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Intermediate effective interactions and dynamical fermion mass generation of QCD

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The functional renormalization group equation is expanded to a two-loop form. This two-loop form equation involves one-loop effective action. An intermediate effective action perspective is adopted toward the one-loop effective action. That is to say, the intermediate effective action could not be of the same form of the bare action and one can make an ansatz to it. Thus by focusing on different high-dimensional operators, effects of the chosen operators can be investigated. QCD through intermediate 4-fermion interactions is investigated. Of the six kinds of 4-fermion interactions generated by one-loop QCD, four kinds generate fermion mass while the other two kinds decrease it. Flow patterns on the $\tilde{m}^2_{\mathrm{phys}} - \tilde{g}^2$ plane are drawn.

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I. INTRODUCTION

Multiloop quantum corrections are notoriously inaccessible. Complexity of contraction of the Dyson series, complexity inherent to the considered theory, and complexity of the multiloop momentum integrals make the calculation extremely complicated. So it is meaningful and necessary if there is an approach that can easily capture or analyze multiloop properties.

The functional renormalization group equation method [1–4] implements Wilson's idea [5] of "integrating a single momentum slice." It is exact, nonperturbative, functional, and compact. There have been many applications of this method in literature; for example, see [6–12].

The functional renormalization group equation (FRGE) usually cannot be solved exactly, so approximation is necessary. It can be treated perturbatively. Expanded up to two-loop order, it involves one-loop effective action. The calculation from the bare action to the two-loop effective action is complicated. To simplify the calculation, one can make approximations during the procedure.

In this article, we will view the involved one-loop effective action as an intermediate effective action. One can make an ansatz to it. By making a different ansatz, a different "route" to, or different part of the two-loop effective action can be investigated.

We will take this perspective to analyze the dynamical chiral symmetry breaking of QCD. Lattice calculations, see [13–19] for example, show that the critical fermion number for chiral symmetry breaking $N_f^\ddagger$ is less than the critical fermion number for asymptotic freedom $N_f^\dagger = 33/2$. So there is a conformal window between $N_f^\ddagger$ and $N_f^\dagger$. QCD at this situation is asymptotically free and chirally symmetric, hence conformal. Lattice calculations indicate that $8 < N_f^\ddagger < 13$.

Many theories have been put up to explain the existence of this conformal window, for example, by an ansatz of all-loop β function [20], mass-dependent β functions [21], critical scaling laws [22], confinement induced gap equation [23], condensation of dynamical chirality [24], gap equation through lattice results [25], and so on.

In [26], we investigated this problem through renormalization flows of the fermion mass and the gauge coupling. We displayed that a theory attracted to an IR-attractive fixed point with a finite fixed dimensionless mass will not show dynamical fermion mass generation. And the critical fermion number can be determined as the turning point for existence/nonexistence of an IR-attractive fixed point. At two-loop order, it is about the lower turning point inhabiting in the β function of QCD, that is, $51 \times 3/19 \approx 8$.

However, there we approximated two-loop results with mainly mathematical considerations. In this article, we will approach two-loop corrections through two-loop FRGE with intermediate effective action ansatzed. The central ingredients are the 4-fermion interactions.

It is well known that 4-fermion interactions are closely related to dynamical chiral symmetry breaking. A lot of work has been done on the Nambu-Jona-Lasinio model [27] or the Thirring theory [28] or other kinds of 4-fermion theories, for example, see [29–32]. So it is reasonable to expect 4-fermion interactions play an important role in dynamical chiral symmetry breaking of QCD.

The structure of this article is as follows. In Sec. II, we elaborate our calculation scheme; in Sec. III, we take the procedure for QCD and derive the renormalization equations; in Sec. IV, we draw flow patterns; finally, in Sec. V, we give our conclusion.

II. TWO-LOOP FRGE

From a bare action S, through adding a regulator term,

$$\Delta S_k = \frac{1}{2}\int_q \varphi(-q)R_k(q)\varphi(q), \tag{1}$$

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the functional renormalization group equation (the Wetterich equation) can be derived

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr}[(\partial_t R_k)\tilde{G}_{\varphi\varphi}], \tag{2}$$

where $\partial_t = k\partial_k$. Γ_k is the scale-dependent effective action. $\tilde{G}_{\varphi\varphi} = \langle\varphi\varphi\rangle - \langle\varphi\rangle\langle\varphi\rangle = (\Gamma_k^{(2)} + R_k)^{-1} \equiv \tilde{\Gamma}_k^{(2)-1}$ is the connected two-point Green function of the regulated theory.

The regulator term is to suppress the low energy modes of the theory and let high energy modes intact. Thus during integration of the bare action S to get the effective action Γ, only high energy modes are integrated.

So there are some conditions on the regulator R_k. First, $R_k(q) \sim 0$ when $q^2 \gg k^2$. Second, $R_k(q) \sim s^2k^2$ for $q^2 \ll k^2$. Effectively, the low energy modes gets a mass suppression of mass s^2k^2. Third, $R_k(q) \sim 0$ when $k^2 \to 0$, to ensure the regulated theory identical to the original one when the regulator term is removed. For a detailed discussion of the regulator, see [26].

The parameter s^2 measures to what degree the low energy modes are suppressed and quantifies the relative strength of the regulator term to the kinetic term. So to completely exclude the effects of low energy modes (if it is necessary), s^2 should be sent to infinity after integration.

In literature, there are other kinds of regulators in use and these regulators can also lead to right answers, such as the optimized regulator [33]. So there are some intriguing aspects about how to choose regulators. However a thorough discussion of this issue is not the theme of this article. In this article, we use the suppression-parametrized regulator with a sloped-step-function profile, see Fig. 1.

