PHYSICAL REVIEW D **88,** 075019 (2013)

New prospects for Higgs compositeness in $h \to Z\gamma$

Aleksandr Azatov,* Roberto Contino,† Andrea Di Iura,‡ and Jamison Galloway§

Dipartimento di Fisica, Università di Roma "La Sapienza" and INFN Sezione di Roma, I-00185 Rome, Italy

(Received 20 September 2013; published 28 October 2013)

We discuss novel effects in the phenomenology of a light Higgs boson within the context of composite models. We show that large modifications may arise in the decay of a composite Nambu-Goldstone boson Higgs to a photon and a Z boson, $h \to Z\gamma$. These can be generated by the exchange of massive composite states of a strong sector that breaks a left-right symmetry, which we show to be the sole symmetry structure responsible for governing the size of these new effects in the absence of Goldstone-breaking interactions. In this paper we consider corrections to the decay $h \to Z\gamma$ obtained either by integrating out vectors at tree level, or by integrating out vectorlike fermions at loop level. In each case, the pertinent operators that are generated are parametrically enhanced relative to other interactions that arise at loop level in the Standard Model such as $h \to gg$ and $h \to \gamma\gamma$. Thus we emphasize that the effects of interest here provide a unique possibility to probe the dynamics underlying electroweak symmetry breaking, and do not depend on any contrivance stemming from carefully chosen spectra. The effects we discuss naturally lead to concerns of compatibility with precision electroweak measurements, and we show with relevant computations that these corrections can be kept well under control in our general parameter space.

DOI: 10.1103/PhysRevD.88.075019 PACS numbers: 12.60.Rc, 12.60.Fr

I. INTRODUCTION

The LHC phase of data taking at 8 TeV is over and a large collection of experimental results on the Higgs boson has been derived. Although data still have to be fully analyzed, a clear picture seems to be emerging: the properties of the newly discovered particle closely resemble those of the Standard Model (SM) Higgs boson. Overall, the quantitative agreement between its measured couplings and the SM predictions is at the 20%–30% level [1,2]. This strongly suggests that the new particle is indeed part of an $SU(2)_L$ doublet H, and that the scale of new physics (NP) must be somewhat higher than the electroweak scale. From this perspective it is important to ask which observables or processes are most sensitive to NP effects and where we may be likely to see deviations from the SM pattern in the future.

It is well known that Higgs processes occurring at loop level in the SM, such as the decays $h \to \gamma\gamma$ and $h \to Z\gamma$, and the gluon-fusion production $gg \to h$, are particularly sensitive probes of weakly coupled extensions of the SM. This is typically not true, however, in theories with a light Higgs boson where the electroweak symmetry breaking (EWSB) dynamics is strong. If the Higgs boson is a composite Nambu-Goldstone (NG) boson of a new strongly interacting sector, parametrically large shifts are expected in the tree-level couplings, while hgg and $h\gamma\gamma$ contact interactions violate the Higgs shift symmetry and are thus suppressed. On the other hand, a similar symmetry suppression does not hold for a $hZ\gamma$ contact interaction.

To make this point more quantitative, contributions to the gg, $\gamma\gamma$, and γZ decay rates induced by the exchange of new particles with mass much larger than the electroweak scale can be conveniently parametrized by local operators. For a Higgs doublet, the leading NP effects are parametrized by dimension-6 operators. A complete characterization of the Higgs effective Lagrangian at the dimension-6 level has been performed in previous studies [3–5]; see Ref. [6] for a recent review. In the basis of the strongly interacting light Higgs (SILH) boson of Ref. [7], the CP-conserving operators relevant for the gg, $\gamma\gamma$, and $Z\gamma$ rates are[1]

$$\begin{aligned} O_g &= \frac{g_S^2}{m_W^2} H^\dagger H G^a_{\mu\nu} G^{a\mu\nu}, \\ O_{HW} &= \frac{ig}{m_W^2} (D^\mu H)^\dagger \sigma^i (D^\nu H) W^i_{\mu\nu} \\ O_\gamma &= \frac{g'^2}{m_W^2} H^\dagger H B_{\mu\nu} B^{\mu\nu}, \\ O_{HB} &= \frac{ig'}{m_W^2} (D^\mu H)^\dagger (D^\nu H) B_{\mu\nu}. \end{aligned} \tag{1.1}$$

