

Phase transitions during cyclic inflation and non-Gaussianity

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Typically, cold inflation with positive vacuum energy density dilutes all matter except the quantum fluctuations, which are stretched outside the Hubble patch during inflation. However, the cyclic-inflation scenario, in which the Universe undergoes many asymmetric cycles leading to an overall exponential growth, operates in the presence of thermal matter. Thus, while some of the nice properties of cold inflation are preserved, in cyclic inflationary models, thermal excitations are continually produced, and there exists a unique opportunity of imprinting interesting thermal history in cosmic microwave background radiation. In particular, we will see that a phase transition can lead to observably large higher-order non-Gaussianities as the critical temperature is approached and that these are increasingly divergent with the order of the correlations.

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I. INTRODUCTION

It is well known that our Universe can inflate over many nonsingular cycles in such a way that every cycle undergoes a slight expansion in the scale factor of the Universe. If the Universe undergoes many cycles, which results in 50–60 e -foldings of inflation, known as cyclic inflation (CI) [1], then it can explain the large-scale homogeneity of the Universe. For the operation of the cyclic inflation scenario, one requires a negative potential energy, $-\Lambda$, along with thermal matter, both relativistic and nonrelativistic, which are expected in any dynamical evolution. For instance, the relativistic particles can come from the Standard Model degrees of freedom and the cold dark matter sector (which would all be relativistic at the energy scale in which we are interested). Of course, if the Universe is locked in the negative potential energy phase, it will keep on inflating. However, as shown in Ref. [2], depending upon the nature of the potential, one can exit primordial inflation through a classical transition from negative to positive potential regions; see also Refs. [3–6] for the study and implementation of similar mechanisms. To our knowledge, the CI model was the first to realize a phenomenologically successful cosmology with negative potential energy¹ and may have

interesting implications for string theory and supergravity, which contains negative vacua [11].

In the present paper, we will argue that the origin of the “relevant” seed perturbations that were created during the cyclic phases, i.e., roughly speaking, during the 6–7 e -foldings of inflation that is seen in the cosmic microwave background (CMB) radiation, are statistical fluctuations that are of thermal nature. In particular, we will look at the two-point correlation function, which is related to the power spectrum, \mathcal{P}_ζ , and the higher-point correlation functions, which are related to the parameters f_{NL} and g_{NL} characterizing the non-Gaussian features in the CMB. The constraints on the f_{NL} from the WMAP7 data for the local type of non-Gaussianity implied $-10 < f_{\text{NL}} < 74$ [12], and PLANCK data significantly tightened the constraint to $f_{\text{NL}} = 2.7 \pm 5.8$ [13]. For the g_{NL} , the constraints are considerably weaker, and both the CMB [14] and large scale structure [15,16] restrict it only to be less than about 10^5 in magnitude.

In a companion paper [17], we provide a comprehensive discussion and derivation of how statistical fluctuations that exist in any thermal ensemble can source fluctuations in the CMB; see also Refs. [18–20]. We will focus on how in the CI scenario the observations of the CMB could reveal to us novel thermal features, such as phase transition, which is impossible to detect in the traditional cold inflationary scenarios. In particular, we will see that if the scale of the negative potential energy is close to the critical temperature of a phase transition, then we can hope to find observable non-Gaussian signatures. Interestingly, the signal increases as one looks at higher-order correlation functions. So, for instance, while we find f_{NL} to be too small to agree with the Planck bounds, g_{NL} as well as τ_{NL} can be

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¹For more recent efforts in this direction, see Ref. [7], though ghost and tachyonic instabilities may develop in this proposal [8]. It is worth stressing that the nonsingular scenario of Refs. [1,2] can be based on ghost-free gravity [9]. See also Ref. [10] for a different attempt at nonlocal reconciliation of AdS and our Universe.

large enough to be observed in the near future. In addition, we expect their amplitude to peak over a certain range of scales, which is a further very distinguishing prediction of the thermal phase transitions we explore here.

So, let us now provide a brief summary of how the cyclic evolution works [1,2,21]. Let us begin our story at the expanding phase of a given cycle with the Universe containing some thermal matter along with a negative cosmological constant.² As our Universe expands, all the matter components dilute and are eventually canceled by the negative cosmological constant causing the Universe to turn around and start contracting. In Einstein's gravity, this leads to an eventual collapse, but, for instance, in an extension of general relativity with infinite higher derivatives [22–26], it is possible to resolve this cosmological singularity, and thus the contraction phase can nonsingularly transition into an expansion phase, thereby completing the cycle. An interesting fact about this class of infinite derivative theories is that, in the ultraviolet regime, it resolves the cosmological singularity,³ but in the far infrared, the theory behaves exactly as that of Einstein's theory of gravity. The bounce happens at a scale close to the Planckian time M_p^{-1} , where M_p is the reduced Planck mass.⁴ Once the Universe departs from the Planckian time scale, i.e., as the Universe expands, we are far away from the bounce, and the dynamics of the universe is governed by Einstein's theory. Since, as will soon become apparent, all the interesting physics that we are going to focus on in this paper occurs near the turnaround, which is far away from the bounce, the details of the bounce mechanism should not matter. Thus, while we do not pretend that our assumption about the smooth resolution of the big bang singularity should be taken lightly, at least, if such a resolution mechanism via bounce does exist in nature around the Planck scale, we fully expect our analysis to be independent of the new physics.

