PHYSICAL REVIEW D **88,** 085008 (2013)

Mass corrections to flavor-changing fermion-graviton vertices in the standard model

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(Received 7 March 2013; published 8 October 2013)

In a previous study, the flavor-changing fermion-graviton interactions have been analyzed in the framework of the standard model, where analytical results for the relevant form factors were obtained at the leading order in the external fermion masses. These interactions arise at one-loop level by the charged electroweak corrections to the fermion-graviton vertex, when the off-diagonal flavor transitions in the corresponding charged weak currents are taken into account. Because of the conservation of the energy-momentum tensor, the corresponding form factors turn out to be finite and gauge invariant when external fermions are on shell. Here we extend this previous analysis by including the exact dependence on the external fermion masses. Complete analytical results are provided for all the relevant form factors to the flavor-changing fermion-graviton transitions.

DOI: 10.1103/PhysRevD.88.085008 PACS numbers: 12.38.Bx, 04.62.+v

I. INTRODUCTION

In a previous analysis [1], following the study of Ref. [2], we have discussed the structure of the perturbative corrections to the graviton-fermion-antifermion ($Tf\bar{f}$) vertex in the standard model (SM), focusing our attention on the flavor diagonal sector. On the other hand, in Ref. [2] the one-loop electroweak corrections which generate the off-diagonal graviton-fermion-antifermion vertex were computed at the leading order in the external fermion masses. These studies address the structure of the interactions between the fermions of the standard model and gravity, beyond leading order in the weak coupling, which have never been presented before in their exact expressions. The choice of an external (classical) gravitational background allows one to simplify the treatment of such interactions where the coupling is obtained by the insertion of the symmetric and improved energy-momentum tensor (EMT) into ordinary correlators of the standard model.

We have addressed some of the main features of the perturbative structure of these corrections, presenting their explicit form, parameterized in terms of a certain set of form factors. We have also discussed some of their radiative properties with regard to their infrared finiteness and renormalizability, the latter being inherited directly from the standard model, when the coupling of the Higgs to the gravitational background is conformal.

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In general, one expects that such corrections are small, although they could become more sizable in theories with a low gravity scale [3–8]. In particular, one can consider the possibility of including, in these constructions, backgrounds which are of dilaton type, with dilaton fields produced by metric compactifications. The same vertices characterize the interaction of a dilaton of a spontaneously broken dilatation symmetry with the ordinary fields of the standard model [9–12]. This second possibility is particularly interesting, in view of the recent discovery of a Higgs-like scalar at the LHC.

Perturbative studies of these vertices have their specific difficulties due to the proliferation of form factors, and the results have to be secured by consistency checks using some relevant Ward identities, which reduce the number of independent contributions and that will be discussed below.

These identities need to be derived from scratch by using the full Lagrangian of the standard model, as discussed in Refs. [1,13]. In this study we are going to reconsider the gravitational form factor of a standard model fermion in the presence of a background graviton in the off-diagonal flavor case, which has been discussed before [2], extending that analysis. One of the goals of this reanalysis is to include all the mass corrections to the related form factors, which have not been given before. These corrections are important in order to proceed, in a follow-up work, with a systematic phenomenological study of their implications. In this respect, mass corrections are important in order to extract the exact behavior of these form factors in the infrared and ultraviolet limits, which may be of experimental interest. We have compared our new results against the previous ones given in Ref. [2] in the limit of massless external fermions and found complete agreement.

Our work is organized as follows. In Sec. II, we give the theoretical framework of the standard model Lagrangian in a curved space-time, assuming as a background metric the usual four-dimensional one. In Sec. III, we discuss the contributions coming from the counterterms of the wave-function renormalization and derive the structures of the unrenormalized and renormalized Ward identities. We have checked that these are satisfied by the reducible set of the 12 factors chosen for the presentation of the final result. These will be explicitly given in Sec. IV, and finally, in Sec. V, we present our conclusions.

II. THE LAGRANGIAN

We follow closely the layout of our previous work [1], where more details concerning the general structure of the action describing the coupling of the standard model to gravity can be found. We just recall, in order to make our treatment self-contained, that the interaction of the standard model fields with gravity is described by the action integral

$$S = -\frac{1}{\kappa^2}\int d^4x\sqrt{-g}R + \int d^4x\sqrt{-g}\mathcal{L}_{\rm SM} \qquad (1)$$

together with a term of improvement

$$S_I = \chi\int d^4x\sqrt{-g}RH^\dagger H, \qquad (2)$$

where R is the scalar curvature and H is the Higgs doublet. The identification of this second term goes back to Ref. [14]. The coefficient χ is an arbitrary parameter, which at the special value $\chi_c \equiv 1/6$ renders the Lagrangian conformally symmetric when the scalar is massless and guarantees its renormalizability at the leading order in the gravitational κ, where $\kappa^2 = 16\pi G_N$, with G_N being the gravitational Newton's constant. For instance, in the case of the Higgs field, this takes place if we drop the quadratic terms in the Higgs potential. As in our previous work, our results are given for an arbitrary χ.

The standard model action $S_{\rm SM}$ is obtained by promoting the ordinary SM Lagrangian to a curved background, which is parameterized by the metric expansion $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$, where $\eta_{\mu\nu} = (+,-,-,-)$ and $h_{\mu\nu}$ denotes the fluctuation of the graviton field around the flat limit. At this order the graviton-matter interactions, which we are going to evaluate in the flavor-changing fermion sector, are described by Green's functions with a single insertion of the energy-momentum tensor

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta}{\delta g^{\mu\nu}}[S_{\rm SM} + S_I]_{g=\eta}. \qquad (3)$$

The complete standard model energy-momentum tensor includes several contributions which can be found in Ref. [13].

III. THE PERTURBATIVE EXPANSION

The interaction of one graviton with two fermions of different flavor is summarized by the vertex function

$$\hat{T}^{\mu\nu} \equiv i\langle f_i, p_i|T^{\mu\nu}(0)|p_j, f_j\rangle \qquad (4)$$

that we intend to study. Here p_j (f_j) and p_i (f_i) indicate the momenta (flavor) of initial and final fermions, respectively. We will restrict to the case of flavor-changing transitions, namely, $f_i \neq f_j$. In order to simplify the results, we will also use the combinations of momenta $p = p_i + p_j$ and $q = p_j - p_i$. The external states are taken on their mass shell, $p_i^2 = m_i^2$ and $p_j^2 = m_j^2$, and can be either leptons or quarks. From now on, we will assume that $m_i \neq m_j$. In the last case, since the EMT is diagonal in color space, the color structure is rather trivial and therefore we omit it.

At tree level the flavor-changing gravitational interaction is absent, so that the leading order contribution comes from the quantum corrections. At one-loop level, instead, we decompose the $\hat{T}^{\mu\nu}$ matrix element as

$$\hat{T}^{\mu\nu} = \hat{T}^{\mu\nu}_W + \hat{T}^{\mu\nu}_{\rm CT}, \qquad (5)$$

where the first term on the right-hand side represents the pure vertex corrections induced by the $W^\pm$ gauge boson and its Goldstone $\phi^\pm$ exchanges, while the last term, $\hat{T}^{\mu\nu}_{\rm CT}$, includes the self-energy diagrams plus the usual counterterms (CT) coming from the wave-function renormalization insertions on the external legs. The inclusion of this last term $\hat{T}^{\mu\nu}_{\rm CT}$ is needed in order to get finite results for the matrix element $\hat{T}^{\mu\nu}$, as it will be extensively discussed in Sec. III A. The finiteness of the result is just a consequence of the nonrenormalization theorem of conserved currents, when applied to the case of a conserved EMT.

We choose to work in the R_ξ gauge, where every massive gauge field is always accompanied by its unphysical longitudinal part. The diagrammatic expansion of $\hat{T}^{\mu\nu}_W$ is depicted in Fig. 1 and is made of one contribution of triangle topology plus contact terms [see Figs. 1(c) and 1(d)] with a fermion and a graviton pinched on the same external point. The Feynman rules are listed in Appendix A. The computation of these diagrams has been performed in dimensional regularization using the on-shell renormalization scheme [15]. In this scheme the renormalization conditions are fixed in terms of the physical parameters of the theory to all orders in perturbation theory. These are the masses of physical particles, the electric charge, and the quark mixing matrix. Moreover, the renormalization conditions on the fields are obtained by requiring a unit residue of the full two-point functions on the physical particle poles. This implies that, in the on-shell renormalization scheme, the $\hat{T}^{\mu\nu}_{\rm CT}$ takes contributions only from the usual counterterms coming from the wave-function renormalization insertions on the external legs.