The operators O_g and O_γ contribute, respectively, to the gg and $\gamma\gamma$ rates, while $Z\gamma$ gets a contribution from both O_γ and the linear combination $O_{HW} - O_{HB}$. The additional operators

*aleksandr.azatov@roma1.infn.it
†roberto.contino@roma1.infn.it
‡jamison.galloway@roma1.infn.it
§andrea.giuseppe.DiIura@roma1.infn.it

[1]We normalize the Wilson coefficients according to the convention of Ref. [6].

$$O_W = \frac{ig}{2m_W^2} D^\nu W^i_{\mu\nu} (H^\dagger \sigma^i \overleftrightarrow{D^\mu} H),$$
$$O_B = \frac{ig'}{2m_W^2} \partial^\nu B_{\mu\nu} (H^\dagger \overleftrightarrow{D^\mu} H) \tag{1.2}$$

do not mediate $h \to Z\gamma$ for on-shell photons, but their sum contributes to the S parameter [8]. By working in unitary gauge and focusing on terms with one Higgs boson, the operators of Eq. (1.1) are rewritten as

$$\mathcal{L} = \frac{c_{gg}}{2} G^a_{\mu\nu} G^{a\mu\nu} \frac{h}{v} + \frac{c_{\gamma\gamma}}{2} \gamma_{\mu\nu} \gamma^{\mu\nu} \frac{h}{v} + c_{Z\gamma} Z_{\mu\nu} \gamma^{\mu\nu} \frac{h}{v}, \tag{1.3}$$

where (by $\bar{c}_i$ we denote the coefficient of the operator O_i of the SILH Lagrangian) [6]

$$c_{gg} = 8(\alpha_s/\alpha_2)\bar{c}_g, \qquad c_{\gamma\gamma} = 8\sin^2\theta_W \bar{c}_\gamma,$$
$$c_{Z\gamma} = -\tan\theta_W [(\bar{c}_{HW} - \bar{c}_{HB}) + 8\sin^2\theta_W \bar{c}_\gamma], \tag{1.4}$$

and $\alpha_2 = \sqrt{2} G_F m_W^2/\pi$. The expression of the partial decay width to $Z\gamma$, including the SM contribution and the correction from Eq. (1.3), is given in Appendix A.

Let us perform a naive estimate of the size of the effects mediated by the operators of Eq. (1.1). By simple dimensional analysis, the Wilson coefficients $\bar{c}_i$ scale as $1/M^2$, where M is the characteristic NP scale. Their contribution to the Higgs decay rates is thus suppressed by a factor $\sim(m_W^2/M^2)$, where $m_W \approx m_h$ is the energy scale of the process. If the operators O_i are generated by the tree-level exchange of new particles with unsuppressed nonminimal couplings to photons and gluons, one naively expects a correction

$$\frac{\Delta\Gamma_{\text{tree}}}{\Gamma_{\text{SM}}} \approx \frac{m_W^2}{M^2} \frac{16\pi^2}{g^2}. \tag{1.5}$$

Unsuppressed nonminimal couplings can arise if the new heavy states are bound states of some new strong dynamics; see for example the recent discussion of Ref. [9]. On the other hand, in weakly coupled UV completions of the SM as well as in some strongly coupled constructions such as holographic Higgs models, the massive states have suppressed higher-derivative couplings. In this case the operators O_i are generated only at the cost of a loop factor $(g_*^2/16\pi^2)$ [7], where g_* denotes the coupling strength of the Higgs boson to the new states:

$$\frac{\Delta\Gamma_{\text{loop}}}{\Gamma_{\text{SM}}} \approx \frac{m_W^2}{M^2} \frac{g_*^2}{g^2} \sim \frac{v^2}{f^2}. \tag{1.6}$$

