Fluctuations of the initial color fields in high-energy heavy-ion collisions

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In the color glass condensate approach to the description of high-energy heavy-ion collisions, one needs to superimpose small random Gaussian distributed fluctuations to the classical background field in order to resum the leading secular terms that result from the Weibel instability, which would otherwise lead to pathological results beyond leading order. In practical numerical simulations, one needs to know this spectrum of fluctuations at a proper time $\tau \ll Q_s^{-1}$ shortly after the collision, in the Fock-Schwinger gauge $\mathcal{A}^\tau = 0$. In this paper, we derive these fluctuations from first principles by solving the Yang-Mills equations linearized around the classical background, with plane wave initial conditions in the remote past. We perform the intermediate steps in light-cone gauge, and we convert the results to the Fock-Schwinger gauge at the end. We obtain simple and explicit formulas for the fluctuation modes.

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I. INTRODUCTION

One of the outstanding theoretical problems in high-energy heavy-ion collisions is the understanding from first principles of the pressure isotropization and possibly the thermalization of the gluonic matter produced in these collisions.

From RHIC and LHC data, there is ample evidence that the expansion and cooling of this matter is well described by relativistic hydrodynamics [1–4] with a very small viscosity (characterized by a viscosity to entropy density ratio, η/s, that is fairly close to the value $1/4\pi$ obtained in the strong coupling limit of some QCD-like theories [5], and that has been conjectured to be a lower bound). This good agreement also suggests that the anisotropy between the transverse and longitudinal (with respect to the collision axis) pressures is not too large, because otherwise the viscous corrections could be important and spoil this agreement. However, understanding from first principles why the hydrodynamics models work so well has proven very challenging until now.

Moreover, there is also a vast amount of data, ranging from deep inelastic scattering to proton-nucleus and nucleus-nucleus collisions, supporting the idea of gluon saturation in high-energy collisions involving hadrons or nuclei [6–8]. In this regime, the gluon density in the projectiles becomes very large, leading to important nonlinear corrections in the evolution of the gluon distribution with energy. These nonlinear effects dynamically generate a dimensionful scale, the saturation momentum Q_s, which controls the scattering [9,10]. The gluon occupation number is nonperturbatively large, of order $1/\alpha_s$, for transverse momenta below Q_s, and decreases rapidly above this scale. The saturation momentum increases with the energy of the collision, to reach values of order $Q_s \approx 1$–2 GeV for nuclei at LHC energies. Since the value of the strong coupling α_s at such scales is around $\alpha_s \approx 0.3$, one may expect to be able to describe these collisions in the color glass condensate (CGC) effective theory [8,11–13], which describes the physics of gluon saturation at weak coupling.[1]

For this reason, the CGC appears to be a well-suited framework in order to try to explain the early isotropization of the system. The state of the system just after such a collision has been calculated at leading order (LO) in α_s in the CGC framework [14], and one finds that its energy-momentum tensor is very anisotropic, with a negative longitudinal pressure exactly opposite to the energy density (a trivial consequence of the fact that the chromo-electric and chromomagnetic fields are parallel to the collision axis just after the collision [15]). At leading order, the subsequent time evolution never leads to the isotropization of the stress tensor [16–20]. But it has been noticed long ago that the CGC result at leading order is insufficient, because of the existence of instabilities in the classical solutions of the Yang-Mills equations [21–24]: some of the higher order (in α_s) corrections grow exponentially fast with time, and soon become larger than the leading order they are supposed to correct. These instabilities are the manifestation in the CGC framework of the well-known Weibel instabilities in plasmas with an anisotropic particle distribution [25–28]. Moreover, a lot of work suggests that these instabilities could play an important role in driving the system towards isotropization and local thermal equilibrium [29–37].

In the CGC formalism, these instabilities spoil the naive estimates of the order of magnitude of contributions [38–40], since these estimates usually keep track only of the powers of α_s, implicitly assuming that all the numerical prefactors remain of order unity at all times. In the presence of unstable modes, this is no longer true: some of these coefficients will grow exponentially in time, leading

[1] Note that "weakly coupled" does not imply "weakly interacting," nor "perturbative," because of the nonperturbatively large gluon occupation number.

to secular divergences when the time goes to infinity—and making the ordinary loop expansion useless after a finite time of order Q_s^{-1}. An improved power counting that tracks these fast growing terms was proposed in Ref. [41], and it was shown [40–42] that one can resum the fastest growing terms by superimposing random Gaussian fluctuations to the initial condition of the classical Yang-Mills equations, and then averaging over these fluctuations. Thanks to this resummation, one completely tames the secular terms, and the validity of the resummed result is extended to larger times.

As a proof of concept, this resummation was implemented numerically in the case of a ϕ^4 scalar field theory. Although very different from a Yang-Mills theory in many respects, this theory has several similar features: it is scale invariant in $3 + 1$ dimensions at the classical level, and its classical solutions have instabilities (here due to parametric resonance). It was shown in Refs. [43–45] that after performing the Gaussian average over the fluctuations of the initial classical field, the system evolves towards the equilibrium equation of state, and that its transverse and longitudinal pressures become equal in the case of a system expanding in the longitudinal direction.

Moreover, the origin of this resummation scheme (and in particular the fact that it includes the exact NLO result) completely prescribes the ensemble of these fluctuations: their spectrum can be obtained by computing a 2-point correlator in the presence of a nontrivial background field (the solution of Yang-Mills equations at leading order). From the analysis of next-to-leading order corrections done in [39], this 2-point function can be constructed as follows

$$\mathcal{G}^{\mu a,\nu b}(x,y) = \sum_{\lambda,c}\int \frac{d^3\boldsymbol{k}}{(2\pi)^3 2k}\, a^{\mu a}_{\boldsymbol{k}\lambda c}(x)\, a^{\nu b *}_{\boldsymbol{k}\lambda c}(y), \qquad (1)$$

where $a^{\mu a}_{\boldsymbol{k}\lambda c}(x)$ is the solution of the Yang-Mills equations linearized around the classical CGC background, whose initial condition at $x^0 = -\infty$ is a plane wave of momentum $\boldsymbol{k}$, polarization λ and color c. However, this calculation has never been done so far.

In Ref. [41], an alternative way of computing these fluctuations was proposed, based on the existence of an inner product between pairs of these fluctuations (written here in terms of the proper time τ, the rapidity η and the transverse coordinate $\boldsymbol{x}_\perp$),

$$(a_1|a_2) \equiv -i\int d^2\boldsymbol{x}_\perp d\eta\, g_{\mu\nu}\delta_{ab}\big(a_1^{\mu a*}(\tau,\boldsymbol{x}_\perp,\eta)e_2^{\nu b}(\tau,\boldsymbol{x}_\perp,\eta) - e_1^{\mu a*}(\tau,\boldsymbol{x}_\perp,\eta)a_2^{\nu b}(\tau,\boldsymbol{x}_\perp,\eta)\big). \qquad (2)$$

e^μ denotes the electrical field associated to the gauge potential a^μ, defined as:

$$e^i \equiv \tau\partial_\tau a_i, \qquad e^\eta \equiv \tau^{-1}\partial_\tau a_\eta. \qquad (3)$$

The above inner product is conserved when the fluctuations evolve over the classical background. It is also easy to check that the modes obtained by evolving plane waves from the remote past form an orthonormal (with respect to the above inner product) basis of the vector space of fluctuations. It was then suggested that one may avoid solving the linearized equations of motion for the fluctuations from the remote past, and that it could be sufficient to find a complete set of modes that obey the equations of motion locally near a proper time $\tau > 0$ just after the collision has taken place, provided that this set of modes also form an orthonormal basis in the above sense. Solving this alternate problem is simpler because one needs only to find *local* solutions of the linearized equations of motion, instead of global solutions with prescribed initial conditions at $x^0 = -\infty$.

The reasoning in [41] was that if one knows a set of orthonormal modes at the time τ, even if it is not the same set as the one originating from the plane waves, it would generate the same Gaussian ensemble of fluctuations provided that the two basis can be related by a unitary transformation. And it is also clear that unitary transformations preserve the inner product defined in Eq. (2). It turns out that there is a caveat in this argument: there are also nonunitary transformations that preserve the inner product. Such a transformation, when applied to a basis of fluctuation modes, will leave all the inner products unchanged (and thus transform an orthonormal basis into another orthonormal basis) but it will lead to a different Gaussian ensemble of fluctuations.

A very simple example of such a transformation is to multiply all the electrical fields by a constant λ, while at the same time dividing the gauge potentials by the same constant. [2]Obviously this transformation does not change the inner product defined in Eq. (2), but it multiplies the variance of the set of Gaussian fluctuations by λ^2 for the electrical fields, and by λ^{-2} for the gauge potentials. Given the existence of these transformations, one cannot be sure that the set of mode functions obtained in [41] leads to the correct[3] fluctuations. Instead, they should be constructed by evolving the plane waves from $x^0 = -\infty$.

In the present paper, we reconsider this question by going back to the original definition of the 2-point function that controls the Gaussian spectrum of fluctuations, i.e., Eq. (1). Using a gauge fixing inspired from Ref. [46], we explicitly solve the linearized Yang-Mills equations over the leading-order classical background field, with plane waves as the initial condition in the remote past. We obtain

[2]More generally, one may note that the inner product defined in Eq. (2) is the complex version of a simplectic product. It is invariant under all the canonical transformations of the fields and their conjugate momenta, that form a superset of the unitary transformations (where one would apply the same unitary rotation both to the gauge potentials and to the electrical fields).

[3]In the special case where the background field vanishes, the modes found in [41] are indeed the correct ones, as they can easily be related to plane waves. The issue exists only for the case of a nontrivial (i.e., nonpure gauge) background field.

rather simple analytical expressions for these solutions, at a proper small positive time $\tau \ll Q_s^{-1}$ (i.e., just after the collision). We provide the results in the Fock-Schwinger (FS) gauge that is commonly employed in the numerical resolution of the Yang-Mills equations, for a choice of quantum numbers which is appropriate for a numerical implementation on a lattice with a fixed spacing in the rapidity η (as opposed to a discretization with a fixed spacing in the longitudinal coordinate z).

The paper is organized as follows. In Sec. II we recall some well-known results for the solution of the classical Yang-Mills equations in the presence of the two color currents that describe the colliding nuclei. Most of the section is devoted to summarizing the derivation of this solution in the $\mathcal{A}^- = 0$ gauge, originally performed in Ref. [46], and on the gauge transformation that one must perform in order to eventually obtain the result in the Fock-Schwinger gauge. In Sec. III, we follow a similar strategy in order to solve the linearized Yang-Mills equations for a small perturbation propagating over this background field. The calculation is subdivided in several stages, corresponding to the successive encounters of the fluctuation with the two nuclei, followed by a final gauge transformation to go from the $\mathcal{A}^- = 0$ gauge to the Fock-Schwinger gauge. The impatient reader may find the final result in Eqs. (69). Section IV is devoted to concluding remarks, and some more technical material is relegated into several Appendixes.

II. CLASSICAL BACKGROUND FIELD

A. General setup of the problem

Before going into the details of our calculation, let us state the problem we need to solve by listing the equations of motion and current conservation constraints that must be satisfied, as well as the boundary conditions that are appropriate in applications to heavy-ion collisions. Here, we list these equations in a generic form that is valid in any gauge. As we shall see later, specific gauge choices may lead to some simplifications.

At leading order in the CGC framework, inclusive observables can be expressed in terms of a gauge field that obeys the classical Yang-Mills equations and that vanishes in the remote past (i.e., before the collision),

$$[\mathcal{D}_\mu, \mathcal{F}^{\mu\nu}] = J^\nu, \qquad [\mathcal{D}_\mu, J^\mu] = 0, \tag{4}$$

$$\lim_{t\to-\infty} \mathcal{F}^{\mu\nu} = 0, \qquad \lim_{t\to-\infty} J^\nu = \delta^{\nu-}\rho_1 + \delta^{\nu+}\rho_2. \tag{5}$$

On the left are the equations obeyed by the gauge potential (or equivalently the field strength $\mathcal{F}^{\mu\nu}$), and on the right are the equations satisfied by the external current J^ν. In the remote past, it is given simply in terms of the two functions ρ_1 and ρ_2 that represent the color charge distribution in the two nuclei before the collision. However, since its conservation equation involves a covariant derivative, this current can be modified during the collision by the radiated gauge fields. This means that in general, one must view the Eq. (4) as coupled equations. This problem has been solved long ago in [14,47]. We just briefly remind the reader of the solution in the rest of this section, and we also discuss an alternate way of solving these equations that has been proposed in [46].

