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Ferromagnetic neutron stars: Axial anomaly, dense neutron matter, and pionic wall

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We show that a chiral nonlinear sigma model coupled to degenerate neutrons exhibits a ferromagnetic phase at high density. The magnetization is due to the axial anomaly acting on the parallel layers of neutral pion domain walls spontaneously formed at high density. The emergent magnetic field would reach the QCD scale $\sim 10^{19}$ [G], which suggests that the quantum anomaly can be a microscopic origin of the magnetars (highly magnetized neutron stars).

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I. MAGNETARS AND HIGH-DENSITY NEUTRON MATTER

The phase diagram of QCD is a mystery to be uncovered. Even though the problem is theoretically well posed with QCD Lagrangian, so far it remains difficult to reveal a specific region of the phase diagram, the region at low temperature and high baryon number density ρ [1].

One of the promising systems where such a high-density region is realized in nature is the deep interior of compact stars, such as neutron stars. Observations of various properties of these stars should give us crucial constraints on high-density matter. In particular, the magnetars, which are considered to be neutron stars with a very strong magnetic field $\sim 10^{15}$ [G] at their surface, are of particular interest [2,3]. The mechanism for generating such a strong magnetic field is so far not established: Among various proposals, including the dynamo formation model and the fossil-field model [2], nuclear ferromagnetism associated with the solidification of the neutron star core, which was originally proposed right after the discovery of the pulsar [4], is an interesting possibility for generating a large intrinsic magnetic field. However, modern quantum many-body calculations on the neutron matter and asymmetric nuclear matter with realistic nuclear force have shown that these systems stay in the liquid phase at high density without having spontaneous ferromagnetic transition [5]. Another interesting possibility is the ferromagnetism of the quark liquid in the central core of a neutron star [6]; a spin-polarized quark phase, similar to that in the low-density electron gas, may be realized in a certain window of baryon density due to the Fock term of the gluon exchange between quarks.

In this paper, we propose a novel mechanism which leads to a *spontaneous* magnetization of the neutron matter, based on the nonlinear chiral Lagrangian of pions coupled to degenerate neutrons. Two basic ingredients are (i) the neutron spin density induced on a pion domain wall in dense matter [7] and (ii) the baryon number induced on the pion domain wall by an external magnetic field through axial anomaly [8]. We show that layers of π^0 domain walls are spontaneously generated by a small seed of an external magnetic field. Then, an intrinsic magnetic field is induced from the layers which have net magnetization. If this mechanism takes place inside the core of a neutron star above a certain threshold density, it acquires a large magnetic field and becomes a magnetar.

II. CHIRAL LAGRANGIAN AND PION DOMAIN WALLS

We use the chiral Lagrangian for low-energy pions and nucleons with the Weinberg parametrization [9]:

$$\mathcal{L} = \bar{N}[i\gamma^\mu(\partial_\mu + i\boldsymbol{\tau}\cdot\boldsymbol{V}_\mu + i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{A}_\mu) - m_{\rm N}]N + \frac{1}{2}|D_\mu\boldsymbol{\phi}|^2 - \frac{1}{2}(m_\pi^2 + \sigma_{\pi N}\bar{N}N)\frac{\boldsymbol{\phi}^2}{1+\boldsymbol{\phi}^2/4f_\pi^2}. \quad (1)$$

Here N is the nucleon field [isospin $SU(2)$ doublet], $\boldsymbol{\phi}$ is the pion (triplet), f_π is the pion decay constant, g_A is the axial charge of the nucleon, and m_π is the pion mass. Also, $\boldsymbol{V}_\mu \equiv \frac{1}{4f_\pi^2}(1+\boldsymbol{\phi}^2/4f_\pi^2)^{-1}(\boldsymbol{\phi}\times\partial_\mu\boldsymbol{\phi})$, $\boldsymbol{A}_\mu \equiv \frac{g_A}{2f_\pi}D_\mu\boldsymbol{\phi}$, with $D_\mu\boldsymbol{\phi} \equiv (1+\boldsymbol{\phi}^2/4f_\pi^2)^{-1}\partial_\mu\boldsymbol{\phi}$.