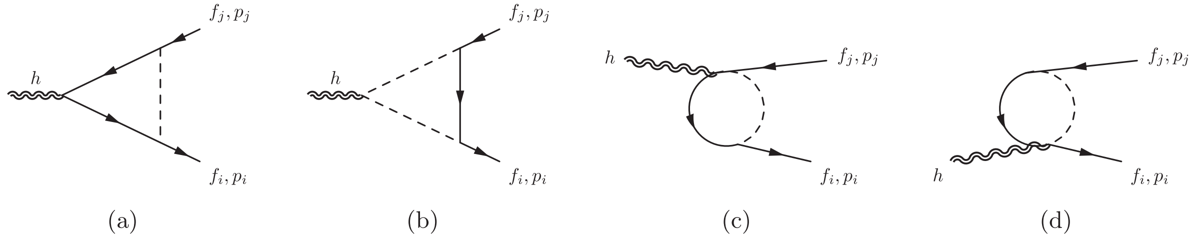


FIG. 1. Diagrams of one-loop SM corrections to the flavor-changing graviton-fermion vertex, where $f_{i,j}$ and $p_{i,j}$ specify the fermion flavors and corresponding momenta, respectively, with $f_i \neq f_j$.

A. Contribution from the wave-function renormalization

As we have just mentioned, the $\hat{T}^{\mu\nu}_W$ matrix element corresponding to the vertex corrections is ultraviolet divergent, and, due to the nonrenormalization theorem of the conserved EMT, it is made finite by adding the contributions from the wave-function renormalization on the external legs, namely, $\hat{T}^{\mu\nu}_{\rm CT}$. This last contribution can be easily determined by using the following method, as illustrated in Ref. [1]. We promote the counterterm SM Lagrangian to a curved background and then extract in the usual way the appropriate renormalized Feynman rules for single insertions of the EMT on the fields of the standard model. The metric is taken to be flat after all the functional differentiations. Then, for the off-diagonal flavor contributions ($i \neq j$) to $\hat{T}^{\mu\nu}_{\rm CT}$, we have

$$\begin{aligned}\hat{T}^{\mu\nu}_{\rm CT} &= i\langle p_i, f_i | T^{\mu\nu}_{\rm CT}(0) | p_j, f_j\rangle \\ &= \frac{i}{4}\bar{u}_i(p_i)\{(\gamma^\mu p^\nu + \gamma^\nu p^\mu)(C^{L+}_{ij}P_L + C^{R+}_{ij}P_R) \\ &\quad + 2\eta^{\mu\nu}[C^{L-}_{ij}(m_i P_L - m_j P_R) \\ &\quad + C^{R-}_{ij}(m_i P_R - m_j P_L)]\}u_j(p_j), \end{aligned} \tag{6}$$

where

$$C^{L\pm}_{ij} = \frac{1}{2}(\delta Z^L_{ij} \pm \delta Z^{L\dagger}_{ij}), \quad C^{R\pm}_{ij} = \frac{1}{2}(\delta Z^R_{ij} \pm \delta Z^{R\dagger}_{ij}), \tag{7}$$

with $\delta Z^{L,R}_{ij}$ being the fermion wave-function renormalization constants. In the on-shell renormalization scheme, which we have chosen for our computation, the renormalization conditions are fixed in terms of the physical parameters of the standard model to all orders in the perturbative expansion. In particular, for the fermion wave-function renormalization constants with $i \neq j$, one obtains

$$\begin{aligned}\delta Z^L_{ij} &= \frac{2}{m_i^2 - m_j^2}\widetilde{\rm Re}\{m_j^2\Sigma^L_{ij}(m_j^2) + m_i m_j \Sigma^R_{ij}(m_j^2) \\ &\quad + (m_i^2 + m_j^2)\Sigma^S_{ij}(m_j^2)\}, \\ \delta Z^R_{ij} &= \frac{2}{m_i^2 - m_j^2}\widetilde{\rm Re}\{m_j^2\Sigma^R_{ij}(m_j^2) + m_i m_j \Sigma^L_{ij}(m_j^2) \\ &\quad + 2m_i m_j \Sigma^S_{ij}(m_j^2)\}.\end{aligned} \tag{8}$$

The symbol $\widetilde{\rm Re}$ gives the real part of the scalar integrals in the self-energies, but it has no effect on the Cabibbo-Kobayashi-Maskawa matrix elements. Its presence yields $\delta Z^\dagger_{ij} = \delta Z_{ij}(m_i^2 \leftrightarrow m_j^2)$. Remember also that if the mixing matrix is real, $\widetilde{\rm Re}$ can obviously be replaced with Re.

For completeness we give the standard model flavor-changing self-energies ($i \neq j$)

$$\begin{aligned}\Sigma^L_{ij}(p^2) &= -\frac{G_F}{4\pi^2\sqrt{2}}\sum_f V_{if}V^\dagger_{fj}[(m_f^2 + 2m_W^2) \\ &\quad \times \mathcal{B}_1(p^2, m_f^2, m_W^2) + m_W^2], \\ \Sigma^R_{ij}(p^2) &= -\frac{G_F}{4\pi^2\sqrt{2}}m_i m_j \sum_f V_{if}V^\dagger_{fj}\mathcal{B}_1(p^2, m_f^2, m_W^2), \\ \Sigma^S_{ij}(p^2) &= -\frac{G_F}{4\pi^2\sqrt{2}}\sum_f V_{if}V^\dagger_{fj}m_f^2\mathcal{B}_0(p^2, m_f^2, m_W^2),\end{aligned} \tag{9}$$

where

$$\begin{aligned}\mathcal{B}_1(p^2, m_0^2, m_1^2) &= \frac{m_1^2 - m_0^2}{2p^2}[\mathcal{B}_0(p^2, m_0^2, m_1^2) \\ &\quad - \mathcal{B}_0(0, m_0^2, m_1^2)] - \frac{1}{2}\mathcal{B}_0(p^2, m_0^2, m_1^2).\end{aligned} \tag{10}$$

We have explicitly checked that the counterterm in Eq. (6) is indeed sufficient to remove all the ultraviolet divergences of the $\hat{T}^{\mu\nu}_W$ matrix element so that $\hat{T}^{\mu\nu}$ is finite, as expected.

B. The Ward identity from the conservation of the EMT

The conservation of the energy-momentum tensor constrains the $\hat{T}^{\mu\nu}$ matrix element reducing the 12 form factors defined above to a smaller subset of six independent contributions. We can derive the Ward identity by imposing the invariance of the one-particle irreducible generating functional—which depends on the external gravitational metric—under a diffeomorphism transformation and then functional differentiating with respect to the fermion fields. We omit the details of this procedure, which has been discussed extensively in [13] and in [1] for the TVV' and the $Tf\bar{f}$ vertices, respectively. The analysis, in this new case, follows similar steps. In momentum space, for the unrenormalized matrix element we obtain the Ward identity

$$q_\mu \hat{T}^{\mu\nu}_W = \bar{u}_i(p_i)\{p_i^\nu \Gamma_{ij}(p_i) - p_j^\nu \Gamma_{ij}(p_j) + \frac{q_\mu}{2}(\Gamma_{ij}(p_i)\sigma^{\mu\nu} - \sigma^{\mu\nu}\Gamma_{ij}(p_j))\}u_j(p_j), \tag{11}$$

where $\sigma^{\mu\nu} = [\gamma^\mu, \gamma^\nu]/4$ and $\Gamma_{ij}(p)$ is the fermion two-point function which is given by

$$\Gamma_{ij}(p) = i[\Sigma^L_{ij}(p^2)\not{p}P_L + \Sigma^R_{ij}(p^2)\not{p}P_R + \Sigma^S_{ij}(p^2)(m_i P_L + m_j P_R)]. \tag{12}$$

The off-diagonal (in flavor space) two-point form factors $\Sigma^{L,R,S}(p^2)$ are explicitly given in Eq. (9). After renormalization, one can show that the Ward identity will take the simpler form

$$q_\mu \hat{T}^{\mu\nu} = 0, \tag{13}$$

which corresponds to the conservation of the energy-momentum tensor.