Now, interactions between relativistic and nonrelativistic species create entropy that can only increase monotonically, thereby breaking the periodicity of the cyclic evolution. In fact, this was precisely what Tolman pointed out in the 1930s, giving rise to Tolman's entropy problem [35,36]. However, this turns out to be a great asset for the CI scenario. As shown in Refs. [1,21], entropy tends to increase by the same factor in every cycle, while the time period of the cycles remain a constant since it is governed by $|\Lambda|$. This means that the Universe must grow by the

²The generalization to a slowly evolving negative potential energy required to obtain the observed spectral tilt and the graceful exit is discussed in Refs. [2,21], respectively.

³In general, the infrared divergence of quantum fluctuations that could otherwise dominate over the thermal fluctuations may also be resolved in nonsingular cosmologies [27].

⁴For some of the other attempts to resolve the big bang/big crunch singularity through bounces when a high-energy scale is reached, see Refs. [28–34].

same factor, say by $(1 + \kappa)$, in every cycle giving rise to an overall inflationary growth; see Fig. 1. Most of the advantages of standard slow-roll inflation is straightforwardly transferred to the CI scenario, including the production of nearly scale-invariant density fluctuations [21]. Tolman's entropy problem also finds an elegant resolution if the Universe is closed [37]. For more details of the cyclic inflationary scenario and how it can solve the different cosmological puzzles (flatness, horizon, and homogeneity), the readers are referred to Refs. [1,2,21]. In this paper, our primary focus would be the computation of the various features in the primordial fluctuations that are present during the CI phase.

The necessary entropy production, for instance, can be realized by considering some massive species interacting with massless particles via scattering and decay processes. In a given cycle, equilibrium can be maintained up to a certain temperature in the expanding branch, below which the massive species falls out of equilibrium. A subsequent out-of-equilibrium decay into radiation creates entropy. The dynamics of the interaction was investigated numerically in detail in Ref. [38] and found to be able to sustain inflationary growth over many cycles as is essential for CI; see also the summary [39]. The exchange dynamics of the particle production can also present various dynamical consequences such as Bose enhancement, Landau damping, dissipation thermalization, and all the various complications of particle production. Such have been investigated in various other contexts [40–46], and their relevance for the CI should be considered in the future, but the basic mechanism has been shown to be successful [38]. In particular, the nonrelativistic particles decay at each cycle

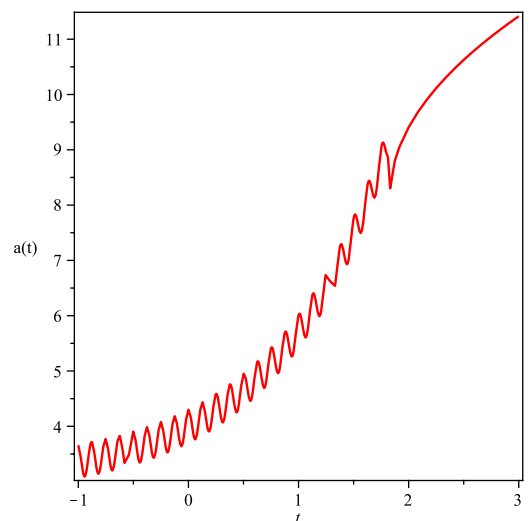


FIG. 1 (color online). Cyclic inflation scenario: The Universe starts in a negative energy phase undergoing asymmetric cyclic evolution. There occurs a last cycle in which the Universe makes a transition from the negative to positive energy phase followed by usual radiation-dominated expansion.

with enough energy and entropy flow to the lighter particles because the cycle contains sufficient nonequilibrium phases in which the heavier particles have a higher effective temperature, and on the other hand, thermalization is sufficiently effective in the high-temperature regimes.

The paper is organized as follows. In Sec. II, we review the evolution of perturbations during the cyclic inflation phase. Sec. III then focuses on the thermal fluctuations in this scenario; in particular, we consider phase transitions and their impact on non-Gaussianity. Tensor modes are found to be insignificant in this context. Section IV briefly concludes. The technical issue of the transition of the modes from the sub- to superhorizon is treated in the Appendix.