In the last identity we have defined $1/f \equiv g_*/M$. Large corrections are thus possible in strongly coupled theories, where the Higgs boson is a composite of the new dynamics and $1 \ll g_* < 4\pi$. However, a composite Higgs is naturally light only if it is a NG boson of a spontaneously broken symmetry $\mathcal{G} \to \mathcal{H}$ of the strong sector. In this case the scale f must be identified with the associated decay constant, and shifts of order $(v/f)^2$ are expected in the tree-level Higgs couplings to SM vector bosons and fermions from the Higgs nonlinear σ-model Lagrangian. At the same time, exact invariance under $\mathcal{G}/\mathcal{H}$ transformations, which include a Higgs shift symmetry $H^i \to H^i + \zeta^i$, forbids the operators O_g and O_γ. For the latter to be generated the shift symmetry must be broken by some weak coupling $g_{\not G}$, so that the naive estimate of $\bar{c}_g$, $\bar{c}_\gamma$, and hence their contribution to the decay rates to gg and $\gamma\gamma$, is further suppressed by an extra factor $(g_{\not G}^2/g_*^2)$. Conversely, the operators O_{HW} and O_{HB} are invariant under the Higgs shift symmetry, and the naive estimate (1.6) holds for the $h \to Z\gamma$ rate. It follows that for a composite NG Higgs boson the largest NP effects are expected to arise from shifts to the tree-level Higgs couplings and from the contact $hZ\gamma$ interaction [7]. From this perspective, a precise measurement of the $Z\gamma$ rate is of crucial importance.

It is the purpose of this paper to quantitatively study the $h \to Z\gamma$ decay rate in the context of composite Higgs models. As implied by the discussion above, the leading effects are captured by neglecting the explicit breaking of the Goldstone symmetry due to weak couplings of the elementary fields to the composite sector. We thus work in this limit in the following as it simplifies the calculations, and concentrate on the contributions of pure composite states within minimal $SO(5)/SO(4)$ theories. The next section contains a brief discussion on the effective operator basis for $SO(5)/SO(4)$ theories and the role played by the P_{LR} parity for the $h \to Z\gamma$ decay. In Sec. III we compute the effective $hZ\gamma$ vertex generated by the tree-level exchange of spin-1 resonances and by the 1-loop exchange of composite fermions. As a byproduct of our $hZ\gamma$ calculation we derive in Sec. IV the correction to the S parameter from loops of pure composite fermions. We report our numerical results and discuss them in Sec. V. Useful formulas are collected in Appendices A, B, C, and D, while Appendix E contains a discussion of different formalisms commonly adopted to describe fermionic resonances. Finally, in Appendix F we describe how the calculation of the 1-loop contribution to $h \to Z\gamma$ from heavy fermions can be performed in full generality in the basis of mass eigenstates, without resorting to the approximation made in the main text.

II. EFFECTIVE LAGRANGIAN FOR $SO(5)/SO(4)$ COMPOSITE HIGGS MODELS AND THE ROLE OF P_{LR}

The effective Lagrangian for $SO(5)/SO(4)$ composite Higgs theories was discussed in Ref. [10], where a complete list of four-derivative operators was given in the formalism of Callan, Coleman, Wess, and Zumino (CCWZ) [11]. Here we closely follow the notation of Ref. [10], although we adopt a different operator basis which is more transparently matched onto the SILH basis

of Ref. [7]. At $O(p^4)$ in the derivative expansion there are seven independent CP-conserving operators[2]

$$\mathcal{L} = \frac{f^2}{4}\,\mathrm{Tr}[d_\mu d^\mu] + \sum_i c_i O_i, \tag{2.2}$$