C. The Ward identity in the reducible basis

Because of the chiral Vector/Axial-Vector nature of the W interactions, we expand the flavor-changing matrix element in terms of 12 invariant amplitudes f_k and tensor operators O_k as

$$\hat{T}^{\mu\nu} = i\frac{G_F}{16\pi^2\sqrt{2}}\sum_{k=1}^{12} f_k(q^2)\bar{u}_i(p_i)O_k^{\mu\nu}u_j(p_j) \tag{14}$$

with the tensor basis given by

$$\begin{aligned}
&O_1^{\mu\nu} = (\gamma^\mu p^\nu + \gamma^\nu p^\mu)P_L, && O_7^{\mu\nu} = \eta^{\mu\nu}M_-,\\
&O_2^{\mu\nu} = (\gamma^\mu q^\nu + \gamma^\nu q^\mu)P_L, && O_8^{\mu\nu} = p^\mu p^\nu M_-,\\
&O_3^{\mu\nu} = \eta^{\mu\nu}M_+, && O_9^{\mu\nu} = q^\mu q^\nu M_-,\\
&O_4^{\mu\nu} = p^\mu p^\nu M_+, && O_{10}^{\mu\nu} = (p^\mu q^\nu + q^\mu p^\nu)M_-,\\
&O_5^{\mu\nu} = q^\mu q^\nu M_+, && O_{11}^{\mu\nu} = \frac{m_i m_j}{m_W^2}(\gamma^\mu p^\nu + \gamma^\nu p^\mu)P_R,\\
&O_6^{\mu\nu} = (p^\mu q^\nu + q^\mu p^\nu)M_+, && O_{12}^{\mu\nu} = \frac{m_i m_j}{m_W^2}(\gamma^\mu q^\nu + \gamma^\nu q^\mu)P_R,
\end{aligned} \tag{15}$$

where $P_{L,R} = (1 \mp \gamma_5)/2$ and $M_\pm \equiv m_j P_R \pm m_i P_L$, and $u_{i,j}(p_{i,j})$ are the corresponding fermion bispinor amplitudes in momentum space.

This is the most general rank-2 tensor basis that can be built out of two momenta, p and q, a metric tensor and Dirac matrices γ^μ and γ^5, whose expression has been given in [2].

The renormalized flavor-changing matrix element $\hat{T}^{\mu\nu}$ satisfies the Ward identity in Eq. (13). This generates homogeneous equations (2) for the renormalized form factors $f_k(q^2)$ given by

$$\begin{aligned}
&p\cdot q f_1(q^2) + q^2 f_2(q^2) = 0, \qquad f_3(q^2) + q^2 f_5(q^2) + p\cdot q f_6(q^2) + \frac{p\cdot q}{2m_W^2}f_{12}(q^2) = 0,\\
&p\cdot q f_4(q^2) + q^2 f_6(q^2) + \frac{p\cdot q}{2m_W^2}f_{11}(q^2) = 0, \qquad f_2(q^2) + f_7(q^2) + q^2 f_9(q^2) + p\cdot q f_{10}(q^2) - \frac{p^2+q^2}{4m_W^2}f_{12}(q^2) = 0,\\
&f_1(q^2) + p\cdot q f_8(q^2) + q^2 f_{10}(q^2) - \frac{p^2+q^2}{4m_W^2}f_{11}(q^2) = 0, \qquad p\cdot q f_{11}(q^2) + q^2 f_{12}(q^2) = 0,
\end{aligned} \tag{16}$$

which allow one to reduce the number of independent contributions to the $\hat{T}^{\mu\nu}$ matrix element to six from the original 12.

IV. FLAVOR-CHANGING FORM FACTORS: EXPLICIT EXPRESSIONS

In this section, we present the explicit expressions of the renormalized form factors f_k appearing in Eq. (14) and using the following notation:

$$f_k(q^2) = \sum_f \lambda_f F_k(q^2, m_f), \tag{17}$$

where we have factorized the term $\lambda_f \equiv V_{fi}V^*_{fj}$ (the external fermions are assumed here to be quarks of down type), with V_{ij} the corresponding CKM matrix element. They have been computed in the on-shell case retaining the full dependence on the internal (m_f, m_W) and external masses (m_i, m_j) and on the virtuality q^2 of the graviton line. They are expressed in terms of the dimensionless ratios $x_S = (m_i^2 + m_j^2)/q^2$, $x_D = (m_j^2 - m_i^2)/q^2$, $x_f = m_f^2/q^2$, and $x_W = m_W^2/q^2$ and of the combination $\lambda = x_D^2 - 2x_S + 1$. We recall that m_f is the mass of the fermion of flavor f running in the loop.

Because of their complexity, we expand our results onto a basis of massive one-, two-, and three-point scalar integrals as

$$F_k(q^2, m_f) = \sum_{l=0}^{7} C_k^l I_l, \tag{18}$$

where

$$I_0 = 1, \quad I_4 = \mathcal{B}_0(q^2, m_f^2, m_f^2),$$
$$I_1 = \mathcal{A}_0(m_f^2) - \mathcal{A}_0(m_W^2), \quad I_5 = B_0(q^2, m_W^2, m_W^2),$$
$$I_2 = \mathcal{B}_0(m_j^2, m_f^2, m_W^2), \quad I_6 = \mathcal{C}_0(m_j^2, q^2, m_i^2, m_f^2, m_W^2, m_W^2),$$
$$I_3 = \mathcal{B}_0(m_i^2, m_f^2, m_W^2), \quad I_7 = \mathcal{C}_0(m_j^2, q^2, m_i^2, m_W^2, m_f^2, m_f^2). \tag{19}$$

We have chosen to give the explicit results for the renormalized form factors F_1, F_3, F_4, F_7, F_8, and F_{11}, while the remaining six can be obtained by exploiting the Ward identities derived in the previous section, which give

$$F_2 = -x_D F_1, \qquad F_5 = -\frac{1}{q^2} F_3 + x_D^2 F_4 + \frac{x_D^2}{m_W^2} F_{11},$$
$$F_6 = -x_D F_4 - \frac{x_D}{2m_W^2} F_{11},$$
$$F_9 = 2\frac{x_D}{q^2} F_1 - \frac{1}{q^2} F_7 + x_D^2 F_8 - \frac{x_S x_D}{m_W^2} F_{11},$$
$$F_{10} = -\frac{1}{q^2} F_1 - x_D F_8 + \frac{x_S}{2m_W^2} F_{11}, \qquad F_{12} = -x_D F_{11}. \tag{20}$$

The explicit form of the coefficients C_k^l appearing in the expansion of the F_k's in Eq. (18), due to their lengthy expressions, can be found in Appendix C. However, as a strong check of our computation, we have computed all 12 form factors $f_i(q^2)$ appearing in Eq. (14) and verified that they satisfy the Ward identity in Eq. (16).

Finally, we remark that the F_3, F_5, F_7, and F_9 form factors show a dependence on the parameter χ which appears in the gravitational coupling of the $\phi^\pm$ Goldstone bosons through the improved energy-momentum tensor.

V. CONCLUSIONS

We have presented the computation of the structure of the gravitational form factors of the standard model fermions in the off-diagonal flavor sector. The analysis has been developed according to our previous study [1], where we have discussed the electroweak corrections in the flavor-conserving case. The work extends a previous investigation [2] of the same flavor-changing vertex in which the external mass dependence has not been included. The exact expressions presented in our work are relevant for a phenomenological study of the small and intermediate momentum behavior of these form factors, which we plan to address in the near future.

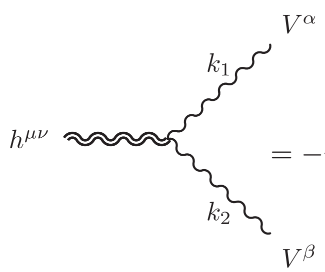
ACKNOWLEDGMENTS

E. G. thanks the PH-TH division of CERN for its kind hospitality during the preparation of this work. This work was supported by the ESF Grant No. MTT60, by the recurrent financing SF0690030s09 project, and by the European Union through the European Regional Development Fund.

APPENDIX A: FEYNMAN RULES

We collect here all the Feynman rules involving a graviton that have been used in this work. All the momenta are incoming.