In Ref. [7], neutrons are integrated with neutron chemical potential, and π^0 domain wall solutions are studied. Here we generalize the system to include the charged pion and obtain the in-medium chiral Lagrangian up to $O(p^2)$ [10]:

$$\begin{aligned}\mathcal{L}_{\rm eff} = &\frac{\alpha}{2}|D_0\phi_3|^2 - \frac{\beta}{2}|D_i\phi_3|^2 + \frac{\tilde\alpha}{2}|D_0\phi_+|^2 - \frac{\tilde\beta}{2}|D_i\phi_+|^2 \\ &- \gamma_0\frac{m_\pi^2}{2}\boldsymbol{\phi}^2/(1+\boldsymbol{\phi}^2/4f_\pi^2) + \gamma_1(\phi_+(D_0\phi_+)^* \\ &- (\phi_+)^*D_0\phi_+) + \gamma_2(\phi_+(D_0\phi_+)^* - (\phi_+)^*D_0\phi_+)^2 \\ &+ \gamma_3|\phi_+D_0\phi_3 - \phi_3D_0\phi_+|^2 + \gamma_4|\phi_+D_i\phi_3 \\ &- \phi_3D_i\phi_+|^2, \end{aligned} \quad (2)$$

with $i = 1, 2, 3$. We have defined the charged pion $\phi_+ \equiv \phi_1 + i\phi_2$, and $D_\mu\phi_+ \equiv (\partial_\mu\phi_+ + i\delta_{\mu 0}\mu_{\rm I}\phi_+)/(1 + |\boldsymbol{\phi}|^2/4f_\pi^2)$, where $\mu_{\rm I}$ is the isospin chemical potential defined as the difference $\mu_{\rm I} \equiv \mu_{\rm p} - \mu_{\rm n}$. The correction due to the background neutrons is in the coefficients α, β, $\tilde{\alpha}$, $\tilde{\beta}$ (and $\gamma_{1,2,3,4}$), which are equal to unity (zero) in the absence of the nucleons. They are given by one-loop calculations (see Ref. [7] for α and β):

$$\beta \equiv 1 - \frac{g_A^2 m_N^2}{4\pi^2 f_\pi^2} \log\left(x + \sqrt{x^2-1}\right),$$

$$\alpha \equiv 1 + \frac{g_A^2 m_N^2}{4\pi^2 f_\pi^2}\left(x\sqrt{x^2-1} - \log\left(x+\sqrt{x^2-1}\right)\right),$$

$$\tilde{\beta} \equiv 1 - \frac{g_A^2 m_N^2}{2\pi^2 f_\pi^2}\int_1^x ds \frac{(s^2-1)^{1/2}(2s^2+4+3(1-x)s)}{3(1-x+2s)(x-1)},$$

$$\tilde{\alpha} \equiv 1 + \frac{g_A^2 m_N^2}{2\pi^2 f_\pi^2}\int_1^x ds \frac{(s^2-1)^{1/2}(2s^2-2+(1-x)s)}{(1-x+2s)(x-1)},$$

$$\gamma_0 \equiv 1 + \frac{\sigma_{\pi N}}{m_\pi^2}\frac{m_N^3}{2\pi^2}\left(x\sqrt{x^2-1} - \log\left(x+\sqrt{x^2-1}\right)\right),$$

$$\gamma_1 \equiv \frac{i m_N^2}{24\pi^2 f_\pi^2}(x^2-1)^{3/2},$$

$$\gamma_2 \equiv \frac{-m_N^2}{128\pi^2 f_\pi^4} x\sqrt{x^2-1},$$

$$\gamma_3 \equiv \frac{m_N^2}{16\pi^2 f_\pi^4}\int_1^x ds \frac{(s^2-1)^{1/2}(2s^2+(1-x)s)}{(1-x+2s)(x-1)},$$

$$\gamma_4 \equiv \frac{m_N^2}{16\pi^2 f_\pi^4}\int_1^x ds \frac{(s^2-1)^{1/2}(2s^2-2+3(1-x)s)}{3(1-x+2s)(x-1)}.$$

Here we consider the pure neutron matter for simplicity, so that $x \equiv \sqrt{k_F^2 + m_N^2}/m_N$, with k_F being the Fermi momentum of the neutron. The neutron number density is related to the Fermi momentum as $\rho_n = k_F^3/(3\pi^2)$. To estimate the influence of the effective mass of the neutron m_N^* qualitatively, we replace m_N with m_N^* in the above expressions.