Equation (2) is a functional equation of Γ_k. It is of one-loop form. It is exact and incorporates all nonperturbative effects. It usually cannot be solved exactly. However, it can be expanded perturbatively. If one expands the effective action as

$$\Gamma = S + \sum_{n=1}^{\infty} \Delta_n \Gamma, \tag{3}$$

substitutes it into Eq. (2), and expands the right-hand side to match the left-hand side, then one can get

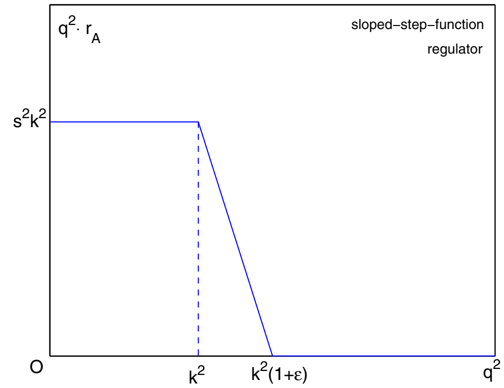

FIG. 1 (color online). Profile of the sloped-step-function regulator.

$$\partial_t \Delta_1 \Gamma = \frac{1}{2} \mathrm{Tr}[(\partial_t R_k)\tilde{G}_0], \tag{4}$$

$$\partial_t \Delta_2 \Gamma = \frac{1}{2} \mathrm{Tr}[(\Delta_1\Gamma)^{(2)}\partial_t \tilde{G}_0]. \tag{5}$$

Here $\Delta_n\Gamma$ denotes the quantum correction at n-loop order only, and $\tilde{G}_0 = [R_k + S^{(2)}]^{-1}$.

Equation (5) involves the one-loop quantum correction,

$$\Delta_1 \Gamma = \frac{1}{2} \mathrm{Tr} \ln \tilde{S}^{(2)}. \tag{6}$$

It is of two-loop form. It can be shown that it is consistent to the ordinary result of perturbative expansion of effective action. This can be done by subsituting Eq. (6) into Eq. (5), then comparing the result with the result of Ref. [34]. In principle, it can be used to calculate two-loop renormalization flows. However there are some subtleties in implementation. Nevertheless, it can be separated and used to capture the specified two-loop effects.

In the following, we use Eq. (5) to discuss the effect of fermion-mass generation of intermediate 4-fermion interactions.

At one-loop order, QCD generates the following six kinds of 4-fermion intermediate effective interactions [of order $(\partial^2)^0$],

$$\begin{aligned}\Delta_1\Gamma_{\mathrm{FF}} = {} & \frac{\Delta\lambda_1}{2}\int(\bar{\psi}^{a_f}t^a\psi^{a_f})(\bar{\psi}^{b_f}t^a\psi^{b_f}) + \frac{\Delta\lambda_2}{2}\int(\bar{\psi}^{a_f}t^a\gamma^\mu\psi^{a_f})(\bar{\psi}^{b_f}t^a\gamma^\mu\psi^{b_f}) \\ & + \frac{\Delta\lambda_3}{2}\int(\bar{\psi}^{a_f}t^a\gamma^\mu\gamma^\nu\psi^{a_f})(\bar{\psi}^{b_f}t^a\gamma^\mu\gamma^\nu\psi^{b_f}) + \frac{\Delta\lambda_4}{2}\int(\bar{\psi}^{a_f}t^a\gamma^\mu\gamma^\nu\gamma^\rho\psi^{a_f})(\bar{\psi}^{b_f}t^a\gamma^\mu\gamma^\nu\gamma^\rho\psi^{b_f}) \\ & + \frac{\Delta\lambda_5}{2}\int(\bar{\psi}^{a_f}t^at^b\psi^{a_f})(\bar{\psi}^{b_f}t^at^b\psi^{b_f}) + \frac{\Delta\lambda_6}{2}\int(\bar{\psi}^{a_f}t^at^b\gamma^\mu\psi^{a_f})(\bar{\psi}^{b_f}t^at^b\gamma^\mu\psi^{b_f}).\end{aligned} \tag{7}$$

By substituting these effective interactions into Eq. (5), corrections to the fermion mass and gauge coupling can be generated.

III. QCD THROUGH 4-FERMION INTERACTIONS

In this section, QCD with intermediate 4-fermion interactions will be considered. The flow equations of fermion mass and gauge coupling will be derived.

We make the following ansatz for the effective action:

$$\begin{aligned}\Gamma_{\rm QCD} = {} & Z_\psi \int_p \bar{\psi}^{a_f}(p)(\not{p} + m)\psi^{a_f}(p) + \frac{1}{2} Z_A \int_p A^a_\mu(-p)\Big(p^2\delta^{\mu\nu} - \Big(1 - \frac{1}{\xi}\Big)p^\mu p^\nu\Big)A^a_\nu(p) \\ & - Z_c \int_p \bar{c}^a(p)p^2 c^a(p) + gZ_\psi\sqrt{Z_A}\int_p\int_{p'} \bar{\psi}^{a_f}(p)\gamma^\mu t^a A^a_\mu(p-p')\psi^{a_f}(p') \\ & + gZ_c\sqrt{Z_A}\int_p\int_{p'} f^{abc}(ip_\mu)\bar{c}^a(p)A^b_\mu(p-p')c^c(p') + gZ_A\sqrt{Z_A}\int_p\int_{p'}\int_q (2\pi)^d\delta^{(d)}(p+p'+q) \\ & \cdot f^{abc}(-iq_\mu)A^a_\nu(q)A^b_\mu(p)A^c_\nu(p') + \frac{1}{4}g^2 Z_A^2 \int_p\int_{p'}\int_q\int_{q'}(2\pi)^d\delta^{(d)}(p+p'+q+q') \\ & \cdot f^{abe}f^{cde}A^a_\kappa(p)A^b_\lambda(p')A^c_\kappa(q)A^d_\lambda(q'). \end{aligned} \tag{8}$$

This ansatz is of the same form of the bare action S. The gauge $\xi = 1$ is used. For simplicity, fermions are assumed to have the same mass. We have used the convention $\{\gamma^\mu, \gamma^\nu\} = -2\delta^{\mu\nu}I_{4\times4}$. This definition of γ matrices differs from the convention $\{\gamma^\mu, \gamma^\nu\} = 2\delta^{\mu\nu}I_{4\times4}$ by a factor i. So in the above expression, the fermion mass is the Wick-rotated mass $m = i\cdot m_{\rm physical}$.

