Peccei-Quinn symmetry from a gauged discrete R symmetry

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The axion solution to the strong CP problem calls for an explanation as to why the Lagrangian should be invariant under the global Peccei-Quinn (PQ) symmetry, $U(1)_{\rm PQ}$, to such a high degree of accuracy. In this paper, we point out that the $U(1)_{\rm PQ}$ can indeed survive as an accidental symmetry in the low-energy effective theory, if the standard model gauge group is supplemented by a gauged and discrete R symmetry, Z_N^R, forbidding all dangerous operators that explicitly break the Peccei-Quinn symmetry. In contrast to similar approaches, the requirement that the Z_N^R symmetry be anomaly-free *forces* us, in general, to extend the supersymmetric standard model by new matter multiplets. Surprisingly, we find a large landscape of viable scenarios that all individually fulfill the current experimental constraints on the QCD vacuum angle as well as on the axion decay constant. In particular, choosing the number of additional multiplets appropriately, the order N of the Z_N^R symmetry can take any integer value larger than 2. This has interesting consequences with respect to possible solutions of the μ problem, collider searches for vectorlike quarks and axion dark matter.

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I. INTRODUCTION

The Peccei-Quinn (PQ) symmetry, $U(1)_{\rm PQ}$, provides us with a very attractive mechanism to solve the strong CP problem in quantum chromodynamics (QCD) [1,2]. Up to now, a convincing explanation for the origin of the PQ symmetry is, however, still pending, since it is a global symmetry and any global symmetry is believed to be broken by quantum gravity effects [3–5]. In order for the PQ symmetry to accidentally survive in the low-energy effective theory, one thus has to arrange for a sufficient suppression of all unwanted operators that explicitly break it. The tight experimental upper bound on the QCD vacuum angle, $\bar{\theta} \lesssim 10^{-10}$ [6], necessitates in particular that this suppression be extremely efficient. One natural way to protect the PQ symmetry is to invoke some gauge symmetry that accidentally forbids all the operators that would break it too severely. In this paper, we point out that, in the context of the supersymmetric standard model, the role of this protective gauge symmetry could be played by a gauged discrete R symmetry, Z_N^R.

Given only the particle content of the minimal supersymmetric standard model (MSSM), any Z_N^R symmetry, except for Z_3^R and Z_6^R, is anomalously broken by $SU(3)_C$ and $SU(2)_L$ instanton effects [7,8].[1] On the supposition that a different Z_N^R symmetry, other than Z_3^R or Z_6^R, might account for the protection of the $U(1)_{\rm PQ}$, we are hence *naturally* led to introduce an extra matter sector canceling the MSSM contributions to the Z_N^R anomalies. For a particular value of N as well as k additional pairs of vector-quark superfields charged under the MSSM gauge group, the requirement that the shift in the QCD vacuum angle induced by PQ-breaking operators be less than 10^{-10} then implies an upper bound on the axion decay constant f_a. By identifying those extensions of the MSSM that yield an upper bound on f_a above the astrophysical lower bound of $f_a \gtrsim 10^9$ GeV [10], we are thus able to single out the values of N and k that are phenomenologically viable. Surprisingly, for each integer value of N larger than 2, a variety of k values is admissible. Here, k can in particular always be chosen such that the unification of the gauge coupling constants still occurs at the perturbative level. Moreover, for $k = 5$, 6 and $k \geq 8$, it is possible to protect the PQ symmetry by means of a Z_4^R symmetry. As we will discuss, this is an especially interesting case, since a Z_4^R may not only explain the origin of the PQ symmetry, but at the same time also allow for a simple solution of the MSSM μ problem.

The very idea to protect the PQ symmetry against gravity effects by means of a gauge symmetry is, of course, not new. Many authors have, for instance, considered extensions of the standard model gauge group $G_{\rm SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$ by some *continuous* symmetry. Early examples of such attempts include models based on the gauge group $G_{\rm SM} \times U(1)'$ [4] or on the group $E_6 \times U(1)'$ [5]. Also extensions of the gauge group by a continuous *and* a discrete symmetry, such as $G_{\rm SM} \times SU(4) \times Z_N$ [11], $SU(3)_C \times SU(3)_L \times U(1)_Y \times Z_{13} \times Z_2$ [12], or $SU(5) \times SU(N) \times Z_N$ [13], have been studied in the literature. Likewise, next to these field-theoretic models, string constructions have been shown to give rise to accidental PQ symmetries. By compactifying the heterotic string on

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[1]We restrict ourselves to generation-independent Z_N^R symmetries, where $N > 2$, that commute with $SU(5)$ and do not consider anomaly cancellation via the Green-Schwarz mechanism [9] coming from string theory, cf. Sec. II B.

Calabi-Yau manifolds [14] or on Z_6-II orbifolds [15], it is, for example, feasible to retain accidental global symmetries in the low-energy effective theory as remnants of exact *stringy* discrete symmetries. All of these approaches, however, rely on rather speculative assumptions about the UV completion of the standard model (SM). In particular, they require in many cases an *ad hoc* extension of the particle content of the standard model that is motivated by the intention to eventually end up with a global PQ symmetry in the first place. In view of this situation, it is thus of great interest to assess what a *minimal* extension of the standard model or the MSSM would look like that still accomplishes a successful protection of the PQ symmetry. The model presented in Ref. [16] might, for instance, be considered a step into this direction, cf. also Ref. [17]. It forgoes any additional continuous symmetry, but only extends G_{SM} by a discrete $Z_{13} \times Z_3$. Still, it comes with a multi-Higgs sector that, while being certainly interesting from a phenomenological point of view, lacks a decisive reason for its origin from a fundamental perspective.[2]

Now, invoking nothing but a discrete Z_N^R symmetry in order to protect the PQ symmetry rests, by contrast, on a very sound conceptional footing. A discrete R symmetry is an often important and sometimes even imperative ingredient to model building and phenomenology in supersymmetry (SUSY). It allows for a solution to the μ problem [19–21], prevents too rapid proton decay [22,23] and forbids a constant term in the superpotential of order the Planck scale which, in scenarios of low-scale SUSY breaking, would otherwise result in a huge negative cosmological constant [24]. The existence of an R symmetry and its potential spontaneous or explicit breaking is furthermore closely linked to the spontaneous breaking of SUSY, irrespectively of whether our present nonsupersymmetric vacuum corresponds to a true [25] or merely metastable ground state [26]. Finally, it is interesting to observe that higher-dimensional supergravity theories such as superstring theory always feature an R symmetry, which might be naturally broken down to its discrete subgroup Z_N^R upon the compactification of the extra dimensions [27]. This last point may again be regarded to be rather speculative, but it does not alter the fact that discrete R symmetries surely play an preeminent role among all conceivable symmetries by which G_{SM} could possibly be extended. In this sense, the main result of this paper is that nothing but the arguably simplest and most natural extra gauge symmetry, namely a gauged and discrete R symmetry Z_N^R, could be responsible for shielding the PQ symmetry from the dangerous effects of gravity.

After having outlined why we are particularly interested in enlarging G_{SM} by a gauged Z_N^R symmetry, we shall present in the next section our minimal extension of the MSSM and explain (i) how the color and weak anomalies of the discrete Z_N^R symmetry force us to introduce new matter multiplets, (ii) how these new matter multiplets acquire masses as well as (iii) how a μ term of order of the soft masses can be generated dynamically. In Sec. III, we will then study the phenomenological constraints on our model and identify the viable combinations of N and k along with upper and lower bounds on the axion decay constant f_a. Finally, we conclude with a summary of our model and a short overview of its phenomenological implications. Two appendices deal with the R charges of the MSSM fields and a slight modification of our model that manages to avoid the axion domain wall problem, respectively.

[2]A multi-Higgs sector may allow for gauge coupling unification close to the PQ-breaking scale, though [18].

II. MINIMAL EXTENSION OF THE MSSM WITH A PQ SYMMETRY

We shall now demonstrate how an anomaly-free discrete R symmetry Z_N^R in combination with an extra matter sector automatically gives rise to a global PQ symmetry. As a preparation, let us first summarize our conventions and assumptions regarding the MSSM sector.

A. Supersymmetric standard model sector

We take the renormalizable MSSM superpotential to be of the following form:

$$W_{\mathrm{MSSM}} = h_{ij}^u \mathbf{10}_i \mathbf{10}_j H_u + h_{ij}^d \mathbf{5}_i^* \mathbf{10}_j H_d + h_{ij}^\nu \mathbf{5}_i^* \mathbf{1}_j H_u + \frac{1}{2} M_i \mathbf{1}_i \mathbf{1}_i, \quad (1)$$

where we have arranged the MSSM chiral quark and lepton superfields into $SU(5)$ multiplets, $\mathbf{10} = (q, u^c, e^c)$ and $\mathbf{5}^* = (d^c, \ell)$. Throughout this paper, we shall assume that the tiny masses of the SM neutrinos are accounted for by the seesaw mechanism [28]. That is why we have also introduced neutrino singlet fields, $\mathbf{1} = (n^c)$, in Eq. (1), next to the actual matter content of the MSSM.[3] Moreover, H_u and H_d is the usual pair of MSSM Higgs doublets, h^u, h^d and h^ν are Yukawa matrices and M denotes the diagonalized Majorana mass matrix for the heavy neutrinos involved in the seesaw mechanism. i and j finally label the three different generations of quarks and leptons, i.e. $i, j = 1, 2, 3$.

We assume the MSSM quark and lepton fields to be unified in $SU(5)$ representations in order to allow for an embedding of the MSSM into a grand unified theory (GUT). Note, however, that taking $SU(5)$ alone to be the full GUT gauge group is problematic. The minimal supersymmetric $SU(5)$ GUT model [22,29] namely fails to give

[3]As this sometimes falls victim to bad jargon, we emphasize that the fermions contained in n^c are *left-handed*. In fact, they are the Hermitian conjugates of the right-handed neutrinos required for the seesaw mechanism.

GUT-scale masses to the colored Higgs triplets that are expected to pair up with the MSSM Higgs doublets in complete $SU(5)$ multiplets. This results in too rapid proton decay and represents what is known as the infamous doublet-triplet splitting problem [30]. In addition to that, the standard way to break $SU(5)$ to $G_{\rm SM}$ by means of a **24**-plet is not compatible with the assumption of an unbroken R symmetry below the GUT scale.[4] Because of that, we shall assume that $SU(5)$ is merely a proper subgroup of the full GUT group, $SU(5) \subset G_{\rm GUT}$. An attractive possibility in this context is unification based on the product group $SU(5) \times U(3)_H$, which can be formulated in an R-invariant fashion [24,32], while solving the doublet-triplet splitting problem in a natural way [33].

Finally, we point out that we define the MSSM to conserve matter parity, P_M, so as to forbid all dangerous baryon and lepton number-violating operators in the renormalizable superpotential. This renders the actual gauge group of the MSSM slightly larger than the one of the standard model, $G_{\rm MSSM} = G_{\rm SM} \times P_M$. One possibility to account for the origin of matter parity is to interpret it as the remnant discrete subgroup of a local $U(1)_{B-L}$ symmetry that is spontaneously broken above the electroweak scale [34]. Here, $B - L$ stands for the difference between baryon number B and lepton L. Assuming the presence of an additional Abelian factor $U(1)_X$ in the GUT gauge group orthogonal to $SU(5)$, it can be expressed in terms of the Abelian GUT charge X and the weak hypercharge Y through the relation $X + 4Y = 5(B - L)$, cf. also Appendix A.

B. Extra matter sector required by a non-anomalous Z_N^R symmetry

As outlined in the Introduction, a discrete R symmetry Z_N^R represents a unique choice when considering possible extensions of the MSSM gauge group. We now perform just such an extension, such that the full gauge group G of our model also features a Z_N^R factor. *A priori*, we allow N, the order of the Z_N^R symmetry, to take any integer value larger than 2. We disregard the case $N = 2$ since a Z_2^R symmetry, i.e. R parity, is not an R symmetry in the actual sense. By including a Lorentz rotation, it can always be reformulated as an ordinary Z_2 parity [21]. On top of that, given only a Z_2^R symmetry, we would also be unable to forbid a constant term in the superpotential, which would result in a cosmological constant of the order of the Planck scale. On the other hand, we point out that, in the case of even N, the Z_N^R symmetry contains R parity as a subgroup, $Z_N^R \supset Z_2^R$ for $N = 4, 6, 8, \ldots$. Depending on the details of the R charge assignments to the particles of our model, this R parity coincides in some cases with the ordinary matter parity P_M. In these cases, we then do not need to additionally impose matter parity by hand, as it is already included in the Z_N^R factor of the gauge group. In all other cases, we rely on the assumption that a spontaneously broken $U(1)_{B-L}$ gauge symmetry gives rise to matter parity at low energies. In summary, the gauge group of our model is, hence, given by

$$G = SU(3)_C \times SU(2)_L \times U(1)_Y \times \begin{cases} Z_N^R \times P_M; & Z_N^R \not\supset P_M \\ Z_N^R; & Z_N^R \supset P_M. \end{cases} \tag{2}$$

1. Gauge anomalies of the Z_N^R symmetry

We attribute the origin of the Z_N^R factor in the gauge group to the presence of a continuous gauged R symmetry at high energies, after the breaking of which Z_N^R remains as a discrete subgroup. Thus being part of the gauge group, it is crucial that the Z_N^R symmetry be anomaly free. The relevant anomaly cancellation conditions are those related to the color as well as to the weak anomaly of the Z_N^R, i.e. the $Z_N^R[SU(3)_C]^2$ and the $Z_N^R[SU(2)_L]^2$ anomaly, respectively. The anomaly coefficients for these two anomalies, $\mathcal{A}_R^{(C)}$ and $\mathcal{A}_R^{(L)}$, are given by [32,35]

$$\begin{aligned} \mathcal{A}_R^{(C)} &= 6 + N_g(3r_{\mathbf{10}} + r_{\mathbf{5}^*} - 4), \\ \mathcal{A}_R^{(L)} &= 4 + N_g(3r_{\mathbf{10}} + r_{\mathbf{5}^*} - 4) + (r_{H_u} + r_{H_d} - 2). \end{aligned} \tag{3}$$

Here, $r_{\mathbf{10}}$, $r_{\mathbf{5}^*}$, $r_{\mathbf{1}}$, r_{H_u} and r_{H_d} denote the R charges of the MSSM matter multiplets and Higgs doublets and $N_g = 3$ is the number of fermion generations in the MSSM.[5] Note that we have assumed the R charges of the matter fields to be generation independent. Otherwise, i.e. in the case of generation-dependent R charges, the R symmetry would suppress some of the entries in the Yukawa matrices h^u and h^d too heavily.[6] We also remark that the R charges are normalized such that the anticommuting superspace coordinate θ carries R charge $r_\theta = 1$. By choosing a different value for the R charge of θ, say, $r'_\theta \neq 1$, we always have the option to collectively rescale all R charges by the common factor r'_θ / r_θ.

Besides the color and the weak anomaly, all further anomalies involving at least one Z_N^R factor also have to vanish in order to render the Z_N^R symmetry fully anomaly free. The anomalies nonlinear in Z_N^R, such as $[Z_N^R]^3$ or $[Z_N^R]^2 U(1)_Y$, are, however, sensitive to heavy,

[4] If we managed to break $SU(5)$ without breaking the R symmetry, we would be left with potentially interesting or dangerous $G_{\rm SM}$-charged exotics whose masses would only receive soft SUSY-breaking contributions [31].

[5] The authors of Ref. [8] have recently made the interesting observation that $N_g \geq 3$ is a *necessary condition* for consistently extending the MSSM gauge group by an anomaly-free discrete R symmetry Z_N^R with $N > 2$.

[6] The Z_N^R symmetry might, however, be embedded into a continuous $U(1)_R$ with *generation-dependent* R charges; cf. Ref. [36], in which this $U(1)_R$ plays the role of a gauged flavor symmetry of the Froggatt-Nielsen type.

fractionally charged states at high energies [37]. Similarly, the gravitational anomaly, $Z_N^R[\text{gravity}]^2$, also receives contributions from light sterile fermions as well as from hidden-sector fermions acquiring large masses of the order of the SUSY-breaking scale in the course of spontaneous SUSY breaking [7]. All of these anomalies hence highly depend on the particle spectrum in the UV and, thus, do not allow us to derive further constraints on our model. In general, the $Z_N^R[U(1)_Y]^2$ anomaly also does not yield a useful condition because the SM hypercharge is not quantized [37,38]. Only if the GUT group is semisimple, such that the normalization of the hypercharge is dictated by the gauge structure, the $Z_N^R[U(1)_Y]^2$ anomaly provides a meaningful constraint on the Z_N^R symmetry as well as on the set of particles charged under it.[7]

Obviously, we only included the contributions from the MSSM sector to the anomaly coefficients in Eq. (3). $\mathcal{A}_R^{(C)}$ and $\mathcal{A}_R^{(L)}$ could, however, still receive corrections $\Delta\mathcal{A}_R^{(C)}$ and $\Delta\mathcal{A}_R^{(L)}$ due to new colored or weakly interacting fermions with masses at or above the electroweak scale. This extra matter would need to be assembled in complete $SU(5)$ multiplets in order not to spoil the unification of the gauge coupling constants. Consequently, extra fermions ought to equally contribute to $\mathcal{A}_R^{(C)}$ and $\mathcal{A}_R^{(L)}$, such that the corresponding corrections are equal to each other, $\Delta\mathcal{A}_R = \Delta\mathcal{A}_R^{(C)} = \Delta\mathcal{A}_R^{(L)}$, and such that the difference between $\mathcal{A}_R^{(C)}$ and $\mathcal{A}_R^{(L)}$ ends up being independent of the properties of the extra matter sector. A minimal necessary condition for rendering the Z_N^R symmetry anomaly free is hence that

$$\mathcal{A}_R^{(L)} - \mathcal{A}_R^{(C)} = r_{H_u} + r_{H_d} - 4 \overset{(N)}{=} 0, \tag{4}$$

where we have introduced the symbol $\overset{(N)}{=}$ as a shorthand notation to denote equality modulo N,

$$a \overset{(N)}{=} b \Leftrightarrow a \bmod N = b \bmod N \Leftrightarrow \exists!\ell \in \mathbb{Z}\colon\ a = b + \ell N. \tag{5}$$

The condition in Eq. (4) is equivalent to $r_{H_u} + r_{H_d} \overset{(N)}{=} 4$. We therefore see that an anomaly-free Z_N^R symmetry automatically suppresses the μ term for the MSSM Higgs doublets.