In Fig. 1, we plot β as a function of ρ_n/ρ_0 for the range $0.5 < m_N^*/m_N < 0.7$, with $\rho_0 = 0.16$ [fm^{-3}] being the normal nuclear matter density. An important feature seen in Fig. 1 is that β is a monotonically decreasing function of ρ_n and vanishes at a certain density above ρ_0. The density at which β becomes zero increases as m_N^* decreases. We note that there are further uncertainties for β originating from in-medium low-energy constants and from the short-range correlation of the neutrons. Therefore, one should consider Fig. 1 only as a qualitative estimate. As for the other coefficients, α and $\tilde{\alpha}$ do not vary much in density, while $\tilde{\beta}$ has a faster decrease within the same approximation.

A classical solution of Eq. (2) is a domain wall of the in-medium neutral pion [7],

$$\phi_3 = \frac{2f_\pi}{\sinh[m_\pi x^3/\sqrt{\beta/\gamma_0}]}, \qquad \phi_1 = \phi_2 = 0. \quad (3)$$

Note that this π^0 domain wall interpolates the vacua $\theta = 0$ and $\theta = 2\pi$, where $\tan\frac{\theta}{2} \equiv |\boldsymbol{\phi}|/2f_\pi$. Interestingly, the domain wall can reduce its weight in the neutron matter: γ_0 stays positive at all x, while β approaches zero as shown in Fig. 1, so that the tension $\mathcal{E}/S$ of the domain wall is significantly reduced for $\beta \to 0$,

$$\mathcal{E}/S = 8\sqrt{\beta\gamma_0} f_\pi^2 m_\pi, \quad (4)$$

where S is the domain wall area. We will show that the parallel layers of the domain walls would populate at high baryon density due to the reduction of the tension and the accumulation of baryon number on the domain wall.

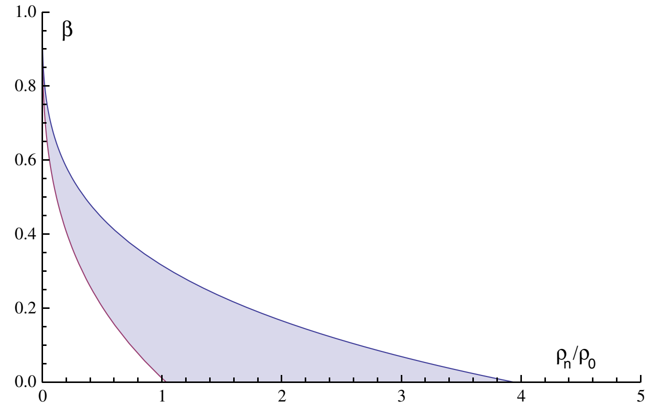

FIG. 1 (color online). β as a function of ρ_n/ρ_0. We use the low-energy constants in the vacuum $f_\pi = 93$ [MeV] and $g_A = 1.27$. The effective mass of the nucleon is taken in the range $0.5 < m_N^*/m_N < 0.7$, which corresponds to the shaded region in the figure.

III. EMERGENT MAGNETIC FIELD ON PIONIC WALLS FROM AXIAL ANOMALY

Let us first discuss how spontaneous magnetization occurs in high-density neutron matter. There are three steps for this to happen: (i) The in-medium domain wall becomes light, and at the same time it acquires finite baryon density due to axial anomaly under an external magnetic field. Then the system with a domain wall becomes energetically favorable to the uniform neutron matter above a certain density. (ii) The domain wall is magnetized due to the spin alignment of surrounding neutrons, so that it creates spontaneous magnetization which enlarges the original magnetic field. (iii) The enhanced magnetic field creates more domain walls. The cycle (i) → (ii) → (iii) → (i) is repeated and leads to a stable configuration with many parallel layers of thin domain walls with a high magnetic field. All the spins are aligned, so the system is ferromagnetic.