The regulator terms are

$$\Delta S_\psi = \int_p\int_q \bar{\psi}^{a_f}(p)\hat{R}^{a_f b_f}_\psi(p,q)\psi^{b_f}(q); \tag{9}$$

$$\Delta S_A = \frac{1}{2}\int_p\int_q A^a_\mu(-p)\hat{R}^{ab,\mu\nu}_A(p,q)A^b_\nu(q); \tag{10}$$

$$\Delta S_c = \int_p\int_q \bar{c}^a(p)\hat{R}^{ab}_c(p,q)c^b(q), \tag{11}$$

with

$$\hat{R}^{a_f b_f}_\psi(p,q) = Z_\psi\delta^{a_f b_f}\delta_{pq}r_\psi(p^2/k^2; s^2)\not{p}, \tag{12}$$

$$\hat{R}^{ab,\mu\nu}_A(p,q) = Z_A\delta^{ab}\delta^{\mu\nu}\delta_{pq}r_A(p^2/k^2; s^2)p^2, \tag{13}$$

$$\hat{R}^{ab}_c(p,q) = -Z_c\delta^{ab}\delta_{pq}r_A(p^2/k^2; s^2)p^2, \tag{14}$$

with $(1 + r_\psi)^2 = (1 + r_A)$.

The functional renormalization group equation can be derived as

$$\partial_t\Gamma = \frac{1}{2}\,\mathrm{Tr}[(\partial_t\hat{R}_A)\tilde{G}_{AA}] - \mathrm{Tr}[(\partial_t\hat{R}_\psi)\tilde{G}_{\psi\bar{\psi}}] - \mathrm{Tr}[(\partial_t\hat{R}_c)\tilde{G}_{c\bar{c}}], \tag{15}$$

where $\tilde{G}_{AA}$, $\tilde{G}_{\psi\bar{\psi}}$, and $\tilde{G}_{c\bar{c}}$ are connected Green functions.

$\Gamma^{(2)}$ has the following structure:

$$\Gamma^{(2)} = \begin{pmatrix} 0 & \Gamma^{(2)}_{\bar{\psi}\psi} & \Gamma^{(2)}_{\bar{\psi}A} & 0 & 0 \\ \Gamma^{(2)}_{\psi\bar{\psi}} & 0 & \Gamma^{(2)}_{\psi A} & 0 & 0 \\ \Gamma^{(2)}_{A\bar{\psi}} & \Gamma^{(2)}_{A\psi} & \Gamma^{(2)}_{AA} & \Gamma^{(2)}_{A\bar{c}} & \Gamma^{(2)}_{Ac} \\ 0 & 0 & \Gamma^{(2)}_{\bar{c}A} & 0 & \Gamma^{(2)}_{\bar{c}c} \\ 0 & 0 & \Gamma^{(2)}_{cA} & \Gamma^{(2)}_{c\bar{c}} & 0 \end{pmatrix}. \tag{16}$$

Since $\tilde{G}\cdot\tilde{\Gamma}^{(2)} = 1$, $\tilde{G}$ can be expressed by inverting $\tilde{\Gamma}^{(2)}$. To derive the flow equations, it can be written that

$$\tilde{G}_{c\bar{c}} \approx \tilde{\Gamma}^{(2)-1}_{\bar{c}c},$$
$$\tilde{G}_{AA} \approx [\tilde{\Gamma}^{(2)}_{AA} - \tilde{\Gamma}^{(2)}_{A\psi}\tilde{\Gamma}^{(2)-1}_{\bar{\psi}\psi}\tilde{\Gamma}^{(2)}_{\bar{\psi}A} - \tilde{\Gamma}^{(2)}_{A\bar{\psi}}\tilde{\Gamma}^{(2)-1}_{\psi\bar{\psi}}\tilde{\Gamma}^{(2)}_{\psi A}]^{-1},$$
$$\tilde{G}_{\psi\bar{\psi}} \approx [\tilde{\Gamma}^{(2)}_{\bar{\psi}\psi} - \tilde{\Gamma}^{(2)}_{\bar{\psi}A}(\tilde{\Gamma}^{(2)}_{AA} - \tilde{\Gamma}^{(2)}_{A\bar{\psi}}\tilde{\Gamma}^{(2)-1}_{\psi\bar{\psi}}\tilde{\Gamma}^{(2)}_{\psi A})^{-1}\tilde{\Gamma}^{(2)}_{A\psi}]^{-1}.$$

Because the right-hand sides of Eqs. (5) and (15) are to be projected onto the left-hand sides, in the above equations terms with extra fields have been discarded. When doing inversion of the right-hand sides later, other such terms are also to be discarded.

The one-loop QCD part has been calculated in [26]. The remaining is the 4-fermion part.

First, we should calculate the generation of 4-fermion interactions of QCD at one loop. The result is

$$\frac{k^{d-2}\Delta\lambda_1}{Z_f^2} = -(\tilde{g}^2)^2 J_N(2,2,4;\tilde{m}^2)\tilde{m}^2 \cdot dC_2(G),$$
$$\frac{k^{d-2}\Delta\lambda_5}{Z_f^2} = -(\tilde{g}^2)^2 J_N(2,2,4;\tilde{m}^2)\tilde{m}^2 \cdot 2d,$$
$$\frac{k^{d-2}\Delta\lambda_3}{Z_f^2} = -(\tilde{g}^2)^2 J_N(2,2,4;\tilde{m}^2)\tilde{m}^2 \cdot \frac{-1}{2}C_2(G),$$
$$\frac{k^{d-2}\Delta\lambda_2}{Z_f^2} = -(\tilde{g}^2)^2 J_N(1,2,2;\tilde{m}^2) \cdot \frac{3d-2}{d}C_2(G),$$
$$\frac{k^{d-2}\Delta\lambda_6}{Z_f^2} = -(\tilde{g}^2)^2 J_N(1,2,2;\tilde{m}^2) \cdot \frac{2(3d-2)}{d},$$
$$\frac{k^{d-2}\Delta\lambda_4}{Z_f^2} = -(\tilde{g}^2)^2 J_N(1,2,2;\tilde{m}^2) \cdot \frac{-1}{2d}C_2(G).$$

For details, see Appendix A. Here $\tilde{m} = m/k$, $\tilde{g}^2 = [\int d\Omega_d/(2\pi)^d]g^2/k^{4-d}$. And $J_N(a,b,c;\tilde{m}^2)$ is a dimensionless integral; see Eq. (22) for definition.