$$\begin{aligned}
O_1 &= \mathrm{Tr}[d_\mu d^\mu]^2, \qquad O_2 = \mathrm{Tr}[d_\mu d_\nu]\mathrm{Tr}[d^\mu d^\nu],\\
O_3^\pm &= \mathrm{Tr}[(E^L_{\mu\nu})^2 \pm (E^R_{\mu\nu})^2],\\
O_4^\pm &= \mathrm{Tr}[(E^L_{\mu\nu} \pm E^R_{\mu\nu}) i[d^\mu, d^\nu]],\\
O_5 &= \sum_{a_L=1}^{3} \mathrm{Tr}(T^{a_L}[d_\mu, d_\nu])^2 - \sum_{a_R=1}^{3} \mathrm{Tr}(T^{a_R}[d_\mu, d_\nu])^2,
\end{aligned} \tag{2.3}$$

where $d_\mu(\pi) = d^{\hat a}_\mu(\pi) T^{\hat a}$, $E^L_\mu(\pi) = E^{a_L}_\mu(\pi) T^{a_L}$, $E^R_\mu(\pi) = E^{a_R}_\mu(\pi) T^{a_R}$ are the CCWZ covariant functions—transforming, respectively, as an adjoint and gauge fields of $SO(4)$—of the NG field $\pi(x) = \pi^{\hat a}(x) T^{\hat a}$. Explicitly,

$$-iU^\dagger(\pi) D_\mu U(\pi) = d_\mu + E^L_\mu + E^R_\mu, \tag{2.4}$$

where $U(\pi) = \exp(i\sqrt{2}\pi(x)/f)$. The field strength $E^{L,R}_{\mu\nu}$ is constructed from a commutator of covariant derivatives $\nabla_\mu = \partial_\mu + i(E^L_\mu + E^R_\mu)$; see Ref. [10] for further details. Here T^{a_L}, T^{a_R} are the generators of the unbroken $SO(4) \sim SU(2)_R \times SU(2)_R$, while those of $SO(5)/SO(4)$ are denoted as $T^{\hat a}$. The SM electroweak vector bosons weakly gauge an $SU(2)_L \times U(1)_Y$ subgroup of $SO(5) \times U(1)_X$, where the $U(1)_X$ does not participate in the dynamical breaking, but is needed to correctly reproduce the hypercharges of the SM fermions. It is convenient to define the tree-level vacuum such that the electroweak group is fully contained in the unbroken $SO(4) \times U(1)_X$, with $Y = T^{3R} + T^X$. The true vacuum will in general be misaligned with this direction by an angle θ due to the radiatively induced potential of the NG bosons.

The operators in Eq. (2.3) have been conveniently defined to be even or odd under a parity P_{LR} which exchanges the $SU(2)_L$ and $SU(2)_R$ comprising the unbroken $SO(4)$. Under P_{LR} the NG bosons transform as $\pi^{\hat a}(x) \to -\eta^{\hat a}\pi^{\hat a}(x)$, with $\eta^{\hat a} = \{1, 1, 1, -1\}$, which implies $d^{\hat a}_\mu \to -\eta^{\hat a} d^{\hat a}_\mu$, $E^L_\mu \leftrightarrow E^R_\mu$ [10]. Ordinary parity is thus the product $P = P_0 \cdot P_{LR}$, where P_0: $(t, \vec{x}) \to (t, -\vec{x})$ is the usual spatial inversion. Under P_{LR}, the operators $O_{1,2}$ and $O^+_{3,4}$ are even, whereas $O^-_{3,4}$ and O_5 are odd. Expanding in the number of NG fields it is easy to match the operators of Eq. (2.3) with the dimension-6 operators of the SILH Lagrangian by noticing that

$$\begin{aligned}
d_\mu &\sim D_\mu H + \cdots\\
E_{\mu\nu} &\sim A_{\mu\nu} + \frac{1}{f^2}[D_\mu(H^\dagger i\overleftrightarrow{D}_\nu H) - (\mu \leftrightarrow \nu)] + \cdots
\end{aligned} \tag{2.5}$$