When extending the CGC to next-to-leading order, one needs to study small perturbations to the gauge field, more specifically those that behave as plane waves before the collision. Because the gauge field is entangled with the current via the conservation equation, this in general leads to a small perturbation to the current as well.[4] A linearization of the above equations around the LO solution gives

$$[\mathcal{D}_\mu, [\mathcal{D}^\mu, a^\nu] - [\mathcal{D}^\nu, a^\mu]] - ig[\mathcal{F}^{\nu\mu}, a_\mu] = j^\nu,$$
$$\lim_{t\to-\infty} a^\mu = \epsilon^\mu e^{ik\cdot x}, \tag{6}$$

and

$$[\mathcal{D}_\mu, j^\mu] - ig[a_\mu, J^\mu] = 0, \qquad \lim_{t\to-\infty} j^\nu = 0. \tag{7}$$

Note that the change j^ν to the current must vanish in the remote past, since this is before the current could possibly have been altered by the plane wave. Depending on the gauge choice, the bracket $[a_\mu, J^\mu]$ may vanish and therefore the perturbation of the current is identically 0. (But in the collision of two projectiles, the current has both nonvanishing J^+ and J^- components, and none of the light-cone gauges can eliminate this term completely).

B. Reminder of standard results

In the CGC description of heavy-ion collisions, the gauge fields are driven by two color currents $J_1^-(x^+, \boldsymbol{x}_\perp)$ and $J_2^+(x^-, \boldsymbol{x}_\perp)$ that describe the color carried by the fast partons of the two projectiles. These currents are proportional to delta distributions $\delta(x^+)$ and $\delta(x^-)$, respectively. Because of the presence of these singular sources in the classical Yang-Mills equations, one starts the numerical resolution of the field equations of motion slightly above the forward light cone, at some small proper time $\tau > 0$. The evolution of the fields is thus free of these singular sources, but the drawback is that one must know the initial value of the gauge potentials and electrical fields at the starting time τ.

These initial conditions were first obtained in Refs. [14,47] from the known values of these fields below the light cone, by a matching procedure that amounts to

[4]This has a simple physical interpretation: in non-Abelian gauge theories, the incoming plane wave a^μ carries a color. A quantum from this wave can be absorbed by one of the charges that contribute to the current J^μ, thereby altering its color, and therefore changing the current itself.

requesting that all the singularities cancel from the solution. At a proper time $\tau = 0^+$ immediately after the collision, the initial conditions read[5]

$$\begin{aligned}\mathcal{A}^\tau_{\rm FS}(\boldsymbol{x}_\perp) &= 0 \text{ (gauge condition)}\\ \mathcal{A}^i_{\rm FS}(\boldsymbol{x}_\perp) &= \alpha^i_1(\boldsymbol{x}_\perp) + \alpha^i_2(\boldsymbol{x}_\perp)\\ \mathcal{A}^\eta_{\rm FS}(\boldsymbol{x}_\perp) &= \frac{ig}{2}[\alpha^i_1(\boldsymbol{x}_\perp), \alpha^i_2(\boldsymbol{x}_\perp)],\end{aligned} \tag{8}$$

where the fields $\alpha_{1,2}$ are the solutions in light-cone gauge of the classical Yang-Mills equations for a single projectile. For the projectile moving in the $-z$ direction, we have

$$\begin{aligned}\alpha^i_1(x^+, \boldsymbol{x}_\perp) &= \frac{i}{g}\mathcal{U}^\dagger_1(x^+, \boldsymbol{x}_\perp)\partial^i\mathcal{U}_1(x^+, \boldsymbol{x}_\perp),\\ \mathcal{U}_1(x^+, \boldsymbol{x}_\perp) &= \mathrm{T}e^{ig\int_{-\infty}^{x^+} dz^+ A^-_1(z^+, \boldsymbol{x}_\perp)},\end{aligned} \tag{9}$$

where A^-_1 (which can be viewed as the gauge potential of that nucleus in Lorenz gauge) is related to the corresponding color current by

$$-\boldsymbol{\nabla}^2_\perp A^-_1(x^+, \boldsymbol{x}_\perp) = J^-_1(x^+, \boldsymbol{x}_\perp). \tag{10}$$

A similar set of equations relates the field α^i_2 to the color current J^+_2 of the second nucleus. Note that the $x^\pm$ dependence of $\alpha^i_{1,2}$ is relevant only inside the support of the color currents. Outside of these (infinitesimal) regions along the light cones, the Wilson lines $\mathcal{U}_{1,2}$ depend only on the transverse coordinate $\boldsymbol{x}_\perp$. This is why in Eq. (8) all the fields have only a transverse dependence. It is sometimes useful in intermediate steps of the calculations to extend the support of these currents to a small but finite range $0 < x^\pm < \epsilon$. The limit $\epsilon \to 0^+$ is always taken at the end of the calculations, and the final answers will all be given for color currents that have an infinitesimal support.

In Ref. [46], the initial conditions (8) have been rederived by doing all the intermediate calculations in light-cone gauge, where crossing the light cones that support the color currents can be handled more easily. Since this is also the gauge choice that we will adopt for the intermediate steps of our calculation of the fluctuations, we reproduce the main steps of [46] in the rest of this section, in order to outline its key features.

C. Solution in the global light-cone gauge $\mathcal{A}^- = 0$

In the first derivation of the initial fields of Eq. (8), different light-cone gauges were used for describing the two projectiles before they collide, by exploiting the fact that there is no causal contact between them until the collision.

The main modification introduced in [46] is to use a unique light-cone gauge, which is employed globally to treat the two projectiles. In this work, we will choose the $\mathcal{A}^- = 0$ gauge condition for this purpose. This choice breaks the symmetry between the two nuclei. For the nucleus moving in the $+z$ direction, the solution of the Yang-Mills equations in Lorenz gauge,

$$-\boldsymbol{\nabla}^2_\perp A^+_2(x^-, \boldsymbol{x}_\perp) = J^+_2(x^-, \boldsymbol{x}_\perp), \tag{11}$$

fulfills the light-cone gauge condition $\mathcal{A}^- = 0$ and therefore does not need to be transformed further. Therefore, we just take

$$\mathcal{A}^+_2 = A^+_2. \tag{12}$$

This is not the case for the nucleus 1, whose gauge potential in Lorenz gauge has a nonzero minus component. We thus need to perform a gauge transformation,

$$\mathcal{A}^\mu_1 = \Omega^\dagger_1 A^\mu_1 \Omega_1 + \frac{i}{g}\Omega^\dagger_1\partial^\mu\Omega_1. \tag{13}$$

The gauge transformation Ω_1 has to be chosen so that it eliminates the minus component, and it turns out that it should be equal to the Wilson line $\mathcal{U}_1$ introduced earlier in Eq. (9). After the transformation, the nonzero components of the field $\mathcal{A}^\mu_1$ are the transverse ones, and moreover they have the form of a transverse pure gauge[6]

$$\mathcal{A}^i_1 = \frac{i}{g}\mathcal{U}^\dagger_1\partial^i\mathcal{U}_1. \tag{14}$$

(This field is of course identical to the α^i_1 defined in Eq. (9). Here we will denote it $\mathcal{A}^i_1$ for consistency with the notation used for the field of the nucleus 2 and to stress the fact that we are now in a different gauge). Note also that the gauge transformation Ω_1 has no incidence on the field of the first nucleus, since it differs from the identity only at $x^+ > 0$. Figure 1 summarizes the structure of the gauge potentials before and after the gauge transformation Ω_1.

A legitimate question that arises is what is the advantage in treating the two nuclei in such an asymmetric fashion? The reason is only technical: many calculations turn out to be simpler in this mixed description. In order to determine the fields just after the collision [i.e., on the blue surface

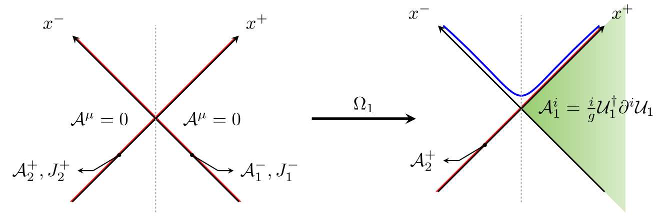

FIG. 1 (color online). The gauge transformation that transforms the Lorenz gauge field A^-_1 into the light-cone gauge field $\mathcal{A}^i_1$. The second nucleus is unaffected by this transformation.

[5]The formulas written without explicit color indices, like Eqs. (8)–(10) in this section, are valid in any representation of the SU(N) algebra.

[6]Note that this is not a global pure gauge, since the gauge rotation Ω_1 has different values at $x^+ < 0$ and $x^+ > \epsilon$.

on the right side of figure (1)], one can independently study what happens on its left and right branches. Indeed, causality prevents the field that travels on the left side of the light cone from interacting with the field that travels on the right side[7] (they travel through regions that are separated by spacelike intervals). The result is the same on the two branches and for infinitesimal x^+ or x^-, it is given by[8] [46]

$$\begin{aligned}\partial^- \mathcal{A}^{+a}(\boldsymbol{x}_\perp) &= (\partial^i \mathcal{U}_2(\boldsymbol{x}_\perp))_{ab} \mathcal{A}_1^{ib}(\boldsymbol{x}_\perp)\\ \mathcal{A}^{ia}(\boldsymbol{x}_\perp) &= \mathcal{U}_{2ab}(\boldsymbol{x}_\perp)\mathcal{A}_1^{ib}(\boldsymbol{x}_\perp)\\ \mathcal{A}^{\pm a}(\boldsymbol{x}_\perp) &= 0.\end{aligned} \tag{15}$$

One sees that $\mathcal{A}^\mu$ only depends on $\boldsymbol{x}_\perp$ on the blue surface of Fig. 1 in the limit where this surface becomes infinitesimally close to the forward light cone. Note also that this solution is not quite symmetric between the nuclei 1 and 2. Indeed, $\partial^- \mathcal{A}^+$ is nonzero, while $\partial^+ \mathcal{A}^-$ is identically zero by virtue of the light-cone gauge condition.

D. Transformation into the Fock-Schwinger gauge

Above the forward light cone, analytical solutions of the classical Yang-Mills equations are not known, and one must resort to numerical techniques. In principle, it would be perfectly doable to solve the equations of motion in the gauge $\mathcal{A}^- = 0$, starting with Eq. (15) as initial conditions.

However, above the forward light cone, the natural coordinates to describe a high-energy collision is the $(\tau, \boldsymbol{x}_\perp, \eta)$ system. And consequently, the Fock-Schwinger gauge condition $\mathcal{A}^\tau = x^- \mathcal{A}^+ + x^+ \mathcal{A}^- = 0$ leads to simpler equations of motion than the light-cone gauge $\mathcal{A}^- = 0$. It is therefore desirable to apply a gauge transformation to the fields of Eq. (15) in order to satisfy the Fock-Schwinger gauge condition.

This transformation can be done in two stages. First of all, let us apply a gauge transformation $\Omega_2 \equiv \mathcal{U}_2$, that changes the field of the second nucleus before the collision from A_2^+ into a transverse pure gauge $\mathcal{A}^i = \alpha_2^i$. By doing this, we arrive at a more symmetric description of the collision, where both nuclei produce a transverse pure gauge field prior to the collision. When applied to the fields of Eq. (15), this transformation gives the following fields at $\tau = 0^+$:

$$\begin{aligned}\partial^- \mathcal{A}^{+a}(\boldsymbol{x}_\perp) &= -ig \mathcal{A}_{2ab}^i(\boldsymbol{x}_\perp)\mathcal{A}_1^{ib}(\boldsymbol{x}_\perp)\\ \mathcal{A}^{ia}(\boldsymbol{x}_\perp) &= \mathcal{A}_1^{ia}(\boldsymbol{x}_\perp) + \mathcal{A}_2^{ia}(\boldsymbol{x}_\perp)\\ \mathcal{A}^{\pm a}(\boldsymbol{x}_\perp) &= 0.\end{aligned} \tag{16}$$

The first of Eq. (16) makes an explicit reference to the components of $\mathcal{A}_2^i$ in the adjoint representation. One can therefore also rewrite it as a commutator,

$$\partial^- \mathcal{A}^+(\boldsymbol{x}_\perp) = ig[\mathcal{A}_1^i(\boldsymbol{x}_\perp), \mathcal{A}_2^i(\boldsymbol{x}_\perp)]. \tag{17}$$

Note that after this first stage, we are still in the light-cone gauge $\mathcal{A}^- = 0$, but with a different choice of the residual gauge fixing compared to Eq. (15). Indeed, since $\mathcal{U}_2$ does not depend on x^+, the gauge transformation generated by $\mathcal{U}_2$ cannot produce a nonzero $\mathcal{A}^-$.