Second, we should calculate the fermion-mass generation of the 4-fermion interactions. Third, we should calculate the coupling corrections which involve 4-fermion interactions.

Finally, we arrive at the following flow equations:

$$\frac{\partial_t \tilde{m}^2}{2\tilde{m}^2} + 1 = -\tilde{g}^2 C_2(r) K_m + \frac{k^{d-2}\Delta\lambda_i}{Z_f^2} H_i, \tag{17}$$

$$\frac{\partial_t \tilde{g}^2}{2\tilde{g}^2} + [g] = -\tilde{g}^2[C_2(r)K_1 + C_2(G)K_2 - N_f C(r)K_3] + \frac{k^{d-2}\Delta\lambda_i}{Z_f^2}T_i. \tag{18}$$

Here $[g]$ is the dimension of g, and

$$K_m = d[J(1,1,4;\tilde{m}^2) + J(0,2,2;\tilde{m}^2)] - \frac{(d-1)(d-2)}{d}J(1,1,3;\tilde{m}^2),$$
$$K_1 = (d-2)[J(1,1,4;\tilde{m}^2) + J(0,2,2;\tilde{m}^2)] - \frac{4(d-2)}{d}J(-1,3,0;\tilde{m}^2) - \frac{(d-1)(d-2)}{d}J(1,1,3;\tilde{m}^2),$$
$$K_2 = -\frac{d-2}{2}[J(1,1,4;\tilde{m}^2) + J(0,2,2;\tilde{m}^2)] + \frac{2(d-2)}{d}J(-1,3,0;\tilde{m}^2) + \frac{3(d-1)}{d}J(1,1,5;\tilde{m}^2) + \frac{2(d-1)}{d}J(0,2,3;\tilde{m}^2)$$
$$- \left[\frac{16(d-2)}{d(d+2)} + \frac{d-14}{2} + \frac{8}{d}\right]J(2,0,6;\tilde{m}^2),$$
$$K_3 = -\frac{8}{d}J(-1,3,-2;\tilde{m}^2) + \frac{16(d+4)}{d(d+2)}J(-2,4,-4;\tilde{m}^2) - \frac{64}{d(d+2)}J(-3,5,-6;\tilde{m}^2),$$
$$H_1 = J(-1,2,0;\tilde{m}^2)\cdot C_2(r)\cdot(-1),$$
$$H_2 = J(-1,2,0;\tilde{m}^2)\cdot C_2(r)d,$$
$$H_3 = J(-1,2,0;\tilde{m}^2)\cdot C_2(r)(d-2)d,$$
$$H_4 = J(-1,2,0;\tilde{m}^2)\cdot C_2(r)[2d^2 - d(d-2)^2],$$
$$H_5 = J(-1,2,0;\tilde{m}^2)\cdot N_f C(r)C_2(r)\cdot 4 - J(-1,2,0;\tilde{m}^2)\cdot\left[C_2(r) - \frac{1}{2}C_2(G)\right]C_2(r),$$
$$H_6 = J(-1,2,0;\tilde{m}^2)\cdot\left[C_2(r) - \frac{1}{2}C_2(G)\right]C_2(r)d,$$

$$T_1 = A \cdot \left[C_2(r) - \frac{1}{2}C_2(G)\right],$$
$$T_2 = A \cdot \left[C_2(r) - \frac{1}{2}C_2(G)\right](d-2) - A \cdot N_f C(r)(-4),$$
$$T_3 = A \cdot \left[C_2(r) - \frac{1}{2}C_2(G)\right][2d - (d-2)^2],$$
$$T_4 = A \cdot \left[C_2(r) - \frac{1}{2}C_2(G)\right](d-2)(10d - 8 - d^2) - A \cdot N_f C(r)(-4)(3d-2),$$
$$T_5 = A \cdot \left[C_2(r) - \frac{1}{2}C_2(G)\right][C_2(r) - C_2(G)],$$
$$T_6 = A \cdot \left[C_2(r) - \frac{1}{2}C_2(G)\right][C_2(r) - C_2(G)](d-2) - A \cdot N_f C(r) C_2(G),$$

$$A \equiv J(-1, 3, 0; \tilde{m}^2) \cdot \tilde{m}^2 \frac{d+2}{d} + J(-2, 3, -2; \tilde{m}^2) \cdot \frac{d-2}{d}.$$

The dimensionless momentum integral $J(a, b, c; \tilde{m}^2)$ is defined by

$$J(a,b,c;\tilde{m}^2) \equiv \int_l \frac{(\partial_t r_A) \cdot k^{2(a+b)-d}(2\pi)^d / \int d\Omega_d}{(l^2)^a [m^2 + l^2(1+r_\psi)^2]^b (1+r_\psi)^c}. \quad (19)$$

This integral with a suppression-parametrized regulator has been discussed in [26]. Note that here the roles of b and c have been exchanged. When $\tilde{m}^2 = 0$ and $b + c/2 - 1 > 0$,

$$J(a, b, c; 0) = \frac{1}{b + \frac{c}{2} - 1}. \quad (20)$$

When $\tilde{m}^2$ is nonzero, it can be evaluated numerically.