2. Constraints on the R charges of the MSSM fields

Next to Eq. (4), the requirement that the first two terms in the superpotential W_{MSSM}, cf. Eq. (1), be in accordance with the Z_N^R symmetry provides us with two further constraints on the R charges of the MSSM fields,

$$2r_{\mathbf{10}} + r_{H_u} \overset{(N)}{=} 2, \qquad r_{\mathbf{5}^*} + r_{\mathbf{10}} + r_{H_d} \overset{(N)}{=} 2. \tag{6}$$

The combination of all three conditions then implies $3r_{\mathbf{10}} + r_{\mathbf{5}}^* \overset{(N)}{=} 0$, which automatically forbids the dangerous dimension-5 operator $\mathbf{10}\,\mathbf{10}\,\mathbf{10}\,\mathbf{5}^*$ in the superpotential, which would otherwise induce too rapid proton decay [23]. Together with matter parity, the anomaly-free Z_N^R symmetry thus bans all baryon and lepton number-violating operators up to dimension 5 except for the operator $\mathbf{5}^* H_u \mathbf{5}^* H_u$, which we, of course, want to retain to be able to explain the small neutrino masses [39]. Finally, in the seesaw extension of the MSSM, we also have to ensure that the last two terms in W_{MSSM} respect the Z_N^R symmetry, which translates into

$$r_{\mathbf{5}*} + r_{\mathbf{1}} + r_{H_u} \overset{(N)}{=} 2, \qquad 2r_{\mathbf{1}} \overset{(N)}{=} 2. \tag{7}$$

Here, as for the second condition, we have assumed zero R charge for the Majorana neutrino mass M, which is to say that we consider its origin to be independent of the mechanism responsible for the spontaneous breaking of R symmetry.

In conclusion, we find that extending the particle content of the MSSM by three neutrino singlets, the five R charges $r_{\mathbf{10}}$, $r_{\mathbf{5}^*}$, $r_{\mathbf{1}}$, r_{H_u} and r_{H_d} are determined by the five conditions in Eqs. (4), (6), and (7). However, due to the fact that all of these conditions only constrain the MSSM R charges up to integer multiples of N, they do not suffice to fix the values of $r_{\mathbf{10}}$, $r_{\mathbf{5}^*}$, $r_{\mathbf{1}}$, r_{H_u} and r_{H_d} uniquely. Instead, for each value of N, there exist exactly ten different possibilities to assign R charges to the MSSM fields. In Appendix A, we derive and discuss these solutions in more detail. In particular, we show that, for any given value of N, the different R charge assignments are related to each other by gauge transformations. First of all, in consequence of the $SU(5)$ invariance of the MSSM Lagrangian, the ten solutions split into two equivalence classes of respectively five solutions. As shown in Appendix A, these two classes are generated by the action of Z_5 transformations on the following two R charge assignments:

$$r_{\mathbf{10}} \overset{(N)}{=} \frac{1}{5} + \ell\frac{N}{2}, \qquad r_{\mathbf{5}^*} \overset{(N)}{=} -\frac{3}{5} + \ell\frac{N}{2}, \qquad r_{\mathbf{1}} \overset{(N)}{=} 1 + \ell\frac{N}{2},$$
$$r_{H_u} \overset{(N)}{=} 2 - \frac{2}{5}, \qquad r_{H_d} \overset{(N)}{=} 2 + \frac{2}{5}, \tag{8}$$

where $\ell = 0$, 1 and where $Z_5 \subset SU(5)$ is the center of $SU(5)$. Furthermore, if matter parity stems from a $U(1)_X$ symmetry that is part of the gauge group at high energies, i.e. if $P_M \subset U(1)_X$, these two solutions are in turn related to each other by a P_M transformation, such that eventually all ten R charge assignments end up being physically equivalent. On the other hand, if matter parity is a subgroup of the Z_N^R symmetry, i.e. if $P_M \subset Z_N^R$, the two solutions in Eq. (8) cannot be related to each other and we are left with two inequivalent classes of solutions. Last but not least, we

[7]We mention in passing that neither of the previously discussed GUT gauge groups, i.e. neither $SU(3) \times U(3)_H$ nor $SU(5) \times U(1)_X$, is semisimple. Assuming one of these two groups to correspond to the GUT gauge group, we are hence not able to make use of the anomaly cancellation condition for the $Z_N^R[U(1)_Y]^2$ anomaly.

remark that, for all values of ℓ and N, all of the R charges in Eq. (8), expect for $r_{\mathbf{1}}$ in some cases, are fractional. In Appendix A, we however show that, for each $N \neq 5, 10, 15, \ldots$, there exists at least one R charge assignment that is equivalent to one of the two assignments in Eq. (8) and which only involves integer-valued R charges. But not only that, we also demonstrate that, in a $U(1)_X$-invariant extension of our model, all R charges in Eq. (8) can always be rendered integer valued by means of a $U(1)_X$ transformation.

3. Anomaly cancellation owing to new matter fields

Irrespectively of the concrete R charges in Eq. (8), the anomaly constraint in Eq. (4) in combination with the two conditions in Eq. (6) immediately implies for the anomaly coefficients in Eq. (3),

$$\mathcal{A}_R^{(C)} \overset{(N)}{=} \mathcal{A}_R^{(L)} \overset{(N)}{=} 6 - 4N_g \overset{(N)}{=} -6. \tag{9}$$

As this result does not rely on either of the two conditions in Eq. (7), it is independent of the fact that we extended the MSSM particle content by three right-handed neutrinos. It rather equally applies in the MSSM as well as in its seesaw extension. But more importantly, it leads us to one of the key observations of this paper: as long as the order N of the Z_N^R symmetry is different from $N = 3$ or $N = 6$, we are forced to introduce a new matter sector in order to cancel the MSSM contributions to the color and the weak Z_N^R anomaly. In this sense, the introduction of new colored and weakly interacting states in our model is not an *ad hoc* measure, but rather a natural consequence of the requirement of an anomaly-free discrete R symmetry.[8]

The simplest way to cancel the MSSM anomalies in Eq. (9) without spoiling the unification of the gauge coupling constants is to introduce k pairs of vector quarks and antiquarks, Q_i and $\bar{Q}_i$, where $i = 1, \ldots, k$, that respectively transform in the $\mathbf{5}$ and $\mathbf{5}^*$ of $SU(5)$.[9] As they transform in complete $SU(5)$ multiplets, the extra quarks and antiquarks yield equal non-MSSM contributions $\Delta\mathcal{A}_R^{(C)}$ and $\Delta\mathcal{A}_R^{(L)}$ to the color and weak anomaly coefficients of the Z_N^R asymmetry. According to Eq. (9), we must require that

$$\Delta\mathcal{A}_R^{(C)} = \Delta\mathcal{A}_R^{(L)} = k(r_Q + r_{\bar{Q}} - 2) \overset{(N)}{=} 6,$$
$$r_{Q\bar{Q}} = r_Q + r_{\bar{Q}} \overset{(N)}{=} 2 + \frac{1}{k}(6 + \ell_Q N), \qquad \ell_Q \in \mathbb{Z}, \tag{10}$$

where r_Q and $r_{\bar{Q}}$ denote the generation-independent R charges of the extra quarks and antiquarks, respectively, and $r_{Q\bar{Q}}$ is the common R charge of the bilinear quark operators $(Q\bar{Q})_i = Q_i\bar{Q}_i$. Just like all other R charges, the R charge $r_{Q\bar{Q}}$ is only defined up to the addition of integer multiples of N. Hence, all inequivalent solutions to the condition in Eq. (10) lie in the interval $[0, N)$. In addition, we observe that, varying ℓ_Q in integer steps, the R charge $r_{Q\bar{Q}}$ changes in steps of $\frac{N}{k}$. Consequently, for each pair of values for N and k, there are k inequivalent choices for $r_{Q\bar{Q}}$,

$$r_{Q\bar{Q}} \overset{(N)}{=} 2 + \frac{1}{k}(6 + \ell_Q N), \qquad \ell_Q = 0, \ldots, k-1. \tag{11}$$

A crucial implication of this result is that, in most cases, the extra quarks and antiquarks are massless as long as the Z_N^R is unbroken. Only for $r_{Q\bar{Q}} = 2$, a supersymmetric and R-invariant mass term is allowed for the extra quark fields in the superpotential. An R charge $r_{Q\bar{Q}}$ of 2 can, however, only be obtained in the case of a Z_3^R or a Z_6^R symmetry,

$$r_{Q\bar{Q}} = 2\colon (N, \ell_Q) = (3, -2),$$
$$(N, \ell_Q) = (6, -1), \qquad k = 1, 2, 3, \ldots, \tag{12}$$

which are just the two Z_N^R symmetries that do not require an extension of the MSSM particle content in the first place. As soon as the introduction of a new matter sector is *mandatory* in order to render the Z_N^R symmetry anomaly free, the new quark fields are therefore guaranteed to be massless. This observation also reflects the self-consistency of our result in Eq. (11). If we had started with the requirement of nonzero contributions from the new quarks to the Z_N^R anomalies and we had found that the new quarks could possibly be massive, our derivation of $r_{Q\bar{Q}}$ would be faulty, since massive quarks would not contribute to the Z_N^R anomalies to begin with.

Of course, the requirement of massless quarks at high energies does not say anything about the masses of the new quarks at low energies, where the Z_N^R is spontaneously broken. The spontaneous breaking of the Z_N^R symmetry might, in fact, even generate masses for the extra quarks of the order of the gravitino mass [41], cf. also Sec. II D 1. A necessary condition for this to happen is that $r_{Q\bar{Q}} = 0$, which can be fulfilled for each value of N as long as k and ℓ_Q are chosen such that $-2k \overset{(N)}{=} 6$. This means in particular that, for $k = 1$ and $N = 4, 8$, the R charge $r_{Q\bar{Q}}$ is always zero. For $N = 3, 4, 6, 8$ and only one pair of extra quark fields, $k = 1$, the generation of a sufficiently large mass for the new quark flavor is therefore not an issue. The new quark either exhibits a supersymmetric mass from the

[8]Again, this statement can be defined down by allowing for anomaly cancellation via the Green-Schwarz mechanism, in the case of which not only Z_3^R and Z_6^R can be rendered anomaly free solely within the MSSM, but also Z_4^R, Z_8^R, Z_{12}^R and Z_{24}^R [7,40]. Moreover, it is worth noting that, in the context of a two-singlet extension of the MSSM, the anomaly-free Z_{24}^R symmetry can be used to successfully protect the PQ symmetry [7].

[9]Transforming in the $\mathbf{5}$ and $\mathbf{5}^*$ of $SU(5)$, the new multiplets Q_i and $\bar{Q}_i$, of course, also contain lepton doublets. From a phenomenological point of view and with regard to the PQ solution of the CP problem, these are however less interesting as compared to the corresponding quark triplets. Because of that, we will refer to Q_i and $\bar{Q}_i$ as the new *quark* and *antiquark* superfields in the following.

outset or it acquires a mass in the course of R symmetry breaking [41]. For completeness, we also mention that, provided $r_{Q\bar{Q}} = 0$, the new quarks could equally acquire masses of the order of the gravitino mass in the course of spontaneous SUSY breaking via the Giudice-Masiero mechanism [19]. A further necessary prerequisite in this case would then be that there exists a coupling of the extra quark fields to the SUSY breaking sector in the Kähler potential.

In our following analysis, we will disregard the two exceptional choices for N and ℓ_Q in Eq. (12) as well as all combinations of N, k and ℓ_Q yielding $r_{Q\bar{Q}} = 0$. Instead, we shall focus on R charges $r_{Q\bar{Q}}$ that imply vanishing masses for the new quarks and antiquarks before and after R symmetry breaking as long as no further fields are introduced. The absence of a supersymmetric mass term is then equivalent to the statement that the renormalizable superpotential of the extra quark sector vanishes completely. This is because all SM singlets solely composed out of the fields Q and $\bar{Q}$ must be combinations of the operator products $Q\bar{Q}$, Q^5 and $\bar{Q}^5$, such that $Q\bar{Q}$ is the *only* conceivable operator which could potentially show up in the renormalizable superpotential. At the renormalizable level, the global flavor symmetry of the extra matter sector by itself, i.e. neglecting its interactions with the other sectors of our model for a moment, is therefore maximally large,

$$U(k)_Q \times U(k)_{\bar{Q}} \cong SU(k)_Q^V \times SU(k)_Q^A \times U(1)_Q^V \times U(1)_Q^A, \tag{13}$$

with $U(k)_Q$ and $U(k)_{\bar{Q}}$ accounting for the flavor rotations of the left-chiral superfields Q_i and $\bar{Q}_i$, respectively. As we will see in Sec. II C 3, the axial Abelian flavor symmetry $U(1)_Q^A$ will play an important role in the identification of the PQ symmetry. Finally, we remark that higher-dimensional operators as well as couplings of the new quarks and antiquarks to other fields explicitly break the flavor symmetry. In order not to spoil the PQ solution to the strong CP problem, these explicit breaking effects must be sufficiently suppressed by means of a protective gauge symmetry. We will return to this point in Sec. III.

C. Extra singlet sector required to render the extra matter massive

In the previous section, we have seen how the requirement of an anomaly-free Z_N^R symmetry forces us to extend the MSSM particle content by new quark fields, Q_i and $\bar{Q}_i$. Except for some special cases, these quark fields are, however, massless as long as the Z_N^R symmetry is unbroken. Extra massless colored and weakly interacting particles are, of course, in conflict with observations, which is why we have to extend our model once more, so as to provide masses to the new quarks and antiquarks.

1. Coupling of the extra matter fields to a new singlet sector

In order to generate sufficiently large mass terms for the quark pairs $(Q\bar{Q})_i$ in the superpotential, we are in need of a SM singlet that acquires a vacuum expectation value (VEV) at least above the electroweak scale. No such singlet exists in the MSSM or its seesaw extension, so we are required to introduce another new field. Let us refer to this field as P and demand that it couples to the quark pairs $(Q\bar{Q})_i$ in the following way[10]:

$$W_Q = \frac{1}{M_{\rm Pl}^{n-1}} \sum_{i=1}^{k} \lambda_i P^n (Q\bar{Q})_i. \tag{14}$$

Here, $M_{\rm Pl} = (8\pi G)^{-1/2} = 2.44 \times 10^{18}$ GeV is the reduced Planck mass and the λ_i denote dimensionless coupling constants, which we assume to be of $\mathcal{O}(1)$. The power n can, *a priori*, be any integer number, $n = 1, 2, \ldots$. Moreover, the coupling in Eq. (14) fixes the R charge r_P of the singlet field P. In order to ensure that it is indeed allowed in the superpotential, we require that

$$\begin{aligned} n r_P + r_{Q\bar{Q}} &= 2 + \ell_P N, \\ r_P &\overset{(N)}{=} \frac{1}{n}(2 - r_{Q\bar{Q}} + \ell_P N), \\ \ell_P &\in \mathbb{Z}. \end{aligned} \tag{15}$$

Making use of our result for the R charge $r_{Q\bar{Q}}$ for the quark pairs, cf. Eq. (11), we then find

$$r_P \overset{(N)}{=} -\frac{6}{nk} + (k\ell_P - \ell_Q)\frac{N}{nk}. \tag{16}$$

Similarly as in the case of the extra quark fields, the R charge r_P is not uniquely determined. For each combination of values for N, n and k, there are instead nk inequivalent solutions to the condition in Eq. (15). These are all of the form given in Eq. (16), with $(k\ell_P - \ell_Q) = 0, 1, \ldots, nk - 1$.

2. Superpotential of the extra singlet sector

So far, the field P does not possess any interactions that would endow it with a nonvanishing VEV. We thus introduce another singlet field X and couple it to the field P, in order to generate a nontrivial F-term potential for the scalar component of P,

$$W_P = \kappa X\left[\frac{\Lambda^2}{2} - f(P, \ldots)\right], \tag{17}$$

where κ is a coupling constant, Λ denotes some mass scale and f stands for a function of P and probably other fields. We assume the scale Λ to carry zero R charge, which directly entails that the singlet field X and the function f

[10]Note that the field P might be part of the hidden sector responsible for the spontaneous breaking of SUSY [42].

must have R charges 2 and 0, respectively. Besides that, we also assume a value for r_P such that none of the operators P, P^2, P^3, XP, XP^2 and X^2P is allowed in the superpotential W_P, i.e. we require r_P to fulfill all of the following relations at once:

$$r_P \overset{(N)}{\neq} 2, \qquad 2r_P \overset{(N)}{\neq} 2, \qquad 3r_P \overset{(N)}{\neq} 2, \qquad 2r_P \overset{(N)}{\neq} 0, \qquad r_P \overset{(N)}{\neq} -2. \tag{18}$$

As we will see shortly, these conditions ensure that W_P ends up featuring a flat direction which can be identified with the axion and its superpartners.

Now, if the function f were merely composed out of powers P^m of the field P, where $m = 3, 4, \ldots$, the R charge of f would only vanish for particular values of r_P and m in the case of particular Z_N^R symmetries. We, however, wish to be able to give masses to the new quarks and antiquarks, irrespectively of the concrete value of N. For that reason, we have to introduce a singlet field $\bar{P}$ carrying the opposite R charge of the field P,

$$r_{\bar{P}} \overset{(N)}{=} -r_P \overset{(N)}{=} \frac{6}{nk} - (k\ell_P - \ell_Q)\frac{N}{nk}, \tag{19}$$

such that we are able to render the function f an R singlet by taking it to be a function of the singlet pair $P\bar{P}$. The superpotential in Eq. (17) can then be fixed to be of the following form:

$$W_P = \kappa X\left[\frac{\Lambda^2}{2} - f(P\bar{P})\right] = \kappa X\left(\frac{\Lambda^2}{2} - P\bar{P}\right) + \cdots, \tag{20}$$

with the dots after the plus sign indicating higher-dimensional nonrenormalizable terms and where, similarly as above, we have assumed that $r_{\bar{P}} = -r_P$ is such that none of the operators $\bar{P}$, $\bar{P}^2$, $\bar{P}^3$, $X\bar{P}$, $X\bar{P}^2$ and $X^2\bar{P}$ is allowed in the superpotential W_P. In addition to the five conditions in Eq. (18), we therefore also have to require that

$$2r_P \overset{(N)}{\neq} -2, \qquad 3r_P \overset{(N)}{\neq} -2. \tag{21}$$

In total, we hence impose seven conditions on the R charge r_P, which, depending on N, allow us to forbid as many as 14 different values for r_P.[11] We now also see that each of the combinations of N, ℓ_Q and k that either result in $r_{Q\bar{Q}} = 0$ or $r_{Q\bar{Q}} = 2$ violates exactly one of these conditions. If $r_{Q\bar{Q}} = 0$, we know that $nr_P \overset{(N)}{=} 2$, such that either P or P^2 is allowed. Similarly, $r_{Q\bar{Q}} = 2$ implies $nr_P \overset{(N)}{=} 0$, such that XP and/or XP^2 is allowed. This means that, in those cases in which we do not depend on an extra singlet sector to generate masses for the extra quarks, Q_i and $\bar{Q}_i$, we would not even succeed in doing so, if we attempted it nonetheless. Finally, we emphasize that, by construction, $X\Lambda^2$ and $XP\bar{P}$ end up being the only renormalizable operators in W_P that are compatible with the Z_N^R symmetry for any value of N. In the following, we shall now show that the new singlet sector consisting of the fields X, P and $\bar{P}$ has the potential to accommodate the *invisible* axion and its superpartners and hence provide a solution of the strong CP problem via the PQ mechanism.

[11]To see this, note that all of our conditions can be written as $r_P \neq a/q + \ell/qN$, where $q \in \{1, 2, 3\}$, $a \in \{-2, 0, 2\}$ and $\ell \in \mathbb{Z}$. The number of different r_P values forbidden by some condition therefore corresponds to its value for q.