As explained in [46], the final step to get the Fock-Schwinger gauge fields is to perform a gauge transform Ω such that

$$\mathcal{A}^\mu = \Omega \mathcal{A}_{\rm FS}^\mu \Omega^\dagger + \frac{i}{g}\Omega \partial^\mu \Omega^\dagger, \tag{18}$$

where the left-hand side is the gauge potential of Eq. (16) in light-cone gauge $\mathcal{A}^- = 0$, and $\mathcal{A}_{\rm FS}^\mu$ the gauge potential in Fock-Schwinger gauge. The $\mu = -$component of these equations should therefore tell us how to choose Ω in order to achieve the desired transformation. Recalling that $\mathcal{A}_{\rm FS}^\pm = \pm x^\pm \mathcal{A}_{\rm FS}^\eta$, and defining also $\mathcal{A}^+ = x^+ \mathcal{A}^\eta$, one then finds

$$\Omega(\tau, \boldsymbol{x}_\perp) = e^{\frac{ig\tau^2}{2}\mathcal{A}^\eta(\boldsymbol{x}_\perp)}. \tag{19}$$

Note that this formula is only valid for very small values of $\tau > 0$, since it has been obtained solely from the knowledge of the value of the gauge fields at $\tau = 0^+$. Applying then this gauge transformation to the other components of the gauge potential, we recover the known results from [14,47], that we have already recalled in Eq. (8).

III. SMALL FLUCTUATIONS AT $\tau = 0^+$

A. Set up of the problem

We now turn to the problem of computing analytically the small fluctuations a^μ on top of the background field, with plane wave initial conditions in the remote past. We will perform most of the calculation in the same $\mathcal{A}^- = 0$ light-cone gauge that was used in the previous section for the background field, and the gauge transformation to obtain finally the fluctuations in the Fock-Schwinger gauge will be performed at the very end. The setup of the problem in this gauge is illustrated in Fig. 2, where we indicate the structure of the background field in each relevant region of space-time.

In this calculation, we will consider only the propagation of the fluctuations on the right part of the space-time diagram in Fig. 2, i.e., waves that encounter first the nucleus 1 and next the nucleus 2. Naturally, there is a

[7]This is not true anymore in the forward light cone, i.e., after the collision, where the fields on the left and on the right can now interact. Therefore, this simplification can only be used to calculate the fields on the surface $\tau = 0^+$.

[8]Here, we have written all the color indices explicitly to avoid possible ambiguities. For instance, the second equation could equivalently be written as

$$\mathcal{A}^i(\boldsymbol{x}_\perp) \equiv \mathcal{A}^{ia}(\boldsymbol{x}_\perp)t^a = \mathcal{U}_2 \mathcal{A}_1^i(\boldsymbol{x}_\perp)\mathcal{U}_2^\dagger,$$

where all the objects in the right-hand side should be in the same representation as the generators t^a.

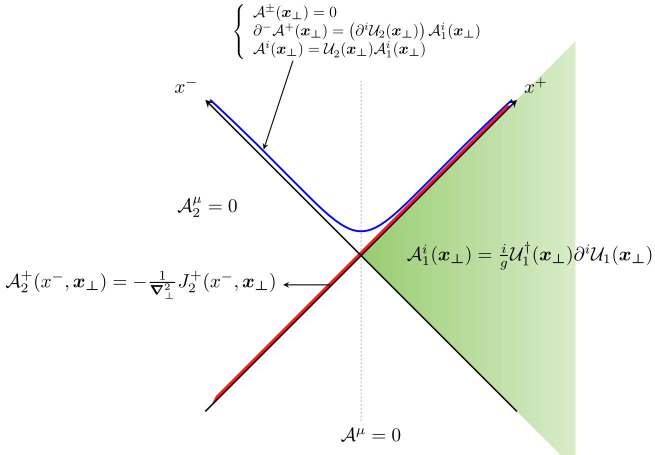

FIG. 2 (color online). Structure of the background field in the light-cone gauge $\mathcal{A}^- = 0$.

second contribution in which this sequence is reversed, but it is easy to guess it by symmetry at the end of the calculation. Note that there is no possibility of cross-talk between these two contributions thanks to causality. This time evolution of a wave starting at $x^0 = -\infty$ can be divided into the following four steps, illustrated in Fig. 3:

(i) evolution in the region $x^\pm < 0$, before the fluctuation encounters any of the nuclei,
(ii) evolution across the trajectory of the first nucleus,
(iii) evolution in the region $x^+ > 0$, $x^- < 0$, between the two nuclei,
(iv) evolution across the trajectory of the second nucleus.

The initial plane wave at $x^0 = -\infty$ is completely characterized by a momentum $\boldsymbol{k}$, a color c, and a polarization λ, and it reads

$$a^{\mu a}_{\boldsymbol{k}\lambda c}(x) \equiv \delta^a_c \epsilon^\mu_{\boldsymbol{k}\lambda} e^{ik\cdot x}. \tag{20}$$

For every momentum $\boldsymbol{k}$, there are two physical polarizations, and we choose their polarization vectors to be mutually orthogonal, $g_{\mu\nu}\epsilon^\mu_{\boldsymbol{k}\lambda}\epsilon^\nu_{\boldsymbol{k}\lambda'} = \delta_{\lambda\lambda'}$. In the rest of this section, we will consistently use the same notation, where the lower indices are the quantum numbers of the initial plane wave at $-\infty$, and the upper indices represent its Lorentz and color structure at the current point x.

B. Step i: evolution in the backward light cone

From now on we will work in the light cone coordinate system.[9] The region $x^\pm < 0$ located below the trajectories of the two nuclei is completely trivial, since none of the nuclei have yet influenced the fluctuation. Thus the equation of motion in this region are simply the free linearized Yang-Mills equations. In the $\mathcal{A}^- = 0$ gauge, the plane waves in this region read

$$\begin{aligned} a^{ia}_{\boldsymbol{k}\lambda c}(x) &= \delta^a_c \epsilon^i_{\boldsymbol{k}\lambda} e^{ik\cdot x} \\ a^{+a}_{\boldsymbol{k}\lambda c}(x) &= \delta^a_c \frac{k^i \epsilon^i_{\boldsymbol{k}\lambda}}{k^-} e^{ik\cdot x} \\ a^{-a}_{\boldsymbol{k}\lambda c}(x) &= 0. \end{aligned} \tag{21}$$

Note that the component ϵ^+ of the polarization vector is constrained by Gauss's law (i.e., the one among the four equations of motion that does not contain the derivative ∂^+, and therefore acts as a constraint at every value of x^-),

$$\partial_\mu a^\mu_{\boldsymbol{k}\lambda c} = 0, \tag{22}$$

which requires $k_\mu \epsilon^\mu = 0$. The two physical polarizations are obtained by choosing the transverse polarization vector ϵ^i, such that $\epsilon^i_{\boldsymbol{k}\lambda}\epsilon^i_{\boldsymbol{k}\lambda'} = \delta_{\lambda\lambda'}$. In the rest of this section, we will often omit the subscripts $\boldsymbol{k}\lambda c$ in the notation for the fluctuation, in order to lighten a bit the notations.

C. Step ii: crossing the trajectory of the first nucleus

The first nontrivial step of the evolution of the fluctuation, represented in Fig. 4, is to cross the trajectory of the first nucleus, on the half-line defined by $x^+ = 0$, $x^- < 0$. Note that here one cannot use the crossing formulas derived in [48], since both the structure of the background field and the gauge condition for the fluctuation are different. The first thing to realize is that the fluctuation has a nonzero a^+ component (since we are in the $\mathcal{A}^- = 0$ gauge), that will induce a precession of the current J_1^- of this nucleus. Therefore, we need to first consider the current conservation equation for the nucleus,

$$\mathcal{D}^{ab}_{1\nu} J_1^{\nu b} = 0. \tag{23}$$

For the background field only, the solution reads

$$\begin{aligned} J_1^{ia} &= J_1^{+a} = 0, \\ J_1^{-a}(x^+, \boldsymbol{x}_\perp) &= \mathcal{U}^\dagger_{1ab}(x^+, \boldsymbol{x}_\perp)\rho_1^b(x^+, \boldsymbol{x}_\perp). \end{aligned} \tag{24}$$

To compute the change of this current j^- induced by the component a^+ of the incoming fluctuation, we need to correct Eq. (23) to linear order, which gives

$$\partial^+ \delta_{ab} j_1^{-b} = ig a^+_{ab} J_1^{-b}. \tag{25}$$

Recalling the fact that J_1^- does not depend on x^-, this equation is solved by

$$j_1^{-a}(x) = -ig J^-_{1ab}(x^+, \boldsymbol{x}_\perp) \frac{1}{\partial^+} a^{+b}(x^+ = 0, x^-, \boldsymbol{x}_\perp). \tag{26}$$

The operator $1/\partial^+$ should be understood as an integration with respect to x^-. We can now write the linearized Yang-Mills equations that drive the evolution of the fluctuation across the infinitesimal region supporting the sources of the first nucleus,

$$\mathcal{D}^{ab}_\mu(\mathcal{D}^{\mu bc} a^{\nu c} - \mathcal{D}^{\nu bc} a^{\mu c}) - ig \mathcal{F}^{\nu\mu ab} a^b_\mu = j^{\nu a}. \tag{27}$$

If $0 < x^+ < \epsilon$ is the range where the sources of this nucleus are nonzero, then the field strength $\mathcal{F}^{\nu\mu}$ of the background

[9] We will translate our expressions in the $(\tau, \eta, \boldsymbol{x}_\perp)$ coordinate system only in Sec. III G, when the fluctuation reaches the forward light cone.

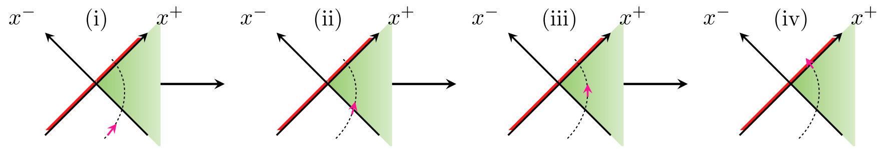

FIG. 3 (color online). The four steps in the evolution of a fluctuation from $x^0 = -\infty$ to the forward light cone. See, in order, Secs. III B, III C, III D, and III E.

gauge potential is identically zero for $x^+ > \epsilon$, while inside the strip $0 < x^+ < \epsilon$, its only nonzero component is

$$\mathcal{F}^{-i} = \partial^- \mathcal{A}_1^i. \tag{28}$$

This allows the following simplifications of Eq. (27):

$$\begin{aligned}
&-\mathcal{D}_{1\mu}^{ab}(\partial^- a^{\mu b}) - ig(\partial^- \mathcal{A}_{1\mu}^{ab})a^{\mu b} = j_1^{-a}\\
&(\delta^{ab} 2\partial^-\partial^+ - \mathcal{D}_1^{iac}\mathcal{D}_1^{icb})a^{+b} - \partial^+(\partial^- a^{+a} - \mathcal{D}_1^{iab}a^{ib}) = 0\\
&(2\delta^{ab}\partial^-\partial^+ - \mathcal{D}_1^{iac}\mathcal{D}_1^{icb})a^{jb} - \partial^-\mathcal{D}_1^{jab}a^{+b}\\
&\quad + \mathcal{D}_1^{iac}\mathcal{D}_1^{jcb}a^{ib} + ig(\partial^-\mathcal{A}_1^{jab})a^{+b} = 0.
\end{aligned} \tag{29}$$

Since we only want to evolve the fluctuation from $x^+ = 0$ to $x^+ = \epsilon$, we are interested only in the terms of these equations that can potentially be of order ϵ^{-1} (i.e., they would behave as $\delta(x^+)$ in the limit $\epsilon \to 0^+$) and therefore lead to a finite variation of the fluctuation. Let us recall that $\mathcal{A}^i$ has a finite jump in this strip, and therefore the derivative $\partial^-\mathcal{A}^i$ behaves as ϵ^{-1}. The induced current j_1^- behaves similarly, since it is proportional to the current J_1^-.