II. BRIEF DISCUSSION ON CREATING PERTURBATIONS FROM CYCLIC INFLATION

In Refs. [1,21], a new way of obtaining nearly scale-invariant fluctuations was proposed in the context of cyclic inflationary models. The underlying mechanism is very similar to the standard inflationary scenarios, but the implementation is very different. To understand how the thermal fluctuations evolve, it is useful to compare the cosmological time scale and the physical wavelength of fluctuations under consideration. Naively, if the physical wavelength is shorter than the cosmological scale (sub-Hubble phase), then the fluctuations are expected to be sourced by thermal statistical fluctuations in the fluid,⁵ but once the wavelengths become larger than the cosmological time scale, thermal correlations cannot be maintained, and the coupled metric and fluid fluctuations are governed by the usual differential equations governing an ideal fluid. This transition is rather similar to the usual transition from the sub- to the super-Hubble phase in inflationary cosmology, after which the amplitude freezes.

In the traditional monotonically expanding or contracting Universe, the cosmological time scale is given by the Hubble radius. According to this reasoning, all modes start out in a “pure sub-Hubble” phase in which the physical wavelengths of fluctuations are always smaller than the cosmological scale, which has a lower bound given by the bounce scale,

$$\tau_b \sim \frac{M_p}{\sqrt{\rho_b}} \sim M_p^{-1}, \quad (1)$$

where $\rho_b \sim M_p^4$ determines the total energy density at the time of bounce. This phase lasts until the physical wavelength of the given comoving mode at the bounce point remains smaller than τ_b . With the gradual increase of the

wavelengths, though, due to the asymmetric growth in the cycles, the modes start to undergo a much more complicated history. The modes now start out as sub-Hubble at the turnaround point but exit the Hubble radius in the contracting phase. It re-enters the sub-Hubble phase near the bounce only to exit the Hubble radius shortly after the bounce. Finally, they cross back into the sub-Hubble domain in the expanding branch.

This cyclic pattern of evolution continues for a while, but again as the Universe undergoes inflationary growth over many cycles, the physical wavelength keeps increasing, so that the times that a given mode spends in the sub-Hubble phases around the turnarounds and the bounces become shorter and shorter. Eventually, there comes a cycle in which the sub-Hubble phases are shorter than the physical wavelength of the modes. From this cycle on, although technically the modes do undergo sub-Hubble evolutions, they are ineffective in establishing thermal correlations, and the modes effectively remain super-Hubble until the rest of the duration of the CI phase. We call this the “last exit.” Thus, the CI scenario mimics the standard inflationary mechanism of freezing the amplitude of perturbations after the Hubble crossing.

Approximately, it is clear that the causal time scale in our scenario is given by the time period of the cycle

$$\tau = \beta \frac{M_p}{\sqrt{\Lambda}}, \quad (2)$$

where $\beta \sim \mathcal{O}(1)$ constant, and therefore the modes must be exiting when their wavelengths $\sim \tau$. In other words, the modes exit very near the turnaround energy density; in the Appendix, we provide a quantitative estimate of this time scale. A more physical way of thinking about the whole process is simply to realize that the cosmological time scale does not correspond to the Hubble radius near turnarounds and bounces but is rather controlled by τ and τ_b , respectively, which in turn are determined by the typical energy densities near these regions. The fact that the Hubble scale does not always correspond to the cosmological scale has indeed been discussed before [47].

We now come to a very important point. Suppose k_1 is a mode that is the first to make the last exit in a given cycle. Then, once the wave number increases by a factor $(1 + \kappa)$, the fluctuations can no longer undergo the last exit in that given cycle because they reenter the thermal sub-Hubble evolution during the following turnaround; see Fig. 2. They therefore must wait for the next cycle for their last exit. What this means is that, if κ is a small parameter, then the last exits always occur at approximately the same energy densities, $\sim \Lambda$, near the turnarounds. We know from our study of perturbations that the energy density (or, equivalently, the temperature in our case) at the exit controls the amplitude of fluctuations that we eventually

⁵For a detailed discussion on the subject, the reader is referred to our companion paper [17].

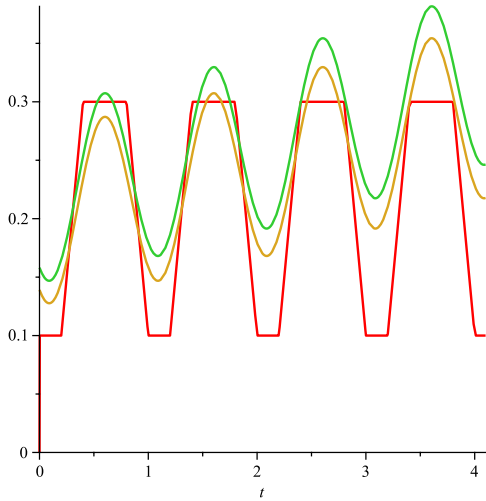


FIG. 2 (color online). The last exit: The curve in the red corresponds to the cosmological time scale, while the green and the yellow curves represent the physical wavelengths of two different comoving modes. The two modes are initially in a mixed phase, but then they make their last exits, after which they evolve as super-Hubble modes. While the green wavelength exits in the third cycle, the yellow curve has to wait for another cycle to make its last exit. The two modes experience identical background cosmology, the yellow curve only lagging behind by a cycle as compared to the green curve.

see in the CMB. This therefore explains why the cyclic inflation model gives rise to an approximate scale-invariant spectrum, preserving one of the most robust predictions of inflation theory.