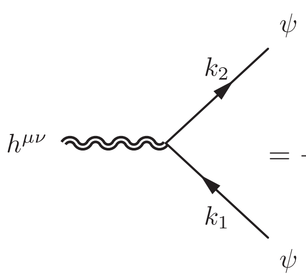
(i) Graviton–gauge boson–gauge boson vertex



$$= -i\frac{\kappa}{2}\left\{ \left(k_1 \cdot k_2 + M_V^2\right) C^{\mu\nu\alpha\beta} + D^{\mu\nu\alpha\beta}(k_1, k_2) + \frac{1}{\xi} E^{\mu\nu\alpha\beta}(k_1, k_2)\right\}, \tag{A1}$$

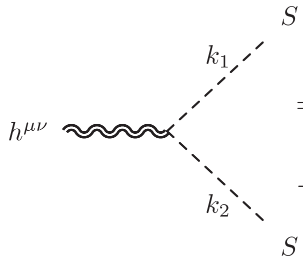
where V stands for the vector gauge boson W.

(ii) Graviton-fermion-fermion vertex



$$= -i\frac{\kappa}{8}\left\{\gamma^\mu (k_1 + k_2)^\nu + \gamma^\nu (k_1 + k_2)^\mu - 2\,\eta^{\mu\nu} (\not{k}_1 + \not{k}_2 - 2m_f)\right\}, \tag{A2}$$

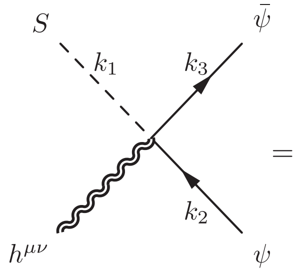
(iii) Graviton-scalar-scalar vertex



$$= i\frac{\kappa}{2}\left\{k_{1\,\rho}\,k_{2\,\sigma}\,C^{\mu\nu\rho\sigma} - M_S^2\,\eta^{\mu\nu}\right\} - i\frac{\kappa}{2}2\chi\left\{(k_1+k_2)^\mu(k_1+k_2)^\nu - \eta^{\mu\nu}(k_1+k_2)^2\right\}, \tag{A3}$$

where S stands for the Goldstone $\phi^\pm$ of the gauge boson W. The first line is the contribution coming from the minimal energy-momentum tensor, while the second is due to the improvement term.

(iv) Graviton-scalar-fermion-fermion vertex

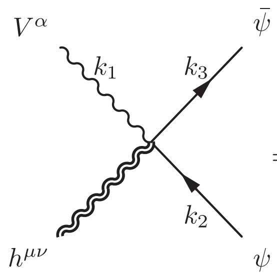


$$= \frac{\kappa}{2}\left(C^L_{S\bar{\psi}\psi}\,P_L + C^R_{S\bar{\psi}\psi}\,P_R\right)\,\eta^{\mu\nu}, \tag{A4}$$

where the coefficients are defined as

$$\begin{aligned} C^L_{\phi^+\bar{\psi}\psi} &= i\frac{e}{\sqrt{2}s_W}\frac{m_{\bar{\psi}}}{m_W}V_{\bar{\psi}\psi}, \qquad C^R_{\phi^+\bar{\psi}\psi} = -i\frac{e}{\sqrt{2}s_W}\frac{m_\psi}{m_W}V_{\bar{\psi}\psi}, \\ C^L_{\phi^-\bar{\psi}\psi} &= -i\frac{e}{\sqrt{2}s_W}\frac{m_{\bar{\psi}}}{m_W}V^*_{\bar{\psi}\psi}, \qquad C^R_{\phi^-\bar{\psi}\psi} = i\frac{e}{\sqrt{2}s_W}\frac{m_\psi}{m_W}V^*_{\bar{\psi}\psi}. \end{aligned} \tag{A5}$$

(v) Graviton–gauge boson–fermion–fermion vertex



$$= -\frac{\kappa}{2}\left(C^L_{V\bar{\psi}\psi}\,P_L + C^R_{V\bar{\psi}\psi}\,P_R\right)C^{\mu\nu\alpha\beta}\gamma_\beta\,, \tag{A6}$$

with

$$C^L_{W^+\bar{\psi}\psi} = i\frac{e}{\sqrt{2}s_W}V_{\bar{\psi}\psi}, \qquad C^L_{W^-\bar{\psi}\psi} = i\frac{e}{\sqrt{2}s_W}V^*_{\bar{\psi}\psi}, \qquad C^R_{W^\pm\bar{\psi}\psi} = 0. \tag{A7}$$

The tensor structures C, D, and E which appear in the Feynman rules defined above are given by

$$\begin{aligned} C_{\mu\nu\rho\sigma} &= \eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma}, \\ D_{\mu\nu\rho\sigma}(k_1,k_2) &= \eta_{\mu\nu}k_{1\sigma}k_{2\rho} - [\eta^{\mu\sigma}k_1^\nu k_2^\rho + \eta_{\mu\rho}k_{1\sigma}k_{2\nu} - \eta_{\rho\sigma}k_{1\mu}k_{2\nu} + (\mu\leftrightarrow\nu)], \\ E_{\mu\nu\rho\sigma}(k_1,k_2) &= \eta_{\mu\nu}(k_{1\rho}k_{1\sigma} + k_{2\rho}k_{2\sigma} + k_{1\rho}k_{2\sigma}) - [\eta_{\nu\sigma}k_{1\mu}k_{1\rho} + \eta_{\nu\rho}k_{2\mu}k_{2\sigma} + (\mu\leftrightarrow\nu)]. \end{aligned} \tag{A8}$$

APPENDIX B: SCALAR INTEGRALS

In this Appendix, we collect the definitions of the scalar integrals appearing in the computation of the matrix element. One-, two-, and three-point functions are denoted, respectively, as $\mathcal{A}_0$, $\mathcal{B}_0$, and $\mathcal{C}_0$ with

$$\mathcal{A}_0(m_0^2) = \frac{1}{i\pi^2}\int d^n l \frac{1}{l^2 - m_0^2}, \qquad \mathcal{B}_0(p_1^2, m_0^2, m_1^2) = \frac{1}{i\pi^2}\int d^n l \frac{1}{(l^2 - m_0^2)((l+p_1)^2 - m_1^2)},$$
$$\mathcal{C}_0(p_1^2, (p_1-p_2)^2, p_2^2, m_0^2, m_1^2, m_2^2) = \frac{1}{i\pi^2}\int d^n l \frac{1}{(l^2 - m_0^2)((l+p_1)^2 - m_1^2)((l+p_2)^2 - m_2^2)}. \tag{B1}$$

APPENDIX C: FORM FACTORS

Here we list the coefficients C_i^j which appear in the expansion of the form factors F_1, F_3, F_4, F_7, F_8, and F_{11} defined in Eq. (18). The remaining ones, as already mentioned, can be computed by using the Ward identities in Eq. (20).

(i) Coefficients C_1^i entering in F_1:

$$\begin{aligned}
C_1^0 &= \frac{q^2}{12\lambda}\{3(x_D^2 - x_S^2) + 6(1-x_S)(x_f - 6x_W) + 16(x_f - x_W)(x_f + 2x_W)\},\\
C_1^1 &= -\frac{2x_f - 3x_S + 4x_W + 3}{6\lambda},\\
C_1^2 &= \frac{q^2}{8\lambda^2 x_D}\{x_D^4(6x_f + x_S - 16x_W - 2) + x_D^3(x_S(-10x_f + 18x_W + 3) + 12(x_f x_W + x_f^2 + x_f - 2x_W^2) - 32x_W - 3)\\
&\quad - x_D^2(x_S^2(20x_f - 46x_W - 6) + x_S(-28x_f x_W - 4x_f(7x_f + 4) + 56x_W^2 + 54x_W + 4) + 24(x_f - x_W)(x_f + 2x_W)\\
&\quad + 6x_f + x_S^3 - 26x_W - 1) + x_D(x_S^2(8x_f + 6x_W + 3) - 2x_S(2x_f x_W + x_f(2x_f + 5) - 4x_W^2 + x_W)\\
&\quad - 8(x_f - x_W)(x_f + 2x_W) - 3x_S^3 + 10x_W) - (1-2x_S)^2(2x_W(2x_f + x_S) + (x_S - 2x_f)^2 - 8x_W^2)\},\\
C_1^3 &= \frac{q^2}{8\lambda^2 x_D}\{x_D^4(-(6x_f + x_S - 16x_W - 2)) + x_D^3(x_S(-10x_f + 18x_W + 3)\\
&\quad + 12(x_f x_W + x_f^2 + x_f - 2x_W^2) - 32x_W - 3) + x_D^2(x_S^2(20x_f - 46x_W - 6)\\
&\quad + x_S(-28x_f x_W - 4x_f(7x_f + 4) + 56x_W^2 + 54x_W + 4) + 24(x_f - x_W)(x_f + 2x_W) + 6x_f + x_S^3 - 26x_W - 1)\\
&\quad + x_D(x_S^2(8x_f + 6x_W + 3) - 2x_S(2x_f x_W + x_f(2x_f + 5) - 4x_W^2 + x_W) - 8(x_f - x_W)(x_f + 2x_W)\\
&\quad - 3x_S^3 + 10x_W) + (1-2x_S)^2(2x_W(2x_f + x_S) + (x_S - 2x_f)^2 - 8x_W^2)\},\\
C_1^4 &= \frac{q^2}{12\lambda^2}\{x_D^2(3(8x_f - 3)x_S - 2x_f(16x_f + 32x_W + 11) + 10x_W + 9) - 3x_S^2(8x_f + 26x_W + 3)\\
&\quad + 4x_S(-7x_f x_W + x_f(13x_f + 5) + 42x_W^2 + 34x_W) + 24(6x_f - 7)x_W^2 + 92x_f x_W - 2x_f(2x_f + 1)(12x_f - 1)\\
&\quad + 9x_S^3 - 96x_W^3 - 68x_W\},\\
C_1^5 &= \frac{q^2}{3\lambda^2}\{x_f(8x_W(x_D^2 + x_S - 2) - 2x_D^2 + 3x_S^2 - 2x_S - 36x_W^2 + 1)\\
&\quad + 4x_W(x_D^2(-3x_S + 4x_W + 5) + (x_S - 3x_W - 2)(3x_S - 2x_W - 1)) - 12x_f^2(x_S - 1) + 12x_f^3\},\\
C_1^6 &= \frac{q^4}{2\lambda^2}\{8x_W^3(-2x_D^2 + 5x_f + 4x_S - 2) - 4x_W^2(x_D^2(-2x_f - 2x_S + 3) + (x_f + x_S)(6x_f + x_S) - 5x_f - 2x_S)\\
&\quad - 2x_W(-4x_f^2(x_D^2 + x_S - 2) + 3x_f(x_D^2(2x_S - 3) - (x_S - 2)x_S) + 2(x_D^2 - x_S^2)(x_D^2 - x_S) + 4x_f^3)\\
&\quad + x_f(2x_f - x_S + 1)(-x_D^2 + (x_S - 2x_f)^2 + 4x_f) - 16x_W^4\},\\
C_1^7 &= \frac{q^4}{8\lambda^2}\{2x_D^2(x_S(x_f(-4x_f + 32x_W + 7) - 8x_W - 2) + (2 - 6x_f)x_S^2 + 2(3 - 16x_f)x_W^2\\
&\quad + 4x_f(4x_f - 9)x_W + x_f(2x_f - 1)(8x_f + 5) + 8x_W + 1) + x_D^4(4x_f - 1) + 16x_W^3(10x_f + 9x_S - 9)\\
&\quad - 4x_W^2(34x_f x_S + 24x_f^2 - 50x_f + 3x_S(9x_S - 16) + 24) + 8x_W(-(x_f + 8)x_S^2 + (4(x_f - 1)x_f + 6)x_S + 6x_f\\
&\quad - 4x_f^2(x_f + 2) + 4x_S^3 - 2) - (x_S - 2x_f)^2(2x_f(x_S - 1) - 8x_f^2 + 3x_S^2 - 4x_S + 2) - 64x_W^4\},
\end{aligned} \tag{C1}$$

(ii) Coefficients C_3^i entering in F_3:

$$
\begin{aligned}
C_3^0 &= \frac{q^2}{6\lambda}\{-x_S(3x_D^2+32x_f-20x_W+3)+2x_f(7x_D^2+4x_W+9)-4x_W(2x_D^2+4x_W+3)+8x_f^2+6x_S^2\},\\
C_3^1 &= \frac{1}{3\lambda(x_S^2-x_D^2)}\{-x_D^2(x_S(6x_f+x_S+12x_W+4)-4x_f-8x_W-3)+x_D^4+x_S(4x_S-3)(2x_f+x_S+4x_W)\},\\
C_3^2 &= \frac{q^2}{4\lambda^2(x_D+x_S)}\{x_D^5(-2x_f+x_S+2x_W+2)+x_D^4(4x_W(2x_f+x_S+1)-2x_fx_S+8x_f^2-2x_f-x_S^2+x_S-16x_W^2)\\
&\quad +x_D^3(2x_W(2x_f(x_S+3)+x_S^2+x_S+2)+4x_f^2x_S+2x_fx_S+12x_f^2-2x_f-8(x_S+3)x_W^2-x_S^3-2x_S^2-5x_S+2)\\
&\quad +x_D^2(2x_S^2(-2x_f+x_W-4)+x_S(2x_f(2x_W+9)+4x_f^2-2x_W(4x_W+9)+5)-4x_fx_W-2x_f(2x_f+5)+x_S^3\\
&\quad +8x_W^2+12x_W-1)+x_D(x_S^3(-8x_f+4x_W-1)+2x_S^2(x_f(4x_W+6)+4x_f^2-x_W(8x_W+9)+2)\\
&\quad -2x_S(x_f(10x_W-3)+10x_f^2+(3-20x_W)x_W+1)+2(5-2x_f)x_W-4x_f(x_f+2)+2x_S^4+8x_W^2)+x_D^6\\
&\quad +x_S^3(-8x_f+4x_W+1)-2x_S(2x_f(5x_W+2)+10x_f^2-5x_W(4x_W+1))+x_S^2(8x_f(x_W+2)+8x_f^2\\
&\quad -2x_W(8x_W+9)-1)+4(x_f-x_W)(x_f+2x_W)+2x_S^4\}+\chi\frac{2q^2}{\lambda}\{2(x_D+1)x_f-(x_D-1)(x_D+x_S)\},\\
C_3^3 &= -\frac{q^2}{4\lambda^2(x_D-x_S)}\{-x_D^5(-2x_f+x_S+2x_W+2)+x_D^4(4x_W(2x_f+x_S+1)-2x_fx_S+8x_f^2-2x_f\\
&\quad -x_S^2+x_S-16x_W^2)+x_D^3(-2x_W(2x_f(x_S+3)+x_S^2+x_S+2)-4x_f^2x_S-2x_fx_S-12x_f^2+2x_f+8(x_S+3)x_W^2\\
&\quad +x_S^3+2x_S^2+5x_S-2)+x_D^2(2x_S^2(-2x_f+x_W-4)+x_S(2x_f(2x_W+9)+4x_f^2-2x_W(4x_W+9)+5)\\
&\quad -2x_f(2x_W+5)-4x_f^2+x_S^3+4x_W(2x_W+3)-1)+x_D(x_S^3(8x_f-4x_W+1)-2x_S^2(x_f(4x_W+6)+4x_f^2\\
&\quad -x_W(8x_W+9)+2)+x_S(x_f(20x_W-6)+20x_f^2-40x_W^2+6x_W+2)+4x_f(x_W+2)+4x_f^2-2x_S^4\\
&\quad -2x_W(4x_W+5))+x_D^6+x_S^3(-8x_f+4x_W+1)-2x_S(2x_f(5x_W+2)+10x_f^2-5x_W(4x_W+1))\\
&\quad +x_S^2(8x_f(x_W+2)+8x_f^2-2x_W(8x_W+9)-1)+4(x_f-x_W)(x_f+2x_W)+2x_S^4\}\\
&\quad -\chi\frac{2q^2}{\lambda}\{2(x_D-1)x_f+(x_D+1)(x_D-x_S)\},\\
C_3^4 &= \frac{q^2}{6\lambda^2}\{x_D^2(-12x_S(2x_f+x_W)+4x_f(3-8x_W)-28x_f^2+3x_S^2+4x_W(3x_W+5)+1)+x_D^4(8x_f-2)\\
&\quad +x_S(4x_f(4x_W-7)+44x_f^2+4x_W(9x_W-1)-1)+(22x_f+2)x_S^2-8(-x_fx_W(9x_W+2)+3x_f^3+2x_f^2\\
&\quad +6x_W^2(x_W+1))+10x_f-3x_S^3-4x_W\},\\
C_3^5 &= -\frac{q^2}{6\lambda^2}\{-x_D^2(4(x_f-10)x_W+2x_f+15x_S(2x_W+1)+44x_W^2-7)+x_D^4(8x_W+4)+x_S^2(-18x_f+28x_W+5)\\
&\quad +x_S(8x_f(x_W+5)+36x_f^2+26x_W(2x_W-3)-4)-4(x_f(-18x_W^2+x_W+5)+6x_f^3+9x_f^2\\
&\quad +2x_W(6x_W^2+x_W-4))+3x_S^3\}+\chi\frac{4q^2}{\lambda}(x_D^2-2x_f-x_S),\\
C_3^6 &= \frac{q^4}{4\lambda^2}\{40x_W^3(-x_D^2+2x_f+x_S)-4x_W^2(-x_D^2(8x_f+4x_S+5)+2x_D^4+4(x_f+2)x_S+4(3x_f^2+x_f-1)+3x_S^2)\\
&\quad +2x_W(x_D^2-2x_f-x_S)(5x_D^2-4(x_f+3)x_S+4x_f(x_f+1)+x_S^2+6)+4x_D^2x_S-x_D^4-2x_D^2-8x_fx_S^3+24x_f^2x_S^2\\
&\quad +24x_fx_S^2-32x_f^3x_S-48x_f^2x_S-24x_fx_S+16x_f^4+32x_f^3+24x_f^2+8x_f+x_S^4-4x_S^3+2x_S^2-32x_W^4\}\\
&\quad -\chi\frac{2q^4}{\lambda}\{x_D^2(2x_W-1)-2x_W(2x_f+x_S)+(x_S-2x_f)^2+4x_f\},\\
C_3^7 &= -\frac{q^4}{4\lambda^2}\{-2x_W(x_D^2+2x_f+x_S-2)+(-2x_f+x_S-1)(x_D^2+2x_f-x_S)+8x_W^2\}\{x_D^2(4x_f-1)\\
&\quad -4x_S(x_f+x_W)+4((x_f-x_W)^2+x_W)+x_S^2\}.
\end{aligned}
\tag{C2}
$$