The first of Eq. (29) has no ∂^+ derivative and can be seen as a constraint at fixed x^-; it is nothing but Gauss's law for the small fluctuation in this gauge. More explicitly, it reads

$$\partial^-(\partial^- a^{+a} - \mathcal{D}_1^{iab}a^{ib}) = 2ig(\partial^-\mathcal{A}_1^{iab})a^{ib} - j_1^{-a}, \tag{30}$$

which implies that the combination $\partial^- a^{+a} - \mathcal{D}^{iab}a^{ib}$ changes by a finite amount when going from $x^+ = 0$ to $x^+ = \epsilon$. Therefore, the second of Eq. (29) does not contain any term proportional to ϵ^{-1}, which implies that a^+ varies infinitesimally between $x^+ = 0$ and $x^+ = \epsilon$. We can now simplify the third equation, by dropping all the terms that are bounded in the limit $\epsilon \to 0$, which leaves only

$$\partial^-\partial^+ a^{ja} = -ig(\partial^-\mathcal{A}_1^{jab})a^{+b}. \tag{31}$$

It is easy to integrate this equation over x^+ from 0 to ϵ, and since a^+ is continuous in the infinitesimal integration domain, it can be taken out of the integral. This leads to

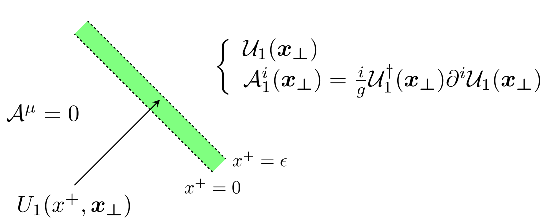

FIG. 4 (color online). Crossing the first nucleus.

$$[a^{ja}]_{x^+=\epsilon} - [a^{ja}]_{x^+=0} = -ig\mathcal{A}_1^{jab}(x^+ = \epsilon, \boldsymbol{x}_\perp)\frac{1}{\partial^+}a_0^{+b}(x), \tag{32}$$

where we have used again the fact that $\mathcal{A}^j$ does not depend on x^-. The subscript 0 in a_0^+ in the right-hand side is used to indicate that this quantity is the free plane wave described in the previous subsection, in Eq. (21).

In order to be complete, we need to calculate also the variation of the derivative $\partial^- a^+$. Indeed, although a^+ itself varies smoothly, this derivative may have a finite change from $x^+ = 0$ to $x^+ = \epsilon$. For that, we integrate Gauss's law over x^+ from 0 to ϵ,

$$\begin{aligned}
&[\partial^- a^{+a}]_{x^+=\epsilon} - [\partial^- a^{+a}]_{x^+=0}\\
&\quad = \int_0^\epsilon \mathrm{d}x^+[\mathcal{D}_1^{iab}\partial^- a^{ib} + ig(\partial^-\mathcal{A}_1^{iab})a^{ib} - j_1^{-a}].
\end{aligned} \tag{33}$$

Using what we have just derived for a^i, and using the equation of motion of the background field, $\mathcal{D}_1^{iab}\partial^-\mathcal{A}_1^{ib} = J_1^{-a}$, we obtain the following result:

$$\begin{aligned}
&[\partial^- a^{+a}]_{x^+=\epsilon} - [\partial^- a^{+a}]_{x^+=0}\\
&\quad = ig\int_0^\epsilon \mathrm{d}x^+(\partial^-\mathcal{A}_1^{iab})\left(a_0^{ib} - \frac{\partial^i}{\partial^+}a_0^{+b}\right)\\
&\quad = ig\mathcal{A}_1^{iab}(x^+ = \epsilon, \boldsymbol{x}_\perp)\left(a_0^{ib} - \frac{\partial^i}{\partial^+}a_0^{+b}\right).
\end{aligned} \tag{34}$$

The formulas (32) and (34), together with the result that a^+ varies smoothly while going from $x^+ = 0$ to $x^+ = \epsilon$, are the central result of this subsection. One can also check Gauss's law at this point, which is a good test of the overall consistency of the solution,

$$\begin{aligned}
&[\partial^-(\partial^- a^{+a} - \mathcal{D}^{iab}a^{ib})]_{x^+=\epsilon}\\
&\quad = ig[\mathcal{D}_1^{iac}\partial^-\mathcal{A}_1^{icb}]_\epsilon\frac{1}{\partial^+}a_0^{+b} + 2ig[\partial^-\mathcal{A}_1^{iab}]_\epsilon a_0^{ib} = 0,
\end{aligned} \tag{35}$$

because the background field $\mathcal{A}_1^i$ is independent of x^+ for $x^+ \geq 0$.

D. Step iii: propagation over the pure gauge $\mathcal{A}_1^i$

In this subsection, we consider the evolution of the fluctuation after it has crossed the trajectory of the first nucleus, and before it reaches the second one. The results of the previous subsection provide the initial conditions for this evolution, and the most direct way to perform the next stage is to write the Green's formula that relates the value of the fluctuation at any point in the quadrant $x^+ > 0$, $x^- < 0$ to this initial data.

Since in this region the Wilson line $\mathcal{U}(\boldsymbol{x}_\perp)$ depends only on $\boldsymbol{x}_\perp$, the background field $\mathcal{A}_1$ is truly a pure gauge, and the linearized equation of motion (27) for the fluctuation can be written as

$$\mathcal{U}^\dagger_{1ac}(\boldsymbol{x}_\perp)(g_{\mu\nu}\square - \partial_\mu\partial_\nu)\mathcal{U}_{1cb}(\boldsymbol{x}_\perp)a^{\mu b}(x) = 0, \quad (36)$$

which means that the gauge-rotated fluctuation $\tilde{a}^{\mu a}(x) \equiv \mathcal{U}_{1ab}(\boldsymbol{x}_\perp)a^{\mu b}(x)$ propagates over the vacuum. One can easily obtain the following Green's formula for this free evolution,[10]

$$\tilde{a}^\mu(x) = i\int_{y^+=0^+} \mathrm{d}y^-\mathrm{d}^2\boldsymbol{y}_\perp\{D_R^{\mu+}(x,y)[\partial_y^\nu\tilde{a}_\nu(y)] - [\partial_\nu^y D_R^{\mu\nu}(x,y)]\tilde{a}^+(y) + D_R^{\mu i}(x,y)\overleftrightarrow{\partial}_y^+\tilde{a}^i(y)\}, \quad (37)$$

where $D_R^{\mu\nu}$ is the free retarded propagator in the light-cone gauge $\mathcal{A}^- = 0$, whose expression in momentum space reads

$$D_R^{\mu\nu} = -\frac{i}{k^2 + ik^0\epsilon}\left(g^{\mu\nu} - \frac{k^\mu n^\nu + k^\nu n^\mu}{n.k + i\epsilon}\right), \quad (38)$$

with $n^+ = 1$, $n^- = n^i = 0$. The following formulas will also prove useful later:

$$\partial_\mu^x D_R^{\mu\nu}(x,y) = -i\delta^{\nu+}\theta(x^+ - y^+)\delta(x^- - y^-)\delta(\boldsymbol{x}_\perp - \boldsymbol{y}_\perp)$$
$$\partial_\mu^y\partial_\mu^x D_R^{\mu\nu}(x,y) = i\delta(x^+ - y^+)\delta(x^- - y^-)\delta(\boldsymbol{x}_\perp - \boldsymbol{y}_\perp). \quad (39)$$

The Green's formula (37) is valid everywhere in the region $x^+ > \epsilon$, $x^- < 0$. One can verify that this formula conserves Gauss's law, as it should. Indeed, since above the $x^+ = \epsilon$ line, $\mathcal{U}_1$ does not depend on x^+, Gauss's law (30) simply becomes

$$\partial^-(\mathcal{D}^\mu_{1ab}a^{\mu b}) = \mathcal{U}_1^{\dagger ab}(\boldsymbol{x}_\perp)\partial^-(\partial^-\tilde{a}^{+b} - \partial^i\tilde{a}^{ib}) = 0, \quad (40)$$

which implies that $\partial_\mu\tilde{a}^\mu$ should be independent of x^+. That this is true can easily be checked thanks to Eqs. (37) and (39).

Some technical results that are necessary in order to calculate $\tilde{a}(x)$ for $x^+ > \epsilon$ are derived in Appendix A. The results can be written in a more compact form by introducing modified polarization vectors defined by

$$\tilde{\epsilon}^i_{\boldsymbol{k}\lambda} \equiv \left(\delta^{ij} - \frac{2k^ik^j}{k^2}\right)\epsilon^j_{\boldsymbol{k}\lambda}. \quad (41)$$

These new polarization vectors satisfy

$$k^i\tilde{\epsilon}^i_{\boldsymbol{k}\lambda} = -k^i\epsilon^i_{\boldsymbol{k}\lambda}, \qquad \sum_{i=1,2}\tilde{\epsilon}^i_{\boldsymbol{k}\lambda}\tilde{\epsilon}^i_{\boldsymbol{k}\lambda'} = \sum_{i=1,2}\epsilon^i_{\boldsymbol{k}\lambda}\epsilon^i_{\boldsymbol{k}\lambda'} = \delta_{\lambda\lambda'}. \quad (42)$$

Thanks to Eq. (37), we obtain

$$\tilde{a}^{ia}_{\boldsymbol{k}\lambda c}(x) = e^{ik^+x^-}\int\frac{\mathrm{d}^2\boldsymbol{p}_\perp}{(2\pi)^2}e^{i\boldsymbol{p}_\perp\cdot\boldsymbol{x}_\perp}\left[e^{i\frac{p^2}{2k^+}x^+}\left(\delta^{ij} - \frac{2p^ip^j}{p^2}\right) + 2p^i\left(\frac{p^j}{p^2} + \frac{k^j}{k^2}\right)\right]\tilde{\mathcal{U}}_1^{ac}(\boldsymbol{p}_\perp + \boldsymbol{k}_\perp)\tilde{\epsilon}^j_{\boldsymbol{k}\lambda}$$

$$\tilde{a}^{+a}_{\boldsymbol{k}\lambda c}(x) = 2k^+e^{ik^+x^-}\int\frac{\mathrm{d}^2\boldsymbol{p}_\perp}{(2\pi)^2}e^{i\boldsymbol{p}_\perp\cdot\boldsymbol{x}_\perp}\left[e^{i\frac{p^2}{2k^+}x^+}\frac{p^i}{p^2} - \left(\frac{p^i}{p^2} + \frac{k^i}{k^2}\right)\right]\tilde{\mathcal{U}}_1^{ac}(\boldsymbol{p}_\perp + \boldsymbol{k}_\perp)\tilde{\epsilon}^i_{\boldsymbol{k}\lambda}, \quad (43)$$

where we denote $k \equiv |\boldsymbol{k}_\perp|$, $p \equiv |\boldsymbol{p}_\perp|$, and where $\tilde{\mathcal{U}}_1(\boldsymbol{k}_\perp)$ is the Fourier transform of $\mathcal{U}_1(\boldsymbol{x}_\perp)$,

$$\tilde{\mathcal{U}}_1(\boldsymbol{k}_\perp) = \int \mathrm{d}^2\boldsymbol{x}_\perp e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}\mathcal{U}_1(\boldsymbol{x}_\perp). \quad (44)$$

Undoing the gauge rotation $\mathcal{U}_1$ to go back to a^μ gives

$$a^{ia}_{\boldsymbol{k}\lambda c}(x) = e^{ik^+x^-}\mathcal{U}_1^{ab\dagger}(\boldsymbol{x}_\perp)\int\frac{\mathrm{d}^2\boldsymbol{p}_\perp}{(2\pi)^2}e^{i\boldsymbol{p}_\perp\cdot\boldsymbol{x}_\perp} \times\left[e^{i\frac{p^2}{2k^+}x^+}\left(\delta^{ij} - \frac{2p^ip^j}{p^2}\right) + 2p^i\left(\frac{p^j}{p^2} + \frac{k^j}{k^2}\right)\right]\tilde{\mathcal{U}}_1^{bc}(\boldsymbol{p}_\perp + \boldsymbol{k}_\perp)\tilde{\epsilon}^j_{\boldsymbol{k}\lambda}, \quad (45)$$

$$a^{+a}_{\boldsymbol{k}\lambda c}(x) = 2k^+e^{ik^+x^-}\mathcal{U}_1^{ab\dagger}(\boldsymbol{x}_\perp)\int\frac{\mathrm{d}^2\boldsymbol{p}_\perp}{(2\pi)^2}e^{i\boldsymbol{p}_\perp\cdot\boldsymbol{x}_\perp}\left[e^{i\frac{p^2}{2k^+}x^+}\frac{p^i}{p^2} - \left(\frac{p^i}{p^2} + \frac{k^i}{k^2}\right)\right]\tilde{\mathcal{U}}_1^{bc}(\boldsymbol{p}_\perp + \boldsymbol{k}_\perp)\tilde{\epsilon}^i_{\boldsymbol{k}\lambda}. \quad (46)$$

These formulas are valid in the entire quadrant $x^+ > \epsilon$, $x^- < 0$. The last step of the evolution is now to let the fluctuation cross the trajectory of the second nucleus. In Appendix B, we perform several consistency checks on the formulas (45) and (46).