The regulator we used for numerical calculation is

$$l^2 \cdot r_A = \frac{s^2}{\epsilon}(k^2 + k^2\epsilon - l^2), \quad (21)$$

for $l^2 \in [k^2, k^2 + k^2\epsilon]$; and $l^2 \cdot r_A$ is constant elsewhere. Shown in Fig. 1 is the profile.

The other momentum integral $J_N(a, b, c; \tilde{m}^2)$ is defined by

$$J_N(a, b, c; \tilde{m}^2) \equiv \int_l \frac{k^{2(a+b)-d}(2\pi)^d / \int d\Omega_d}{(l^2)^a [m^2 + l^2(1+r_\psi)^2]^b (1+r_\psi)^c}. \quad (22)$$

As there is no factor $\partial_t r_A$ in the numerator, one can take the limit $s^2 \to \infty$ before integration. When $2(a+b) - d > 0$,

$$J_N(a, b, c; 0) = \frac{1}{2(a+b) - d}. \quad (23)$$

IV. MASS GENERATION AND DECREASE

In this section, the effects of 4-fermion interactions will be considered. The physical mass $\tilde{m}^2_{\text{physical}} = -\tilde{m}^2$ will be resumed. Renormalization flows in the $\tilde{m}^2_{\text{phys}} - \tilde{g}^2$ plane will be plotted. In the following, $d = 4$ and $N_c = 3$.

Let us first consider the QCD one-loop results without intermediate interactions. The flow patterns for the Wick-rotated mass and gauge coupling have been presented in [26]. Here we draw the flow patterns for the physical mass and gauge coupling.

Shown in Fig. 2 are the results. The left panel is typical for $N_f < 33/2$, while the right is typical for $N_f > 33/2$. No matter what N_f is, in the infrared direction, dimensionless mass always increases. Actually, K_m is always positive, so the dimensional mass also increases.

Now let us focus on the fermion mass generation effect of 4-fermion interactions. From the flow equation (17), one sees that only $\Delta\lambda_5$ and $\Delta\lambda_6$ contribute negatively to the fermion mass in the infrared direction. These two interactions lead to fermion mass decrease.

Shown in Figs. 3 and 4 are the flow patterns of one-loop QCD together with the effects of all six kinds of intermediate interactions.

Figure 3 is typical for $N_f > 33/2$. One sees that all the flows go to an IR-attractive fixed point. At this point, $\tilde{m}^2_{\text{phys}}$ is finite, which means $m^2_{\text{phys}} = k^2\tilde{m}^2_{\text{phys}}$ is zero. That is to say, there is no fermion mass generation, or dynamical chiral symmetry breaking. So it indicates that an IR-attractive fixed point with a finite $\tilde{m}^2_{\text{phys}}$ could induce ceasing of dynamical chiral symmetry breaking.

Figure 4 is typical for $N_f < 33/2$. Though in the infrared, the dimensionless mass is also finite, all the

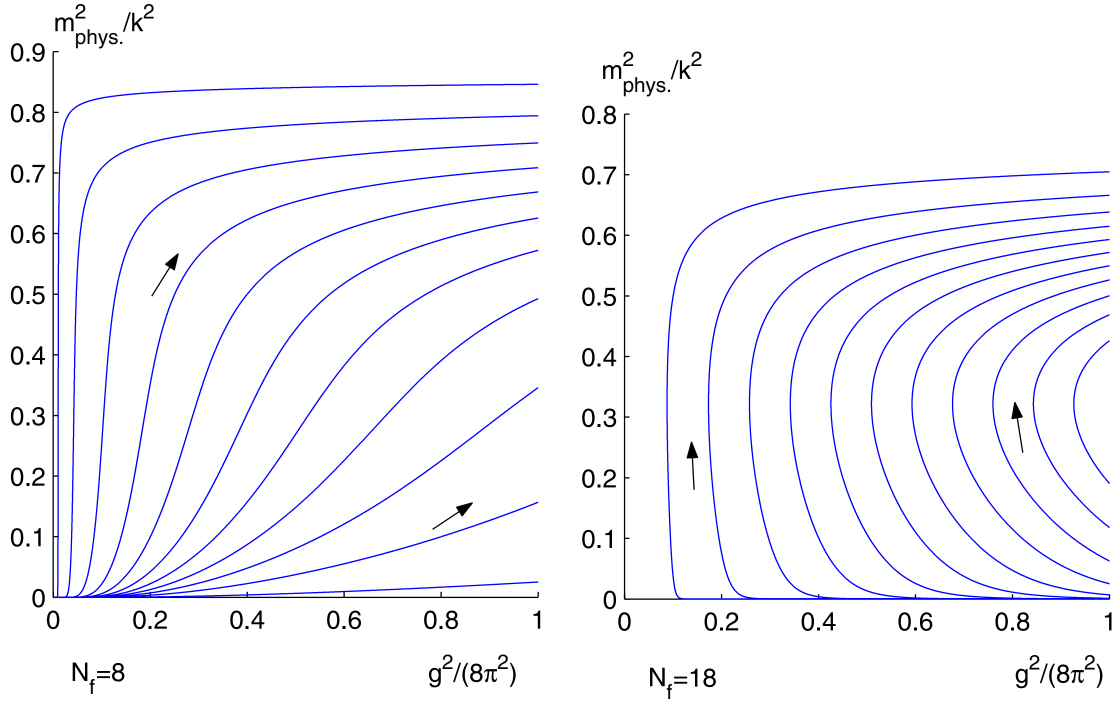

FIG. 2 (color online). Flow patterns of one-loop QCD. Arrows point to infrared.

flows eventually go beyond perturbative region. So it cannot be interpreted that there is no fermion mass generation.

The conformal window is not captured. There is only one number (33/2) that defines the flow patterns. By far, we have only considered one-loop corrections to the gauge coupling. As has been shown in [26], with two-loop corrections counted in, the conformal window turns up.