3. *Identification of the PQ symmetry*

Evidently, the superpotential in Eq. (20) exhibits a global $U(1)$ symmetry, *viz.* it is invariant under a global phase rotation of the fields P and $\bar{P}$. Let us refer to this symmetry as $U(1)_P$ and stipulate that the two singlets P and $\bar{P}$ respectively carry charge $q_P^{(P)} = 1$ and $q_{\bar{P}}^{(P)} = -1$ under it. The $U(1)_P$ symmetry is explicitly broken by the coupling of the singlet operator P^n to the quark pairs $(Q\bar{Q})_i$ in the superpotential in Eq. (14). At the same time, this coupling also breaks the $U(1)_Q^A$ symmetry in the extra quark sector. Altogether, the coupling between the new quark sector and the new singlet sector reduces the number of global Abelian symmetries from three to two,

$$U(1)_P \times U(1)_Q^V \times U(1)_Q^A \to U(1)_{\mathrm{PQ}} \times U(1)_Q^V. \tag{22}$$

The operators $(Q\bar{Q})_i$ are invariant under $U(1)_Q^V$ transformations, which is why the global vectorial symmetry in the quark sector survives the introduction of the superpotential in Eq. (14). The other global symmetry leaving the coupling in Eq. (14) invariant corresponds to some linear combination of $U(1)_P$, $U(1)_Q^V$ and $U(1)_Q^A$. It is this symmetry that we shall identify with the PQ symmetry. In the remainder of this paper, we will now investigate under which circumstances it may be successfully protected against the effects of higher-dimensional operators.

Before continuing, let us, however, reiterate once more for clarity: $U(1)_P$, $U(1)_Q^V$ and $U(1)_Q^A$ are accidental global symmetries of the new singlet and quark sectors at the renormalizable level that arise due to our particular choice of R charges. Neither of them manages to survive as an exact symmetry in the full low-energy effective theory. To begin with, the coupling between the two new sectors in Eq. (14) breaks $U(1)_P \times U(1)_Q^V \times U(1)_Q^A$ to its subgroup $U(1)_{\mathrm{PQ}} \times U(1)_Q^V$. This residual symmetry is, in turn, explicitly broken by other higher-dimensional operators. The dimension-6 operators Q^5 and $\bar{Q}^5$, for instance, explicitly break the vectorial Abelian symmetry in the new quark sector. The crucial question which we will have to address in the following therefore is how severe the explicit breaking of the PQ symmetry turns out to be and whether it remains sufficiently small enough, so that our model can still explain a QCD vacuum angle $\bar{\theta}$ of less than $\mathcal{O}(10^{-10})$.

Up to now, we are unable to specify the PQ charges of the new quark and antiquark fields separately, as the superpotential in Eq. (14) only contains the quark product operators $(Q\bar{Q})_i$. Demanding that the PQ charges of the singlet fields P and $\bar{P}$ coincide with their $U(1)_P$ charges, all we can say is that the operators $(Q\bar{Q})_i$ must carry a total PQ charge of $-n$. For the time being, we may thus work with the following PQ charges:

$$q_P = 1, \qquad q_{\bar{P}} = -1, \qquad q_Q \in \mathbb{R}, \\ q_{\bar{Q}} = -n - q_Q, \qquad q_{Q\bar{Q}} = -n. \tag{23}$$

The PQ charges of the MSSM fields q_i, where i now runs over $i = q, u^c, d^c, \ell, e^c, n^c, H_u, H_d$, are subject to constraints deriving from the Yukawa couplings in the superpotential W_{MSSM} in Eq. (1). The first two terms in W_{MSSM} yield the following three conditions:

$$\begin{aligned} q_{u^c} + q_q + q_{H_u} &= 0, \\ q_{d^c} + q_q + q_{H_d} &= 0, \\ q_{e^c} + q_\ell + q_{H_d} &= 0, \end{aligned} \tag{24}$$

the first two of which combine to give $q_{u^c} + q_{d^c} + 2q_q + q_{H_u} + q_{H_d} = 0$. As we will see in Sec. II D, the PQ charges of the two MSSM Higgs doublets must sum to zero, $q_{H_u} + q_{H_d} = 0$, implying that

$$q_{u^c} + q_{d^c} + 2q_q = 0. \tag{25}$$

The total PQ charge of all MSSM quark fields hence vanishes, such that the color anomaly of the PQ symmetry ends up receiving contributions only from the extra matter sector and none from the MSSM sector, cf. also Eq. (29) further below.

Having derived this important result, we would still like to know which values the MSSM PQ charges can actually take. Forgetting for a moment about the neutrino singlets required for the seesaw mechanism, the answer is clearly all values compatible with the three conditions in Eq. (24). The PQ charges q_i can then, for instance, be parametrized in terms of $q_q, q_\ell, q_{H_u} \in \mathbb{R}$. Moreover, we note that in the course of electroweak symmetry breaking the Yukawa couplings in W_{MSSM} turn into mass terms for the MSSM matter fields, breaking the PQ symmetry unless $q_{H_u} = -q_{H_d} = 0$. In this particular case, the PQ symmetry can be identified as a linear combination of $U(1)_B$ and $U(1)_L$, the global Abelian symmetries associated with baryon number B and lepton number L. This result is a useful crosscheck, since $U(1)_B$ and $U(1)_L$ are the *unique* accidental global symmetries of the standard model. In the seesaw extension of the MSSM, the conditions in Eq. (24) are supplemented by two further conditions deriving from the last two terms in W_{MSSM},

$$q_{n^c} + q_\ell + q_{H_u} = 0, \qquad 2q_{n^c} = 0, \tag{26}$$

eliminating the PQ charge q_ℓ as a free parameter. Upon extending the MSSM by three neutrino singlet fields, the PQ charges q_i can therefore be parametrized by only two charges, $q_q, q_{H_u} \in \mathbb{R}$. Setting q_{H_u} to zero now renders the PQ symmetry proportional to $U(1)_B$, which is, of course, expected, since the $U(1)_L$ is explicitly broken by the Majorana mass term in W_{MSSM}. The only relation among the PQ charges q_i relevant for our further analysis is Eq. (25). Without loss of generality, we are thus free to take q_q, q_ℓ and q_{H_u} to be zero, so that $q_i = 0$ for all fields i. The field content of our model as well as our assignment of the PQ charges are hence similar as in the KSVZ axion model proposed by Kim [43] as well as by Shifman, Vainshtein and Zakharov [44].

4. Spontaneous breaking and color anomaly of the PQ symmetry

In the true vacuum of the scalar potential corresponding to the superpotential W_P in Eq. (20), the singlet field X vanishes and the PQ symmetry is spontaneously broken,[12]

$$\langle X\rangle = 0, \qquad \langle P\rangle = \frac{\Lambda}{\sqrt{2}}\exp\left(\frac{A}{\Lambda}\right), \\ \langle \bar{P}\rangle = \frac{\Lambda}{\sqrt{2}}\exp\left(-\frac{A}{\Lambda}\right), \qquad \phi \subset A, \qquad \phi = \frac{1}{\sqrt{2}}(b + ia), \tag{27}$$

where the chiral superfield A represents the axion multiplet, which consists of the pseudoscalar axion a, the scalar saxino b and the fermionic axino $\tilde{a}$. The various factors of $\sqrt{2}$ in Eq. (27) serve two purposes. First, they render the kinetic term of the axion canonically normalized; second, they ensure that the scalar mass eigenstate that actually breaks the PQ symmetry, $p_+ = \frac{1}{\sqrt{2}}(p + \bar{p}^*)$, where p and $\bar{p}$ are the complex scalars contained in P and $\bar{P}$, acquires a VEV $\langle p_+\rangle = \Lambda$.

Before continuing, we remark that, in the special case of a Z_4^R symmetry, also a cubic term in the singlet field X is allowed in the superpotential W_P,

$$N = 4:\ W = \kappa X\left(\frac{\Lambda^2}{2} - P\bar{P}\right) - \lambda_X X^3 + \cdots, \tag{28}$$

where λ_X is some dimensionless coupling constant of $\mathcal{O}(1)$. In this case, the field configuration in Eq. (27) no longer represents the unique vacuum of the scalar potential corresponding to W_P. At $\langle P\rangle = \langle\bar{P}\rangle = 0$ and $\langle X\rangle = \sqrt{\kappa/(6\lambda_X)}\Lambda$, the scalar potential exhibits another local minimum. Because of the linear term in X in the scalar potential, $V \supset -\kappa m_{3/2}\Lambda^2 X$, this vacuum then has a negative energy density, the absolute value of which is much larger than the energy density of the PQ-breaking vacuum

[12] Spontaneous R symmetry breaking results in a tadpole term for X in the scalar potential, $V \supset -\kappa m_{3/2}\Lambda^2 X$. Besides that, X also couples to other fields of our model, cf. Sec. II D 3, such that its VEV eventually turns out to be of the order of the gravitino mass rather than zero, $\langle X\rangle \sim m_{3/2}$.

in Eq. (27). On top of that, the PQ-preserving vacuum and the PQ-breaking vacuum are separated from each other by a potential barrier which mainly arises due to the SUSY-invariant F-term contributions to the scalar potential from the fields P and $\bar{P}$. Consequently, there exists no flat direction connecting the alternative vacuum with our PQ-breaking vacuum, which is why we do not have to worry about the stability of the latter one. We merely have to assume that, in the course of the cosmological evolution, our universe has settled in the vacuum in Eq. (27) rather than in the alternative vacuum. In fact, this is a very plausible assumption, if we believe that the field X is stabilized at $\langle X\rangle = 0$ during inflation due to a large positive Hubble-induced mass. In such a situation, the scalar field configuration might then be automatically driven towards the PQ-breaking vacuum during inflation.

In order to solve the strong CP problem, it is necessary that the PQ symmetry has a color anomaly. Thanks to our derivation of the PQ charges of all colored matter fields in the previous subsection, we are now able to calculate the anomalous divergence of the axial PQ current $J^\mu_{\rm PQ}$ and show that it is nonzero,

$$\partial_\mu J^\mu_{\rm PQ} = \mathcal{A}_{\rm PQ}\frac{\alpha_s}{8\pi}\,{\rm Tr}[G_{\mu\nu}\tilde{G}^{\mu\nu}], \qquad \mathcal{A}_{\rm PQ} = kq_{Q\bar{Q}} + q_{u^c} + q_{d^c} + 2q_q = -nk, \tag{29}$$

where we have introduced $\mathcal{A}_{\rm PQ}$ as the anomaly coefficient of the $U(1)_{\rm PQ}[SU(3)_C]^2$ anomaly. This color anomaly of the PQ symmetry induces an extra term in the effective Lagrangian [1,45],

$$\mathcal{L}^{\rm eff}_{\rm QCD} \supset \left(\bar{\theta} - \frac{a}{f_a}\right)\frac{\alpha_s}{8\pi}{\rm Tr}[G_{\mu\nu}\tilde{G}^{\mu\nu}], \quad f_a = \frac{\sqrt{2}\Lambda}{|\mathcal{A}_{\rm PQ}|}, \tag{30}$$

with f_a denoting the axion decay constant. In consequence of this coupling of the axion a to the gluon field strength $G_{\mu\nu}$, an effective nonperturbative potential for the axion is generated,

$$V^{\rm eff}_a = \Lambda^4_{\rm QCD}\left[1 - \cos\left(\bar{\theta} - \frac{a}{f_a}\right)\right], \tag{31}$$

the minimum of which is located at $\langle a\rangle = f_a\bar{\theta}$. Shifting a by its VEV $\langle a\rangle$ then cancels the $\bar{\theta}$ term in Eq. (30), thereby rendering the QCD Lagrangian CP invariant. Our singlet sector consisting of the fields X, P and $\bar{P}$ hence entails a manifestation of the PQ solution to the strong CP problem.

An important detail to note is that it is the scale f_a, rather than Λ, which determines the strength of all low-energy interactions of the axion [46]. This is also the reason why experimental constraints on the axion coupling are always formulated as bounds on f_a and not on Λ. Requiring, for instance, that astrophysical objects such as supernovae or white dwarfs do not lose energy too fast due to axion emission allows one to put a lower bound of $\mathcal{O}(10^9)$ GeV [10] on f_a. Meanwhile, cosmology restricts the possible range of f_a values from above. In order to prevent cold axions from overclosing the universe, f_a must be at most of $\mathcal{O}(10^{12})$ GeV [47,48], hence leaving open the following phenomenologically viable window for the axion decay constant,

$$10^9\ {\rm GeV} \lesssim f_a \lesssim 10^{12}\ {\rm GeV}. \tag{32}$$

Furthermore, as evident from the effective axion potential in Eq. (31), the nonperturbative QCD instanton effects break the PQ symmetry to a global and discrete $Z_{N_{\rm DW}}$ symmetry, where $N_{\rm DW} = |\mathcal{A}_{\rm PQ}| = nk$, commonly referred to as the domain wall number, counts the number of degenerate axion vacua. If the breaking of the PQ symmetry occurs after inflation, this vacuum structure of the axion potential implies the formation of axion domain walls during the QCD phase transition, thereby leading to a cosmological disaster [48,49]. One obvious solution to this domain wall problem is to impose that inflation takes place after the spontaneous breaking of the PQ symmetry, such that the axion field is homogenized across the entire observable universe.[13] Alternatively, one may attempt to construct an axion model with $N_{\rm DW} = 1$, in which case the axion domain walls collapse under their boundary tension soon after their formation [51]. In Appendix B, we present a slight modification of our model that just yields $N_{\rm DW} = 1$ and which hence allows for a solution of the axion domain wall problem even if the spontaneous breaking of the PQ symmetry takes place after inflation. To be clear about that, as for our actual model with $N_{\rm DW} = nk$, we will however assume in the following that the PQ symmetry is broken sufficiently early before the end of inflation. A necessary condition for this to happen is that the PQ-breaking scale Λ exceeds the reheating temperature $T_{\rm RH}$ after inflation, $T_{\rm RH} \lesssim \Lambda$. Such a hierarchy between the two scales Λ and $T_{\rm RH}$ then also ensures that the PQ symmetry is not restored after inflation due to finite-temperature effects, which would otherwise bring us back to the axion domain wall problem. Finally, the fact that $T_{\rm RH}$ is bounded from above by Λ also explains why we are allowed to neglect all finite-temperature corrections to the scalar potential of the PQ-breaking sector. In general, such corrections could change the vacuum structure of our model; but given that the temperature T remains below Λ at all times, we can rest assured that this is not the case.

5. Mass scale of the extra matter sector

As anticipated, the spontaneous breaking of the PQ symmetry furnishes the extra quarks and antiquarks with

[13] In this case, perturbations in the axion field amplified during inflation may result in too large isocurvature contributions to the temperature fluctuations seen in the cosmic microwave background. A variety of solutions to this isocurvature perturbation problem have however been proposed in the literature, cf. for instance Ref. [50] and references therein, which is why we will not consider it any further.

TABLE I. Values of k leading to unwanted operators in W_P, the superpotential of the new singlet sector, cf. Eq. (20). This table does not indicate for which Z_N^R symmetries only one extra quark pair is problematic. For $n = 1$, these are the symmetries with $N = 3, 4, 5, 6, 7, 8, 10, 12, 14, 16, 20$; for $n = 2$, it is the symmetries with $N = 5, 7, 10, 11, 14, 22$. In addition, independently of n, we disregard the possibility of only one extra quark pair for $N = 3, 4, 6, 8$ in any case, cf. Sec. II B 3.

	Z_3^R	Z_4^R	Z_5^R	Z_6^R	Z_8^R	Z_9^R	Z_{10}^R	Z_{12}^R	Z_{18}^R
$n = 1$	2, 3, 6, 9	2, 3		2, 3, 6, 9	2, 3			3, 9	
$n = 2$	2, 6		2	2, 6		3	2		3

Dirac masses m_{Q_i}, which can be read off from the superpotential W_Q in Eq. (14) after expanding the singlet field P around its VEV,

$$m_{Q_i} = \frac{\lambda_i}{M_{\rm Pl}^{n-1}}\left(\frac{\Lambda}{\sqrt{2}}\right)^n \simeq \left(\frac{\lambda_i}{1}\right)\left(\frac{k}{4}\right)^n\left(\frac{f_a}{10^{10}\ \mathrm{GeV}}\right)^n \times \begin{cases} 2.0\times 10^{10}\ \mathrm{GeV}; & n=1 \\ 6.6\times 10^{2}\ \mathrm{GeV}; & n=2 \\ 3.6\times 10^{-5}\ \mathrm{GeV}; & n=3 \\ \cdots; & n=4 \end{cases}. \tag{33}$$

For $n \geq 3$, our model thus predicts k new quark multiplets with masses below the electroweak scale, which is, of course, inconsistent with experiments. Hence, the only viable values for n are $n = 1$ and $n = 2$. From a phenomenological point of view, the $n = 2$ case is certainly more interesting as it features new colored states with masses possibly within the range of collider experiments. On the other hand, if no heavy quarks should be found at or above the TeV scale, our model would not automatically be ruled out. Falling back to the $n = 1$ case, the extra vector quarks can always be decoupled from the physics at the TeV scale, thereby leaving still some room for the realization of our extension of the MSSM.

For both viable values of n, we can now ask how many new quark flavors we are allowed to introduce, i.e. which values k can possibly take. Recall that in Sec. II C 2 we required the R charge r_P to fulfill all of the seven conditions in Eqs. (18) and (21). Given the explicit expression for r_P in terms of n and k in Eq. (16), this requirement then directly translates into a set of k values that, depending on the values of n and N, we are not allowed to employ, cf. Table I. In Sec. III A, we will derive further restrictions on the set of allowed k values based on the requirement that the unification of the SM gauge couplings ought to occur at the perturbative level.

D. Generation of the MSSM μ term

In absence of any new physics beyond the MSSM, one might expect the supersymmetric mass of the MSSM Higgs doublets to be of the order of the Planck scale, $\mu \sim M_{\rm Pl}$. Such a large μ value would then require a miraculous cancellation between the supersymmetric and the soft SUSY-breaking contributions to the MSSM Higgs scalar potential, given that one ought to end up with Higgs VEVs $\langle H_{u,d}\rangle = v_{u,d}$ close the electroweak scale. This puzzle, i.e. the question why μ should be of the same order as the soft Higgs masses, represents the infamous μ problem. As we have seen in the previous section, an anomaly-free discrete R symmetry Z_N^R forbids the μ term in the MSSM superpotential, thus solving the μ problem halfway through. What remains to be done is to demonstrate how the μ term emerges with the right order of magnitude once the Z_N^R has been spontaneously broken.

1. μ term from spontaneous R symmetry breaking

In the special case of a Z_4^R symmetry, the R charges of H_u and H_d sum to zero, $r_{H_u} + r_{H_d} \overset{(4)}{=} 4 \overset{(4)}{=} 0$, cf. Eq. (4), such that a μ term of the correct magnitude can be easily generated in the course of spontaneous R symmetry breaking [41]. This mechanism is based on two ingredients: (i) the observation that, for $r_{H_u} + r_{H_d} = 0$, the operator $H_u H_d$ can be accommodated with some $\mathcal{O}(1)$ coefficient g_H' in the Kähler potential, $K \supset g_H' H_u H_d$, as well as (ii) the fact that, during spontaneous R symmetry breaking, the superpotential acquires a nonzero VEV $\langle W\rangle = W_0$,[14] where $W_0/M_{\rm Pl}^2$ can be identified with the gravitino mass, $m_{3/2} = W_0/M_{\rm Pl}^2$. At low energies, the Higgs operator in the Kähler potential then induces an effective superpotential $W_\mu = g_H' m_{3/2} H_u H_d$, which is nothing but the desired μ term with $\mu = g_H' m_{3/2}$. Besides that, an additional contribution to the μ term may be generated in the course of spontaneous SUSY breaking, if the Kähler potential should contain a coupling between the operator $H_u H_d$ and the hidden SUSY breaking sector [19]. In the remainder of this section, we will now mostly focus on Z_N^R symmetries with $N \neq 4$.