E. Step iv: crossing the trajectory of the second nucleus

The propagation of the fluctuation through the second nucleus, depicted in Fig. 5, is very similar to the situation

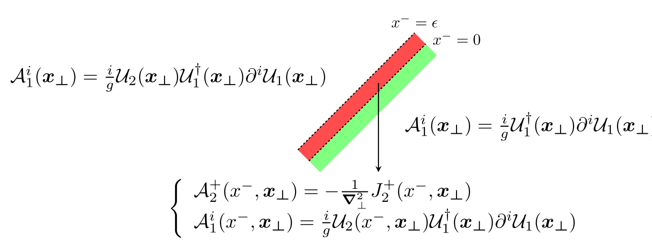

FIG. 5 (color online). Crossing the second nucleus.

[10]The derivation can be found in Ref. [39].

studied in [48]. The only difference is that in [48], the gauge potential of the nucleus had only an $\mathcal{A}^+$ component, proportional to $\delta(x^-)$. In the present situation, the nucleus also has a nonzero $\mathcal{A}^i$. Despite this important difference, the calculation can be done in a very similar way as in [48]. In particular, the fact that we are in the $\mathcal{A}^- = 0$ light-cone gauge prevents any precession of the color current J_2^+ of the nucleus. The linearized Yang-Mills equations for the fluctuation therefore take a simpler form, without any source term in the right-hand side,[11]

$$\mathcal{D}_\mu^{ab}(\mathcal{D}_{bc}^\mu a^{\nu c} - \mathcal{D}_{bc}^\nu a^{\mu c}) - ig\mathcal{F}_{ab}^{\nu\mu} a_\mu^b = 0. \quad (47)$$

For the component $\nu = -$, using the fact that $\mathcal{A}_1^i$ does not depend on x^+, this gives Gauss's law,

$$\partial^-(\partial^- a^{+a} - \mathcal{D}_{1ab}^i a^{ib}) = 0. \quad (48)$$

For $\nu = i$, keeping only terms that have a singular behavior in ϵ^{-1}, we get

$$\partial^- \mathcal{D}_{2ab}^+ a^{ib} = 0, \quad (49)$$

which can be solved by

$$a_{\boldsymbol{k}\lambda c}^{ia}(x^- = \epsilon, x^+, \boldsymbol{x}_\perp) = \mathcal{U}_{2ab}(x^-, \boldsymbol{x}_\perp) a_{\boldsymbol{k}\lambda c}^{ib}(x^- = 0, x^+, \boldsymbol{x}_\perp). \quad (50)$$

Finally, for $\nu = +$, the equation of motion reads

$$\begin{aligned}
&(2\partial^- \mathcal{D}_{2ab}^+ - \mathcal{D}_{1ac}^i \mathcal{D}_{1cd}^i) a^{+b} \\
&\quad - (\partial^- \mathcal{D}_{2ab}^+ a^{+b} - \mathcal{D}_{1ac}^i \mathcal{D}_{2cb}^+ a^{ib}) \\
&\quad + g f^{abc}(\partial^+ \mathcal{A}_{1c}^i - ig\mathcal{A}_{2cd}^+ \mathcal{A}_{1d}^i - \partial^i \mathcal{A}_{2c}^+) a^{ib} = 0.
\end{aligned} \quad (51)$$

Since $\partial^+ \mathcal{A}_1^{ic} = ig\mathcal{A}_{2cd}^+ \mathcal{A}_1^{id}$ in the strip $0 < x^- < \epsilon$, the previous equation can be simplified into

$$\mathcal{D}_{2ab}^+ a^{+b} = ig(\partial^i \mathcal{A}_{2ab}^+) \frac{1}{\partial^-} a^{ib}. \quad (52)$$

The solution of this equation is known (see [48]), and agrees with Gauss's law,

$$\begin{aligned}
&a_{\boldsymbol{k}\lambda c}^{+a}(x^- = \epsilon, x^+, \boldsymbol{x}_\perp) \\
&\quad = \mathcal{U}_{2ab}(\boldsymbol{x}_\perp) a_{\boldsymbol{k}\lambda c}^{+b}(x^- = 0, x^+, \boldsymbol{x}_\perp) \\
&\qquad + (\partial^i \mathcal{U}_{2ab}(\boldsymbol{x}_\perp)) \frac{1}{\partial^-} a_{\boldsymbol{k}\lambda c}^{ib}(x^- = 0^-, x^+, \boldsymbol{x}_\perp).
\end{aligned} \quad (53)$$

It turns out in the end that the equations (50) and (53) are identical to the crossing formulas of [48], despite the presence of a nonvanishing $\mathcal{A}_1^i$. Equations (50) and (53) provide the value of the fluctuation in the light-cone gauge $\mathcal{A}^- = 0$ on the right branch of the light cone, just after the collision. Our next task will be to convert these expressions into the Fock-Schwinger gauge.

F. From light-cone gauge to Fock-Schwinger gauge

Like for the background field itself, the first stage in this process is to go back to the situation where both nuclei are described by transverse pure gauges before the collision. To do so, we first perform a gauge transform $\mathcal{U}_2^\dagger$ which trivially affects the small fluctuations,

$$a^\mu \to \alpha^\mu \equiv \mathcal{U}_2 a^\mu \mathcal{U}_2^\dagger. \quad (54)$$

After this first gauge transformation, the fluctuations on the right branch of the light cone, therefore, read

$$\begin{aligned}
\alpha_{\boldsymbol{k}\lambda c}^{+a}(x^- = \epsilon) &= a_{\boldsymbol{k}\lambda c}^{+a}(x^- = 0) \\
&\quad - ig\mathcal{A}_{2ab}^i(\boldsymbol{x}_\perp) \frac{1}{\partial^-} a_{\boldsymbol{k}\lambda c}^{ib}(x^- = 0) \\
\alpha_{\boldsymbol{k}\lambda c}^{ia}(x^- = \epsilon) &= a_{\boldsymbol{k}\lambda c}^{ia}(x^- = 0).
\end{aligned} \quad (55)$$

In order to check Gauss's law at this point, one should recall that after this transformation the transverse background gauge potential is now $\mathcal{A}^i = \mathcal{A}_1^i + \mathcal{A}_2^i$, and therefore the covariant derivative in Eq. (48) should be modified accordingly.

To go to the Fock-Schwinger gauge, we must perform one last gauge transformation W, in analogy with the transformation of Eq. (18) for the background field. The crucial point to note is that W must differ from Ω (where Ω is the gauge transformation used to transform the background field into the Fock-Schwinger gauge), since the fluctuation depends on η. But since the fluctuation is small compared to the background field, W should be close to Ω, $W \equiv \Omega + ig\omega$, with ω of order unity. The action of this gauge transformation on the background field and on the small fluctuations can be split as follows:

$$\begin{aligned}
\mathcal{A}^\mu &= \Omega \mathcal{A}_{\rm FS}^\mu \Omega^\dagger + \frac{i}{g} \Omega \partial^\mu \Omega^\dagger \\
\alpha^\mu &= \Omega \alpha_{\rm FS}^\mu \Omega^\dagger + \Omega \partial^\mu \omega + ig(\omega \mathcal{A}_{\rm FS}^\mu \Omega^\dagger - \Omega \mathcal{A}_{\rm FS}^\mu \omega),
\end{aligned} \quad (56)$$

where the light-cone gauge quantities are in the left-hand side, and the Fock-Schwinger gauge quantities carry a FS subscript. When $\tau \to 0^+$, the gauge rotation Ω goes to 1, and therefore the transformation of the fluctuation simply becomes

$$\alpha^{\mu a} = \alpha_{\rm FS}^{\mu a} + \mathcal{D}_{\rm FS}^{\mu ab} \omega^b, \quad (57)$$

where $\mathcal{D}_{\rm FS}$ is the covariant derivative constructed with the background field near $\tau = 0^+$ in the Fock-Schwinger gauge. Like in the case of the background field, we will obtain ω by requesting that $\alpha^- = 0$.

Like for the background field, let us parametrize the $\pm$ components of the fluctuations as follows:

[11] $\mathcal{D}^\mu \equiv \partial^\mu - ig\mathcal{A}^\mu$ is the generic notation for the covariant derivative in light-cone gauge, while $\mathcal{D}_1^i \equiv \partial^i - ig\mathcal{A}_1^i$ and $\mathcal{D}_2^+ \equiv \partial^+ - ig\mathcal{A}_2^+$ are built specifically with the fields $\mathcal{A}_1^i$ and $\mathcal{A}_2^+$, respectively.

$$\alpha^{\pm}_{\rm FS} \equiv \pm x^{\pm}\alpha^{\eta}_{\rm FS}, \qquad \alpha^{+} \equiv x^{+}\alpha^{\eta}. \tag{58}$$

From the component $\mu = -$ of Eq. (57), we get

$$x^{-}\alpha^{a\eta}_{\rm FS} = \partial^{-}\omega^{a} - ig\mathcal{A}^{-ab}_{\rm FS}\omega^{b}, \tag{59}$$

and in terms of the coordinates τ, η, we obtain

$$\tau\alpha^{a\eta}_{\rm FS} = \partial_\tau\omega^a + \frac{1}{\tau}\partial_\eta\omega^a + ig\tau\mathcal{A}^{ab\eta}_{\rm FS}\omega^b. \tag{60}$$

Injecting this into the $\mu = +$ equation and using $x^+ = \tau e^{\eta}/\sqrt{2}$ gives

$$\omega^a(\tau,\eta,\boldsymbol{x}_\perp) = \frac{1}{\sqrt{2}}\int_0^\tau {\rm d}\tau' e^{-\eta}\alpha^{+a}(\tau',\eta,\boldsymbol{x}_\perp), \tag{61}$$

where we recall that α^+ is given by Eqs. (55) and (46). In terms of this ω, the gauge transformation formulas for the fluctuation can also be written as

$$\begin{aligned}\alpha^{ia}_{\rm FS} &= \alpha^{ia} - \mathcal{D}^{i}_{ab}\omega^b,\\ \alpha^{\eta a}_{\rm FS} &= \frac{1}{2}\alpha^{\eta a} + \frac{ig}{2}\mathcal{A}^{\eta}_{ab}\omega^b + \frac{1}{\tau^2}\partial_\eta\omega^a.\end{aligned} \tag{62}$$

(We recall that all the fields and fluctuations without the FS subscript are in the light-cone gauge $\mathcal{A}^- = 0$.) It turns out that the terms $\mathcal{D}^i_{ab}\omega^b$ and $\frac{ig}{2}\mathcal{A}^\eta_{ab}\omega^b$ will not contribute at lowest nonzero order in τ.

G. Expression in terms of the conjugate momentum to η

Equations (45), (46), (55), (61), and (62) provide the value in Fock-Schwinger gauge, just above the forward light cone, for any fluctuation that started as a free plane wave in the remote past. However, for practical uses of these fluctuations in heavy-ion collisions, the numerical implementation will be performed on a lattice that has a fixed extent in the rapidity η, which is more appropriate for the description of a system in rapid expansion in the longitudinal direction.

Anticipating the use of this system of coordinates, it would be desirable to have an ensemble of fluctuations labeled by a quantum number which is the conjugate momentum of rapidity (that we shall denote ν in the following), instead of the conjugate momentum k_z of the Cartesian longitudinal coordinate z. Obviously, since the two sets of fluctuations both are a basis of the vector space of all fluctuations, there must be a linear transformation to obtain one from the other. The transformation that goes from the basis of fluctuations labeled by k_z to the basis of fluctuations labeled by ν reads

$$\alpha^{\mu}_{\boldsymbol{k}_\perp\nu\lambda c}(x) = \int_{-\infty}^{+\infty} dy e^{i\nu y}\alpha^{\mu}_{\boldsymbol{k}\lambda c}(x), \tag{63}$$

where $y \equiv \log(k^+/k^-)/2$ is the momentum rapidity. Indeed, using the fact that the problem is invariant under boosts in the longitudinal direction, the fluctuation $\alpha^{\mu}_{\boldsymbol{k}\lambda c}(x)$ must depend on y and η only via the difference $y-\eta$. Changing the integration variable y in favor of $y' \equiv y - \eta$, we readily see that the η dependence of the left-hand side of Eq. (63) is of the form $\exp(i\nu\eta)$. This shows that this transformation indeed leads to fluctuations that have a well-defined conjugate momentum ν to the rapidity η.