From the above discussion, a rather distinguishing feature of the cyclic inflation model also emerges, which was highlighted in Ref. [21]. The fact that in a given cycle only a certain range of modes make their last exit means that there should exist small wiggles in the primordial spectrum of the form

$$\mathcal{P}_\Phi = \mathcal{A}^2 S(k), \quad (3)$$

where \mathcal{A} is strictly a constant if we have a negative cosmological constant but will vary slowly if we have a $V(\phi)$ giving rise to an overall tilt in the spectrum, while $S(k)$ is a periodic function in $\ln k$:

$$S(k(1 + \kappa)) = S(k). \quad (4)$$

The precise shape of $S(k)$ is determined by how the temperature rises as the modes exit in a give cycle, which in turn depends on the thermodynamic properties of the fluid. While our previous analysis provided some tantalizing hints for such oscillatory features, the Planck data would be able to shed more light. For us, the important point is that the oscillatory feature can be used in conjunction with the non-Gaussian features, which we will soon discuss, to

differentiate our model from other inflationary scenarios that also produce large non-Gaussianities.

III. STATISTICAL THERMAL FLUCTUATIONS AND CMB

In many extensions of the Standard model, we expect our Universe to have undergone phase transitions at high temperatures; these include grand unified theories (supersymmetric and nonsupersymmetric) as well as string theory. Could these phase transitions have left any observable imprint in our sky? In this section, we will provide a simple example of thermal phase transition that can leave detectable signals in the CMB. First, the phase transition parametrization will be described; then, we will apply the results of the companion paper [17] to compute the resulting bispectrum and trispectrum. Finally, gravitational wave production is discussed briefly.

A. Phase transition

Phase transitions are characterized by critical exponents. For our purpose, the most relevant is the one associated with the heat capacity, commonly denoted by α :

$$\lim_{T \rightarrow T_c} C_V \sim \left| \frac{T - T_c}{T_c} \right|^\alpha. \quad (5)$$

$\alpha = -1$ corresponds to a first-order phase transition, where the heat capacity diverges. We are going to restrict ourselves to higher-order transitions with $\alpha > -1$ and consider the case in which the temperature of the last exit is below the critical temperature. A prototype partition function corresponding to critical behavior such as Eq. (5) is given by

$$p = \frac{T \ln Z}{V} = \frac{1}{3} g T^4 - \Lambda + \mu T_0^4 \left[\left(1 - \frac{T}{T_c} \right)^{\alpha+2} - 1 \right]. \quad (6)$$

The two terms in the first line correspond to the pressure coming from the usual massless degrees of freedom and the negative cosmological constant that is required in the CI model to provide the turnarounds. The term in the second parentheses contain the physics of the phase transition; incorporating some multiplicative modulating functions of temperature should not change the results in any significant way, as we are only going to be interested in temperatures very close to the critical temperature, T_c . The parameter g is the number of effective massless degrees of freedom times $(\pi^2/30)$, and the parameter μ controls the contribution of the thermal density component undergoing the phase transition; we consider the regime $g \gg \mu$. The temperature at the turnover is T_0 , and it provides the overall energy scale. In terms of these parameters, the cosmological constant is given by

$$\begin{aligned}\Lambda &= \rho(T \rightarrow 0) = -p(T \rightarrow 0) \\ &= -T_0^4 \left[g + \mu - \mu \left(1 + \frac{T_0}{T_c} \right)^{\alpha+1} \left(1 + (1 + \alpha) \frac{T_0}{T_c} \right) \right],\end{aligned}\quad (7)$$

and, as mentioned, it is negative in the CI scenario.

Noting the thermodynamic relation

$$\rho(T) = -\frac{1}{V} \frac{\partial(\ln Z)}{\partial \beta} = T \frac{dp(T)}{dT} - p(T), \quad (8)$$

one obtains

$$\rho = gT^4 + \Lambda - \mu T_0^4 \left[\left(1 - \frac{T}{T_c} \right)^{\alpha+1} \left(1 + (1 + \alpha) \frac{T}{T_c} \right) - 1 \right], \quad (9)$$

$$\frac{C_V}{V} = 4gT^3 + \mu \frac{T_0^4 T}{T_c^2} (1 + \alpha)(2 + \alpha) \left(1 - \frac{T}{T_c} \right)^\alpha. \quad (10)$$

One readily checks that this is consistent with the density vanishing at the turnaround $T = T_0$ and reducing to the cosmological term at the zero temperature. We plot the equation of state in Fig. 3.