We now explain the mechanism of inducing the baryon charge following Ref. [8]. In a background constant magnetic field, the QCD axial anomaly term in the chiral Lagrangian reads

$$\mathcal{L}_{WZW} = \frac{ie}{16\pi^2} A_0^{(B)} B_3 \,\mathrm{tr}[\tau_3(U\partial_3 U^\dagger + \partial_3 U^\dagger U)], \quad (5)$$

where $A_0^{(B)}$ is the temporal component of a gauge potential for the baryon number symmetry, B_3 is the background magnetic field along x^3, and $U \equiv \cos\theta + i\boldsymbol{\tau}\cdot\hat{\boldsymbol{\phi}}\sin\theta$ with $\hat{\boldsymbol{\phi}} = \boldsymbol{\phi}/|\boldsymbol{\phi}|$. Since the pion-dependent part can be evaluated (for $\phi_1 = \phi_2 = 0$) as

$$\mathrm{tr}[\tau_3(U\partial_3 U^\dagger + \partial_3 U^\dagger U)] = -4i\frac{D_3\phi_3}{f_\pi} = -4i\partial_3\theta, \quad (6)$$

we immediately see that the domain wall, which interpolates $\theta = 0$ and $\theta = 2\pi$, can obtain a baryon charge per unit area [8],

$$N_B/S = eB_3/2\pi. \quad (7)$$

If the domain wall is not parallel to the magnetic field, the induced baryon charge is reduced to $B_i\hat{n}_i$, where $\hat{n}$ is the unit vector perpendicular to the domain wall. Note that the formula in Eq. (7) is valid even for $\beta \neq 1$.

Combining Eq. (7) with Eq. (4), the domain wall energy per unit baryon charge is

$$\mathcal{E}/N_B = \frac{16\pi f_\pi^2 m_\pi}{eB_3}\sqrt{\beta\gamma_0}. \quad (8)$$

The system with a domain wall is more favorable than the uniform neutron matter when the domain wall energy per baryon becomes smaller than the neutron chemical potential [8], i.e., $B_3 > \sqrt{\beta\gamma_0}\times(16\pi f_\pi^2 m_\pi/\mu_\mathrm{n}) \sim \sqrt{\beta\gamma_0}\times 10^{19}$ [G]. Here, the factor β, which was not taken into account in Ref. [8], is important. Due to this β, the threshold magnetic field for the domain wall creation is parametrically small compared to that of Ref. [8]. This fact, together with the induced spin on the domain wall, leads to the spontaneous generation of the magnetic field with a small seed. Such a mechanism has not been proposed before. Since $\beta(\rho_n)$ is a monotonically decreasing function of ρ_n, an adiabatic increase of the density ρ_n inevitably hits the critical value of β at which the domain wall is created.

Once the pionic wall is formed, neutron spins on the wall align in the direction perpendicular to the domain wall [7]. The spin density of the neutrons is a spatial part of the axial current $j_i^{(A)} = \langle\bar{\psi}_n\gamma_i\gamma_5\psi_n\rangle$. It was evaluated in Ref. [7], in the same approximation, as

$$s_3/S = 2\pi(\beta - 1)f_\pi^2, \quad (9)$$

which is the third component of the neutron spin density per unit area of the domain wall.

We note here that neutrons have a magnetic moment $\mu = ges_3/2m_\mathrm{N}$, where $g \sim -3.8$ is the neutron g factor. So the total magnetic moment (which is the magnetization M) per unit volume at $\beta \sim 0$ is

$$M = \frac{\pi|g|ef_\pi^2}{m_\mathrm{N}}\frac{1}{d}, \quad (10)$$

where d is the separation between adjacent domain walls. Therefore, the domain wall phase is ferromagnetic.

The magnetization M is larger for smaller separation d among the domain walls. This d is intimately related to the induced baryon density due to the domain wall, and in fact this is a driving force for developing a strong magnetic field. From Eq. (7), we know that the averaged baryon number density induced by the domain walls is

$$\rho_\mathrm{dw} = \frac{eB_3}{2\pi}\frac{1}{d}. \quad (11)$$

The total baryon number density ρ is given as a sum of ρ_dw and the remaining neutron density ρ_n. (This ρ_n cannot vanish, since the β correction needs background neutrons.) Combining Eq. (11) with Eq. (10), we obtain

$$M = \frac{2\pi^2|g|f_\pi^2\rho_\mathrm{dw}}{m_\mathrm{N}B_3}. \quad (12)$$

Once the background B_3 becomes larger, the domain wall energy cost [Eq. (8)] becomes smaller. So more domain walls parallel to the original ones are created more easily, and the domain wall separation d becomes smaller. Then, from Eq. (10), the magnetization M increases and helps the B_3 to increase. So, this system is a self-enhancement mechanism of the magnetic field. Equilibrium can be reached at $B_3 = M$, which is our critical induced magnetic field,

$$B_3 = \sqrt{2\pi^2|g|f_\pi^2\rho_\mathrm{dw}/m_N} \sim 3\times 10^{19}\ [\mathrm{G}], \quad (13)$$

supposing a typical value for $\rho \sim \rho_\mathrm{dw} \sim \rho_n$. The value of the magnetic field is quite large ($\sqrt{B_3} \sim 10^2$ [MeV]) and close to the QCD deconfinement scale around which our approximation breaks down [11]. The magnetization is expected to stop increasing somewhere before reaching this value.