(iii) Coefficients C_4^i entering in F_4:

$$
\begin{aligned}
C_4^0 =& \frac{2}{3\lambda^2}\{x_f(x_D^2 - 7x_S + 10x_W + 6) - x_W(7x_D^2 - 19x_S + 20x_W + 12) + 10x_f^2\},\\
C_4^1 =& \frac{2}{3q^2\lambda^2(x_D^2 - x_S^2)}\{x_D^2(6x_f x_S - 10x_f + 12x_S x_W - 2x_S^2 + x_S - 20x_W - 3) + 2x_D^4 - (x_S - 3)x_S(2x_f + x_S + 4x_W)\},\\
C_4^2 =& \frac{1}{\lambda^3(x_D + x_S)}\{x_D^4(x_f(x_S + 4x_W + 2) + 4x_f^2 + x_W(-9x_S - 8x_W + 4)) + x_D^3(x_f(2(x_S + 6)x_W + x_S^2 - 2x_S + 7)\\
&+ 2x_f^2(x_S + 6) - x_W(4(x_S + 6)x_W + 7x_S^2 - 10x_S + 5)) + 2x_D^2(x_f(2x_S(-4x_S + 5x_W + 6) - 7x_W - 4)\\
&+ x_f^2(10x_S - 7) + x_W(5x_S(3x_S - 4x_W - 4) + 14x_W + 7)) + x_D(-x_f(x_S(x_S(12x_S - 16x_W - 15) + 22x_W + 6)\\
&+ 8x_W + 3) + 2x_f^2(x_S(8x_S - 11) - 4) + x_W(x_S(x_S(24x_S - 32x_W - 29) + 44x_W + 2) + 16x_W + 7))\\
&- 2x_D^5 x_W + x_W(2x_f(2(x_S - 4)x_S + 1) + x_S(6(x_S - 2)x_S + 7)) + x_f(x_S(4x_f(x_S - 4) - 2x_S^2 + 2x_S - 3) + 2x_f)\\
&- 4(2(x_S - 4)x_S + 1)x_W^2\},\\
C_4^3 =& \frac{1}{\lambda^3(x_D - x_S)}\{-x_D^4(x_f(x_S + 4x_W + 2) + 4x_f^2 + x_W(-9x_S - 8x_W + 4)) + x_D^3(x_f(2(x_S + 6)x_W + x_S^2 - 2x_S + 7)\\
&+ 2x_f^2(x_S + 6) - x_W(4(x_S + 6)x_W + 7x_S^2 - 10x_S + 5)) + 2x_D^2(x_f(2x_S(4x_S - 5x_W - 6) + 7x_W + 4)\\
&+ x_f^2(7 - 10x_S) + x_W(5x_S(-3x_S + 4x_W + 4) - 7(2x_W + 1))) + x_D(-x_f(x_S(x_S(12x_S - 16x_W - 15)\\
&+ 22x_W + 6) + 8x_W + 3) + 2x_f^2(x_S(8x_S - 11) - 4) + x_W(x_S(x_S(24x_S - 32x_W - 29) + 44x_W + 2)\\
&+ 16x_W + 7)) - 2x_D^5 x_W - x_W(2x_f(2(x_S - 4)x_S + 1) + x_S(6(x_S - 2)x_S + 7)) + x_f(x_S(2x_S(-2x_f + x_S - 1)\\
&+ 16x_f + 3) - 2x_f) + 4(2(x_S - 4)x_S + 1)x_W^2\},\\
C_4^4 =& \frac{1}{6\lambda^3}\{x_D^2(x_S(36x_f - 6x_W - 9) - 2(2x_f x_W + 5x_f(8x_f + 3) + 6x_W^2) + 6x_S^2 + 40x_W + 11) - 4x_D^4(2x_f + 1)\\
&- x_S^2(10x_f + 108x_W + 5) + 2x_S(2(-58x_f x_W + 25x_f^2 + x_f + 81x_W^2) + 77x_W - 1) - 4(6(13 - 15x_f)x_W^2\\
&- 59x_f x_W + x_f(5x_f(6x_f + 1) - 2) + 60x_W^3 + 20x_W) + 3x_S^3\},\\
C_4^5 =& \frac{1}{6\lambda^3}\{-x_D^2(-3x_S(8x_f - 26x_W + 3) + 50x_f(2x_W + 1) + 24x_f^2 + 6x_S^2 - 4x_W(55x_W + 4) + 11) + 4x_D^4(8x_W + 1)\\
&- 20x_W^2(18x_f + 13x_S - 2) + 2(2x_S - 1)x_W(50x_f + 7x_S + 8) + 42x_f x_S^2 - 132x_f^2 x_S - 56x_f x_S + 120x_f^3\\
&+ 156x_f^2 + 40x_f - 3x_S^3 + 5x_S^2 + 2x_S + 240x_W^3\},\\
C_4^6 =& \frac{q^2}{4\lambda^3}\{x_D^4(2x_W(8x_f - 4x_S + 5) + 4x_f - 2x_S - 56x_W^2 + 3) + 2x_D^2(x_S^2(-6x_f + 13x_W - 2) + x_S(-5(4x_f + 5)x_W\\
&+ 12x_f^2 + 16x_f + 52x_W^2 + 1) + 2(5(12x_f + 1)x_W^2 + (5 - 6x_f)x_f x_W - x_f(4x_f(x_f + 3) + 7) - 50x_W^3) + x_S^3\\
&+ 10x_W - 1) - 2x_S^3(8x_f + 5x_W + 1) + x_S^2(x_f(28 - 44x_W) + 72x_f^2 + 4(6 - 11x_W)x_W + 2) - 4x_S(6x_f^2(5 - 7x_W)\\
&+ x_f(60x_W^2 - 26x_W + 4) + 32x_f^3 + x_W(10(1 - 5x_W)x_W + 3)) + 8(50x_f x_W^3 + 2(1 - 15x_f^2)x_W^2\\
&- x_f(2x_f(5x_f + 9) + 7)x_W + x_f(x_f + 1)(2x_f(5x_f + 4) + 1) - 20x_W^4) + x_S^4\},\\
C_4^7 =& \frac{q^2}{4\lambda^3}\{x_D^4(4x_f(4x_f - 2x_W + 1) + 2x_S - 2x_W - 3) - 2x_D^2(x_S^2(6x_f - 3x_W - 2) + x_S(10x_f(2x_f - 4x_W - 1)\\
&+ 17x_W + 1) + 6(4x_f - 3)x_W^2 + 2x_f(6x_f + 25)x_W - 4x_f(10x_f^2 + x_f - 2) + x_S^3 + 4x_W^3 - 16x_W - 1)\\
&+ 8x_W^3(50x_f + 37x_S - 36) - 12x_W^2(8(4x_f - 3)x_S + 4(x_f(5x_f - 9) + 3) + 15x_S^2) + 2x_W((26x_f - 34)x_S^2\\
&+ (28x_f(3x_f - 2) + 22)x_S + 44x_f - 8x_f^2(5x_f + 9) + 19x_S^3 - 8)\\
&- (x_S - 2x_f)^2(-20x_f^2 + (x_S - 2)x_S + 2) - 160x_W^4\}.
\end{aligned}
\tag{C3}
$$