B. Non-Gaussianity

The expressions for calculating the power spectrum and the non-Gaussianity parameters from statistical fluctuations are derived in our companion paper [17]. Here, we summarize the results:

$$\mathcal{P}_\zeta = \gamma^2 \sqrt{3} A^2(T) \frac{T^2 \rho'}{M_p^3 \sqrt{\rho}}, \quad (11)$$

$$f_{\text{NL}} = \left(\frac{1}{\gamma} \right) \frac{1}{A(T)} \left[\frac{5\rho(2\rho' + T\rho'')}{24T(\rho')^2} \right], \quad (12)$$

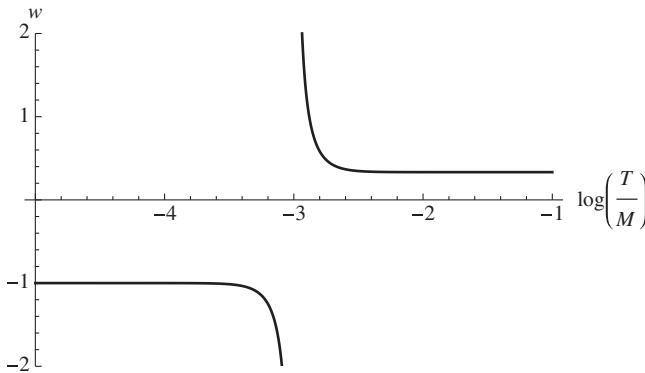


FIG. 3. The equation of state of the fluid as a function of temperature when $T_c = 0.1M_p$ and $T_0 = 0.01M_p$. We use $\alpha = 0.1$, $g = 100$, and $\mu = 0.1$. At high temperatures, the radiation dominates, and $w = 1/3$; at low temperatures, the cosmological constant is important, and $w = -1$. The divergence occurs at $T = T_0$, where the density vanishes.

$$g_{\text{NL}} = \left(\frac{1}{\gamma^2} \right) \frac{1}{A^2(T)} \left[\frac{25\rho^2[3(\rho' + T\rho'') + T^2\rho''']}{243T^2(\rho')^3} \right], \quad (13)$$

where

$$A(T) \equiv \frac{9 + 3w + 2r}{6(1 + w)} \quad (14)$$

$$r \equiv -\frac{3}{2} \left[1 + \frac{(1 + w)\rho(2\rho' + T\rho'')}{T\rho'^2} \right] \quad (15)$$

$$w = \frac{p}{\rho} \quad \text{and} \quad \gamma = 2\sqrt{2}\pi^{3/4} \approx 6.7. \quad (16)$$

We have used the value of γ that corresponds to a Gaussian window function [17] and have set $\Omega = 1$ since Eq. (9) already incorporated the energy coming from the cosmological constant, radiation, and the thermal matter undergoing phase transition.⁶

Before trying to compute and plot these different quantities, let us try to understand the physics with the help of some approximations. To begin with, we have six parameters: T_e , T_c , T_0 , μ , g , and α , where T_e is the exit temperature. However, not all of these parameters are independent or important. First, we can choose g to be the number appropriate for the Standard Model, $g \sim (\pi^2/30)\mathcal{O}(100)$. The value we use for numerics is $g = 100$. We note in passing that one of the virtues of the CI model is that there is no need for reheating, and the Universe could have only contained the Standard Model degrees of freedom all along. Also, our computation of the exit temperature in terms of T_0 in the Appendix was done for radiative matter sources, and since $\mu \ll g$, the result remains applicable to our case; as also seen in Fig. 3, the radiative matter dominates the system when $T > T_0$. Hence, it is justified to fix

$$T_0 = 0.8T_e. \quad (17)$$

To consider a definite and relevant phase transition, we can use the critical exponent corresponding to ϕ^4 theory. It has been computed/measured using various methods, and the results converge to $\alpha \approx 0.1$; see, for instance, Ref. [48]. This is the value we use for numerical examples unless otherwise specified.

We are thus left with three parameters: T_0 , μ , and T_c . Instead of the latter, we shall refer to

$$\Delta \equiv (T_e - T_c)/T_c. \quad (18)$$

However, the amplitude of CMB fluctuations is known, and this can be used to constrain one of the parameters; a convenient choice is T_0 , which gives the overall scale of the cycles. We show in Fig. 4 solutions for the temperature solved by matching the amplitude of the spectrum with the observed PLANCK value. To conclude, our predictions depend upon two parameters: the contribution of the matter

⁶In the CI scenario, one also requires the presence of non-relativistic species [21], but it is always present in small amounts and, in fact, decays before the onset of the contraction phase.