In addition to the neutron spin alignment, the WZW term [Eq. (5)] itself may provide a magnetization [12] of the same sign. Details will be presented in our forthcoming paper [13].

Finally, we briefly comment on the stability of the domain walls. Our domain wall is topologically trivial, because the target space of the nonlinear sigma model is $SU(2) \simeq S^3$, on which the two vacua $\theta = 0, 2\pi$ are the same point (south pole); see Figs. 2 and 3. Therefore, the domain walls can be created spontaneously, but on the other hand, they can decay through the fluctuations along $\phi_{1,2}$ directions. Indeed, it was shown that the domain wall is stable only for $B_3 > 10^{19}$ [G] at $\beta = \tilde{\beta} = 1$ by analyzing its local and global stability [8]. As we have shown after Eq. (13), the above condition is replaced by $B_3 > \sqrt{\beta\gamma_0}\times 10^{19}$ [G]. This implies that the global stability is guaranteed for smaller B_3 when $\beta \to 0$. The analysis of the local stability is more involved, especially where $\beta > 0$ and $\tilde{\beta} < 0$ (see Fig. 1): In this case, the charged pion condensation $\phi_+ = a\exp(-i\mu_\pi t + i\vec{k}\cdot\vec{x})$ should be considered together with the π^0 domain wall. We leave the analysis to our future work [14].

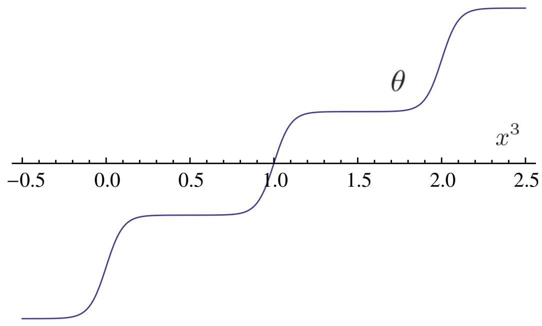

FIG. 2 (color online). The solution of multiple domain walls written by $\theta(x^3)$. Each wall interpolates adjacent vacua, $\theta = 0, 2\pi, 4\pi, \ldots$. For the exact solution, see for example Ref. [17].

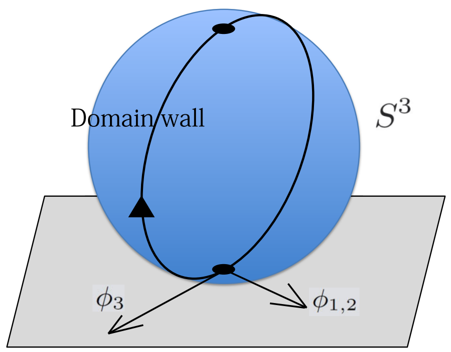

FIG. 3 (color online). The topological path of the neutral pion domain wall. The sphere represents the target space S^3 of the sigma model. On S^3, the thick line with an arrow represents the domain wall solution [Eq. (3)]. It rounds the sphere but is topologically trivial. The plane beneath the sphere is the parameterization space of ϕ_3 and $\phi_{1,2}$. There is a one-to-one correspondence between a point on the sphere and a point on the plane.

IV. IMPLICATION FOR MAGNETARS

A schematic picture of the core region of the neutron star is shown in Fig. 4. Only at the core region, because of the high density (or rather, the large value of the neutron chemical potential), the domain walls are present. At the boundary of the domain walls, neutrons drip from the wall boundary so that the total baryon number is conserved [15]. As the domain wall layers (the ferromagnetic region) are present only at the core of the neutron star, the magnetic field does not reach the value in Eq. (13) at the surface of the neutron star, but it would be strong enough to explain the magnetars.