(iv) Coefficients C_7^i entering in F_7:

$$
\begin{aligned}
C_7^0 &= \frac{q^2 x_D}{6\lambda}\{3x_D^2 + x_S(4x_f - 4x_W - 6) - 8(x_f - x_W)(x_f + 2x_W) - 4x_f + 4x_W + 3\},\\
C_7^1 &= \frac{2x_D}{3\lambda(x_D^2 - x_S^2)}\{x_D^2(-(x_f + x_S + 2x_W - 1)) + x_S(x_S(-2x_f + x_S - 4x_W - 1) + 6(x_f + 2x_W)) - 3(x_f + 2x_W)\},\\
C_7^2 &= \frac{q^2}{4\lambda^2(x_D + x_S)}\{-x_D^4(x_f(-2x_S + 4x_W + 2) + 4x_f^2 + 2x_W(6x_S - 4x_W + 1) - 1) + x_D^3(3x_S^2(4x_f - 4x_W - 1)\\
&\quad + x_S(34x_W - 6(4x_f x_W + 4x_f^2 + x_f - 8x_W^2)) + 16(x_f - x_W)(x_f + 2x_W) + 2x_f - 18x_W + 1)\\
&\quad + x_D^2(x_S^3(8x_f - 8x_W - 2) + x_S^2(-4x_f(4x_W + 1) - 16x_f^2 + 4x_W(8x_W + 9) + 1) - 2x_S(3x_f + x_W + 1)\\
&\quad + 16(x_f - x_W)(x_f + 2x_W) + 6x_f - 10x_W + 1) + x_D(x_S(x_S(-24x_f + 8x_W - 7) + 8(x_f - x_W)(x_f + 2x_W)\\
&\quad + 22x_f + 6x_S^2 - 2x_W + 2) - 2(2x_f + x_W)) + x_D^5(-2x_f - 8x_W + 1) + (1 - 2x_S)^2(-2x_f + x_S + 2x_W)\\
&\quad \times (x_S - 2(x_f + 2x_W))\} - \chi\frac{2q^2}{\lambda}\{2x_f(x_D + 2x_S - 1) - x_D x_S + x_D^2 + x_D - 2x_S^2 + x_S\},\\
C_7^3 &= \frac{q^2}{4\lambda^2(x_D - x_S)}\{-x_D^4(x_f(-2x_S + 4x_W + 2) + 4x_f^2 + 2x_W(6x_S - 4x_W + 1) - 1) + x_D^3(3x_S^2(-4x_f + 4x_W + 1)\\
&\quad + x_S(6(4x_f x_W + 4x_f^2 + x_f - 8x_W^2) - 34x_W) - 16(x_f - x_W)(x_f + 2x_W) - 2x_f + 18x_W - 1)\\
&\quad + x_D^2(x_S^3(8x_f - 8x_W - 2) + x_S^2(-4x_f(4x_W + 1) - 16x_f^2 + 4x_W(8x_W + 9) + 1) - 2x_S(3x_f + x_W + 1)\\
&\quad + 16(x_f - x_W)(x_f + 2x_W) + 6x_f - 10x_W + 1) + x_D(2(2x_f + x_W) - x_S(x_S(-24x_f + 8x_W - 7)\\
&\quad + 8(x_f - x_W)(x_f + 2x_W) + 22x_f + 6x_S^2 - 2x_W + 2)) + x_D^5(2x_f + 8x_W - 1) + (1 - 2x_S)^2(-2x_f + x_S + 2x_W)\\
&\quad \times (x_S - 2(x_f + 2x_W))\} + \chi\frac{2q^2}{\lambda}\{x_D(-2x_f + x_S - 1) + x_D^2 - (2x_S - 1)(x_S - 2x_f)\},\\
C_7^4 &= -\frac{q^2 x_D}{6\lambda^2}(-2x_f + x_S - 4x_W - 1)\{x_D^2(8x_f - 2) - 2x_S(2x_f + 6x_W + 1) + 12(x_f - x_W)^2 - 4x_f + 3x_S^2 + 12x_W + 1\},\\
C_7^5 &= \frac{q^2 x_D}{6\lambda^2}\{-48x_W^3 + 4x_W^2(-8x_D^2 + 18x_f + 7x_S + 1) + (-2x_f + x_S - 1)(x_D^2 - 4(3x_f + 2)x_S\\
&\quad + 4(3x_f(x_f + 1) + 1) + 3x_S^2) + 2\lambda x_W(-8x_f + 4x_S + 9)\} + \chi\frac{4q^2 x_D}{\lambda}(2x_f - x_S + 1),\\
C_7^6 &= -\frac{q^4 x_D}{4\lambda^2}\{2x_D^2(-x_S(x_W(8x_f + 4x_W + 23) - 2) + x_W(x_f(8x_W - 2) + 8x_f^2 + 2(7 - 8x_W)x_W + 9) + 2x_S^2 x_W - 1)\\
&\quad + x_D^4(12x_W - 1) - 2x_S^3(4x_f + 3x_W + 2) + x_S^2(4(5x_f x_W + 6x_f(x_f + 1) + x_W^2) + 42x_W + 2)\\
&\quad - 4x_S(2x_f^2(x_W + 6) + x_f(6 - 4x_W(x_W + 1)) + 8x_f^3 + x_W(2(5 - 3x_W)x_W + 7)) + 8((10x_f + 1)x_W^3\\
&\quad - 2(x_f + 1)(3x_f - 1)x_W^2 - x_f(2x_f^2 + x_f + 2)x_W + x_f(x_f + 1)(2x_f(x_f + 1) + 1) - 4x_W^4) + x_S^4 + 4x_W\}\\
&\quad + \chi\frac{2q^4 x_D}{\lambda}\{-x_D^2 + (x_S - 2x_f)(-2x_f + x_S + 2x_W) + 4x_f - 2x_W\},\\
C_7^7 &= \frac{q^4 x_D}{4\lambda^2}(-2x_f + x_S - 4x_W - 1)(2x_f + x_S - 2x_W - 1)\{x_D^2(4x_f - 1) - 4x_S(x_f + x_W) + 4((x_f - x_W)^2 + x_W) + x_S^2\}.
\end{aligned}
\tag{C4}
$$