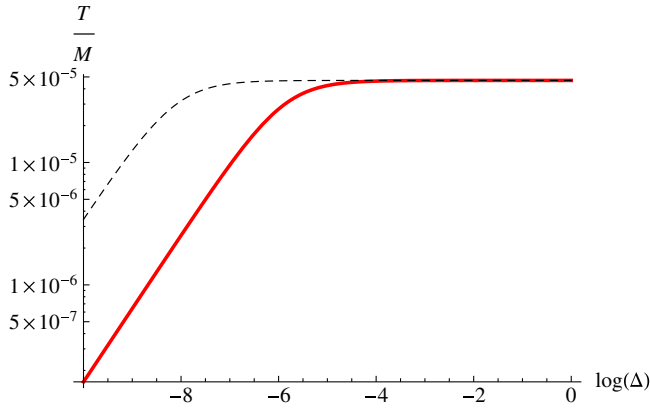


FIG. 4 (color online). The turnover temperature T_0 of the fluid as a function of Δ . For the red solid line, $\mu = 0.1$, and for black dashed line, $\mu = 1/1000$.

subject to phase transition, μ , and the proximity to the critical temperature, Δ .

The essential feature of our results is large higher-order non-Gaussianity. They contain more and more derivatives of $\rho(T)$ and therefore become more and more divergent as T approaches T_c . Accordingly, the f_{NL} parameter remains typically small, as seen in Fig. 5. The parameter approaches a constant as $T \rightarrow T_c$ and another constant far away from the critical temperature. The next-order non-Gaussianity g_{NL} is already much more interesting: the parameter diverges as $T \rightarrow T_c$. To understand this, note that the leading term in this limit, from Eq. (13), is due to the third derivative of the density that goes like $\rho''' \sim \Delta^{\alpha-2}$. However, one should also take into account the divergent terms in the denominator, which is given by $A^2 \sim r^2 \sim \rho'^2 \sim \Delta^{2(1-\alpha)}$, as seen from Eqs. (14) and (15). The result is that the non-Gaussianity diverges as $g_{\text{NL}} \sim \Delta^{-\alpha}$.

We illustrate this in the Fig. 6 where the non-Gaussianity is logarithmically plotted as a function of μ and Δ . While the fact that we only get an interesting signal for $\Delta \leq 10^{-4}$ may appear to be a fine-tuning, that is not so. We remind the readers that, to be consistent with the observed spectral tilt

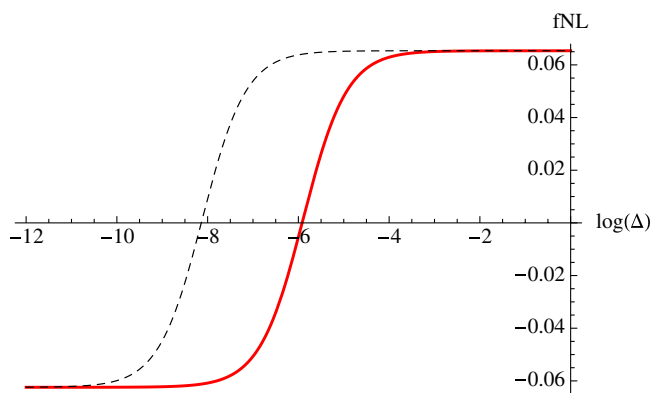


FIG. 5 (color online). Non-Gaussianity f_{NL} as a function of Δ . For the red solid line, $\mu = 0.1$, and for black dashed line, $\mu = 1/1000$.

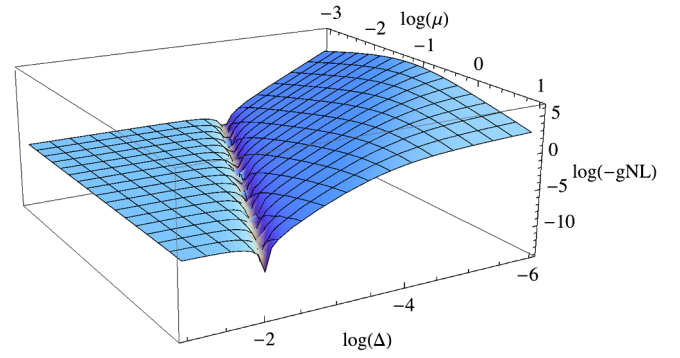


FIG. 6 (color online). Non-Gaussianity g_{NL} as a function of Δ and μ . The parameter is negative and diverges as $T \rightarrow T_c$.

of the spectrum, T_0 and hence T_e have to vary. For $n_s \sim 0.95$, T_e varies by approximately 30%, which means, as long as T_c lies within this range, we can observe a signal. In fact, based on our results, it seems that rather than having a broad g_{NL} spectrum, we expect a sharp peak in k , which is another distinctive feature of the thermal signal. The higher-order non-Gaussianities will peak at the wavelengths that cross the horizon as the phase transition occurs.