[14]Since W carries R charge $r_W = 2$, the VEV $\langle W\rangle$ breaks the Z_N^R completely; R parity, which potentially remains as an unbroken subgroup of the Z_N^R, is not an actual discrete R symmetry, cf. Sec. II B. A possible mechanism to generate a constant term in the superpotential is the condensation of hidden gauginos, such that $W_0 = \langle \mathcal{W}_\alpha \mathcal{W}^\alpha\rangle$ [52]. Alternatively, the VEV of the superpotential might originate from the condensation of hidden-sector quarks $\tilde{Q}$, such that $W_0 = \langle (\tilde{Q}\tilde{Q})^n\rangle$. In Appendix A of Ref. [53], we present an exemplary model illustrating how such a quark condensate could potentially be generated by means of strong gauge dynamics in some hidden sector.

2. Contributions to the μ term from spontaneous PQ breaking

Next, we note that sometimes already the spontaneous breaking of the PQ symmetry entails the generation of a supersymmetric mass term for the MSSM Higgs doublets H_u and H_d, which, however, turns out to be too small in all viable cases. The origin for this contribution to the MSSM μ term is the following higher-dimensional operators in the tree-level superpotential,

$$W_\mu = \left(C_\mu^{(p)} \frac{P^p}{M_{\rm Pl}^{p-1}} + C_\mu^{(\bar p)} \frac{\bar P^{\bar p}}{M_{\rm Pl}^{\bar p -1}} \right) H_u H_d, \tag{34}$$

with $C_\mu^{(p)}$ and $C_\mu^{(\bar p)}$ denoting dimensionless coupling constants of $\mathcal{O}(1)$. Of course, these couplings are only allowed if they are compatible with the Z_N^R symmetry, which is the case given that

$$p r_P \overset{(N)}{=} -2 \quad \text{and/or} \quad \bar p r_{\bar P} \overset{(N)}{=} -2. \tag{35}$$

We shall now assume for a moment that at least one of these two conditions can be satisfied. In case only the first or the second condition can be fulfilled, let q denote the corresponding value of p or $\bar p$. If both conditions can be satisfied simultaneously, q shall denote the smaller of the two possible powers, $q = \min\{p, \bar p\}$. The spontaneous breaking of the PQ symmetry then induces a supersymmetric mass μ for H_u and H_d, which looks very similar to the Dirac masses m_{Q_i} for the extra quarks and antiquarks in Eq. (33),

$$\mu = \frac{C_\mu^{(q)}}{M_{\rm Pl}^{q-1}} \left(\frac{\Lambda}{\sqrt{2}} \right)^q \simeq \left(\frac{C_\mu^{(q)}}{1} \right) \left(\frac{k}{4} \right)^q \left(\frac{f_a}{10^{10}\ \text{GeV}} \right)^q \times \begin{cases} 2.0 \times 10^{10}\ \text{GeV}; & q = 1 \\ 6.6 \times 10^{2}\ \text{GeV}; & q = 2 \\ 3.6 \times 10^{-5}\ \text{GeV}; & q = 3 \\ \cdots; & q = 4. \end{cases} \tag{36}$$

For $q = 1$, the generated μ term is, hence, dangerously large; for $q = 2$ it is of the desired order of magnitude; and for $q \geq 3$ it is drastically too small. On the other hand, given our restrictions on the R charge r_P in Eqs. (18) and (21), we know that q has to be at least $q = 4$. This means that, for $q = 1, 2, 3$, *all* possible R charges r_P fulfilling at least one of the two conditions in Eq. (35) lead to an unwanted operator in W_P, the superpotential of the extra scalar sector, cf. Eq. (20). We thus conclude that the spontaneous breaking of the PQ symmetry does not suffice to generate a μ term of the right order of magnitude. For the last time, we are therefore led to extend our model.

3. Singlet extension of the MSSM Higgs sector

Extensions of the MSSM aiming at generating the μ term dynamically usually couple the MSSM Higgs doublets to another chiral singlet S, which acquires a VEV of the order of the soft Higgs masses in the course of electroweak symmetry breaking. We will now adopt this approach and introduce a chiral singlet field S with R charge $r_S = -2$, in order to allow for the operator SH_uH_d in the superpotential. As we will see in the following, this operator usually indeed yields a μ term of the right order of magnitude, i.e. a value for the μ parameter of the order of the soft SUSY-breaking scale.

Given the fact that the superpotential carries R charge $r_W = 2$, the relation between the gravitino mass and the VEV of the superpotential, $m_{3/2} = W_0/M_{\rm Pl}^2$, implies that $m_{3/2}$ should be regarded as a spurious field also carrying R charge $r_{3/2} = 2$. After spontaneous R symmetry breaking, the superpotential of the field S hence contains the following terms:

$$N \neq 4:\ W_S \supset g_H H_u H_d S + m_{3/2}^2 S + m_S S^2 (+\lambda_S S^3), \tag{37}$$

where m_S denotes a supersymmetric mass for the singlet field S and where the term in parentheses is only allowed in the case of a discrete Z_8^R symmetry. In this section, we explicitly exclude the possibility of a Z_4^R symmetry, because in this case the μ term is already generated in the course of R symmetry breaking, cf. Sec. II D 1. Besides that, for a Z_4^R symmetry, the R charge of the field S would be equivalent to $r_S = 2$, such that a tadpole term of the order of the Planck scale would be allowed in the superpotential. Such a large tadpole would then severely destabilize the electroweak scale. By contrast, all other Z_N^R symmetries successfully prevent the appearance of a dangerously large tadpole term. In fact, the only tadpole that we are able to generate for $N \neq 4$ arises from the spontaneous breaking of R symmetry and is of the size of the gravitino mass, cf. Eq. (37).

Assuming a discrete Z_3^R or Z_6^R symmetry, the R charge of S^2 is equivalent to $r_s = 2$ and m_S is expected to be very large, $m_S = g_S M_{\rm Pl}$, where g_S is a dimensionless constant of $\mathcal{O}(1)$ in general. For all other Z_N^R symmetries, the S mass term is only allowed if, similarly as for the gravitino mass, m_S is interpreted as a spurious field, now with R charge 6 instead of 2. On the supposition that only a single dynamical process is responsible for the generation of $m_{3/2}$ and m_S, the S mass then turns out to be heavily suppressed,

$$N = 3, 6:\ m_S = g_S M_{\rm Pl}, \qquad g_S \sim 1; \qquad N \neq 3, 6:\ m_S \sim \frac{m_{3/2}^3}{M_{\rm Pl}^2}. \tag{38}$$

For $N = 3$, 6, the large supersymmetric mass m_S hence leads to a very small VEV of the field S, thereby causing our attempt to dynamically generate the MSSM μ term to fail. Only in the case that, for one reason or another, the parameter g_S is severely suppressed, $g_S \ll 1$, such that $m_S \ll M_{\rm Pl}$, a Z_3^R or a Z_6^R symmetry may still be considered viable. Otherwise, Z_N^R symmetries with $N = 3, 6$ should be regarded disfavored within the context of our model.[15] By contrast, in the case of all other symmetries, i.e. Z_N^R symmetries with $N = 5$ or $N \geq 7$, the mass of the field S is completely negligible, which is why we will omit from now on. Thus, as far as the generation of the μ term in our model is concerned, we will assume the following terms in the superpotential:

$$W_S \supset \begin{cases} g_H H_u H_d S + m_{3/2}^2 S + m_S S^2; & N = 3, 6 \\ g_H' m_{3/2} H_u H_d; & N = 4 \\ g_H H_u H_d S + m_{3/2}^2 S + \lambda_S S^3; & N = 8 \\ g_H H_u H_d S + m_{3/2}^2 S; & N \neq 3, 4, 6, 8. \end{cases} \tag{39}$$

Together with the scalar masses and couplings in the soft SUSY-breaking Lagrangian, the interactions in Eq. (39) result in a scalar potential that is minimized for $\langle H_{u,d} \rangle = v_{u,d}$ and $\langle S \rangle = \mu / g_H$, whereby our solution to the μ problem is completed. The actual value of the μ parameter depends in a complicated way on the couplings in the superpotential W_S as well as on the soft parameters for the fields $H_{u,d}$ and S. For our purposes, it will however suffice to treat μ as an effectively free parameter that is allowed to vary within some range.

Some of the expressions for W_S in Eq. (39) are reminiscent of the superpotential of other extensions of the MSSM that successfully generate the μ term by means of a singlet field S. For instance, assuming a discrete Z_8^R symmetry and neglecting the tadpole term, the superpotential in Eq. (37) corresponds to the Higgs superpotential of the next-to-minimal supersymmetric standard model (NMSSM).[16] Conversely, assuming a Z_N^R symmetry with $N \neq 3, 4, 6, 8$ and taking the tadpole term into account, the superpotential W_S coincides with the effective Higgs superpotential of the new MSSM (nMSSM) [55] as well as with the effective Higgs superpotential of the PQ-invariant extension of the NMSSM (PQ-NMSSM) [56]. While in the nMSSM, the shape of the S superpotential is fixed by means of a discrete R symmetry, similarly as in our model, the PQ-NMSSM invokes a PQ symmetry by hand in order to ensure the absence of further couplings of the singlet field S. On the other hand, the PQ-NMSSM features a PQ singlet field, similar to our singlet fields P and $\bar{P}$, which couples to the field S. By contrast, such a PQ-breaking field is absent in the nMSSM. But as the mixing between the singlet S and the PQ-breaking sector is always suppressed by powers of the PQ scale Λ, this has basically no effect on the low-energy phenomenology of the Higgs and neutralino sectors.

As for the expected low-energy signatures of these two sectors, our model thus makes the same predictions as the PQ-NMSSM and the nMSSM. This means, in particular, that our model predicts a fifth neutralino mostly consisting of the singlino, which only receives a small mass from mixing with the neutral Higgsinos. Among all superparticles that either directly belong to the MSSM or that at least share some renormalizable interaction with it, the singlino-like neutralino is hence expected to be the lightest. Furthermore, at small values of $\tan\beta$, the decay of the standard model-like Higgs boson into two singlino-like neutralinos might represent an important Higgs decay mode [57]. Such a scenario is already constrained by the search for invisible Higgs decays by the ATLAS experiment at the LHC [58] and will be further tested as data taking at the LHC is resumed.[17] Another interesting feature of our model is that, independently of $\tan\beta$, the Higgs boson mass receives positive corrections of the order of a few GeV from singlino loops, provided that the Higgsinos are lighter than all other superparticles of the MSSM. Finally, we mention that our model features a series of interesting implications for cosmology [56,60].

The operators on the right-hand side of Eq. (37) are the only terms in the superpotential of the singlet field S playing a role in the generation of the μ term. Besides that, the field S participates, of course, also in a series of other interactions. As the field X carries the same R charge as the gravitino mass, $r_X = r_{3/2} = 2$, the tadpole term in Eq. (37) has, in particular, to be supplemented by the operators $m_{3/2} X S$ and $X^2 S$. The full superpotential of the field S thus reads

$$N \neq 4: \quad W_S = g_H H_u H_d S + m_{3/2}^2 S + g_X m_{3/2} X S + g_{X^2} X^2 S (+ m_S S^2)(+\lambda_S S^3) + \cdots. \tag{40}$$

Here, g_X and g_{X^2} are again dimensionless coupling constants of $\mathcal{O}(1)$ and the dots after the plus sign indicate higher-dimensional nonrenormalizable terms. Given the fact that the field X does not carry any PQ charge, the coupling between X and S immediately implies that the field S also does not transform under PQ rotations, $q_S = 0$. This proves in turn our statement in Sec. II C 3 that the PQ charges of H_u and H_d must sum to zero, $q_{H_u} + q_{H_d} = 0$.

Another important consequence of the operators $m_{3/2} X S$ and $X^2 S$ in Eq. (40) is that, at the supersymmetric level, the scalar field VEVs in Eq. (27) no longer represent the

[15] Interestingly, these are just the two anomaly-free Z_N^R symmetries of the MSSM. Now we see that they are most likely not compatible with the generation of the μ term by means of an additional singlet field S. This justifies once more our approach to extend the particle content of the MSSM by a new quark sector in such a way that the gauge anomalies of the Z_N^R symmetry are always canceled, independently of the value of N.

[16] For reviews of the NMSSM, cf. for instance Ref. [54].

[17] We can easily evade the LHC constraints on the invisible Higgs decay mode if we equip the singlino with a Dirac mass term. To this end, we need to supplement the field S by a gauge singlet $\bar{S}$ with R charge $r_{\bar{S}} = -r_S = 2$, such that $W \supset M_S S \bar{S}$ with $M_S \sim m_{3/2}$ as is the case in the Dirac NMSSM [59].

unique vacuum configuration. The PQ-breaking vacuum, in which $\langle P\bar{P}\rangle = \Lambda^2/2$, is now continuously connected to a family of degenerate vacua, all of which are characterized by the fact that they fulfill the condition $\langle P\bar{P}\rangle - g_X/\kappa m_{3/2}\langle S\rangle = \Lambda^2/2$. However, this vacuum degeneracy is fortunately lifted by the soft SUSY breaking masses for the scalar fields P, $\bar{P}$ and S, such that, also in the presence of the operators $m_{3/2}XS$ and X^2S, the vacuum configuration of interest, i.e. $\langle P\bar{P}\rangle = \Lambda^2/2$ together with $\langle S\rangle \sim m_{3/2}$ and $\langle X\rangle \sim m_{3/2}$, corresponds to a local minimum. Besides that, the new interactions between S and X also lead to a second local minimum at $\langle X\rangle \sim m_{3/2}^{1/3}\Lambda^{2/3}$, $\langle XS\rangle \sim \Lambda^2$ and $\langle P\bar{P}\rangle = 0$. The energy of this vacuum is, however, much higher than the one of the PQ-breaking vacuum and hence, we expect the fields P, $\bar{P}$, S and X to settle in the PQ-breaking vacuum at low energies,

$$\langle P\rangle = \frac{\Lambda}{\sqrt{2}}e^{A/\Lambda}, \qquad \langle \bar{P}\rangle = \frac{\Lambda}{\sqrt{2}}e^{-A/\Lambda}, \qquad \langle S\rangle \sim m_{3/2}, \qquad \langle X\rangle \sim m_{3/2}. \tag{41}$$

Just as in the case of a Z_4^R symmetry, this vacuum might also be automatically reached if during inflation X is stabilized close to its origin thanks to a large positive Hubble-induced mass term. Moreover, we do not expect thermal effects to play a significant role in the selection of the low-energy vacuum as long as the reheating temperature $T_{\rm RH}$ is smaller than the PQ-breaking scale Λ, cf. our discussion at the end of Sec. II C 4.

4. Decay of the extra matter fields into MSSM particles

The extension of the MSSM Higgs sector by the singlet field S completes the field content of our model. We are therefore almost ready to turn to the phenomenological constraints on our model and discuss which values of N, n and k allow for a sufficient protection of the PQ symmetry. But before we are able to do so, we have to take care of one last detail: the new quarks and antiquarks are thermally produced in the early universe, which potentially results in serious cosmological problems. If the extra quarks are stable, they might be produced so abundantly that they overclose the universe. On the other hand, if they are unstable, their late-time decays might alter the primordial abundances of the light elements produced during big bang nucleosynthesis (BBN), so that these are no longer in accordance with the observational data. To avoid these problems, we require a coupling between the extra quark sector and the MSSM fields, such that the extra quarks quickly decay after their production. So far, we only had to fix the R charge $r_{Q\bar{Q}}$ of the quark pair operator $Q\bar{Q}$, cf. Eq. (11). The R charges r_Q and $r_{\bar{Q}} = r_{Q\bar{Q}} - r_Q$ of the individual quarks and antiquarks have by contrast remained unspecified up to now. By choosing a particular value for the R charge r_Q, we are therefore now able to pinpoint the operator by means of which the extra quarks shall couple to the MSSM.

Under the SM gauge group, the antiquark fields $\bar{Q}_i$ transform in the same representation as the MSSM $\mathbf{5}_i^*$ multiplets. An obvious possibility to couple the new quarks to the MSSM thus is to allow for the operator $\bar{Q}_i\mathbf{10}_jH_d$ in the superpotential, in which case the antiquarks ought to carry the same R charge as the $\mathbf{5}_i^*$ multiplets, $r_{\bar{Q}} = r_{\mathbf{5}}^*$. The only way in which the extra antiquark fields then distinguish themselves from the MSSM $\mathbf{5}_i^*$ multiplets is their coupling to the extra quark fields Q_i. More precisely, starting out with a superpotential containing the operators $P^nQ_i(\bar{Q}_i' + \mathbf{5}_i^{*\prime})$ and $(\bar{Q}_i' + \mathbf{5}_i^{*\prime})\mathbf{10}_jH_d$, we can always perform a field transformation $(\bar{Q}_i', \mathbf{5}_i^{*\prime}) \to (\bar{Q}_i, \mathbf{5}_i^*)$, such that, *by definition*, the MSSM $\mathbf{5}_i^*$ multiplets do not couple to the extra quark fields Q_i and only the operators $P^nQ_i\bar{Q}_i$ and $(\bar{Q}_i + \mathbf{5}_i^*)\mathbf{10}_jH_d$ remain in the superpotential. The operator $\bar{Q}_i\mathbf{10}_jH_d$ then mixes the quarks and leptons respectively contained in the $\bar{Q}_i$ and $\mathbf{5}_i^*$ multiplets, which potentially gives rise to dangerous flavor-changing neutral-current (FCNC) interactions. In the case of very heavy extra quarks, i.e. for $n = 1$, we however do not have to worry about FCNC processes as these are always automatically suppressed by the large quark masses. Only for $n = 2$, we have to pay attention that the mixing between the MSSM fermions and the new matter fields does not become too large. For extra quarks with masses around 1 TeV, we have for instance to require that the Yukawa coupling constants belonging to the operator $\bar{Q}_i\mathbf{10}_jH_d$ are at most of $\mathcal{O}(10^{-2})$ [61]. This is a rather mild constraint, which may be easily satisfied in a large class of flavor models.