(v) Coefficients C_8^i entering in F_8:

$$
\begin{aligned}
C_8^0 &= \frac{10x_D}{3\lambda^2}(x_f - x_W)(-2x_f + x_S - 4x_W - 1),\\
C_8^1 &= \frac{2x_D}{3q^2\lambda^2(x_D^2 - x_S^2)}\{x_D^2(4x_f - 5x_S + 8x_W + 5) + x_S(5x_S(-2x_f + x_S - 4x_W - 1) + 12(x_f + 2x_W)) - 6(x_f + 2x_W)\},
\end{aligned}
$$

$$C_8^2 = \frac{x_W - x_f}{\lambda^3(x_D + x_S)}\{x_D^4(4x_f - x_S + 8x_W + 6) + x_D^3(9x_S(2x_f - x_S + 4x_W + 2) - 16x_f - 32x_W - 7)$$
$$- 2x_D^2(-2x_S^2(3x_f + 6x_W + 2) + 9x_f + 3x_S^3 + x_S + 18x_W + 2) + x_D(x_S(x_S(24x_f - 12x_S + 48x_W + 17)$$
$$- 10(3x_f + 6x_W + 1)) + 4x_f + 8x_W + 1) + 2x_D^5 - (2x_S - 1)^3(x_S - 2(x_f + 2x_W))\},$$
$$C_8^3 = \frac{x_f - x_W}{\lambda^3(x_D - x_S)}\{x_D^4(-4x_f + x_S - 8x_W - 6) + x_D^3(9x_S(2x_f - x_S + 4x_W + 2) - 16x_f - 32x_W - 7)$$
$$+ 2x_D^2(-2x_S^2(3x_f + 6x_W + 2) + 9x_f + 3x_S^3 + x_S + 18x_W + 2) + x_D(x_S(x_S(24x_f - 12x_S + 48x_W + 17)$$
$$- 10(3x_f + 6x_W + 1)) + 4x_f + 8x_W + 1) + 2x_D^5 + (2x_S - 1)^3(x_S - 2(x_f + 2x_W))\},$$
$$C_8^4 = \frac{x_D}{6\lambda^3}\{x_D^2((13 - 16x_f)x_S + 32x_f(x_f + 2x_W) + 26x_f - 34x_W - 13) + x_S^2(2x_f + 120x_W + 19)$$
$$- 2x_S(-56x_f x_W + 2x_f(x_f + 2) + 150x_W^2 + 86x_W + 3) + 2(-2x_f(90x_W^2 + 44x_W + 1) + 60x_f^3 - 14x_f^2$$
$$+ x_W(30x_W(4x_W + 5) + 43) + 1) - 15x_S^3\},$$
$$C_8^5 = \frac{x_D}{6\lambda^3}\{4x_W^2(-16x_D^2 + 90x_f - 13x_S + 29) + (-2x_f + x_S - 1)(-13x_D^2 - 4(15x_f + 1)x_S$$
$$+ 60x_f(x_f + 1) + 15x_S^2 + 2) + 2\lambda x_W(-16x_f + 8x_S - 9) - 240x_W^3\},$$
$$C_8^6 = \frac{q^2 x_D}{4\lambda^3}\{2x_D^2(x_S(12(2x_f x_W - x_f + x_W^2) + 7x_W - 4) - 6(4x_f + 3)x_W^2 - 6x_f(4x_f + 5)x_W + 12x_f(x_f + 1)$$
$$+ x_S^2(3 - 6x_W) + 48x_W^3 - 5x_W + 2) + x_D^4(4x_W - 1) + 2x_S^3(20x_f + 7x_W + 4) - 2x_S^2(18x_f(x_W + 2) + 60x_f^2$$
$$+ (13 - 6x_W)x_W + 2) + 4x_S(-6x_f^2(x_W - 8) + 12x_f(-3x_W^2 + x_W + 1) + 40x_f^3 + x_W(2(x_W - 3)x_W + 5))$$
$$+ 4(-2(50x_f + 13)x_W^3 + 6(2x_f(5x_f + 4) + 1)x_W^2 + (2x_f^2(10x_f + 9) - 1)x_W$$
$$- 4x_f(x_f + 1)(5x_f(x_f + 1) + 1) + 40x_W^4) - 5x_S^4\},$$
$$C_8^7 = \frac{q^2 x_D}{4\lambda^3}\{2x_D^2(x_S(-2x_f(18x_W + 5) + 15x_W + 4) + (6x_f - 3)x_S^2 + 6(8x_f - 3)x_W^2 - 3(2x_f(4x_f - 9) + 5)x_W$$
$$- 24x_f^3 + 8x_f - 2) + x_D^4(1 - 4x_f) - 2x_S^3(2x_f + 25x_W + 4) + 4x_S(72(x_f - 1)x_W^2 - 3(2(x_f - 3)x_f + 5)x_W$$
$$+ 4x_f^3 - 2x_f - 70x_W^3) + x_S^2(x_f(8 - 36x_W) + 90x_W(2x_W + 1) + 4) + 8(5(7 - 10x_f)x_W^3 + 6(x_f - 1)(5x_f - 3)x_W^2$$
$$+ x_f(2x_f + 3)(5x_f - 3)x_W + 2(2 - 5x_f)x_f^3 + 20x_W^4) + 5x_S^4 + 20x_W\}. \quad \text{(C5)}$$

(vi) Coefficients C_{11}^i entering in F_{11}:

$$C_{11}^0 = \frac{q^2 x_W}{6\lambda}\{2x_f - 3x_S + 28x_W + 3\},$$
$$C_{11}^1 = \frac{x_W}{3\lambda(x_D^2 - x_S^2)}\{x_S^2 - x_D^2 + 6(1 - x_S)(x_f + 2x_W)\},$$
$$C_{11}^2 = \frac{q^2 x_W}{4\lambda^2 x_D(x_D + x_S)}\{x_D^4(-2x_f + x_S + 10x_W + 2) - x_D^3(x_S(6x_f - 40x_W - 5) + 8(x_f - x_W)(x_f + 2x_W)$$
$$+ x_S^2 + 28x_W + 2) - x_D^2(2x_W(2x_f(x_S + 4) - 15x_S^2 + 11x_S + 8) + 4x_f x_S^2 + 4x_f^2 x_S - 8x_f x_S + 16x_f^2 + 6x_f$$
$$- 8(x_S + 4)x_W^2 + x_S^3 + 1) + x_D(x_S^2(24x_f - 2x_W + 6) - 2x_S(10x_f x_W + x_f(10x_f + 11) - 20x_W^2 + 4x_W + 1)$$
$$+ 4(x_f x_W + x_f^2 + x_f - 2x_W^2) - 7x_S^3 - 2x_W) + x_D^5 - (1 - 2x_S)^2(2x_W(2x_f + x_S) + (x_S - 2x_f)^2 - 8x_W^2)\},$$
$$C_{11}^3 = -\frac{q^2 x_W}{4\lambda^2 x_D(x_D - x_S)}\{-x_D^4(-2x_f + x_S + 10x_W + 2) - x_D^3(x_S(6x_f - 40x_W - 5) + 8(x_f - x_W)(x_f + 2x_W)$$
$$+ x_S^2 + 28x_W + 2) + x_D^2(2x_W(2x_f(x_S + 4) - 15x_S^2 + 11x_S + 8) + 4x_f x_S^2 + 4x_f^2 x_S - 8x_f x_S + 16x_f^2 + 6x_f$$
$$- 8(x_S + 4)x_W^2 + x_S^3 + 1) + x_D(x_S^2(24x_f - 2x_W + 6) - 2x_S(10x_f x_W + x_f(10x_f + 11) - 20x_W^2 + 4x_W + 1)$$
$$+ 4(x_f x_W + x_f^2 + x_f - 2x_W^2) - 7x_S^3 - 2x_W) + x_D^5 + (1 - 2x_S)^2(2x_W(2x_f + x_S) + (x_S - 2x_f)^2 - 8x_W^2)\},$$

$$C_{11}^4 = \frac{q^2 x_W}{6\lambda^2}\{x_D^2(8x_f - 2) - 2x_S(2x_f + 15x_W + 1) + 12(x_f - 4x_W)(x_f - x_W) - 4x_f + 3x_S^2 + 30x_W + 1\},$$
$$C_{11}^5 = \frac{q^2 x_W}{3\lambda^2}\{-2x_D^2(8x_W + 1) - 2x_S(6x_f - 4x_W + 1) + 12(4x_f x_W + x_f^2 + x_f - 5x_W^2) + 3x_S^2 + 8x_W + 1\},$$
$$C_{11}^6 = \frac{q^4 x_W}{2\lambda^2}\{-12x_W^2(-2x_D^2 + 6x_f + x_S + 1) - 2x_W(x_D^2(4x_f - 2x_S + 3) + (4x_f - 2)x_S - 4x_f(3x_f + 2) + x_S^2) + (-2x_f + x_S - 1)(x_D^2 - (x_S - 2x_f)^2 - 4x_f) + 40x_W^3\},$$
$$C_{11}^7 = \frac{q^4 x_W}{4\lambda^2}\{4x_W(x_D^2(4x_f - 1) + 4x_f(3x_f + x_S) - 8x_f + x_S(3x_S - 4) + 2) - (2x_f + x_S - 1)(x_D^2(4x_f - 1) + (x_S - 2x_f)^2) - 36x_W^2(2x_f + x_S - 1) + 32x_W^3\}. \quad \text{(C6)}$$

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