Now let us consider the gauge coupling correction effect of the 4-fermion interaction. At $\tilde{m}^2_{\rm phys} = 0$, the flow equation (18) becomes

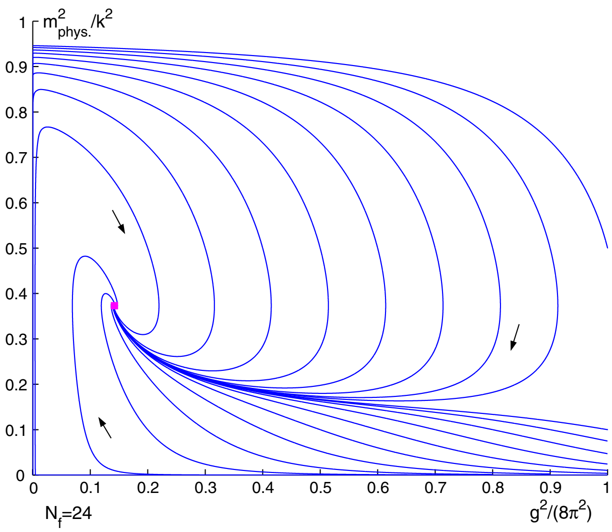

FIG. 3 (color online). Flow pattern for $N_f = 24$. Arrows point to infrared.

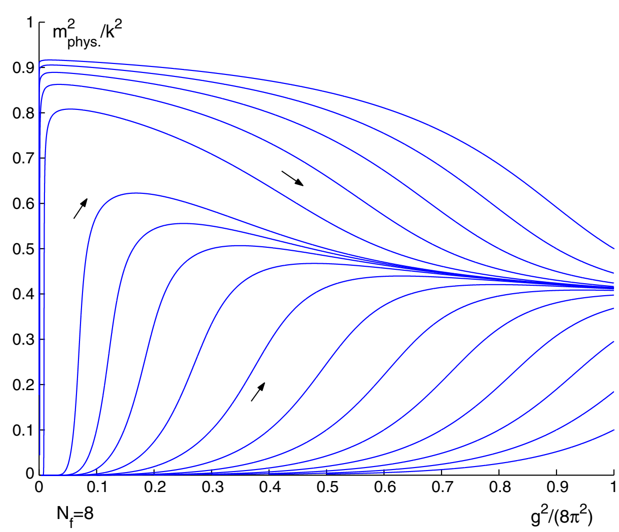

FIG. 4 (color online). Flow pattern for $N_f = 8$. Arrows point to infrared.

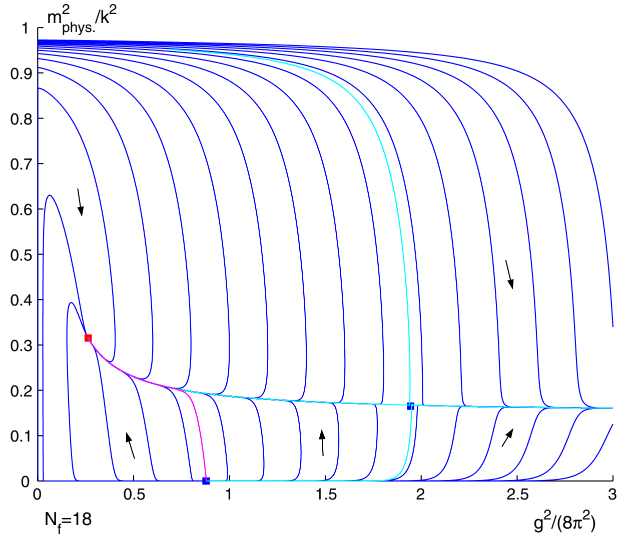

FIG. 5 (color online). Flow pattern of $N_f = 18$ with gauge coupling correction of 4-fermion interactions considered. Arrows point to infrared.

$$\frac{\partial_t \tilde{g}^2}{2\tilde{g}^2} = -\tilde{g}^2\left(\frac{11}{2} - \frac{N_f}{3}\right) - (\tilde{g}^2)^2 \cdot \frac{41}{72}. \quad (24)$$

The first term in the right-hand side is the one-loop result. The second term is the two-loop corrections through the 4-fermion interactions. The coefficient of this term is negative and independent of N_f, which means when $\tilde{g}^2$ is large enough, $\tilde{g}^2$ always increases (in the IR direction) no matter what N_f is.

At this circumstance, the flow pattern for $N_f < 33/2$ is the same as in Fig. 4. There is no IR-attractive fixed point. The flow pattern for $N_f > 33/2$ is shown in Fig. 5. Three nontrivial fixed points turn up. When $\tilde{g}^2$ is large enough, $\tilde{g}^2$ always increases in the two patterns. But in Fig. 5, there is an IR-attractive fixed point in the region with small $\tilde{g}^2$. In this region, flows are attracted to this fixed point. That is to say, there is no dynamical fermion mass generation for these theories.

The conformal window is still not captured. There is still only one number (33/2) that defines the flow patterns. One can see the reason by comparing Eq. (24) with the right two-loop equation

$$\frac{\partial_t \tilde{g}^2}{2\tilde{g}^2} = -\tilde{g}^2\left(\frac{11}{2} - \frac{N_f}{3}\right) - (\tilde{g}^2)^2 \cdot \left(\frac{51}{2} - \frac{19N_f}{6}\right). \quad (25)$$

In the above equation, the coefficient of the two-loop term is dependent of N_f and provides a second turning point for the β function. This turning point is $51 \times 3/19 \approx 8$. As has been demonstrated in [26], theories with the fermion number falling between this one and the former one (33/2) also show no dynamical fermion mass generation. So the second turning point can be regarded as the critical fermion number for chiral symmetry (at two-loop order approximation).