Coupling the new quark sector to the MSSM via the operator $\bar{Q}_i\mathbf{10}_jH_d$ is therefore certainly a viable option. The R and PQ charges of the extra quarks and antiquarks are then given by

$$r_Q \overset{(N)}{=} r_{Q\bar{Q}} - r_{\bar{Q}}, \qquad r_{\bar{Q}} \overset{(N)}{=} r_{\mathbf{5}^*}; \qquad q_Q = q_{Q\bar{Q}} - q_{\bar{Q}}, \qquad q_{\bar{Q}} = q_{\mathbf{5}^*}. \tag{42}$$

Making use of our results for $r_{\mathbf{5}^*}$ and $r_{Q\bar{Q}}$ in Eqs. (8) and (11), we find for r_Q and $r_{\bar{Q}}$,

$$r_Q \overset{(N)}{=} \frac{13}{5} + \frac{6}{k} + \left[\frac{\ell_Q}{k} - \frac{\ell}{2}\right]N, \qquad r_{\bar{Q}} \overset{(N)}{=} -\frac{3}{5} + \ell\frac{N}{2}. \tag{43}$$

Likewise, employing our results for $q_{Q\bar{Q}}$ in Eq. (23) and setting $q_{\mathbf{5}^*}$ to 0, we obtain for q_Q and $q_{\bar{Q}}$,

$$q_Q = -n, \qquad q_{\bar{Q}} = 0. \tag{44}$$

Finally, we also note that the operator $\bar{Q}_i\mathbf{10}_jH_d$ explicitly breaks the vectorial global symmetry in the extra quark sector, such that the PQ symmetry remains as the only global Abelian symmetry,

$$U(1)_{\rm PQ} \times U(1)_Q^V \to U(1)_{\rm PQ}. \tag{45}$$

Given our choice for the MSSM PQ charges in Sec. II C 3, we are now eventually able to determine the relation between the generators of the three global Abelian symmetries $U(1)_P$, $U(1)_Q^V$ and $U(1)_Q^A$ on the one hand and the PQ generator on the other hand. Denoting these generators by P, V, A and PQ, respectively, we find

$$\mathrm{PQ} = P - \frac{n}{2}(V + A). \tag{46}$$

In order to avoid the above constraint on the Yukawa couplings associated with $\bar{Q}_i \mathbf{10}_j H_d$ in the case $n = 2$, one may alternatively consider couplings of the new quark fields to the MSSM via higher-dimensional operators. Naively, there are three different choices for such an operator, namely $S\bar{Q}_i\mathbf{10}_jH_d$, $P\bar{Q}_i\mathbf{10}_jH_d$ and $\bar{P}\bar{Q}_i\mathbf{10}_jH_d$. Replacing the singlet fields S, P and $\bar{P}$ in these operators by their respective VEVs, all of them turn again into $\bar{Q}_i\mathbf{10}_jH_d$, now, however, with coupling constants that are naturally suppressed compared to unity. Allowing for any of these operators rather than $\bar{Q}_i\mathbf{10}_jH_d$, we therefore do not have to fear dangerous FCNC processes due to the mixing between the $\bar{Q}_i$ and $\mathbf{5}_i^*$ multiplets. Meanwhile, $S\bar{Q}_i\mathbf{10}_jH_d$ and $P\bar{Q}_i\mathbf{10}_jH_d$ do not represent viable operators by means of which the new quarks could couple to the MSSM after all. In the case of $S\bar{Q}_i\mathbf{10}_jH_d$, the extra quarks do not decay sufficiently fast in the early universe. The operator $S\bar{Q}_i\mathbf{10}_jH_d$ furnishes the new quarks with two-body and three-body decay channels, the partial decay rates of which can roughly be estimated as

$$\begin{aligned}\Gamma(\bar{Q}_i \to q_jH_d, e_j^cH_d) &\sim \frac{1}{8\pi}\left(\frac{\mu/g_H}{M_{\mathrm{Pl}}}\right)^2 m_{Q_i} \\ &\sim \begin{cases} 10^2\ \mathrm{s}^{-1}; & m_{Q_i} = 10^{10}\ \mathrm{GeV} \\ 10^{-5}\ \mathrm{s}^{-1}; & m_{Q_i} = 1\ \mathrm{TeV}, \end{cases} \\ \Gamma(\bar{Q}_i \to Sq_jH_d, Se_j^cH_d) &\sim \frac{1}{128\pi^3}\frac{m_{Q_i}^3}{M_{\mathrm{Pl}}^2} \\ &\sim \begin{cases} 10^{14}\ \mathrm{s}^{-1}; & m_{Q_i} = 10^{10}\ \mathrm{GeV} \\ 10^{-7}\ \mathrm{s}^{-1}; & m_{Q_i} = 1\ \mathrm{TeV}. \end{cases}\end{aligned} \tag{47}$$

Here, we have set the VEV of the scalar field S to $\langle S\rangle = \mu/g_H = 1$ TeV. For $n = 2$, the extra quarks thus decay only after BBN, which begins at a cosmic time of around 1 s and lasts for roughly 10^3 s. Moreover, if we choose the R charge r_Q, such that $P\bar{Q}_i\mathbf{10}_jH_d$ is contained in the superpotential, also $P^{n-1}Q_i\mathbf{5}_j^*$ is allowed. Unlike in our first case, in which we considered $\bar{Q}_i\mathbf{10}_jH_d$, this operator cannot be simply eliminated by a field redefinition. Together with $P^n(Q\bar{Q})_i$, it instead leads to an unacceptably strong mixing between the $\bar{Q}_i$ and $\mathbf{5}_i^*$ multiplets.

The only remaining option therefore is to allow for $\bar{P}\bar{Q}_i\mathbf{10}_jH_d$. In this case, the superpotential also features $P^{n+1}Q_i\mathbf{5}_j^*$, which cannot be transformed away as well, but which fortunately results in the mixing between the $\bar{Q}_i$ and $\mathbf{5}_j^*$ multiplets being suppressed by a factor of $\mathcal{O}(\Lambda/M_{\mathrm{Pl}})$. Furthermore, $\bar{P}\bar{Q}_i\mathbf{10}_jH_d$ gives rise to two-body decays of the extra quarks at a fast rate. After replacing the scalar field $\bar{P}$ by $\Lambda/\sqrt{2}$, we obtain

$$\begin{aligned}\Gamma(\bar{Q}_i \to q_jH_d, e_j^cH_d) &\sim \frac{1}{16\pi}\left(\frac{\Lambda}{M_{\mathrm{Pl}}}\right)^2 m_{Q_i} \\ &\sim \begin{cases} 10^{16}\ \mathrm{s}^{-1}; & m_{Q_i} = 10^{10}\ \mathrm{GeV} \\ 10^{10}\ \mathrm{s}^{-1}; & m_{Q_i} = 1\ \mathrm{TeV}, \end{cases}\end{aligned} \tag{48}$$

where we have chosen the PQ-breaking scale Λ such that it respectively results in $m_{Q_i} = 10^{10}$ GeV or $m_{Q_i} = 1$ TeV, if n is set to 1 or 2, cf. Eq. (33). Similarly to $S\bar{Q}_i\mathbf{10}_jH_d$, the operator $\bar{P}\bar{Q}_i\mathbf{10}_jH_d$ also entails three-body decays, which, however, always proceed at a slower rate than the corresponding two-body decays, cf. Eq. (47). A coupling of the extra quarks to the MSSM via $\bar{P}\bar{Q}_i\mathbf{10}_jH_d$ is hence a viable alternative to the coupling via $\bar{Q}_i\mathbf{10}_jH_d$. A particular advantage of this coupling is that we do not have to require suppressed Yukawa couplings, if $n = 2$. On the other hand, the charges of the extra antiquarks now do not coincide any more with the charges of the MSSM $\mathbf{5}_i^*$ multiplets. The R and PQ charges of the new quark and antiquark fields are instead given by

$$\begin{aligned} r_Q &\overset{(N)}{=} r_{Q\bar{Q}} - r_{\bar{Q}}, & r_{\bar{Q}} &\overset{(N)}{=} r_{\mathbf{5}^*} - r_{\bar{P}}; \\ q_Q &= q_{Q\bar{Q}} - q_{\bar{Q}}, & q_{\bar{Q}} &= -q_{\bar{P}}. \end{aligned} \tag{49}$$

Our results for $r_{\mathbf{5}^*}$, $r_{Q\bar{Q}}$ and $r_{\bar{P}}$ in Eqs. (8), (11), and (19) therefore provide us with

$$\begin{aligned} r_Q &\overset{(N)}{=} \frac{13}{5} + \frac{(n+1)6}{nk} - \left[\frac{\ell}{2} + \frac{\ell_P}{n} - \frac{(n+1)\ell_Q}{nk}\right]N, \\ r_{\bar{Q}} &\overset{(N)}{=} -\frac{3}{5} - \frac{6}{nk} + \left[\frac{\ell}{2} + \frac{\ell_P}{n} - \frac{\ell_Q}{nk}\right]N. \end{aligned} \tag{50}$$

Similarly, making use of the fact that $q_{\bar{P}} = -1$ and $q_{Q\bar{Q}} = -n$, cf. Eq. (23), we find for q_Q and $q_{\bar{Q}}$,

$$q_Q = -n - 1, \qquad q_{\bar{Q}} = 1. \tag{51}$$

Combining this result with our choice for the MSSM PQ charges in Sec. II C 3, the relation between the four Abelian generators P, V, A and PQ now turns out to be

$$\mathrm{PQ} = P - V - \frac{n}{2}(V + A). \tag{52}$$

These findings complete the construction of our model. To sum up, in this section, we have introduced (i) the field content of the MSSM along with three generations of right-handed neutrinos, (ii) k pairs of extra quarks and antiquarks in order to render the discrete R symmetry anomaly free,

TABLE II. Summary of the possible charge assignments in our model assuming that the extra quarks couple to the MSSM via the operator $\bar{P}\bar{Q}_i\mathbf{10}_jH_d$. If the extra quarks should instead couple to the MSSM via $\bar{Q}_i\mathbf{10}_jH_d$, the values given in Eqs. (43) and (44) must be used for the R and PQ charges of the fields Q_i and $\bar{Q}_i$. The $\mathbf{2}_L$ in the column indicating the $SU(5)$ representations denote $SU(2)_L$ doublets. All R charges are only defined up to the addition of integer multiples of N. The MSSM R charges can additionally be changed by acting on them with Z_5 transformations. $N \geq 3$; $n = 1, 2$; $k \geq 1$; ℓ; ℓ_P and ℓ_Q are all integers.

	$SU(5)$	P_M	Z_N^R	$U(1)_{\mathrm{PQ}}$
(q, u^c, e^c)	$\mathbf{10}$	$-$	$\frac{1}{5} + \ell\frac{N}{2}$	0
(d^c, ℓ)	$\mathbf{5}^*$	$-$	$-\frac{3}{5} + \ell\frac{N}{2}$	0
(n^c)	$\mathbf{1}$	$-$	$1 + \ell\frac{N}{2}$	0
H_u	$\mathbf{2}_L$	$+$	$2 - \frac{2}{5}$	0
H_d	$\mathbf{2}_L$	$+$	$2 + \frac{2}{5}$	0
Q	$\mathbf{5}$	$-$	$\frac{13}{5} + \frac{(n+1)6}{nk} - [\frac{\ell}{2} + \frac{\ell_P}{n} - \frac{(n+1)\ell_Q}{nk}]N$	$-n-1$
$\bar{Q}$	$\mathbf{5}^*$	$-$	$-\frac{3}{5} - \frac{6}{nk} + [\frac{\ell}{2} + \frac{\ell_P}{n} - \frac{\ell_Q}{nk}]N$	1
P	$\mathbf{1}$	$+$	$-\frac{6}{nk} + (k\ell_P - \ell_Q)\frac{N}{nk}$	1
$\bar{P}$	$\mathbf{1}$	$+$	$\frac{6}{nk} - (k\ell_P - \ell_Q)\frac{N}{nk}$	-1
X	$\mathbf{1}$	$+$	2	0
S	$\mathbf{1}$	$+$	-2	0

(iii) an additional singlet sector in order to provide masses to the new quarks and antiquarks, and (iv) a singlet field S in order to dynamically generate the MSSM μ term. The charges of all these fields are summarized in Table II.

III. PHENOMENOLOGICAL CONSTRAINTS

The MSSM extension presented in the previous section is subject to a variety of phenomenological constraints. As we have already seen in Sec. II C 5, the positive integer n can, for instance, only be 1 or 2, since otherwise the extra quarks would always have masses below the electroweak scale. Besides that, i.e. besides the lower bound on the masses of the new quarks, we also have to ensure (i) that, despite our extension of the MSSM particle content, the unification of the SM gauge coupling constants still occurs at the perturbative level, (ii) that operators explicitly breaking the PQ symmetry do not induce shifts in the QCD vacuum angle larger than 10^{-10} as well as (iii) that the axion decay constant takes a value within the experimentally allowed window, cf. Eq. (32). In the next two subsections, we will now discuss these constraints in turn and show how they allow us to single out the phenomenologically viable combinations of N, n and k along with corresponding upper and lower bounds on f_a.

A. Gauge coupling unification

The new quark and antiquark fields contribute to the beta functions of the SM gauge coupling constants and thus cause a change in the value g_{GUT} at which these coupling constants unify at high energies. The more extra quark pairs we add to the MSSM particle content, the higher g_{GUT} turns out to be, which provides us with a means to constrain the allowed number of extra quark pairs k from above. For given masses m_{Q_i} of the new quarks, we define the maximal viable number of extra quark pairs $k_{\max}$ such that

$$\begin{aligned} g_{\mathrm{GUT}}(m_{Q_i}, k = k_{\max}) &\leq \sqrt{4\pi}, \\ g_{\mathrm{GUT}}(m_{Q_i}, k = k_{\max} + 1) &> \sqrt{4\pi}, \\ k_{\max} &= k_{\max}(m_{Q_i}). \end{aligned} \tag{53}$$

In order to determine $k_{\max}$ in dependence of the heavy quark mass spectrum, we make the simplifying approximation that all new quark flavors have the same mass, $M_Q = m_{Q_i}$, where $M_Q = (\Lambda/\sqrt{2}/M_{\mathrm{Pl}})^n M_{\mathrm{Pl}}$, cf. Eq. (33). At the same time, we assume that all superparticles share a common soft SUSY breaking mass M_{SUSY} of 1 TeV. When solving the renormalization group equations of the SM gauge couplings for energy scales μ ranging from the Z boson mass $M_Z = 91.2$ GeV to the GUT scale $M_{\mathrm{GUT}} = 2 \times 10^{16}$ GeV, we then have to distinguish between two different scenarios:

(i) If $M_Q > M_{\mathrm{SUSY}}$, we use the SM one-loop beta functions for $M_Z \leq \mu < M_{\mathrm{SUSY}}$, the MSSM one-loop beta functions for $M_{\mathrm{SUSY}} \leq \mu < M_Q$ and the two-loop beta functions of the MSSM plus the extra quark multiplets in the Novikov-Shifman-Vainshtein-Zakharov (NSVZ) scheme [62] for $M_Q \leq \mu \leq M_{\mathrm{GUT}}$.

(ii) If $M_Q \leq M_{\mathrm{SUSY}}$, we use the SM one-loop beta functions for $M_Z \leq \mu < M_Q$, the one-loop beta functions of the standard model plus the extra fermionic quarks for $M_Q \leq \mu < M_{\mathrm{SUSY}}$ and the two-loop beta functions of the MSSM plus the extra quark multiplets, i.e. plus the extra fermionic *and scalar* quarks, in the NSVZ scheme for $M_{\rm SUSY} \le \mu \le M_{\rm GUT}$.

Given the solutions of the renormalization group equations, we are able to determine $k_{\max}$ as a function of M_Q according to Eq. (53). The relation between the PQ scale Λ and the axion decay constant f_a in Eq. (30) then provides us with $k_{\max}$ as a function of f_a. The result of our calculation is presented in Fig. 1, which displays $k_{\max}$ as a function of f_a for $n = 1$ and $n = 2$, respectively.

Moreover, we note that collider searches for heavy down-type quarks are capable of placing a lower bound $M_Q^{\min}$ on the quark mass scale M_Q. As M_Q decreases with f_a and k, cf. Eq. (33), this lower bound on M_Q readily translates into a lower bound $k_{\min}$ on k,

$$\begin{aligned} M_Q(f_a, n, k = k_{\min}) &\ge M_Q^{\min}, \\ M_Q(f_a, n, k = k_{\min} - 1) &< M_Q^{\min} \\ k_{\min} &= k_{\min}(f_a, n). \end{aligned} \tag{54}$$

Assuming that the new quarks primarily couple to the SM quarks of the third generation via the operator $\bar{Q}_i \mathbf{10}_j H_d$, such as in the model discussed in Ref. [63], the ATLAS experiment at the LHC has recently reported a lower bound of 590 GeV on the heavy quark mass scale [64]. In the following, we will adopt this value for $M_Q^{\min}$, although we remark that smaller values of M_Q might still be viable, if the new quarks should predominantly couple to the first or second generation of the SM quarks rather than to the third generation. Conversely, an even larger mass range could in principle be excluded using the present data, if the new quarks should couple to the MSSM via the operator $\bar{P}\bar{Q}_i \mathbf{10}_j H_d$ rather than via the operator $\bar{Q}_i \mathbf{10}_j H_d$. In this case, the new quarks would be long-lived, thereby leaving very distinct signatures in collider experiments. In this section, we, however, assume a coupling via the operator $\bar{Q}_i \mathbf{10}_j H_d$ and set $M_Q^{\min}$ to 590 GeV. Solving Eq. (54) for $k_{\min}$, we then find $k_{\min}$ as a function of f_a, cf. Fig. 1. For $n = 1$ and all values of f_a of interest, $k_{\min}$ is always 1. On the other hand, for $n = 2$ and $f_a \lesssim 3 \times 10^{10}$ GeV, the minimal possible number of quark pairs rapidly grows as we go to smaller and smaller values of f_a.

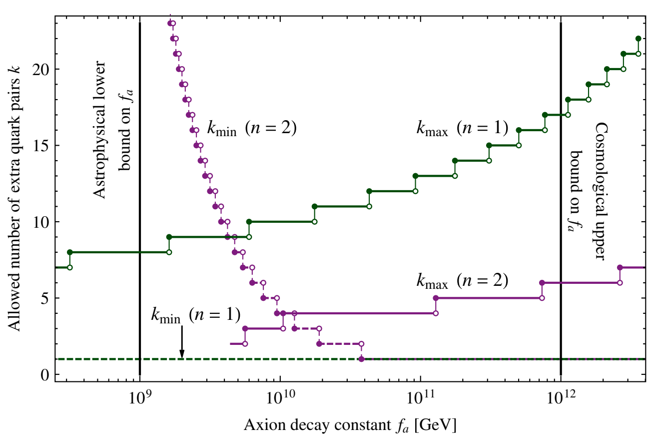

FIG. 1 (color online). Constraints on the number of extra quark pairs k for $n = 1$ and $n = 2$, respectively. The lower bounds are due to the experimental lower bound on the mass of new heavy down-type quarks; the upper bounds derive from the requirement of perturbative gauge coupling unification.