The dependence on the critical exponent very near the transition is depicted in Fig. 7. In brief, for positive α , whenever we are close enough to the critical temperature, the g_{NL} is large. In general, the non-Gaussianity corresponding to the N -point function diverges near the critical temperature as $(1 - T/T_c)^{\alpha(3-N)}$, so the higher-order non-Gaussianities can be significantly larger than the lower-order ones. This property stems from the diverging behavior of the higher derivatives of the density ρ and can provide a smoking-gun signal for these models.

C. Gravity waves

To close the section of CMB constraints, let us investigate the gravitational wave production in the setup. For extensive thermal systems, there are no off-diagonal stress energy

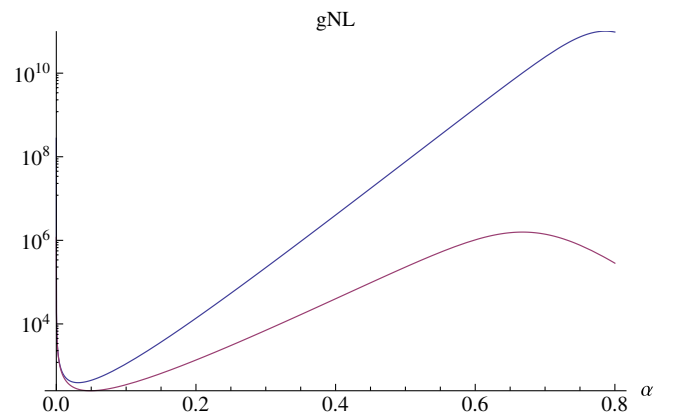


FIG. 7 (color online). Non-Gaussianity g_{NL} as a function of the critical exponent α when $\mu = 1$. For the upper line, $\Delta = 10^{-15}$, and for the lower one, $\Delta = 10^{-10}$. The monotonic scaling behavior comes apparent only very near the transition.

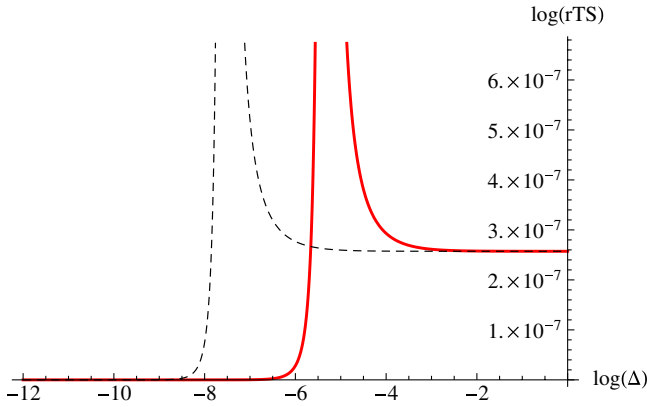


FIG. 8 (color online). The scalar-to-tensor ratio as a function of Δ . For the red solid line, $\mu = 0.1$, and for black dashed line, $\mu = 1/1000$.

components, and, as a result, there are no source terms for the gravitational waves. The gravity wave spectrum is therefore governed by the quantum vacuum fluctuations similar to what happens in the standard scalar field driven inflationary paradigm, assuming that the gravity is indeed quantum [49]. The gravity wave power spectrum is thus given by

$$\mathcal{P}_h = \frac{1}{4\pi^3} \left(\frac{H}{M_p} \right)^2 = \frac{1}{12\pi^3} \frac{\rho}{M_p^4}, \quad (19)$$

where ρ corresponds to the energy density of the thermal fluid at the time of the last exit. Therefore, the tensor-to-scalar ratio is given by

$$r_{t/s} \equiv \frac{\mathcal{P}_h}{\mathcal{P}_\zeta} = \frac{1}{3\sqrt{3}\pi^3\gamma^2} \left(\frac{1+w}{2+w+r} \right)^2 \frac{\rho^{\frac{3}{2}}}{T^2 M_p^4} \left(\frac{\partial \rho}{\partial T} \right)^{-1}. \quad (20)$$

Again, the expression has to be evaluated at the Hubble crossing, $k = aH$. The results are plotted in Fig. 8. We see that the amplitude of the tensor modes is typically too small to be observed. For particular parameter combinations though, the scalar-to-tensor ratio diverges. However, the cyclic inflation to occur sufficiently near these singular points to obtain an appreciable amount of gravitational waves would seem to require considerable fine-tuning. Hence, we regard negligible gravitational wave contribution a robust prediction of the scenario.

IV. CONCLUSIONS

We computed the different features of the CMB in the cyclic inflation scenario by applying the formalism developed for cosmological perturbations arising from statistical thermal fluctuations in our companion paper [17].