The magnetic field at the surface of the neutron star is smaller than that of the domain wall core, as $B_{\rm surface} = B_{\rm core}(R_{\rm core}/R_{\rm NS})^3$ at the north pole. $R_{\rm core}$ is the radius of the domain wall core (assumed to be spherical and to have a homogeneous ferromagnetism inside), and $R_{\rm NS}$ is the radius of the neutron star. The average of the magnetic field magnitude on the neutron star surface is $B_{\rm average} =$

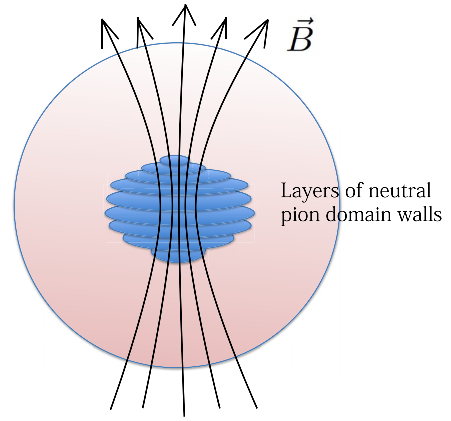

FIG. 4 (color online). A schematic figure of the neutron star with domain wall layers at the core. Scales should not be taken seriously.

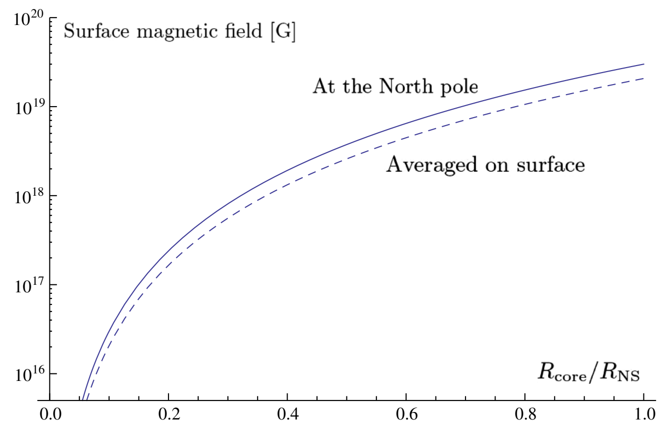

FIG. 5 (color online). The magnetic field at the neutron star surface. $R_{\rm core}$ is the radius of the domain wall layer core (assumed to be spherical), and $R_{\rm surfce}$ is the radius of the neutron star. At the core, the critical magnetic field [Eq. (13)] is assumed. For the $1.41M_\odot$ neutron star with the standard APR equation of state [18], $R_{\rm core}/R_{\rm NS} = 0.1$ corresponds to the critical density of the pionic wall formation, $3.2\rho_0$.

$0.69B_{\rm surface}$. The core-radius dependence of the surface magnetic field is shown in Fig. 5. If the core is sufficiently small such that $R_{\rm core}/R_{\rm NS} \sim 1/10$, the surface magnetic field may reduce to $B_{\rm surface} \sim \mathcal{O}(10^{16})$ [G].

The mechanism suggests that there are two kinds of neutron stars: one which reaches the critical density and has the domain wall layer structure, and the other which does not have it. The former would have a strong magnetic field, but the latter would not have it. It is interesting that the recent data [3] of the magnitude of the magnetic field on the surfaces of neutron stars show two categories, magnetars and the others.

It is important to construct more realistic models with nuclear forces, as our model uses free neutrons. For example, the neutron superfluidity can coexist with the domain wall, since at higher densities the spin-aligned neutron pairing $^3P_2$ is known to be favored. Furthermore, the structure of the solitonic core of neutron stars would influence the equation of state and may be sensitive to the mass-radius map of the neutron stars. A large magnetic field $\geq 10^{19}$ [G] may lead to a gravitational instability: For example, an order-of-magnitude estimate given in Ref. [16] shows the maximum possible magnetic field, $B_{\rm max} \sim 10^{19}$ [G]. The precise value of $B_{\rm max}$, however, depends on the shape of the magnetic flux lines, the equation of state of dense matter, and the spatial structure of our domain wall in the neutron star core. If the orientation of the solitonic core is different from the rotation axis of the neutron star, it would be a source of gravitational waves. All details need to be explored to match the observations of the neutron stars. We hope that our mechanism may survive various corrections and explain observations.

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