In summary, we conclude that, for each value of f_a, the requirements of perturbative gauge coupling unification as well as the lower bound on the mass of heavy down-type quarks provide us with a range of possible k values, cf. Eqs. (53) and (54),

$$k_{\min}(f_a, n) \le k \le k_{\max}(f_a, n). \tag{55}$$

Turning this statement around, we can say that, for given values of n and k, our two phenomenological constraints imply a lower bound on f_a,

$$f_a \ge \max\{f_a^{\min,\rm p}, f_a^{\min,\rm m}\}, \tag{56}$$

where $f_a^{\min,\rm p}$ and $f_a^{\min,\rm m}$ are defined such that

$$\begin{aligned} g_{\rm GUT}(f_a^{\min,\rm p}, n, k) &= \sqrt{4\pi}, \\ M_Q(f_a^{\min,\rm m}, n, k) &= M_Q^{\min}, \\ f_a^{\min,\rm i} &= f_a^{\min,\rm i}(k, n), \qquad i = p, m. \end{aligned} \tag{57}$$

In addition to that, we know from astrophysical and cosmological observations that the axion decay constant must not be smaller than $\mathcal{O}(10^9)$ GeV and not be larger than $\mathcal{O}(10^{12})$ GeV, cf. Eq. (32), so that we are eventually led to imposing the following lower and upper bounds on f_a:

$$\begin{aligned} f_a^{\min} &\le f_a \le 10^{12}\ \text{GeV}, \\ f_a^{\min} &= \max\{10^9\ \text{GeV}, f_a^{\min,\rm p}, f_a^{\min,\rm m}\}. \end{aligned} \tag{58}$$

B. Shifts in the QCD vacuum angle

Given the particle content and charge assignments of our model, it is easy to construct operators that explicitly break the PQ symmetry. Instead of an exact symmetry, the PQ symmetry therefore merely ends up being an approximate symmetry, which poses a threat to the PQ solution of the strong *CP* problem. Most PQ-breaking operators induce a shift in the VEV of the axion field, such that the $\bar{\theta}$ term in the QCD Lagrangian is no longer completely canceled. The magnitudes of these shifts in $\langle a \rangle$ differ from operator to operator and depend in addition on the axion decay constant f_a, the gravitino mass $m_{3/2}$ as well as on the scalar VEVs $\langle S \rangle$ and $\langle X \rangle$ in some cases. In this section, we will now investigate for which Z_N^R symmetries, which choices of n and k as well as which values of f_a the total shift in the axion VEV remains small enough, such that the shifted $\bar{\theta}$ angle does not exceed the upper experimental bound, $\bar{\theta} \lesssim 10^{-10}$.

1. PQ-breaking operators in the superpotential

All PQ-breaking operators in the superpotential inducing a shift in $\langle a\rangle$ are of the following form:[18]

$$W \supset \frac{CP^p\bar{P}^{\bar{p}}}{p!\bar{p}!h!s!x!M_{\rm Pl}^c}(H_uH_d)^h m_{3/2}^m S^s X^x,\quad c = p+\bar{p}+h+m+s+x-3,\qquad p\neq\bar{p}, \tag{59}$$

where C is a $\mathcal{O}(1)$ constant and where the powers of the various fields have to be chosen such that

$$r_P(p-\bar{p}) + 4h + 2m - 2s + 2x \overset{(N)}{=} 2. \tag{60}$$

Our intention behind explicitly dividing the operators in Eq. (59) by the factorials of the powers p, $\bar{p}$, h, s and x is to eventually obtain maximally conservative bounds on the axion decay constant. Fortunately, we do not have to consider all possible combinations of p, $\bar{p}$, h, m, s and x in the following. For instance, if some operator involving powers $(p,\bar{p})$ with $\max\{p,\bar{p}\} > \min\{p,\bar{p}\} > 0$ is allowed in the superpotential, the same operator with $(p,\bar{p})$ being either replaced by $(p-\bar{p},0)$ or $(0,\bar{p}-p)$ is also allowed. The shift in $\langle a\rangle$ induced by this second operator is then enhanced compared to the shift induced by the original operator by a factor of $\mathcal{O}(M_{\rm Pl}^q/\Lambda^q)$, where $q = 2\min\{p,\bar{p}\}$. Consequently, we are allowed to solely focus on PQ-breaking operators in the following that either involve some power of P *or* some power of $\bar{P}$. For a similar reason, we do not have to care about operators involving some power of H_uH_d. Given an operator with powers $h\geq 1$ and $m\geq 0$, we can always write down a similar operator in which (h,m) is replaced by $(0,m+2h)$. This is possible because $(H_uH_d)^h$ and $m_{3/2}^{2h}$ have the same R charge up to an integer multiple of N. Now assuming that $m_{3/2}^2$ is larger than $\langle H_uH_d\rangle = v_uv_d$, the operator with powers $(0,\ m+2h)$ always yields a larger shift in $\langle a\rangle$ than the operator with powers (h,m). Furthermore, the same game as with the fields P and $\bar{P}$ can also be played with $m_{3/2}$ and the fields S and X. Operators with powers (m,s,x) satisfying the relation $s > m+x \geq 0$ can always be traded for operators with powers $(0,\ s-m-x,\ 0)$. The shift in $\langle a\rangle$ due to these alternative operators is then enhanced compared to the shift due to the original operators by a factor of $\mathcal{O}(M_{\rm Pl}^{2(m+x)}/(m_{3/2}^m\langle S\rangle^{m+x}\langle X\rangle^x))$. In the end, we therefore only have to consider the following set of PQ-breaking operators:

$$W \supset \frac{CP^p}{p!M_{\rm Pl}^c}\left[\frac{1}{s!}S^s,\frac{1}{x!}m_{3/2}^m X^x\right]\Big|(P,p)\leftrightarrow(\bar{P},\bar{p}). \tag{61}$$

Each of the operators in Eq. (61) results in PQ-breaking terms in the scalar potential. Among these PQ-breaking contributions to the scalar potential, one class of terms derives from the F terms of the fields S and X,

$$F_S = \frac{C}{M_{\rm Pl}^c}\left[\frac{s}{p!s!}P^pS^{s-1},\frac{s}{\bar{p}!s!}\bar{P}^{\bar{p}}S^{s-1}\right] + F_S^0,$$
$$F_X = \frac{C}{M_{\rm Pl}^c}\left[\frac{x}{p!x!}P^p m_{3/2}^m X^{x-1},\frac{x}{\bar{p}!x!}\bar{P}^{\bar{p}} m_{3/2}^m X^{x-1}\right] + F_X^0, \tag{62}$$

where we have introduced F_S^0 and F_X^0 to denote the contributions to F_X and F_S deriving from PQ-invariant operators in the superpotential. Given the superpotential in Eq. (40) and taking into account the various supergravity effects induced by the constant term in the superpotential, $W_0 = m_{3/2}M_{\rm Pl}$, we are able to estimate of what order of magnitude we expect F_S^0 and F_X^0 to be,

$$F_S^0 = \mathcal{O}(m_{3/2}^2, v_uv_d, m_{3/2}\langle X\rangle, \langle X^2\rangle,\ldots),\quad F_X^0 = \mathcal{O}(m_{3/2}^2, m_{3/2}\langle S\rangle, \langle XS\rangle,\ldots), \tag{63}$$

with the dots denoting further contributions to F_S^0 and F_X^0 that only arise in the case of certain Z_N^R symmetries. The VEVs of the fields S and X are both of the order of the gravitino mass, such that the leading contributions to F_S^0 and F_X^0 can eventually be estimated as

$$F_S^0 = \mathcal{O}(m_{3/2}^2),\qquad F_X^0 = \mathcal{O}(m_{3/2}^2). \tag{64}$$

The mixing between F_S^0 and F_X^0 and the PQ-breaking contributions to F_S and F_X in Eq. (63) then gives rise to the following PQ-breaking terms in the scalar potential:

$$V \supset m_{3/2}^2\frac{CP^p}{p!M_{\rm Pl}^c}\left[\frac{s}{s!}S^{s-1},\frac{x}{x!}m_{3/2}^mX^{x-1}\right] + \text{H.c.}\Big|(P,p)\leftrightarrow(\bar{P},\bar{p}). \tag{65}$$

A second important class of PQ-breaking terms in the scalar potential are the A terms which derive from the mixing between the operators in Eq. (61) and the VEV of the superpotential W_0,

$$V \supset \frac{W_0}{M_{\rm Pl}^2}\frac{CP^p}{p!M_{\rm Pl}^c}\left[\frac{(p+s-3)}{s!}S^s,\frac{(p+x-3)}{x!}m_{3/2}^mX^x\right] + \text{H.c.}\Big|(P,p)\leftrightarrow(\bar{P},\bar{p}). \tag{66}$$

For a given operator in the superpotential with powers (p,s) or $(\bar{p},s)$, the largest PQ-breaking term in the scalar potential hence corresponds to

$$V \supset m_{3/2}\frac{CP^p}{p!s!M_{\rm Pl}^c}\max\{sm_{3/2},|p+s-3|S\}S^{s-1} + \text{H.c.}\Big|(P,p)\leftrightarrow(\bar{P},\bar{p}). \tag{67}$$

[18] PQ-breaking operators that do not involve any power of P or $\bar{P}$ (for instance, Q^5 or $\bar{Q}^5$) do not induce a shift in the axion VEV and are therefore irrelevant for our purposes.

Similarly, the largest term induced by an operator with powers (m, p, x) or $(m, \bar{p}, x)$ is given by[19]

$$V \supset m_{3/2} \frac{CP^p}{p!x!M_{\rm Pl}^c} \max\{x m_{3/2}, |p + x - 3|X\} m_{3/2}^m X^{x-1} + \text{H.c.} |(P, p) \leftrightarrow (\bar{P}, \bar{p}). \quad (68)$$

Next, we replace all scalar fields in these two operators by their VEVs,

$$P \to \frac{\Lambda}{\sqrt{2}} \exp\left(i\frac{a}{\sqrt{2}\Lambda}\right), \qquad \bar{P} \to \frac{\Lambda}{\sqrt{2}} \exp\left(-i\frac{a}{\sqrt{2}\Lambda}\right), \qquad S \to \langle S\rangle, \qquad X \to \langle X\rangle. \quad (69)$$

This provides us with contributions to the axion potential all of which are of the following form:

$$\Delta V_a = \frac{1}{2} M^4 \left[\exp\left(i\frac{pa}{\sqrt{2}\Lambda}\right) + \text{H.c.}\right] = M^4 \cos\left(p\frac{a}{\sqrt{2}\Lambda}\right), \quad (70)$$

where, for the terms in the scalar potential in Eqs. (67) and (68), the mass scale M is respectively to be identified as[20]

$$\begin{aligned} P^p S^s &: M^4 \to \frac{2Cm_{3/2}}{p!s!M_{\rm Pl}^c}\left(\frac{\Lambda}{\sqrt{2}}\right)^p M_S \langle S\rangle^{s-1}, \\ & M_S = \max\{s m_{3/2}, |p+s-3|\langle S\rangle\}, \\ P^p m_{3/2}^m X^x &: M^4 \to \frac{2Cm_{3/2}}{p!x!M_{\rm Pl}^c}\left(\frac{\Lambda}{\sqrt{2}}\right)^p M_X m_{3/2}^m \langle X\rangle^{s-1}, \\ & M_X = \max\{x m_{3/2}, |p+x-3|\langle X\rangle\}. \end{aligned} \quad (71)$$

2. *PQ-breaking operators in the Kähler potential and the effective potential*

Next to the PQ-breaking operators in the superpotential, we also have to take into account the PQ-breaking contributions to the Kähler potential K. It is, however, easy to show that the PQ-breaking terms in the scalar potential induced by the Kähler potential can at most be as large as the terms induced by the superpotential. Given some PQ-breaking term $K_{\not{PQ}} \subset K$, its largest contribution to the scalar potential is given by

$$V \supset \frac{C'}{M_{\rm Pl}^2} |W_0|^2 K_{\not{PQ}} = C' m_{3/2}^2 K_{\not{PQ}}, \qquad C' \sim \mathcal{O}(1). \quad (72)$$

The operator $K_{\not{PQ}}$ is either holomorphic from the outset or it is accompanied by a holomorphic term in the Kähler potential $K_{\not{PQ}}^l$ that follows from $K_{\not{PQ}}$ by performing the following replacements:

$$P^\dagger \to \bar{P}, \qquad \bar{P}^\dagger \to P, \qquad S^\dagger \to X, \qquad X^\dagger \to S, \qquad (H_u H_d)^\dagger \to S^2. \quad (73)$$

Furthermore, we know that, in order to be consistent with the Z_N^R symmetry, the R charge of $K_{\not{PQ}}$ must be zero. As the gravitino mass carries R charge 2, the holomorphicity of $K_{\not{PQ}}^{(l)}$ in combination with its vanishing R charge thus directly implies that $m_{3/2} K_{\not{PQ}}^{(l)}$ is one of the allowed operators in the superpotential. The A term deriving from $m_{3/2} K_{\not{PQ}}^{(l)}$ is then exactly of the same order of magnitude as the term in the scalar potential induced by $K_{\not{PQ}}$, cf Eq. (66). We therefore do not have to take care of the PQ-breaking terms in the Kähler potential explicitly. By studying the effects on the axion VEV related to the PQ-breaking operators in the superpotential, we automatically cover all relevant effects on the axion VEV related to the Kähler potential.

So far, we have only discussed PQ-breaking terms in the tree-level scalar potential. Below the heavy quark mass threshold, interactions at the loop level give rise to further PQ-breaking terms in the *effective* scalar potential. These higher-dimensional terms are then no longer solely suppressed by the Planck scale, but partly also by the heavy quark mass scale M_Q,

$$V_{\rm eff} \supset \frac{1}{M_{\rm Pl}^c M_Q^d} \frac{CP^p}{p!s!x!} m_{3/2}^m S^s X^x, \quad c + d = p + m + s + x - 4, \quad d > 0 |(P, p) \leftrightarrow (\bar{P}, \bar{p}), \quad (74)$$

where the coupling constant C is in general now also field dependent. We might therefore worry that some of these effective operators could yield larger shifts in the axion VEV than the actual tree-level operators that we have considered up to now. By imposing the requirement that the radiatively induced terms in the scalar potential must vanish in the limit $M_Q \to 0$,

$$M_Q \to 0: V_{\rm eff} \supset \frac{1}{M_{\rm Pl}^c M_Q^d} \frac{CP^p}{p!s!x!} m_{3/2}^m S^s X^x \to 0, \quad |(P, p) \leftrightarrow (\bar{P}, \bar{p}), \quad (75)$$

one can however show that the factor M_Q^{-d} in Eq. (74) is always canceled by factors contained in the coupling constant C, such that all of the effective terms can eventually be rewritten as

$$V_{\rm eff} \supset \frac{1}{M_{\rm Pl}^{c'}} \frac{C' P^{p'}}{p'!s'!x'!} m_{3/2}^{m'} S^{s'} X^{x'}, \quad c' = p' + m' + s' + x' - 4, \qquad C' \in \mathbb{R}, \quad |(P, p') \leftrightarrow (\bar{P}, \bar{p}'). \quad (76)$$

The radiative corrections in the effective potential are hence *not* enhanced with respect to the terms in the

[19] Note that, in Eqs. (67) and (68), we have implicitly absorbed the sign of $(p + s - 3)$ and $(p + x - 3)$ in C.

[20] The expressions for M^4 corresponding to the operators $\bar{P}^{\bar{p}} S^s$ and $\bar{P}^{\bar{p}} m_{3/2}^m X^x$ look exactly the same.

tree-level scalar potential. We conclude that, for our purposes, it will suffice to only consider the PQ-breaking terms in the scalar potential induced by the superpotential. A separate treatment of Kähler-induced effects or radiative corrections is not necessary.

3. Upper bounds on the axion decay constant

The ΔV_a terms in the scalar potential, cf. Eq. (70), disturb the effective QCD instanton-induced potential $V_a^{\rm eff}$, cf. Eq. (31), such that the axion potential is no longer minimized by $f_a\bar\theta$,

$$\left.\frac{d(V_a+\Delta V_a)}{da}\right|_{a=\langle a\rangle}=0,\qquad \langle a\rangle = f_a(\bar\theta+\Delta\bar\theta). \tag{77}$$

This shift in the axion VEV directly translates into a nonzero value $\Delta\bar\theta$ of the QCD vacuum angle. Making use of our results for $V_a^{\rm eff}$ and ΔV_a in Eqs. (31) and (70), we obtain for $\Delta\bar\theta$

$$\begin{aligned}\Delta\bar\theta &= \Delta\bar\theta_0 \sin\left(\frac{p}{|A_{\rm PQ}|}\bar\theta\right)+\mathcal{O}((\Delta\bar\theta_0)^2),\\ \Delta\bar\theta_0 &= \frac{p}{|A_{\rm PQ}|}\frac{M^4}{\Lambda_{\rm QCD}^4}.\end{aligned} \tag{78}$$

According to the experimental upper bound on the QCD vacuum angle, $\Delta\bar\theta_0$ must not be larger than 10^{-10},

$$\Delta\bar\theta_0 \le \Delta\bar\theta_0^{\rm max} = 10^{-10},\qquad M^4 \le \frac{|A_{\rm PQ}|}{p}\Delta\bar\theta_0^{\rm max}\Lambda_{\rm QCD}^4, \tag{79}$$

which results in an upper bound on the mass scale M. Combining this constraint with our expressions for M in Eq. (71), we are able to derive an upper bound on the axion decay constant f_a for each of the PQ-breaking operators in the superpotential,

$$\begin{aligned}&P^pS^s\colon f_a^{\max,S}\\ &\quad=\left[2^{p-1}p!s!\frac{\Delta\bar\theta_0^{\rm max}}{C}\frac{|A_{\rm PQ}|^{1-p}}{p}\frac{\Lambda_{\rm QCD}^4 M_{\rm Pl}^c}{M_S\langle S\rangle^{s-1}m_{3/2}}\right]^{1/p},\\ &P^p m_{3/2}^m X^x\colon f_a^{\max,X}\\ &\quad=\left[2^{p-1}p!x!\frac{\Delta\bar\theta_0^{\rm max}}{C}\frac{|A_{\rm PQ}|^{1-p}}{p}\frac{\Lambda_{\rm QCD}^4 M_{\rm Pl}^c}{M_X\langle X\rangle^{x-1}m_{3/2}^{m+1}}\right]^{1/p},\end{aligned} \tag{80}$$

with the bounds corresponding to $\bar P^{\bar p}S^s$ and $\bar P^{\bar p}m_{3/2}^m X^x$ being of exactly the same form.

For given values of N, n, k and r_P, a multitude of different PQ-breaking operators might be allowed in the superpotential, all of which imply an upper limit on f_a. Let us denote the most restrictive among these upper limits by $f_a^{\max,\bar\theta}$,

$$f_a^{\max,\bar\theta} = \min\{\text{all } f_a^{\max,S}, \text{all } f_a^{\max,X}\}. \tag{81}$$

Together with the constraints on f_a in Eq. (58), we thus find the following total lower and upper limits on the axion decay constant f_a,

$$\begin{aligned}f_a^{\rm min} &\le f_a \le f_a^{\rm max},\\ f_a^{\rm min} &= \max\{10^9\ \text{GeV}, f_a^{\rm min,p}, f_a^{\rm min,m}\},\\ f_a^{\rm max} &= \min\{10^{12}\ \text{GeV}, f_a^{\max,\bar\theta}\}.\end{aligned} \tag{82}$$

By virtue of this result, we are now able to identify the phenomenologically viable combinations of N, n, k and r_P. The corresponding criterion is nothing but the requirement that there has to be an allowed window of possible values for f_a,

$$f_a^{\rm min} < f_a^{\rm max} \Rightarrow (N,n,k,r_p)\text{viable}. \tag{83}$$

To determine the allowed combinations of N, n, k and r_P, we compute $f_a^{\rm min}$ and $f_a^{\rm max}$ for

$$\begin{aligned}N &= 3,4,\ldots,12;\qquad n=1,2;\\ k &= 1,2,\ldots,k_{\max}(10^{12}\ \text{GeV},n);\\ r_P &= r_P(N,n,k,\ell_Q,\ell_P),\end{aligned} \tag{84}$$

where $k_{\max}(10^{12}\ \text{GeV},1)=17$ and $k_{\max}(10^{12}\ \text{GeV},2)=6$ and where r_P as a function of N, n, k, ℓ_Q and ℓ_P is given in Eq. (16),[21] and check whether or not the criterion in Eq. (83) is fulfilled. In doing so, we set all dimensionless coupling constants to 1 and use a common value of 1 TeV for the gravitino mass and the scalar VEVs,

$$m_{3/2}=1\ \text{TeV},\quad \langle S\rangle = 1\ \text{TeV},\quad \langle X\rangle = 1\ \text{TeV}. \tag{85}$$

Larger values of $m_{3/2}$, $\langle S\rangle$ and $\langle X\rangle$ would lead to more stringent bounds on f_a, which means that the bounds that we obtain should be regarded as conservative. For the nonperturbative scale of QCD, we employ the $\overline{\rm MS}$ value above the bottom-quark mass threshold, $\Lambda_{\rm QCD}\simeq 213$ MeV [65].