However Eq. (24) does not capture the second turning point. As a result, the conformal window is not captured. In our calculation, only the lowest terms of 4-fermion interactions in the derivative expansion are considered. By adding momentum-dependent 4-fermion interactions the N_f-dependent corrections may be captured. This calls for future investigation.

V. CONCLUSION AND DISCUSSION

In conclusion, dynamical fermion mass generation of QCD was investigated through two-loop FRGE with intermediate 4-fermion interactions.

Two-loop quantum corrections are complicated to obtain. The intermediate effective action approach toward the two-loop FRGE provides a convenient tool to analyze and capture two-loop effects.

One-loop QCD generates six kinds of 4-fermion interactions. Four kinds contribute positively to the fermion mass in the IR direction while the other two kinds contribute negatively. The net effect is fermion mass decrease in the IR direction when dimensionless mass is large.

Renormalization flow patterns were drawn on the $\tilde{m}^2_{\text{phys}} - \tilde{g}^2$ plane. It was displayed that, when chiral symmetry is preserved ($N_f > 33/2$), all flows go to an IR-attractive fixed point. When chiral symmetry is broken ($N_f < 33/2$), the IR-attractive fixed point does not exist. This provided a mechanism for the ceasing of dynamical fermion-mass generation of QCD.

However, the conformal window was not captured as 4-fermion interactions do not play a dominate role in all the corrections to the gauge coupling, because we had considered the simplest 4-fermion interactions. By adding momentum-dependent 4-fermion interactions, the conformal window might be captured.

We considered the simplest situation of the fermion masses, i.e., all the fermions have the same mass. To be more realistic, it could be interesting to consider with different fermion masses in the future.

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APPENDIX A: SOME DETAILS

1. Generation of 4-fermion interactions at one loop

Generation of 4-fermion interactions at one loop is done through carrying out Eq. (6). The following terms contribute to 4-fermion interactions:

$$
\begin{aligned}
&-\operatorname{Tr}(\tilde{\Gamma}^{(2)}_{\bar{\psi}A}\tilde{\Gamma}^{(2)-1}_{AA,\mathrm{kin}}\tilde{\Gamma}^{(2)}_{A\psi})\tilde{\Gamma}^{(2)-1}_{\bar{\psi}\psi,\mathrm{kin}}(\tilde{\Gamma}^{(2)}_{\bar{\psi}A}\tilde{\Gamma}^{(2)-1}_{AA,\mathrm{kin}}\tilde{\Gamma}^{(2)}_{A\psi})\tilde{\Gamma}^{(2)-1}_{\bar{\psi}\psi,\mathrm{kin}}\cdot\left(-\frac{1}{4}\right),\\
&-\operatorname{Tr}(\tilde{\Gamma}^{(2)}_{\bar{\psi}A}\tilde{\Gamma}^{(2)-1}_{AA,\mathrm{kin}}\tilde{\Gamma}^{(2)}_{A\bar{\psi}})\tilde{\Gamma}^{(2)-1}_{\psi\bar{\psi},\mathrm{kin}}(\tilde{\Gamma}^{(2)}_{\psi A}\tilde{\Gamma}^{(2)-1}_{AA,\mathrm{kin}}\tilde{\Gamma}^{(2)}_{A\psi})\tilde{\Gamma}^{(2)-1}_{\bar{\psi}\psi,\mathrm{kin}}\cdot\left(-\frac{1}{4}\right),\\
&+\frac{1}{2}\operatorname{Tr}(\tilde{\Gamma}^{(2)}_{A\psi}\tilde{\Gamma}^{(2)-1}_{\bar{\psi}\psi,\mathrm{kin}}\tilde{\Gamma}^{(2)}_{\bar{\psi}A})\tilde{\Gamma}^{(2)-1}_{AA,\mathrm{kin}}(\tilde{\Gamma}^{(2)}_{A\psi}\tilde{\Gamma}^{(2)-1}_{\bar{\psi}\psi,\mathrm{kin}}\tilde{\Gamma}^{(2)}_{\bar{\psi}A})\tilde{\Gamma}^{(2)-1}_{AA,\mathrm{kin}}\cdot\left(-\frac{1}{4}\right),\\
&+\frac{1}{2}\operatorname{Tr}(\tilde{\Gamma}^{(2)}_{A\psi}\tilde{\Gamma}^{(2)-1}_{\bar{\psi}\psi,\mathrm{kin}}\tilde{\Gamma}^{(2)}_{\bar{\psi}A})\tilde{\Gamma}^{(2)-1}_{AA,\mathrm{kin}}(\tilde{\Gamma}^{(2)}_{A\bar{\psi}}\tilde{\Gamma}^{(2)-1}_{\psi\bar{\psi},\mathrm{kin}}\tilde{\Gamma}^{(2)}_{\psi A})\tilde{\Gamma}^{(2)-1}_{AA,\mathrm{kin}}\cdot\left(-\frac{1}{4}\right),\\
&+\frac{1}{2}\operatorname{Tr}(\tilde{\Gamma}^{(2)}_{A\bar{\psi}}\tilde{\Gamma}^{(2)-1}_{\psi\bar{\psi},\mathrm{kin}}\tilde{\Gamma}^{(2)}_{\psi A})\tilde{\Gamma}^{(2)-1}_{AA,\mathrm{kin}}(\tilde{\Gamma}^{(2)}_{A\psi}\tilde{\Gamma}^{(2)-1}_{\bar{\psi}\psi,\mathrm{kin}}\tilde{\Gamma}^{(2)}_{\bar{\psi}A})\tilde{\Gamma}^{(2)-1}_{AA,\mathrm{kin}}\cdot\left(-\frac{1}{4}\right),\\
&+\frac{1}{2}\operatorname{Tr}(\tilde{\Gamma}^{(2)}_{A\bar{\psi}}\tilde{\Gamma}^{(2)-1}_{\psi\bar{\psi},\mathrm{kin}}\tilde{\Gamma}^{(2)}_{\psi A})\tilde{\Gamma}^{(2)-1}_{AA,\mathrm{kin}}(\tilde{\Gamma}^{(2)}_{A\bar{\psi}}\tilde{\Gamma}^{(2)-1}_{\psi\bar{\psi},\mathrm{kin}}\tilde{\Gamma}^{(2)}_{\psi A})\tilde{\Gamma}^{(2)-1}_{AA,\mathrm{kin}}\cdot\left(-\frac{1}{4}\right).
\end{aligned}
$$

Here "kin" indicates the kinetic part (including the mass term for fermion).