We have now all the ingredients to compute the final form of the fluctuations.[12] After a straightforward but tedious calculation, we obtain the following formulas for the fluctuations and the corresponding electrical fields,

$$\begin{aligned}\alpha^{\rm R\it ia}_{{\rm FS}\boldsymbol{k}_\perp\nu\lambda c}(\tau,\eta,\boldsymbol{x}_\perp) &= F^{+,ia}_{\boldsymbol{k}_\perp\nu\lambda c}(\tau,\eta,\boldsymbol{x}_\perp)\\ e^{\rm R\it ia}_{{\rm FS}\boldsymbol{k}_\perp\nu\lambda c}(\tau,\eta,\boldsymbol{x}_\perp) &= -i\nu F^{+,ia}_{\boldsymbol{k}_\perp\nu\lambda c}(\tau,\eta,\boldsymbol{x}_\perp)\\ \alpha^{\rm R\eta\it a}_{{\rm FS}\boldsymbol{k}_\perp\nu\lambda c}(\tau,\eta,\boldsymbol{x}_\perp) &= \frac{1}{2+i\nu}\mathcal{D}^{iab}_{\rm FS}F^{+,ib}_{\boldsymbol{k}_\perp\nu\lambda c}(\tau,\eta,\boldsymbol{x}_\perp)\\ e^{\rm R\eta\it a}_{{\rm FS}\boldsymbol{k}_\perp\nu\lambda c}(\tau,\eta,\boldsymbol{x}_\perp) &= -\mathcal{D}^{iab}_{\rm FS}F^{+,ib}_{\boldsymbol{k}_\perp\nu\lambda c}(\tau,\eta,\boldsymbol{x}_\perp),\end{aligned} \tag{64}$$

(the superscript R indicates that we have only the contribution due to the wave that propagates on the right of the light cone) where we denote

$$\begin{aligned}&F^{+,ia}_{\boldsymbol{k}_\perp\nu\lambda c}(\tau,\eta,\boldsymbol{x}_\perp)\\ &\quad\equiv\Gamma(-i\nu)e^{+\frac{\nu\pi}{2}}e^{i\nu\eta}\mathcal{U}^{\dagger}_{1ab}(\boldsymbol{x}_\perp)\tilde{\epsilon}^{j}_{\boldsymbol{k}\lambda}\int\frac{{\rm d}^2\boldsymbol{p}_\perp}{(2\pi)^2}e^{i\boldsymbol{p}_\perp\cdot\boldsymbol{x}_\perp}\\ &\qquad\times\tilde{\mathcal{U}}_{1bc}(\boldsymbol{p}_\perp+\boldsymbol{k}_\perp)\left(\frac{p_\perp^2\tau}{2k_\perp}\right)^{+i\nu}\left[\delta^{ji}-\frac{2p^j_\perp p^i_\perp}{p^2_\perp}\right].\end{aligned} \tag{65}$$

In order to write the η components of the gauge potential and of the electrical field as a covariant derivative acting on the function $F^{+,ia}_{\boldsymbol{k}_\perp\nu\lambda c}(\tau,\eta,\boldsymbol{x}_\perp)$, we have used the following identity:

$$\mathcal{D}^{i}_{1ab}\mathcal{U}^{\dagger}_{1bc} = \mathcal{D}^{i}_{2ab}\mathcal{U}^{\dagger}_{2bc} = 0. \tag{66}$$

These formulas (to be completed by Eq. (67), which give the result when the fluctuations have propagated on the other side of the light cone) are the central result of this work. They provide analytical expressions for fluctuations with plane wave initial conditions in the remote past, after they have propagated over the classical background field created in a heavy-ion collision, in the Fock-Schwinger gauge and with a set of quantum numbers appropriate for a discretization on a lattice with a fixed spacing in the rapidity η. Note that in the first formula, we have written

[12]The following integral is also useful in order to perform the integration over the rapidity y,

$$\int {\rm d}y e^{i\nu y}e^{y-\eta}e^{i\frac{\tau p^2}{2k}e^{\eta-y}} = -ie^{i\nu\eta}\left(\frac{p}{k}\right)^{i\nu+1}\Gamma(-1-i\nu)e^{\frac{\nu\pi}{2}}\left(\frac{\tau p}{2}\right)^{1+i\nu}.$$

α^η (with the Lorentz index up). The corresponding α_η (with the Lorentz index down) is obtained by multiplying by $-\tau^2$ and therefore becomes very small when $\tau \to 0^+$.

In the limit $\tau \to 0^+$, the fluctuations of the potentials and electrical fields behave in the same manner as their counterpart in the background field, except for the transverse electrical field e^i. The background field has a transverse electrical field $\mathcal{E}^i$ that vanishes like τ^2, while its fluctuations e^i go to a nonzero limit when $\tau \to 0^+$ for all the modes $\nu \neq 0$.

The dependence of these fluctuations on the classical background field is known explicitly, and is entirely contained in the Wilson lines that appear in the function $F^{+,ia}_{\boldsymbol{k}_\perp \nu\lambda c}(\tau, \eta, \boldsymbol{x}_\perp)$, and in the covariant derivatives that appear in some of the Eq. (64). From this function, it is easy to obtain all the components of the fluctuations and the corresponding electrical fields thanks to Eq. (64). The numerical evaluation of $F^{+,ia}_{\boldsymbol{k}_\perp \nu\lambda c}(\tau, \eta, \boldsymbol{x}_\perp)$ is rather straightforward, since it only involves a pair of Fourier transforms.[13]

H. Contribution from the propagation on the left

Equation (64) have been derived by evolving the small fluctuations in the right part of the light cone (crossing first the nucleus 1, and then crossing the nucleus 2). To this contribution should be added the contribution obtained by the other ordering of the encounters with the two nuclei, i.e., when the fluctuation propagates on the left side of the light cone. This extra contribution is completely independent from the one we have just calculated, since by causality they cannot talk to each other.

This new contribution can be obtained by repeating the same steps as the ones employed so far, but now working in the $\mathcal{A}^+ = 0$ gauge. This leads to

$$\begin{aligned}
\alpha^{\mathrm{L}ia}_{\mathrm{FS}\boldsymbol{k}_\perp \nu\lambda c}(\tau, \eta, \boldsymbol{x}_\perp) &= F^{-,ia}_{\boldsymbol{k}_\perp \nu\lambda c}(\tau, \eta, \boldsymbol{x}_\perp)\\
e^{\mathrm{L}ia}_{\mathrm{FS}\boldsymbol{k}_\perp \nu\lambda c}(\tau, \eta, \boldsymbol{x}_\perp) &= i\nu F^{-,ia}_{\boldsymbol{k}_\perp \nu\lambda c}(\tau, \eta, \boldsymbol{x}_\perp)\\
\alpha^{\mathrm{L}\eta a}_{\mathrm{FS}\boldsymbol{k}_\perp \nu\lambda c}(\tau, \eta, \boldsymbol{x}_\perp) &= -\frac{1}{2-i\nu}\mathcal{D}^{iab}_{\mathrm{FS}} F^{-,ib}_{\boldsymbol{k}_\perp \nu\lambda c}(\tau, \eta, \boldsymbol{x}_\perp)\\
e^{\mathrm{L}\eta a}_{\mathrm{FS}\boldsymbol{k}_\perp \nu\lambda c}(\tau, \eta, \boldsymbol{x}_\perp) &= \mathcal{D}^{iab}_{\mathrm{FS}} F^{-,ib}_{\boldsymbol{k}_\perp \nu\lambda c}(\tau, \eta, \boldsymbol{x}_\perp),
\end{aligned} \tag{67}$$

where the superscript L indicates that this is the partial wave that has propagated on the left part of the light cone, and where we now denote

$$\begin{aligned}
&F^{-,ia}_{\boldsymbol{k}_\perp \nu\lambda c}(\tau, \eta, \boldsymbol{x}_\perp)\\
&\quad \equiv \Gamma(+i\nu) e^{-\frac{\nu\pi}{2}} e^{i\nu\eta}\, \mathcal{U}^\dagger_{2ab}(\boldsymbol{x}_\perp)\tilde{\epsilon}^j_{\boldsymbol{k}\lambda} \int \frac{d^2\boldsymbol{p}_\perp}{(2\pi)^2} e^{i\boldsymbol{p}_\perp\cdot \boldsymbol{x}_\perp}\\
&\qquad \times \tilde{\mathcal{U}}_{2bc}(\boldsymbol{p}_\perp + \boldsymbol{k}_\perp)\left(\frac{p_\perp^2 \tau}{2k_\perp}\right)^{-i\nu}\left[\delta^{ji} - \frac{2p^j_\perp p^i_\perp}{p^2_\perp}\right].
\end{aligned} \tag{68}$$

I. Complete result

Let us finally add up the results of Eqs. (64) and (67) in order to obtain the complete value of the fluctuations just above the forward light cone,

$$\begin{aligned}
\alpha^{ia}_{\mathrm{FS}\boldsymbol{k}_\perp \nu\lambda c}(\tau, \eta, \boldsymbol{x}_\perp) &= F^{+,ia}_{\boldsymbol{k}_\perp \nu\lambda c}(\tau, \eta, \boldsymbol{x}_\perp) + F^{-,ia}_{\boldsymbol{k}_\perp \nu\lambda c}(\tau, \eta, \boldsymbol{x}_\perp)\\
e^{ia}_{\mathrm{FS}\boldsymbol{k}_\perp \nu\lambda c}(\tau, \eta, \boldsymbol{x}_\perp) &= -i\nu\Big(F^{+,ia}_{\boldsymbol{k}_\perp \nu\lambda c}(\tau, \eta, \boldsymbol{x}_\perp)\\
&\qquad - F^{-,ia}_{\boldsymbol{k}_\perp \nu\lambda c}(\tau, \eta, \boldsymbol{x}_\perp)\Big)\\
\alpha^{\eta a}_{\mathrm{FS}\boldsymbol{k}_\perp \nu\lambda c}(\tau, \eta, \boldsymbol{x}_\perp) &= \mathcal{D}^{iab}_{\mathrm{FS}}\Bigg(\frac{F^{+,ib}_{\boldsymbol{k}_\perp \nu\lambda c}(\tau, \eta, \boldsymbol{x}_\perp)}{2+i\nu} -\\
&\qquad \frac{F^{-,ib}_{\boldsymbol{k}_\perp \nu\lambda c}(\tau, \eta, \boldsymbol{x}_\perp)}{2-i\nu}\Bigg)\\
e^{\eta a}_{\mathrm{FS}\boldsymbol{k}_\perp \nu\lambda c}(\tau, \eta, \boldsymbol{x}_\perp) &= -\mathcal{D}^{iab}_{\mathrm{FS}}\Big(F^{+,ia}_{\boldsymbol{k}_\perp \nu\lambda c}(\tau, \eta, \boldsymbol{x}_\perp)\\
&\qquad - F^{-,ia}_{\boldsymbol{k}_\perp \nu\lambda c}(\tau, \eta, \boldsymbol{x}_\perp)\Big),
\end{aligned} \tag{69}$$

with the functions $F^{+,ia}_{\boldsymbol{k}_\perp \nu\lambda c}$ and $F^{-,ia}_{\boldsymbol{k}_\perp \nu\lambda c}$ defined in Eqs. (65) and (68), respectively. These formulas are the analogue for gluons of the Eq. (14) of Ref. [49], that had been derived for the quark mode functions. Analogous formulas are also known for leptons in the QED electromagnetic background created by two colliding electrical charges [50].

J. Various checks

The most obvious check one can perform is that the fluctuations given by Eq. (69) satisfy the equations of motion at lowest order in τ, i.e., in an infinitesimal domain $\tau \ll Q_s^{-1}$ above the forward light cone,

$$\begin{aligned}
\frac{1}{\tau}\partial_\tau\left(\frac{1}{\tau}\partial_\tau\right)\alpha^a_{\mathrm{FS}\eta} + \frac{i\nu}{\tau^2}\mathcal{D}^{iab}_{\mathrm{FS}}\alpha^{ib}_{\mathrm{FS}} &= 0,\\
\left[\frac{1}{\tau}\partial_\tau(\tau\partial_\tau) + \frac{\nu^2}{\tau^2}\right]\alpha^{ia}_{\mathrm{FS}} &= 0.
\end{aligned} \tag{70}$$

A more stringent test is to check whether Gauss's law is still satisfied, because it involves a delicate interplay between the background field and the fluctuation. We indeed find that

$$\partial_\eta e^\eta_{\mathrm{FS}} - \mathcal{D}^i_{\mathrm{FS}} e^i_{\mathrm{FS}} = 0. \tag{71}$$

Note that the terms in $ig\mathcal{E}^\eta_{\mathrm{FS}}\alpha_{\mathrm{FS}\eta} - ig\mathcal{E}^i_{\mathrm{FS}}\alpha^i_{\mathrm{FS}}$, that are normally part of the Gauss's law for a fluctuation in the Fock-Schwinger gauge, are of higher order in τ, and do not play a role here. Moreover, the terms in $\tau^{\pm i\nu}$ in the solution

[13] Even if the formulas proposed in Ref. [41] were not affected by the caveat raised in the Introduction, they would be more difficult to evaluate numerically since they require that one solves a large eigenvalue problem. Moreover, the separation between the physical and unphysical modes was problematic in Ref. [41]. In contrast, the approach followed in the present paper gives directly the physical modes.

satisfy independently Gauss's law, as they should since they have evolved independently on each side of the light cone.