4. Phenomenologically viable scenarios

Restricting ourselves to the parameter values specified in Eqs. (84) and (85), we find in total 1068 viable combinations of N, n, k and r_P, where 936 of these solutions belong to the case $n=1$ and 132 solutions to the case $n=2$. In Tables III and IV, we indicate how many solutions we respectively obtain for the individual values of k and N under study. In Table V, we list all viable combinations of N, n, k and r_P for all k values up to $k=6$. In summary, we conclude that our minimal extension of the MSSM apparently gives rise to a large landscape of viable scenarios. It is in particular surprising and intriguing that the order N of the Z_N^R symmetry can take any value, as long as the number of extra quark pairs k is chosen

[21] In total, we thus scan 1950 different combinations of N, n, k and r_P. Out of these combinations, 1530 belong to the case $n=1$, whereas 430 belong to the case $n=2$.

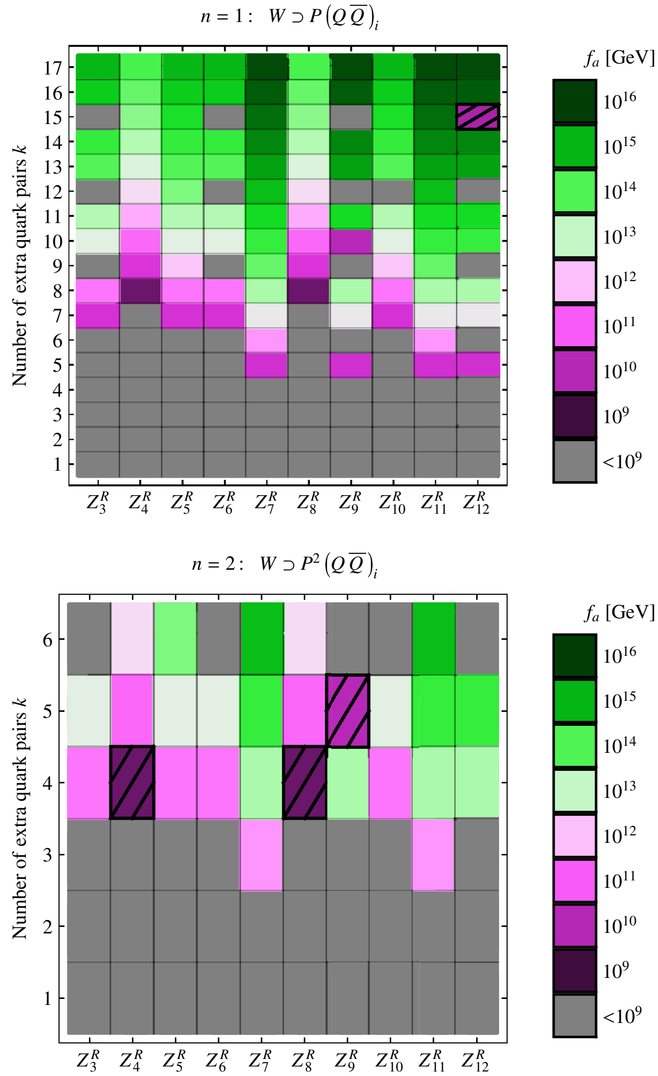

FIG. 2 (color online). Upper bounds $f_a^{\max,0}$ on the axion decay constant f_a according to the requirement that the shift in the QCD vacuum angle $\bar{\theta}$ induced by PQ-breaking operators not be larger than 10^{-10}, cf. Eqs. (81) and (86). Both plots are based on $m_{3/2} = 1$ TeV, $\langle S \rangle = \mu/g_H = 1$ TeV and $\langle X \rangle = 1$ TeV. At the same time, all dimensionless coupling constants have been set to 1. The black diagonal lines indicate that 10^9 GeV $\leq f_a^{\max,0} \leq f_a^{\min}$, cf. Eq. (58).

appropriately. A comprehensive phenomenological study of this landscape of possible solutions is beyond the scope of this paper. In the following, we shall thus restrict ourselves to a few interesting observations, illustrating what kind of questions one might be able to answer based on the full numerical data describing the landscape.

We observe for instance that, for all possible combinations of N, n and k, there exists either no viable r_P value at all or at least two different values. It is therefore interesting to ask which of the various possible r_P values for given N, n and k yields the least stringent upper bound on f_a,

$$f_a^{\max,0}(N, n, k) = \max_{r_P} \{ f_a^{\max,\bar{\theta}}(N, n, k, r_P) \}. \tag{86}$$

This maximal upper bound can then be regarded as the most conservative constraint on f_a for the respective combinations of N, n and k. The two panels of Fig. 2 present $f_a^{\max,0}$ as a function of N and k for $n = 1$ and $n = 2$, respectively. Apart from four exceptions, $f_a^{\max,0}$ interestingly always exceeds $f_a^{\min}$ as long as it is larger than 10^9 GeV,

$$(N, n, k) \neq (4, 2, 4), (8, 2, 4), (9, 2, 5), (12, 1, 15): f_a^{\max,0} \geq 10^9 \text{ GeV} \Rightarrow f_a^{\max,0} > f_a^{\min}. \tag{87}$$

Only for $(N, n, k) = (4, 2, 4), (8, 2, 4), (9, 2, 5), (12, 1, 15)$, $f_a^{\max,0}$ is smaller than $f_a^{\min}$, which renders these four cases phenomenologically unviable. This is indicated in Fig. 2 by the diagonal black lines crossing out the respective squares.

A further question that one might be interested in is which of the viable scenarios are compatible with the assumption of axion dark matter. In case inflation takes place after the spontaneous breaking of the PQ symmetry, the only contribution to the relic axion density stems from the vacuum realignment of the zero-momentum mode of the axion field during the QCD phase transition [47]. The present value of the axion density parameter $\Omega_a^0 h^2$ can then be estimated as [48]

$$\Omega_a^0 h^2 \sim 0.50 \left(\frac{\bar{\theta}_i^2}{\pi^2/3} \right) \left(\frac{f_a}{10^{12}\text{ GeV}} \right)^{7/6}, \tag{88}$$

where $\bar{\theta}_i \in (-\pi, \pi]$ denotes the initial misalignment angle of the axion field before the onset of the QCD phase transition, $\bar{\theta}_i = a(t_i)/f_a$. In the derivation of Eq. (88), it is assumed that $\bar{\theta}_i$ is constant across the entire observable universe as well as that the axion relic density is not diluted after its generation by some form of late-time entropy production. If all possible values of $\bar{\theta}_i$ are equally likely, we expect that $\langle \bar{\theta}_i^2 \rangle = \pi^2/3$. By comparing the expression for $\Omega_a^0 h^2$ in Eq. (88) with the density parameter of cold dark matter (CDM), which has recently been determined

TABLE III. Numbers of viable scenarios for individual values of k, including as well all scenarios with a Z_3^R or a Z_6^R symmetry. For $k \leq 6$, the respective numbers of solutions for the two cases $n = 1$ and $n = 2$ are indicated in the format $(\#|_{n=1}, \#|_{n=2})$.

k	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
#	(0, 4)	(0, 38)	(16, 54)	(4, 36)	49	54	36	62	101	36	120	110	56	132	160

TABLE IV. Numbers of viable scenarios for individual values of N in the format $(\#|_{n=1}, \#|_{n=2})$.

N	3	4	5	6	7	8	9	10
#	(76, 8)	(104, 16)	(91, 13)	(76, 8)	(103, 14)	(104, 16)	(90, 6)	(91, 13)
N	11	12						
#	(109,22)	(92,16)						

TABLE V. Viable values of r_P in units of $1/(nk)$ and in dependence of N, n and k for all k values up to $k = 6$. We also include the r_P values for $(N, n, k) = (4, 2, 4)$, (8, 2, 4), (9, 2, 5), which are actually phenomenologically unviable if we believe in the perturbative unification of the gauge coupling constants. For these combinations of N, n and k, we namely find 10^9 GeV $\leq f_a^{\max,0} \leq f_a^{\min}$, cf. Eq. (87) and Fig. 2.

(n, k)	Z_3^R	Z_4^R	Z_5^R	Z_6^R
(2, 4)	{3, 9, 15, 21}	{2, 6, 10, 14, 18, 22, 26, 30}	{9, 19, 29, 39}	{6, 18, 30, 42}
(2, 5)	{3, 9, 21, 27}	{2, 6, 14, 18, 22, 26, 34, 38}	{9, 19, 29, 39, 49}	{6, 18, 42, 54}
(2, 6)		{2, 10, 14, 22, 26, 34, 38, 46}	{19, 29, 49, 59}	
(n, k)	Z_7^R	Z_8^R	Z_9^R	Z_{10}^R
(1, 5)	{1, 8, 22, 29}		{3, 12, 21, 39}	
(1, 6)	{1, 29}			
(2, 3)	{1, 29}			
(2, 4)	{1, 15, 29, 43}	{2, 10, 18, 26, 34, 42, 50, 58}	{3, 21, 39, 57}	{14, 34, 54, 74}
(2, 5)	{1, 29, 43, 57}	{2, 18, 26, 34, 42, 58, 66, 74}	{3, 12, 21, 39, 48, 57, 66, 84}	{14, 34, 54, 74, 94}
(2, 6)	{1, 29, 43, 71}	{2, 10, 26, 34, 50, 58, 74, 82}	{3, 57}	{14, 34, 74, 94}
(n, k)	Z_{11}^R		Z_{12}^R	
(1, 5)	{16, 27, 38, 49}		{6, 18, 42, 54}	
(1, 6)	{5, 49}			
(2, 3)	{5, 49}			
(2, 4))	{5, 27, 38, 49, 71, 82}		{6, 18, 30, 42, 54, 66, 78, 90}	
(2, 5)	{16, 27, 38, 49, 71, 82, 93, 104}		{6, 18, 42, 54, 66, 78, 102, 114}	
(2, 6)	{5, 38, 49, 71, 82, 115}			

very precisely by the PLANCK satellite, $\Omega^0_{\rm CDM} h^2 \simeq 0.1199$ [66], we see that, for an axion decay constant f_a of $\mathcal{O}(10^{12})$ GeV, cold axions may completely account for the relic density of dark matter. For $f_a \gtrsim 10^{12}$ GeV, the axion density exceeds the measured abundance of dark matter, which is nothing but the cosmological upper bound on f_a which we introduced in Sec. II C 3.

Setting f_a to 10^{12} GeV, we can now ask how large a QCD vacuum angle $\bar\theta$ we expect to be induced by the PQ-breaking operators in the respective viable scenarios. In the case of those scenarios for which we found that $f_a^{\min} < f_a^{\max,\bar\theta} < 10^{12}$ GeV, the induced QCD vacuum angle, of course, turns out to be larger than 10^{-10}, i.e. only scenarios in which $f_a^{\min} < 10^{12}$ GeV $< f_a^{\max,\bar\theta}$ are compatible with the requirement of axion dark matter. In total, we find 861 of such scenarios. Among these, 763 belong to the case $n = 1$ and 98 to the case $n = 2$. Analogously to the upper bounds on the axion decay constant, for which we introduced $f_a^{\max,0}$, cf. Eq. (86), we would also like to know which R charge r_P for given N, n and k yields the smallest QCD vacuum angle,

$$f_a = 10^{12}\ \text{GeV:}\ \bar\theta^0(N, n, k) = \min_{r_P}\{\bar\theta(N, n, k, r_P)\},\qquad \bar\theta(N, n, k, r_P) = \max\{\text{all}\ \Delta\bar\theta_0\}, \tag{89}$$

where the shifts in the QCD vacuum angle $\Delta\bar\theta_0$ are to be calculated according to Eq. (78). The angles $\bar\theta^0$ then represent the most conservative lower bounds on $\bar\theta$ for the

TABLE VI. All combinations of N, n and k which, in the case of axion dark matter, i.e. for $f_a = 10^{12}$ GeV, result in a lower bound $\bar\theta^0$ on the theta angle between 10^{-15} and 10^{-10}, cf. Eq. (89).

(n, N, k)	(1, 4, 12)	(1, 5, 9)	(1, 7, 7)	(1, 8, 12)	(1, 9, 7)
$10^{10}\bar\theta^0$	1×10^{-4}	3×10^{-1}	2×10^{-4}	1×10^{-4}	2×10^{-4}
(n, N, k)	(1, 10, 9)	(1, 11, 7)	(1, 12, 7)	(2, 4, 6)	(2, 8, 6)
$10^{10}\bar\theta^0$	3×10^{-1}	2×10^{-4}	2×10^{-4}	1×10^{-4}	1×10^{-4}

respective combinations of N, n and k. For 96 combinations of N, n and k, splitting into 79 combinations corresponding to $n = 1$ and 17 combinations corresponding to $n = 2$, the angle $\bar{\theta}^0$ does not exceed 10^{-10}. But only for a few of these solutions, $\bar{\theta}^0$ falls into a range that might be experimentally accessible in the not so far future. For instance, only for ten solutions we find values of $\bar{\theta}^0$ between 10^{-15} and 10^{-10}, cf. Table VI. Provided that dark matter is really composed out of axions, these ten scenarios can then be tested in experiments aiming at measuring a nonzero value of the QCD vacuum angle.

IV. CONCLUSIONS AND DISCUSSION

The PQ solution of the strong *CP* problem requires an anomalous global Abelian symmetry, $U(1)_{\rm PQ}$. On the other hand, any global symmetry is expected to be explicitly broken by quantum gravity effects. In this paper, we have pointed out that imposing a gauged and discrete R symmetry, Z_N^R, one is able to retain a PQ symmetry of high enough quality as an approximate and accidental symmetry in the low-energy effective theory. The reasoning behind the construction of our model was the following: In order to render the Z_N^R symmetry anomaly free, it is, in general, necessary to extend the particle content of the MSSM by new matter multiplets. Except for some special cases, these new particles are *a priori* massless, which calls for a further extension of the spectrum by an extra singlet sector that is capable of generating masses for the new particles. As we were able to show, the new matter and singlet sectors then exhibit several global Abelian symmetries, a linear combination of which can be identified as the PQ symmetry. In addition to that, for all Z_N^R symmetries apart from Z_4^R, we supplemented the MSSM Higgs sector by an additional chiral singlet S, so as to allow for a dynamical generation of the MSSM μ term.

The presence of the extra matter multiplets and the singlet S in our model entail a potentially rich phenomenology in collider experiments. Depending on the nature of the coupling between the extra matter and singlet sectors, the new particles might either have masses in the TeV or multi-TeV range or they might be very heavy, with their masses being close to the scale of PQ symmetry breaking. In the former case, our model is being directly probed by searches for heavy vectorlike quarks at the LHC. At the same time, the phenomenology of the Higgs sector of our model is similar to the one in the PQ-NMSSM or in the nMSSM. Next to the four ordinary neutralinos, we expect a fifth, very light neutralino, the singlino $\tilde{S}$, which receives its mass only from mixing with the neutral Higgsinos. The singlino may play an important role in the decay of the standard model-like Higgs boson and contribute to the relic density of dark matter.

In order to single out the phenomenologically viable variants of our model, we imposed four phenomenological constraints. We required (i) the masses of the new quarks to exceed the lower experimental bound on the mass of heavy down-type quarks, $M_Q^{\rm min} = 590$ GeV, (ii) the unification of the standard model gauge couplings to still occur at the perturbative level, $g_{\rm GUT} \lesssim \sqrt{4\pi}$, (iii) the shift in the QCD vacuum angle induced by higher-dimensional PQ-breaking operators to remain below the upper experimental bound, $\bar{\theta} < 10^{-10}$, as well as (iv) the axion decay constant to take a value within the experimentally allowed window, 10^9 GeV $\lesssim f_a \lesssim 10^{12}$ GeV. To our surprise, we found a large landscape of possible scenarios, all compatible with these four constraints. In particular, we showed that, for an appropriately chosen number of extra matter multiplets, the order N of the Z_N^R symmetry can take any integer value larger than 2. Besides that, for each viable scenario, we derived an upper bound on the axion decay constant based on the requirement that QCD vacuum angle must not exceed 10^{-10}. In many cases, these upper bounds turned out to be larger than 10^{12} GeV, thereby rendering the corresponding scenarios compatible with the assumption of axion dark matter. For these scenarios, we then estimated the expected value of the QCD vacuum angle, in case dark matter should really be composed out of axions. A measurement of a nonzero theta angle in combination with a confirmation of axion dark matter would therefore allow for a highly nontrivial experimental test of our model.

We also emphasized the virtues of the special case of a Z_4^R symmetry. In the case of a Z_4^R symmetry, the MSSM μ term can be easily generated in the course of spontaneous R symmetry breaking, such that there is no need to introduce an additional chiral singlet. As a consequence of that, the scalar potential does not exhibit a flat direction in the supersymmetric limit, so that we do not have to rely on the soft SUSY breaking masses to stabilize the PQ-breaking vacuum, as is the case for all other Z_N^R symmetries. Moreover, a Z_4^R is the only discrete R symmetry that allows for MSSM R charges consistent with the assumption of $SO(10)$ unification, cf. Appendix A.

Finally, we mention that our study needs be extended into several directions. First of all, it is necessary to embed our extension of the MSSM into a grander model that explains the origin of the Z_N^R symmetry and provides some guidance as to the number of extra matter multiplets and the exact nature of their couplings. Likewise, it is important to further explore the cosmological implications of our model. One open question, for instance, is the generation and composition of dark matter in terms of axions, saxions, neutralinos and/or gravitinos in dependence of our model parameters. Besides that, it would be interesting to make contact between our model and R-invariant scenarios of inflation [67]. Altogether, our model promises to give rise to a rich phenomenology that can be probed at colliders and in astrophysical and cosmological observations. Future experiments will thus be able to test the intriguing possibility that the PQ symmetry, required for the PQ solution of the strong *CP* problem, is indeed an accidental consequence of a gauged and discrete R symmetry.

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APPENDIX A: POSSIBLE *R* CHARGES OF THE MSSM FIELDS

In Sec. II B, we derive five constraints on $r_{\mathbf{10}}$, $r_{\mathbf{5}^*}$, $r_{\mathbf{1}}$, r_{H_u} and r_{H_d}, the R charges of the MSSM matter and Higgs multiplets, cf. Eqs. (4), (6), and (7). As these conditions only hold up to the addition of integer multiples of N, they do not suffice to fix the values of the MSSM R charges uniquely. In this Appendix, we now show that, for each value of N, there exist exactly ten different R charge assignments for the MSSM fields that comply with all constraints. Moreover, we also discuss under which circumstances these solutions are equivalent to each other.