2. Corrections through 4-fermion interactions

Corrections through 4-fermion interactions are done by carrying out Eq. (5).

Two-loop corrections of fermion mass through 4-fermion intermediate interactions are as follows:

$$-\operatorname{Tr}(\partial_t\hat{R}_\psi)\cdot(-1)\tilde{\Gamma}^{(2)-1}_{\bar{\psi}\psi,\mathrm{kin}}\tilde{\Gamma}^{(2)}_{\bar{\psi}\psi,\mathrm{FF}}\tilde{\Gamma}^{(2)-1}_{\bar{\psi}\psi,\mathrm{kin}}.$$

Here $\tilde{\Gamma}^{(2)}_{\bar{\psi}\psi,\mathrm{FF}}$ indicates the part of $\tilde{\Gamma}^{(2)}_{\bar{\psi}\psi}$ derived from 4-fermion interactions.

Two-loop corrections of fermion-gluon vertex through 4-fermion intermediate interactions include

$$
\begin{aligned}
&-\operatorname{Tr}(\partial_t\hat{R}_\psi)\tilde{\Gamma}^{(2)-1}_{\bar{\psi}\psi,\mathrm{kin}}\tilde{\Gamma}^{(2)}_{\bar{\psi}\psi,\mathrm{FF}}\tilde{\Gamma}^{(2)-1}_{\bar{\psi}\psi,\mathrm{kin}}\tilde{\Gamma}^{(2)}_{\bar{\psi}\psi,\mathrm{QCD}}\tilde{\Gamma}^{(2)-1}_{\bar{\psi}\psi,\mathrm{kin}},\\
&-\operatorname{Tr}(\partial_t\hat{R}_\psi)\tilde{\Gamma}^{(2)-1}_{\bar{\psi}\psi,\mathrm{kin}}\tilde{\Gamma}^{(2)}_{\bar{\psi}\psi,\mathrm{QCD}}\tilde{\Gamma}^{(2)-1}_{\bar{\psi}\psi,\mathrm{kin}}\tilde{\Gamma}^{(2)}_{\bar{\psi}\psi,\mathrm{FF}}\tilde{\Gamma}^{(2)-1}_{\bar{\psi}\psi,\mathrm{kin}}.
\end{aligned}
$$

Here $\tilde{\Gamma}^{(2)}_{\bar{\psi}\psi,\mathrm{QCD}}$ indicates the part of $\tilde{\Gamma}^{(2)}_{\bar{\psi}\psi}$ derived from the QCD fermion-gluon interaction.

The above terms seem simple, but actually they are tedious. Because, for example, there are 12 terms in $\tilde{\Gamma}^{(2)}_{\bar{\psi}\psi,\mathrm{FF}}$, so it is too lengthy to write all terms explicitly.

APPENDIX B: CONVENTIONS AND IDENTITIES

With the convention $\{\gamma^\mu,\gamma^\nu\}=-2\delta^{\mu\nu}I_{4\times4}$ and d-dimensional Euclidian spacetime,

$$\gamma^\mu\gamma^\mu=(-d)I_{4\times4}, \tag{B1}$$

$$\gamma^\mu\gamma^\rho\gamma^\mu=(d-2)\gamma^\rho, \tag{B2}$$

$$\gamma^\mu\gamma^\rho\gamma^\nu+\gamma^\nu\gamma^\rho\gamma^\mu=2\delta^{\mu\nu}\gamma^\rho-2\delta^{\rho\nu}\gamma^\mu-2\delta^{\rho\mu}\gamma^\nu, \tag{B3}$$

$$\not{l}\not{l}=(-l^2)I_{4\times4}, \tag{B4}$$

$$\operatorname{Tr}(\gamma^\mu\gamma^\nu)=-4\delta^{\mu\nu}. \tag{B5}$$

Color matrix products include

$$t^at^bt^a=\left[C_2(r)-\frac{1}{2}C_2(G)\right]t^b, \tag{B6}$$

$$t^at^bt^et^at^b=\left[C_2(r)-\frac{1}{2}C_2(G)\right][C_2(r)-C_2(G)]t^e, \tag{B7}$$

$$f^{abc}t^bt^c=\frac{1}{2}iC_2(G)t^a. \tag{B8}$$

For fundamental representation of $SU(N_c)$,

$$t^at^b=\frac{1}{2N_c}\delta^{ab}+\frac{1}{2}(if^{abc}+d^{abc})t^c, \tag{B9}$$

$$t^at^b+t^bt^a=\frac{1}{N_c}\delta^{ab}+d^{abc}t^c, \tag{B10}$$

$$d^{acd}d^{bcd}=\frac{N_c^2-4}{N_c}\delta^{ab}, \tag{B11}$$

$$d^{abc}t^bt^c=\left[2C_2(r)-\frac{1}{2}C_2(G)-\frac{1}{N_c}\right]t^a, \tag{B12}$$

$$d^{aac}t^c=0, \tag{B13}$$

$$f^{abe}d^{cde}=0, \tag{B14}$$

$$t^et^at^bt^e=\frac{1}{4N_c}\delta^{ab}C_2(G)+t^at^b\left[C_2(r)-\frac{1}{2}C_2(G)\right], \tag{B15}$$

$$\operatorname{Tr}(t^at^bt^c)=\frac{1}{2}if^{abc}C(r), \tag{B16}$$

$$f^{auv}f^{bvw}f^{cwu}=\frac{1}{2}C_2(G)f^{abc}. \tag{B17}$$

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