Finally, one can again compute the inner product of two of the fluctuations we have obtained in (69). The term in $\alpha^*_{\mathrm{FS}\eta} e^\eta_{\mathrm{FS}} - e^{\eta *}_{\mathrm{FS}} \alpha_{\mathrm{FS}\eta}$ does not contribute at lowest order in τ, and therefore the inner product simply reads

$$(\alpha^{\mathrm{FS}}_{\boldsymbol{k}_\perp \nu\lambda c}|\alpha^{\mathrm{FS}}_{\boldsymbol{k}'_\perp \nu'\lambda' d}) = i\int \mathrm{d}^2\boldsymbol{x}_\perp \mathrm{d}\eta (\alpha^{ia*}_{\mathrm{FS}\boldsymbol{k}_\perp \nu\lambda c}(\tau,\eta,\boldsymbol{x}_\perp) e^{ia}_{\mathrm{FS}\boldsymbol{k}'_\perp \nu'\lambda' d}(\tau,\eta,\boldsymbol{x}_\perp) - e^{ia*}_{\mathrm{FS}\boldsymbol{k}_\perp \nu\lambda c}(\tau,\eta,\boldsymbol{x}_\perp) \alpha^{ia}_{\mathrm{FS}\boldsymbol{k}'_\perp \nu'\lambda' d}(\tau,\eta,\boldsymbol{x}_\perp)). \quad (72)$$

Using the fact that $\nu|\Gamma(i\nu)|^2(e^{\pi\nu} - e^{-\pi\nu}) = 2\pi$, we find

$$(\alpha^{\mathrm{FS}}_{\boldsymbol{k}_\perp \nu\lambda c}|\alpha^{\mathrm{FS}}_{\boldsymbol{k}'_\perp \nu'\lambda' d}) = 4\pi(2\pi)^3\delta(\nu-\nu')\delta(\boldsymbol{k}_\perp - \boldsymbol{k}'_\perp)\delta_{\lambda\lambda'}\delta_{cd}. \quad (73)$$

To prove that this is indeed the correct answer, let us recall what this inner product should be before we made the transformation $k_z \to \nu$,

$$(\alpha^{\mathrm{FS}}_{\boldsymbol{k}\lambda c}|\alpha^{\mathrm{FS}}_{\boldsymbol{k}'\lambda' d}) = 2|k^0|(2\pi)^3\delta(\boldsymbol{k}-\boldsymbol{k}')\delta_{\lambda\lambda'}\delta_{cd}. \quad (74)$$

Using

$$2|k^0|(2\pi)^3\delta(\boldsymbol{k}-\boldsymbol{k}') = 2(2\pi)^3\delta(\boldsymbol{k}_\perp - \boldsymbol{k}'_\perp)\delta(y-y'), \quad (75)$$

and applying the transformation $\int \mathrm{d}y\mathrm{d}y' e^{i(\nu y - \nu' y')}$ to the right-hand side, we find that the inner product in the basis where we use the quantum number ν instead of k_z should indeed be given by Eq. (73).

K. Numerical implementation

From Eqs. (69), (65), and (68), it is obvious that the most difficult and time-consuming part in evaluating numerically these fields is the computation of the functions $F^{+,ia}_{\boldsymbol{k}_\perp \nu\lambda c}$ and $F^{-,ia}_{\boldsymbol{k}_\perp \nu\lambda c}$. Let us list here the main steps in their computation.

(i) Compute the Wilson lines $\mathcal{U}_{1,2}$, which represent the color charge content of the two colliding nuclei in the McLerran-Venugopalan model. This is very easy for the SU(2) gauge group, and a little more involved for SU(3).

(ii) Compute the Fourier transform (over the transverse coordinate $\boldsymbol{x}_\perp$) $\tilde{\mathcal{U}}_{1,2}$ of these Wilson lines. Since space is discretized on a lattice, this is a discrete Fourier transform, for which there are some very efficient algorithms.[14]

(iii) The integration over $\boldsymbol{p}_\perp$ in Eqs. (65) and (68) is also a discrete Fourier transform.

This is the work that needs to be done to compute one of the mode functions, with given quantum numbers $\boldsymbol{k}_\perp$, ν, λ, c. When computing a generic perturbation to the gauge potential, one must sum over all these mode functions with random weights. Since the η, ν dependence of the mode functions is in $\exp(i\nu\eta)$, the sum over the index ν can also be viewed as a discrete Fourier transform. For computing N_{conf} configurations of these fluctuating fields, on a lattice that has $L \times L$ sites in the transverse direction, and N sites in the η direction, the computational cost scales as

$$N_{\mathrm{conf}} \times N\log(N) \times L^4 \log(L). \quad (76)$$

This is the estimate for a straightforward implementation. A more careful examination of how the various steps of the calculation depend on each other leads to a better algorithm, whose cost scales as

$$N\log(N) \times L^4 \times (A\log(L) + BN_{\mathrm{conf}}), \quad (77)$$

with A and B two constants. For large L and/or N_{conf}, this is significantly better than (76).

The only subtlety arises when discretizing the first order differential operators $\mathcal{D}^i$, ∂^i and the corresponding momenta such as p^i, k^i that enter in (69). They can be discretized either as backward or forward finite differences. The choice between the two is arbitrary, and is completely determined by what kind of discretization is chosen for the derivatives in the linearized Gauss law.

IV. CONCLUSIONS AND OUTLOOK

In this work, we have performed an explicit calculation of the small fluctuations that must be superimposed to the classical CGC field in order to resum the unstable modes of the Yang-Mills equations. The calculation has been done from first principles, by solving the evolution equation for small fluctuations on top of the classical background field, with plane wave initial conditions in the remote past.

Although the intermediate steps of the evolution are done in the light-cone gauge $\mathcal{A}^- = 0$, the final results are given in the Fock-Schwinger gauge $\mathcal{A}^\tau = 0$. Moreover, they are also given in terms of the quantum number ν, Fourier conjugate of the rapidity η, which is conserved when the background field in independent of rapidity. Fluctuations expressed in terms of ν are also more suitable for a numerical implementation on a lattice with a fixed spacing in η. Our final formulas, Eq. (69), are valid just after the collision, at proper times $\tau \ll Q_s^{-1}$. By construction, they obey the linearized Yang-Mills equations with the correct initial condition at $x^0 = -\infty$, and satisfy Gauss's law. They have also been checked to satisfy the expected orthonormality conditions, when we compute the inner product defined in Eq. (2).

These formulas also turn out to be very compact and are straightforward to implement numerically. They will be essential in the study of the behavior at early times of the strong color fields produced in high-energy heavy-ion

[14]Naive algorithms for the discrete Fourier transform of an array of size L scale as L^2, while the efficient implementations scale as $L\log(L)$.

collisions. Indeed, it has been noticed a long time ago that certain modes are subject to the Weibel instability and therefore have an exponential growth in time, and that this effect may play a crucial role in the isotropization and thermalization (the basic idea being that it could be fast thanks to the exponential growth of these modes). In the CGC framework, these modes first appear at NLO, where they can give corrections that become sizable (as large as the LO contribution) in a short time. It is therefore important to compute these NLO corrections in order to reliably describe the behavior at early times of the fields produced in a collision. And as explained in the introduction, one needs the mode functions derived in this paper in order to perform this calculation.

Note that with mode functions that differ from the ones derived here, one would still trigger the Weibel instabilities since any randomly chosen fluctuations are likely to have a nonzero projection on some of the unstable modes. However, the timescale for the growth of the fields depends on the initial amplitude of the fluctuations, in particular their amplitude relative to that of the background field (this is were the dependence on the coupling constant g comes from, since the background is of order $1/g$ while the fluctuations are of order 1). This timescale also depends on the relative amplitude of the various mode functions, since the growth rate of the Weibel instability depends on the quantum numbers $\boldsymbol{k}_\perp$ and ν. Therefore, in order to assess correctly the early time evolution of the system, it is necessary to use the mode functions calculated here, that have been constructed in order to guarantee an accurate result up to NLO.

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APPENDIX A: USEFUL FORMULAS IN THE DERIVATION OF (45) AND (46)

This appendix provides some formulas that are useful in order to compute the fluctuation $\tilde{a}^\mu$ via the Gree n's formula (37),

$$i\int_{y^+=0^+} dy^- d^2\boldsymbol{y}_\perp D_R^{ji}(x,y)\overleftrightarrow{\partial}_y^+ e^{iky}\alpha(\boldsymbol{y}_\perp) = \delta^{ij} e^{ik^+x^-}\int\frac{d^2\boldsymbol{p}_\perp}{(2\pi)^2}\tilde{\alpha}(\boldsymbol{p}_\perp+\boldsymbol{k}_\perp)e^{i\boldsymbol{p}_\perp\cdot\boldsymbol{x}_\perp}e^{i\frac{p^2}{2k^+}x^+}, \quad \text{(A1)}$$

$$i\int_{y^+=0^+} dy^- d^2\boldsymbol{y}_\perp D_R^{i+}(x,y)e^{iky}\alpha(\boldsymbol{y}_\perp) = -ie^{ik^+x^-}\int\frac{d^2\boldsymbol{p}_\perp}{(2\pi)^2}\tilde{\alpha}(\boldsymbol{p}_\perp+\boldsymbol{k}_\perp)e^{i\boldsymbol{p}_\perp\cdot\boldsymbol{x}_\perp}\frac{p^i}{p^2}(1-e^{i\frac{p^2}{2k^+}x^+}), \quad \text{(A2)}$$

$$i\int_{y^+=0^+} dy^- d^2\boldsymbol{y}_\perp D_R^{+i}(x,y)\overleftrightarrow{\partial}_y^+ e^{iky}\alpha(\boldsymbol{y}_\perp) = 2k^+e^{ik^+x^-}\int\frac{d^2\boldsymbol{p}_\perp}{(2\pi)^2}\tilde{\alpha}(\boldsymbol{p}_\perp+\boldsymbol{k}_\perp)e^{i\boldsymbol{p}_\perp\cdot\boldsymbol{x}_\perp}\frac{p^i}{p^2}(1-e^{i\frac{p^2}{2k^+}x^+}), \quad \text{(A3)}$$

$$i\int_{y^+=0^+} dy^- d^2\boldsymbol{y}_\perp[\partial_\mu^y D_R^{+\mu}(x,y)]\tilde{a}^+(y) = -\tilde{a}^+(x^+=0^+), \quad \text{(A4)}$$

$$i\int_{y^+=0^+} dy^- d^2\boldsymbol{y}_\perp D_R^{++}(x,y)e^{iky}\alpha(\boldsymbol{y}_\perp) = ie^{ik^+x^-}\int\frac{d^2\boldsymbol{p}_\perp}{(2\pi)^2}\tilde{\alpha}(\boldsymbol{p}_\perp+\boldsymbol{k}_\perp)e^{i\boldsymbol{p}_\perp\cdot\boldsymbol{x}_\perp}\frac{2k^+}{p^2}(1-e^{i\frac{p^2}{2k^+}x^+}), \quad \text{(A5)}$$

where $\tilde{\alpha}$ denotes the transverse Fourier transform of the function $\alpha(\boldsymbol{y}_\perp)$,

$$\tilde{\alpha}(\boldsymbol{k}_\perp) \equiv \int d^2\boldsymbol{y}_\perp e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{y}_\perp}\alpha(\boldsymbol{y}_\perp). \quad \text{(A6)}$$

In order to obtain these formulas, one should replace the free retarded propagator by its Fourier representation, and perform the integral over the energy in the complex plane thanks to the theorem of residues.

APPENDIX B: VARIOUS CHECKS OF EQS. (45) AND (46)

First of all, one can check that the Gauss's law, given in Eq. (40), is indeed satisfied by Eqs. (45) and (46). From these formulas, one can readily see that

$$\partial_\mu\tilde{a}^{\mu a}_{\boldsymbol{k}\lambda c}(x) = 2e^{ik^+x^-}e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}\left(i\frac{\partial^i\partial^i k^j\boldsymbol{\epsilon}^j_{\boldsymbol{k}\lambda}}{k^2}-\partial^i\boldsymbol{\epsilon}^i_{\boldsymbol{k}\lambda}\right)\mathcal{U}_1^{ac}(\boldsymbol{x}_\perp). \quad \text{(B1)}$$

Using Eq. (66), which also implies

$$\mathcal{D}^i_{1ac}(\boldsymbol{x}_\perp)\mathcal{A}^i_{1cb}(\boldsymbol{x}_\perp) = \frac{i}{g}\mathcal{U}^\dagger_{1ac}(\boldsymbol{x}_\perp)\partial^i\partial^i\mathcal{U}_{1cb}(\boldsymbol{x}_\perp) \quad \text{(B2)}$$

and $\mathcal{D}^\mu_{1ac}(\boldsymbol{x}_\perp)a^{\mu c} = \partial_\mu\tilde{a}^{\mu a}$, we obtain (40).