In particular, we explored the implications of phase transitions during the inflationary expansion. We found that the non-Gaussianity corresponding to the N -point function of thermal fluctuations diverge near the critical temperature T_c as $(1 - T/T_c)^{\alpha(3-N)}$, where α is the critical exponent. The higher-order correlations will be drastically amplified over a given range of scales. This generic feature

is an imprint quite unique to the physics of thermal phase transitions. Observationally, the higher-order correlations are much more difficult to constrain, and since they are not typically huge in cold inflation models, the attention has been almost exclusively upon the bispectrum and trispectrum. However, as we predict that the amplitude of the signal is increasingly divergent for higher-order non-Gaussianities, our findings can motivate the study of previously neglected completely new signatures.

Coupled with the firm prediction of negligible tensor amplitude and subtle wiggly features in the spectra [21], the structure of non-Gaussianities can provide a robust test of the cyclic inflation scenario.

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APPENDIX: TEMPERATURE OF THE LAST EXIT

While intuitively it is clear that, once the wavelengths become larger than the period of the oscillations, the modes should essentially behave as zero modes and follow the super-Hubble evolution, in any nonsingular cyclic model, there are always two points within a cycle, the bounce and the turnaround, at which the Hubble radius is infinite. So, according to the naive definition, all modes are back in the sub-Hubble phase. Of course, the crucial point is that the larger the wavelength of the modes, the shorter the time was that they spent in this sub-Hubble phase. Very conservatively, these modes need to spend at least a minimum of $\Delta t \sim \lambda/c$ time to have any ‘‘sub-Hubble’’ evolution at all. For instance, if the time the mode spends is less than λ/c , there is no way any thermal correlation could be established stretching a wavelength λ . Thus, although technically all modes are sub-Hubble during the turnaround and the bounce, that is of little relevance. Once a mode reaches a particular physical wavelength, it makes its last exit and starts behaving as a super-Hubble mode for the cycles to follow. Let us, in fact, calculate this wavelength/temperature of the last exit point.

Let us denote the scale factor and temperature when a given mode, k , exits the Hubble radius in the contraction phase as a_k and T_k , respectively:

$$a_k H(a_k) = k. \quad (A1)$$

Next, let us calculate the time that this mode spends in the sub-Hubble phase. Ignoring any asymmetry, this is given by

$$\Delta t = \int dt = 2 \int_1^{a_k} \frac{da}{aH}, \quad (A2)$$

where we have chosen the convention that $a = 1$ corresponds to the turnaround in question, so that $H(1) = 0$. The smaller the k , the closer a_k is to one, and therefore Δt is

shorter, as expected. Now, in order for any thermal correlation to reestablish, $\Delta t > \lambda = a/k$. Clearly, then there exists a value of k below which this condition can never be satisfied. The largest k for which thermal correlation can be reestablished is then given by

$$\Delta t = \frac{a_k}{k}. \quad (\text{A3})$$

The above is an implicit equation for k via Eq. (A1). Using the above equation, for any given thermodynamic fluid, one can calculate the exit temperature, k , and a_k . Let us now demonstrate this for radiation.

For radiation in the presence of a negative cosmological constant, we have

$$\begin{aligned} H &= \frac{\sqrt{\Lambda}}{\sqrt{3}M_p} \sqrt{a^{-4} - 1} \Rightarrow k = a_k \frac{\sqrt{\Lambda}}{\sqrt{3}M_p} \sqrt{a_k^{-4} - 1} \\ &= \frac{\sqrt{\Lambda}}{\sqrt{3}M_p} \sqrt{a_k^{-2} - a_k^2} \quad \text{and} \\ \Delta t &= \frac{\pi}{2} - \tan^{-1} \left(\frac{a_k^2}{\sqrt{1 - a_k^4}} \right). \end{aligned} \quad (\text{A4})$$

Equation (A3) can now be solved numerically to obtain a_e , the scale factor at which the last exit starts to occur. Approximately, we find

$$a_e \approx 0.8 \Rightarrow T_e \approx 1.25T_0, \quad (\text{A5})$$

where T_0 is the temperature at the turnaround point. More physically, all those modes for which the physical wavelength at the turnaround point is larger than a certain wavelength,

$$\lambda > \frac{\sqrt{3}M_p}{\sqrt{\Lambda}} \frac{a_e}{\sqrt{1 - a_e^4}} \approx 1.04\tau, \quad (\text{A6})$$

never undergo any thermal sub-Hubble evolution at all; they must have had their last exits in some previous cycle.

One can carry out a similar analysis during the bounce, since the time scale of the bounce is much shorter than the turnaround; any mode which does not undergo any sub-Hubble phase near the turnaround also has no sub-Hubble evolution during the bounce.

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