabla_\mu + i\nabla_\mu l/\varepsilon) = \sum_{m=0}^{2r} \varepsilon^{-2r+m} A_m(x, \nabla_\mu, \nabla_\mu l). \qquad (5.11)$$

Substituting these expansions into Eq. (5.7) and collecting terms of the same order in ε, one gets a system of equations for the coefficients σ_m which can be solved recursively.

The diagonal matrix elements of the heat kernel are then given by the relation

$$\langle x|e^{-tH}|x\rangle = \sum_{m=0}^{\infty} \int \frac{d^n k}{(2\pi)^n \sqrt{g}} \int_C \frac{id\lambda}{2\pi} e^{-t\lambda}[\sigma_m](x, k, \lambda). \qquad (5.12)$$

One finds from the recursion relations satisfied by the coefficients σ_m that these coefficients are generalized homogeneous functions in the variables (k, λ),

$$[\sigma_m](x, tk, t^{2r}\lambda)] = t^{-(m+2r)}[\sigma_m](x, k, \lambda)]. \qquad (5.13)$$

Hence it follows that the heat-kernel expansion coefficients are obtained from those of the Laplace transform of the resolvent kernel,

$$E_m(x|H) = \int \frac{d^n k}{(2\pi)^n \sqrt{g}} \int_C \frac{id\lambda}{2\pi} e^{-t\lambda}[\sigma_m](x, k, \lambda). \qquad (5.14)$$

Conversely, the resolvent kernel may be obtained from the heat kernel $K(x, x', t)$ by the Laplace transform

$$G(x, x', \lambda) = \int_0^{\infty} e^{t\lambda} K(x, x', t) dt, \qquad (5.15)$$

and the Green function $\mathcal{G}(x, x')$ is equal to $G(x, x', \lambda = 0)$.

The advantage of this approach is that it gives an algorithm to calculate the coefficients $E_m(x|H)$ and it can be generalized to the case of nonminimal operators. Nonminimal second-order operators are indeed a very interesting class of operators, whose general form is

$$H^{\mu\nu} = -g^{\mu\nu}\Box + a\nabla^\mu\nabla^\nu + X^{\mu\nu}. \qquad (5.16)$$

Here ∇^μ is the covariant derivative, including both the Levi-Civita connection and the gauge connection. The tensor $X^{\mu\nu}$ is a matrix in the internal indices. The parameter a may assume all real values; in particular, for $a = 0$ the operator reduces to a minimal one. Indeed, the gauge-field operator for Yang-Mills theory falls into this class,

$$H_{YM}^{\mu\nu} = -g^{\mu\nu}\Box + \left(1 - \frac{1}{\alpha}\right)\nabla^\mu\nabla^\nu + R^{\mu\nu}. \qquad (5.17)$$

This can also be expressed as an operator acting on 1-forms,

$$H(\alpha) = \delta d + \frac{1}{\alpha} d\delta. \qquad (5.18)$$

In the case $\alpha = 1$ it reduces to the Laplace-Beltrami operator, whose action on 1-forms $\varphi_\nu dx^\nu$ is given by a Bochner-Lichnerowicz formula,

$$\left(\left(\delta d + \frac{1}{\alpha} d\delta\right)\varphi\right)_\mu = (-\delta_\mu{}^\nu\Box + R_\mu{}^\nu)\varphi_\nu. \qquad (5.19)$$

We also recall that Endo [16] obtained a formula which makes it possible to express the integrated heat kernel for a generic value of the parameter α in terms of that relative to the minimal case $\alpha = 1$, i.e.,

$$K_{\mu\nu'}^{(\alpha)}(\tau) = K_{\mu\nu'}^{(1)}(\tau) + \mathrm{i} \int_\tau^{\tau/\alpha} dy \nabla_\mu \nabla^\lambda K_{\lambda\nu'}^{(1)}(y). \qquad (5.20)$$

Unfortunately, the heat-kernel expansion corresponding to our gauge-field operator does not exist in the singular case $\alpha \to 0$, i.e., the Landau choice of gauge parameter. In fact in this limit only some heat-kernel coefficients are finite, while the others diverge. A possible way out might be to remove the divergent parts of such coefficients. More precisely, the authors of Ref. [17] evaluated the ghost propagator for Yang-Mills in de Sitter space [see Eqs. (2.5) and (2.6) therein]. The authors of Ref. [17] argued that, since ghost fields occur only in internal loops and couple to the gauge field through a derivative coupling, the divergent term in the ghost propagator does not contribute to the calculation of n-point functions of gauge fields. Hence they proposed that one should use the effective ghost propagator obtained by subtracting the divergent contribution. We are currently trying to understand whether such a subtraction procedure can also be advocated for both gluon and ghost propagators when Yang-Mills theory is studied in a generic curved spacetime in the presence of a Gribov horizon.

VI. INTERPRETATION AND CONCLUDING REMARKS

Within our scheme, the full shift to the zero-energy level is

$$\varepsilon_1 + \varepsilon_2 = -\frac{R}{6} + \frac{1}{3} \frac{\int \gamma^{\mu\nu} \mathrm{Tr}(\omega_{,\mu}\omega_{,\nu})}{\int \mathrm{Tr}(\omega^2)}. \qquad (6.1)$$

If it were just for the $-\frac{R}{6}$ term, we might argue as follows [4]: if the scalar curvature R is positive, the Gribov horizon moves inward, and hence it is reached at a higher energy and the gauge-field propagator should be more suppressed. By contrast, if R is negative, the horizon moves outward, and the energy is such that field fluctuations reaching the horizon should be lower. In other words, the gauge-field propagator should be less suppressed in the infrared if $R < 0$. For these conclusions to remain qualitatively the same, we should study the conditions $\varepsilon_1 + \varepsilon_2 < 0$ and $\varepsilon_1 + \varepsilon_2 > 0$, respectively. The former is fulfilled provided that

$$2\frac{\int \gamma^{\mu\nu}\,\mathrm{Tr}(\omega_{,\mu}\omega_{,\nu})}{\int \mathrm{Tr}(\omega^2)} < R, \tag{6.2}$$

while the latter is satisfied in the opposite case, i.e., if

$$2\frac{\int \gamma^{\mu\nu}\,\mathrm{Tr}(\omega_{,\mu}\omega_{,\nu})}{\int \mathrm{Tr}(\omega^2)} > R. \tag{6.3}$$

For each choice of curved Riemannian background, one has to check which of the two conditions above is satisfied. It should be stressed that it is hard to obtain an estimate of the integral on the left-hand side of Eqs. (6.2) and (6.3), because the zero mode ω, and hence the integral itself, depends on the gauge connection at the Gribov horizon. Such a difficulty becomes clearer if one bears in mind that the Gribov horizon is not precisely localizable, not even in the flat case (where it is known that, in a first approximation, it is an ellipsoid [4]). Nevertheless, we hope that the scheme here proposed, with the explicit computational recipe provided, will lead to further progress on the understanding of the Gribov phenomenon in curved spaces. It would also be interesting to study the extension to curved spacetime of the scheme proposed in Ref. [18] for the elimination of infinitesimal Gribov ambiguities in non-Abelian gauge theories.

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