Another nontrivial check is to compute the inner product on the $x^- = 0$ surface (i.e., before the fluctuation traverses the trajectory of the second nucleus). This inner product reads

$$(a_{\boldsymbol{k}\lambda c}|a_{\boldsymbol{k}'\lambda' d}) = -i\int_{x^-=0} \mathrm{d}^2\boldsymbol{x}_\perp \mathrm{d}x^+ a^{ia*}_{\boldsymbol{k}\lambda c}\overleftrightarrow{\partial}^- a^{ia}_{\boldsymbol{k}'\lambda' d}. \quad \text{(B3)}$$

That is calculated by dividing the integration on x^+ in three pieces: $-\infty < x^+ < 0$, $0 < x^+ < \epsilon$, and $\epsilon < x^+ < +\infty$. Note that the second range gives a finite contribution, despite its infinitesimal size, because $\partial^-\mathcal{A}_1^i$ behaves as ϵ^{-1}. Doing this calculation is tedious but straightforward. Using the following identities,

$$k^+\delta(k^+ - k'^+) = k^-\delta(k^- - k'^-) = |k^0|\delta(k_z - k'_z)$$
$$(\text{for } k^2 = k'^2 = 0)$$

$$\int \frac{\mathrm{d}^2\boldsymbol{p}_\perp}{(2\pi)^2}\tilde{\mathcal{U}}^\dagger_{1cb}(\boldsymbol{p}_\perp + \boldsymbol{k}_\perp)\tilde{\mathcal{U}}_{1bd}(\boldsymbol{p}_\perp + \boldsymbol{k}'_\perp) = \delta_{cd}(2\pi)^2\delta(\boldsymbol{k}_\perp - \boldsymbol{k}'_\perp), \quad \text{(B4)}$$

a (somewhat lengthy) calculation gives

$$(a_{\boldsymbol{k}\lambda c}|a_{\boldsymbol{k}'\lambda' d}) = \delta_{\lambda\lambda'}\delta_{cd}(2\pi)^3 2|k^0|\delta(\boldsymbol{k} - \boldsymbol{k}') + 4g\frac{\epsilon^-_{\boldsymbol{k}\lambda}\epsilon^-_{\boldsymbol{k}'\lambda'}}{kk'}\int_0^\epsilon \mathrm{d}x^+\mathrm{d}^2\boldsymbol{x}_\perp \times e^{i(\boldsymbol{k}_\perp - \boldsymbol{k}'_\perp)\cdot\boldsymbol{x}_\perp}J^-_{1cd}(x^+, \boldsymbol{x}_\perp). \quad \text{(B5)}$$

The right-hand side of this inner product has a somewhat unexpected term, proportional to the integral of the color current of the first nucleus. As we shall see now, this term is correct and is the consequence of the fact that we are in a gauge where the incoming wave induces a change in this current (because it has a nonzero a^+ component that induces a precession of J_1^-). This induced current enters in the equation of motion for the fluctuation itself and produces this extra term in the inner product.

Quite generally, in a gauge where such an induced current may appear, the equation of motion of the fluctuation reads

$$\mathcal{D}^{ab}_\mu(\mathcal{D}^{\mu bc}a^{\nu c}_{\boldsymbol{k}\lambda c} - \mathcal{D}^{\nu bc}_\beta a^{\mu c}_{\boldsymbol{k}\lambda c}) - ig\mathcal{F}^\nu{}_{\mu ab}a^{\mu b}_{\boldsymbol{k}\lambda c} = j^{\nu a}_{\boldsymbol{k}\lambda c}. \quad \text{(B6)}$$

Because of the induced current in the right-hand side, the variation of the inner product between two (locally spacelike) surfaces Σ_1 and Σ_2 may be nonzero. More specifically, one has

$$(a_{\boldsymbol{k}\lambda c}|a_{\boldsymbol{k}'\lambda' d})_{\Sigma_2} - (a_{\boldsymbol{k}\lambda c}|a_{\boldsymbol{k}'\lambda' d})_{\Sigma_1} = \int_\Omega d^4x\Big(a^{+a*}_{\boldsymbol{k}\lambda c}j^{-a}_{\boldsymbol{k}'\lambda' d} - j^{+a*}_{\boldsymbol{k}\lambda c}a^{+a}_{\boldsymbol{k}'\lambda' d}\Big), \quad \text{(B7)}$$

where Ω is the four-dimensional domain comprised between the surfaces Σ_1 and Σ_2. In other words, the inner product is conserved only if there are no induced currents between the two surfaces on which it is calculated.

In the situation of interest to us here, the surface Σ_1 is entirely located below the backward light cone, and the surface Σ_2 is the plane $x^- = 0$ (just below the trajectory of the second nucleus). The first term in the right-hand side of Eq. (B5) is nothing but $(a_{\boldsymbol{k}\lambda c}|a_{\boldsymbol{k}'\lambda' d})_{\Sigma_1}$. In the right-hand side of Eq. (B7), one can perform analytically the integral over x^-, which gives the extra term in the right-hand side of Eq. (B5).

APPENDIX C: VACUUM SOLUTIONS

In this appendix, we derive the transformation from the $\mathcal{A}^- = 0$ gauge to the Fock-Schwinger gauge in the case of fluctuations propagating in the vacuum (i.e., when the background field is zero). In this situation, the fluctuations in light-cone gauge are completely trivial, of the form

$$\alpha^i = \epsilon^i e^{ik\cdot x}, \qquad \alpha^+ = \frac{k^i\epsilon^i}{k^-}e^{ik\cdot x}. \quad \text{(C1)}$$

The transformation to Fock-Schwinger gauge can be done via Eq. (62), simplified here thanks to the absence of background field,

$$\alpha^i_{\mathrm{FS}} = \alpha^i - \partial^i\omega, \qquad \alpha^\eta_{\mathrm{FS}} = \frac{1}{2}\alpha^\eta + \frac{1}{\tau^2}\partial_\eta\omega, \quad \text{(C2)}$$

with

$$\alpha^\eta = \frac{\alpha^+}{x^+}, \qquad \omega = \int_0^\tau \mathrm{d}\tau'\frac{\tau'}{2}\alpha^\eta. \quad \text{(C3)}$$

One obtains easily the following explicit expression for ω:

$$\omega(\tau, \eta, \boldsymbol{x}_\perp) = -i\frac{k^i\epsilon^i}{k_\perp^2}e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}\frac{e^{y-\eta}(e^{ik_\perp\tau\cosh(y-\eta)} - 1)}{\cosh(y-\eta)}. \quad \text{(C4)}$$

This leads to

$$\alpha^i_{\mathrm{FS}}(\tau, \eta, \boldsymbol{x}_\perp) = \epsilon^j e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}\Bigg[\delta^{ij}e^{ik_\perp\tau\cosh(y-\eta)} - \frac{k^ik^j}{k_\perp^2}\frac{e^{y-\eta}(e^{ik_\perp\tau\cosh(y-\eta)} - 1)}{\cosh(y-\eta)}\Bigg], \quad \text{(C5)}$$

$$\alpha^\eta_{\mathrm{FS}}(\tau, \eta, \boldsymbol{x}_\perp) = \frac{k^i\epsilon^i}{k_\perp\tau}e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}\frac{1}{\cosh(y-\eta)}\Bigg[e^{ik_\perp\tau\cosh(y-\eta)} + i\frac{e^{ik_\perp\tau\cosh(y-\eta)} - 1}{k_\perp\tau\cosh(y-\eta)}\Bigg]. \quad \text{(C6)}$$

The final step is to go from the quantum number k_z to the Fourier conjugate of rapidity, ν. This is achieved by a Fourier transform of the y dependence,

$$f(y) \to g(\nu) \equiv \int dy e^{iy\nu}f(y). \quad \text{(C7)}$$

After this transformation, Eqs. (C5) and (C6) become, respectively,

$$\alpha^i_{\mathrm{FS}}(\tau, \eta, \boldsymbol{x}_\perp) = \pi e^{-\frac{\pi\nu}{2}}\epsilon^j e^{i(\nu\eta - \boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp)}\Bigg[i\Big(\delta^{ij} - \frac{k^ik^j}{k_\perp^2}\Big)H^{(1)}_{i\nu}(k_\perp\tau) - \nu\frac{k^ik^j}{k_\perp^2}\int_0^\tau\frac{d\tau'}{\tau'}H^{(1)}_{i\nu}(k_\perp\tau')\Bigg] \quad \text{(C8)}$$

and

$$\alpha_{\mathrm{FS}\eta}(\tau,\eta,\boldsymbol{x}_\perp)=\pi e^{-\frac{\pi\nu}{2}}k^j\epsilon^j e^{i(\nu\eta-\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp)}\int_0^\tau d\tau'\tau' H^{(1)}_{i\nu}(k_\perp\tau'), \tag{C9}$$

where $H^{(1)}_{i\nu}$ is the Hankel function defined in terms of the Bessel functions as $H^{(1)}_{i\nu}\equiv J_{i\nu}+iY_{i\nu}$. One can check that the two vacuum solutions[15] found in Ref. [41] [Eqs. (A2) and (B1)] can be rewritten as linear combinations of the present Eqs. (C8) and (C9).

Note that if we had started from the vacuum plane wave solutions in the $\mathcal{A}^+=0$ light-cone gauge instead, we would have $\alpha'^\eta=-\alpha'^-/x^-$ and the function ω' (we denote with a prime all the quantities obtained from this alternate starting point) that defines the transformation to the Fock-Schwinger gauge would be

$$\omega'(\tau,\eta,\boldsymbol{x}_\perp)=-i\frac{k^i\epsilon'^i}{k_\perp^2}e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}\frac{e^{\eta-y}(e^{ik_\perp\tau\cosh(y-\eta)}-1)}{\cosh(y-\eta)}. \tag{C10}$$

[15] The issue in Ref. [41], which we raised in the Introduction, is only with the solutions in the presence of a nontrivial background field, which is the case of interest in heavy-ion collisions. Note that in Ref. [51], the authors took as initial conditions the vacuum fluctuations of Ref. [41] and rescaled them in order to obtain a prescribed gluon occupation number at their starting time (which is much larger than Q_s^{-1}).

Consequently, the vacuum fluctuations in the Fock-Schwinger gauge would be replaced by

$$\alpha'^i_{\mathrm{FS}}(\tau,\eta,\boldsymbol{x}_\perp)=\epsilon'^j e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}\Bigg[\delta^{ij}e^{ik_\perp\tau\cosh(y-\eta)}-\frac{k^ik^j}{k_\perp^2}\frac{e^{\eta-y}(e^{ik_\perp\tau\cosh(y-\eta)}-1)}{\cosh(y-\eta)}\Bigg], \tag{C11}$$

$$\alpha'^\eta_{\mathrm{FS}}(\tau,\eta,\boldsymbol{x}_\perp)=-\frac{k^i\epsilon'^i}{k_\perp\tau}e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}\frac{1}{\cosh(y-\eta)}\times\Bigg[e^{ik_\perp\tau\cosh(y-\eta)}+i\frac{e^{ik_\perp\tau\cosh(y-\eta)}-1}{k_\perp\tau\cosh(y-\eta)}\Bigg]. \tag{C12}$$

Recalling that the polarization vectors ϵ^i in the $\mathcal{A}^-=0$ gauge and ϵ'^i in the $\mathcal{A}^+=0$ gauge are related by

$$\epsilon'^i=\Big(\delta^{ij}-2\frac{k^ik^j}{k_\perp^2}\Big)\epsilon^j, \tag{C13}$$

it is trivial to see that α^μ_{FS} and $\alpha'^\mu_{\mathrm{FS}}$ differ by a gauge transformation that does not depend on τ. In other words, they correspond to two different ways of fixing the residual gauge freedom in the Fock-Schwinger gauge. This is why the vacuum limit ($\mathcal{U}_{1,2}\to 1$) of Eq. (69) does not give precisely Eqs. (C8) and (C9). Indeed, Eq. (69) correspond to fixing this residual gauge freedom independently on the two branches of the light cone.

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