1. *R* charge assignments consistent with all constraints

To begin with, let us rewrite the five conditions in Eqs. (4), (6), and (7) as follows:

$$\begin{aligned} r_{H_u} + r_{H_d} &= 4 + \ell_1 N, \\ 2r_{\mathbf{10}} + r_{H_u} &= 2 + \ell_2 N, \\ r_{\mathbf{5}^*} + r_{\mathbf{10}} + r_{H_d} &= 2 + \ell_3 N, \\ r_{\mathbf{5}*} + r_{\mathbf{1}} + r_{H_u} &= 2 + \ell_4 N, \\ 2r_{\mathbf{1}} &= 2 + \ell_5 N, \end{aligned} \tag{A1}$$

where we have made use of the relation in Eq. (5) and with $\ell_i \in \mathbb{Z}$ for all $i = 1, \ldots, 5$. Solving this system of linear equations for the R charges $\boldsymbol{r} = (r_{\mathbf{10}}, r_{\mathbf{5}^*}, r_{\mathbf{1}}, r_{H_u}, r_{H_d})^T$ yields

$$\begin{pmatrix} r_{\mathbf{10}} \\ r_{\mathbf{5}^*} \\ r_{\mathbf{1}} \\ r_{H_u} \\ r_{H_d} \end{pmatrix} \overset{(N)}{=} \begin{pmatrix} \frac{1}{5} \\ -\frac{3}{5} \\ 1 \\ 2-\frac{2}{5} \\ 2+\frac{2}{5} \end{pmatrix} + \tilde{\ell}\frac{N}{10}\begin{pmatrix} 1 \\ -3 \\ 5 \\ -2 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 & 0 \\ 1 & -2 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \ell_1 \\ \ell_2 \\ \ell_3 \\ \ell_4 \\ \ell_5 \end{pmatrix} N, \tag{A2}$$

with $\tilde{\ell} = -2\ell_1 + 4\ell_2 + 2\ell_3 - 2\ell_4 + \ell_5 \in \mathbb{Z}$. As indicated by the $\overset{(N)}{=}$ symbol in Eq. (A2), all R charges are only defined modulo N. Thus, after picking explicit values for the ℓ_i, we always have to take all R charges modulo N, such that $0 \le r_i < N$ for all fields i. At the same time, the last summand on the right-hand side of Eq. (A2) does nothing but shifting the charges $r_{\mathbf{5}^*}$, $r_{\mathbf{1}}$, r_{H_u} and r_{H_d} by integer multiples of N. Its effect is hence always nullified by the modulo N operation, allowing us to omit it in the following. Furthermore, we observe that the entries of the second column vector on the right-hand side of Eq. (A2) correspond to the X charges of the MSSM multiplets.[22] We shall therefore denote this vector by $\boldsymbol{X}$, such that

$$\boldsymbol{r} \overset{(N)}{=} \boldsymbol{r}_0 + \tilde{\ell}\frac{N}{10}\boldsymbol{X}, \qquad \boldsymbol{r}_0 = \left(\frac{1}{5}, -\frac{3}{5}, 1, 2-\frac{2}{5}, 2+\frac{2}{5}\right)^T, \\ \boldsymbol{X} = (1, -3, 5, -2, 2)^T. \tag{A3}$$

This result illustrates that, for any given N, there are indeed ten different possible R charge assignments $\boldsymbol{r}$. Independently of the concrete value of N, the assignment $\boldsymbol{r}_0$ always represents a solution to the conditions in Eqs. (4), (6), and (7). All other solutions can be constructed from $\boldsymbol{r}_0$ by adding multiples of $\frac{N}{10}\boldsymbol{X}$ to it. Here, the fact that the R charges r_i are only defined modulo N implies that all R charge assignments corresponding to values of $\tilde{\ell}$ that differ from each other by integer multiples of 10 are equivalent to each other. The ten possible solutions for the MSSM R charges then follow from Eq. (A3) by setting $\tilde{\ell}$ to $\tilde{\ell} = 0, 1, 2, \ldots, 9$.

Among all viable R charge assignments that can be obtained from Eq. (A3), there are several which are particularly interesting. For instance, for $N = 4$, it is possible to assign R charges to the MSSM fields in such a way that they are consistent with the assumption of $SO(10)$ unification. In this case, the GUT gauge group contains $SO(10)$ as a subgroup, $G_{\rm GUT} \supset SO(10) \supset SU(5)$, and the MSSM matter and Higgs fields are unified in $SO(10)$ multiplets, such that $r_{\mathbf{10}} = r_{\mathbf{5}^*} = r_{\mathbf{1}}$ and $r_{H_u} = r_{H_d}$. For $N = 4$ and $\tilde{\ell} = 2, 7$, these two relations can indeed be realized,

$$\begin{aligned} N = 4:\ \tilde{\ell} = 2:\ \boldsymbol{r} &= (1, 1, 1, 0, 0), \\ \tilde{\ell} = 7:\ \boldsymbol{r} &= (3, 3, 3, 0, 0). \end{aligned} \tag{A4}$$

In the case of $N = 4$, the R charge of the superpotential is equivalent to -2. Hence, given any viable R charge assignment, reversing the signs of all R charges and applying the modulo N operation, so that all R charges lie again in the interval $[0, N)$, provides one with another viable R charge assignment. The two solutions for $\boldsymbol{r}$ in Eq. (A4) are related

[22] X denotes the charge corresponding to the Abelian symmetry $U(1)_X$, which is the subgroup of $U(1)_{B-L} \times U(1)_Y$ that commutes with $SU(5)$. In terms of $B - L$ and the weak hypercharge Y, it is given as $X = 5(B - L) - 4Y$.

to each other in just this way, implying that they are in fact equivalent. In Refs. [7,40], the discrete Z_4^R symmetry with R charges $\boldsymbol{r} = (1, 1, 1, 0, 0)$ has been discussed in more detail. Allowing for anomaly cancellation via the Green-Schwarz mechanism, this symmetry has in particular been identified as the *unique* discrete R symmetry of the MSSM that may be rendered anomaly free without introducing any new particles and which, at the same time, commutes with $SO(10)$ and forbids the μ term in the superpotential. Finally, we point out that the two R charge assignments in Eq. (A4) only feature integer-valued R charges. We mention in passing that, in fact, for each value of N that is not an integer multiple of 5 there is at least one viable R charge assignment that only involves integer-valued R charges. This is a direct consequence of our result for $\boldsymbol{r}$ in Eq. (A3) and the fact that all R charges in $\boldsymbol{r}_0$ are integer multiples of $\frac{1}{5}$.

2. Relationship between the different *R* charge assignments

The form of our result for $\boldsymbol{r}$ in Eq. (A3) reflects the symmetries of the MSSM superpotential that commute with $SU(5)$. Among these symmetries, there is in particular a Z_{10} subgroup of $U(1)_X$. To see this, notice that the MSSM superpotential without the Majorana mass term for the neutrino singlets $\mathbf{1}_i$ is invariant under $U(1)_X$ transformations. The Majorana mass term, however, carries X charge 10 and thus breaks the $U(1)_X$ symmetry to its Z_{10} subgroup. Our solutions for the MSSM R charges are therefore related to each other by Z_{10} transformations, which also explains why we have found exactly ten different solutions for each value of N. This result is independent of the question of whether or not we assume the $U(1)_X$ symmetry to be part of the gauge group above some high energy scale. We will address this question shortly, but before we do that, we remark that the Z_{10} subgroup of $U(1)_X$ is not the only symmetry of the MSSM superpotential that commutes with $SU(5)$. By definition, the center of $SU(5)$, a discrete Z_5 symmetry, also commutes with all $SU(5)$ elements. Under this Z_5 symmetry, the MSSM multiplets $\mathbf{10}_i$, $\mathbf{5}_i^*$, H_u and H_d carry charges 1, 2, 3 and 2, while all SM singlets have zero charge. At the same time, all SM singlets of our model transform trivially under the Z_5 subgroup of the Z_{10} contained in $U(1)_X$. The Z_5 center of $SU(5)$ is hence equivalent to this $U(1)_X$ subgroup,

$$SU(5) \supset Z_5 \cong Z_5 \subset Z_{10} \subset U(1)_X. \tag{A5}$$

Therefore, independently of whether $U(1)_X$ is gauged or not, the Z_5 subgroup of Z_{10} always has to be treated as a gauge symmetry, as it is also contained in $SU(5)$. Under the action of this gauged Z_5 symmetry, the R charge assignments in Eq. (A3) split into two equivalence classes of respectively five solutions. The R charge assignments corresponding to $\tilde{\ell} = 2, 4, 6, 8$ can all be generated by acting with Z_5 transformations on the R charge assignment corresponding to $\tilde{\ell} = 0$. Similarly, the R charge assignments corresponding to $\tilde{\ell} = 1, 3, 7, 9$ can all be generated by acting with Z_5 transformations on the R charge assignment corresponding to $\tilde{\ell} = 5$. All viable R charge assignments are hence physically equivalent to one of the following two solutions, cf. Eq. (8):

$$r_{\mathbf{10}} \overset{(N)}{=} \frac{1}{5} + \ell\frac{N}{2}, \qquad r_{\mathbf{5}^*} \overset{(N)}{=} -\frac{3}{5} + \ell\frac{N}{2}, \qquad r_{\mathbf{1}} \overset{(N)}{=} 1 + \ell\frac{N}{2},$$
$$r_{H_u} \overset{(N)}{=} 2 - \frac{2}{5}, \qquad r_{H_d} \overset{(N)}{=} 2 + \frac{2}{5}, \tag{A6}$$

where $\ell = 0, 1$. These two remaining R charge assignments are related to each other by transformations under the quotient group Z_{10}/Z_5, which is nothing but a simple Z_2 parity.

Whether the two solutions in Eq. (A6) are also equivalent to each other depends on the nature of this Z_2 parity. If $U(1)_X$ is part of the gauge group at high energies, its Z_{10} subgroup is a gauge symmetry at low energies. Dividing the center of $SU(5)$ out of this Z_{10}, we are then left with a gauged Z_2 parity, which can be identified as matter parity, $P_M = Z_{10}/Z_5$. The transformations relating the two solutions in Eq. (A6) to each other are then gauge transformations and both solutions end up being equivalent. On the other hand, if $U(1)_X$ is not gauged and matter parity is contained in the Z_N^R symmetry, $P_M \subset Z_N^R$, the Z_{10} subgroup of $U(1)_X$ is also only a global symmetry. The Z_2 parity transformations relating the two solutions in Eq. (A6) to each other are then global transformations, rendering these two R charge assignments physically inequivalent. In conclusion, we hence arrive at the following picture:

$$\begin{aligned} &P_M = Z_{10}/Z_5 \subset U(1)_X\text{: all ten solutions equivalent,}\\ &P_M \subset Z_N^R\text{: two equivalence classes containing respectively five solutions.}\end{aligned} \tag{A7}$$

3. *R* charges in a $U(1)_X$-invariant extension of the MSSM

If matter parity is a subgroup of the $U(1)_X$, our model as presented in Sec. II is not yet complete, as it still lacks an explanation for the spontaneous breaking of $U(1)_X$ at some high energy scale. In the last subsection of this Appendix, we shall thus illustrate by means of a minimal example how our model could be embedded into a $U(1)_X$-invariant extension of the MSSM.

The seesaw extension of the MSSM is not invariant under $U(1)_X$ transformations because of the lepton number-violating Majorana mass term in the superpotential $W_{\rm MSSM}$, cf. Eq. (1). Imagine, however, that this Majorana mass term derives from the Yukawa interaction of the neutrino singlets $\mathbf{1}_i$ with some chiral singlet Φ carrying $B-L$ charge -2 that acquires a nonvanishing VEV $\Lambda_{B-L}/\sqrt{2}$ at the GUT scale,

$$W_{\rm MSSM} \supset \frac{1}{2} M_i \mathbf{1}_i \mathbf{1}_i \to \frac{1}{\sqrt{2}} h_i^n \Phi \mathbf{1}_i \mathbf{1}_i + \lambda T\left(\frac{\Lambda_{B-L}^2}{2} - \Phi\bar{\Phi}\right). \tag{A8}$$

Here, T and $\bar{\Phi}$ are two further SM singlets with $B-L$ charges 0 and 2, respectively. The field T carries R charge $r_T = 2$, while Φ and $\bar{\Phi}$ have opposite R charges, $r_\Phi = -r_{\bar{\Phi}}$. The diagonal matrix h^n denotes a fourth Yukawa matrix and λ is a dimensionless coupling constant. This replacement of the Majorana mass term evidently renders the superpotential $U(1)_X$ invariant, which allows us to enlarge the gauge group of our model by a $U(1)_X$ factor. Above the GUT scale, the gauge group hence contains the following subgroup:

$$G_{\rm GUT} \supset [SU(5) \times U(1)_X \times Z_N^R]/Z_5, \tag{A9}$$

where the Z_5 symmetry dividing $SU(5)\times U(1)_X$ corresponds to the center of $SU(5)$ and, at the same time, to a subgroup of $U(1)_X$, cf. Eq. (A5). To prevent it from appearing twice in the gauge group, it has to be divided out once. At energies around Λ_{B-L}, the Higgs fields Φ and $\bar{\Phi}$ acquire nonvanishing VEVs, whereby they spontaneously break $U(1)_X/Z_5$ to matter parity P_M,

$$\langle\Phi\rangle, \quad \langle\bar{\Phi}\rangle \to \frac{\Lambda_{B-L}}{2}, \qquad U(1)_X \to Z_{10}, \qquad P_M = Z_{10}/Z_5. \tag{A10}$$

Having introduced the fields Φ, $\bar{\Phi}$ and T and modified the superpotential as in Eq. (A8), we have successfully embedded our model into a $U(1)_X$-invariant extension of the MSSM. Let us now discuss the set of possible R charge assignments in this extended model. Next to the five R charges of the MSSM fields, r_Φ, the R charge of the Higgs field Φ, now represents a further, sixth independent R charge. All six R charges are again subject to five constraints, which are almost identical to those in Eq. (A1). The only difference now is that the condition deriving from the neutrino Majorana mass term has to be modified, so as to account for the presence of the field Φ,

$$2r_{\mathbf{1}} = 2 + \ell_5 N \to 2r_{\mathbf{1}} + r_\Phi = 2 + \ell_5 N. \tag{A11}$$

This replacement entails a shift of all viable R charge assignments, cf. Eq. (A3), proportional to r_Φ, which itself remains undetermined, in the direction of the vector $\boldsymbol{X}$,

$$\boldsymbol{r} \overset{(N)}{=} \boldsymbol{r}_0 + \tilde{\ell}\frac{N}{10}\boldsymbol{X} \to \boldsymbol{r} \overset{(N)}{=} \boldsymbol{r}_0 + \frac{1}{10}(-r_\Phi + \tilde{\ell}N)\boldsymbol{X}. \tag{A12}$$

For $r_\Phi = 0$, we hence recover exactly the same solutions as in Eq. (A3). On the other hand, for $r_\Phi \neq 0$, all solutions are shifted by $-\frac{r_\Phi}{10}\boldsymbol{X}$. The *universal* solution $\boldsymbol{r}_0$, which always satisfies the conditions in Eq. (A1), irrespectively of the value of N, turns in particular into $\boldsymbol{r}_0 - \frac{r_\Phi}{10}\boldsymbol{X}$. Given the fact that the field Φ carries X charge -10, these shifts are readily identified as $U(1)_X$ gauge transformations acting on the MSSM R charges as well as on the R charge r_Φ. The form of our result in Eq. (A12) is hence a direct consequence of the $U(1)_X$ invariance of our extended model. As expected, any R charge assignment is only uniquely defined up to arbitrary $U(1)_X$ gauge transformations. Before closing this section, we remark that we are able to use this observation to render all R charges of the universal solution integer valued. Performing a $U(1)_X$ transformation such that $r_\Phi = -8$, we obtain for the MSSM R charges

$$r_\Phi = -8\!: \boldsymbol{r}_0 - \frac{r_\Phi}{10}\boldsymbol{X} = (1, -3, 5, 0, 4)^T. \tag{A13}$$

APPENDIX B: SOLUTION TO THE AXON DOMAIN WALL PROBLEM

If the PQ-breaking sector only exhibits a single vacuum, i.e. if $N_{\rm DW} = 1$, the axion domain wall problem [49] does not exist from the outset, whatever the thermal history of the universe is [51]. In this Appendix, we now illustrate how our model may be easily modified in such a way that it ends up having a unique PQ-breaking vacuum.

The simplest way to have a unique vacuum is to couple only one pair of additional quarks to the singlet field P, as is done in the original KSVZ axion model. For $n = 1$, we may for instance impose the following superpotential, cf. Eq. (14):

$$W_Q = \lambda_1 P(Q\bar{Q})_1, \quad \lambda_1 \sim O(1). \tag{B1}$$

The other $k-1$ quark pairs are then supposed to obtain masses in consequence of the spontaneous breaking of R symmetry [41]. Given the coupling in Eq. (B1) and requiring vanishing R charges for all quark pairs that do not couple to the singlet field P, the R and PQ charges of P, $\bar{P}$ as well as of the new quarks and antiquarks can be fixed as listed in Table VII.

Let us now discuss whether the PQ symmetry can be a *good* accidental symmetry. First of all, in the continuous R

TABLE VII. R and PQ charge assignments in a model with a single PQ-breaking vacuum. All R charges are only defined up to the addition of integer multiples of N. As far as the PQ mechanism is concerned, the R charges of the extra quark fields Q_i can be chosen arbitrarily. They may, however, be further constrained by requiring appropriate couplings between the new quarks and antiquarks and the fields of the MSSM, cf. Sec. II D 4.

	Q_1	$\bar{Q}_1$	Q_i $(i>1)$	$\bar{Q}_i$ $(i>1)$	P	$\bar{P}$
Z_N^R	r_{Q_1}	$6+2k-r_{Q_1}$	r_{Q_i}	$-r_{Q_i}$	$-4-2k$	$4+2k$
$U(1)_{\rm PQ}$	q_Q	$-1-q_Q$	0	0	1	-1

symmetry limit, the global $U(1)_P$, $U(1)_Q^V$ and $U(1)_Q^A$ symmetries are *almost* exact accidental symmetries of the extra singlet and extra quark sectors, respectively. The $U(1)_P$ symmetry is, however, always explicitly broken by the operator $Pm_{3/2}^{k+1}$ in the superpotential. Likewise, given the charge assignments in Table VII, we see that the operator $\bar{P}S^{k+1}$ is always allowed in the superpotential, even in the continuous R symmetry limit. Therefore, the number of extra quark pairs k should be large enough in order to ensure that the PQ symmetry is not broken too severely. After performing an analysis similar to the one in Sec. III, we find that

$$k \gtrsim 3.3 + 0.12 \ln\left(\frac{\langle S\rangle}{1\ \mathrm{TeV}}\right) + 0.028 \ln\left(\frac{m_{3/2}}{1\ \mathrm{TeV}}\frac{\Lambda}{10^{12}\ \mathrm{GeV}}\right) \tag{B2}$$

is required in order to keep the QCD vacuum angle below 10^{-10}. Consequently, at least $k = 4$ extra quark pairs are needed. With this setup, the PQ symmetry becomes a good accidental symmetry for sufficiently large N, as is the case in the model discussed in the main body of this paper.

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