where the gauge fields entering into the field strength $A_{\mu\nu}$ and the covariant derivative D_μ are those of $SU(2)_L \times U(1)_Y$. From Eq. (2.5) one can see that $O_3^\pm$ and $O_4^\pm$ correspond, respectively, to $O_W \pm O_B$ and $O_{HW} \pm O_{HB}$. The leading terms in the expansion of O_1, O_2, and O_5 are instead of dimension 8, meaning that these operators do not have a counterpart in the SILH Lagrangian of Ref. [7]. The exact relations and the connection between our basis and that of Ref. [10] are reported in Appendix B. Notice that there is no operator in Eq. (2.3) corresponding to O_γ and O_g, since the latter explicitly breaks the $SO(5)$ global symmetry. It then follows that the only (CP-conserving) operator which gives an $hZ\gamma$ contact coupling is O_4^-:

$$c_{Z\gamma} = g^2 \sin^2\theta c_4^-. \tag{2.6}$$

Notice also that only O_3^+ contributes to the S parameter:

$$S = -32\pi \sin^2\theta c_3^+. \tag{2.7}$$

It is not an accident that the $hZ\gamma$ coupling follows from a P_{LR}-odd operator. Since the Abelian $U(1)_X$ subgroup factorizes with respect to the nonlinearly realized $SO(5)$, the photon and Z fields enter into the operators of Eq. (2.3) only through the weak gauging of the $U(1)_L \times U(1)_R$ subgroup. By formally assigning the transformation rules $W^3_\mu \leftrightarrow B_\mu$, $g \leftrightarrow g'$, the P_{LR} symmetry is exact even after turning on the neutral gauge fields. By the above rules, the Z field is odd while the photon and the Higgs boson are even under P_{LR}, so that the decay $h \to Z\gamma$ can be mediated only by an odd operator. By the same argument, S is an even quantity under LR exchange (it is proportional to the coefficients of the unitary-gauge operator $W^3_{\mu\nu}B^{\mu\nu}$), and it is consistently induced by the P_{LR}-even operator O_3^+.

It is interesting to notice that in the limit of unbroken $SO(5)$ symmetry the renormalization group (RG) running of c_4^-, as well as that of the coefficient of any $O(p^4)$ odd operator, vanishes due to the P_{LR} parity. While the effective operators are generated at some high-energy scale M by the exchange of massive states, the Wilson coefficients appearing in the expressions of low-energy observables, like in Eq. (2.6) for the $Z\gamma$ decay rate, must be evaluated at the typical scale of the process, $\mu \ll M$. The running of the Wilson coefficients from M down to μ originates from 1-loop logarithmically divergent diagrams constructed with $O(p^2)$ vertices,

[2]There are additionally four CP-odd operators that can be written as

$$\begin{aligned}
\tilde O_3^\pm &= \mathrm{Tr}[(\tilde E^L_{\mu\nu} E^{L\mu\nu} \pm \tilde E^R_{\mu\nu} E^{R\mu\nu})^2],\\
\tilde O_4^\pm &= \mathrm{Tr}[(\tilde E^L_{\mu\nu} \pm \tilde E^R_{\mu\nu}) i[d^\mu, d^\nu]],
\end{aligned} \tag{2.1}$$

where $\tilde E_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma} E^{\rho\sigma}$. In particular $\tilde O_4^-$ contributes to $h \to Z\gamma$. Although these operators can be included straightforwardly, in this paper we will focus on the CP-conserving ones for simplicity.

$$c_i(\mu) = c_i(M) + \frac{b_i}{16\pi^2}\log\frac{M}{\mu}, \tag{2.8}$$

where b_i are $O(1)$ numbers. Since, however, P_{LR} is an accidental symmetry of the $O(p^2)$ Lagrangian [10], it follows that there cannot be any running from loops of NG bosons in the case of P_{LR}-odd operators, i.e. $b_i = 0$.[3] An RG evolution is in general induced by loops of transverse vector bosons,[4] since the weak gauging explicitly breaks P_{LR} at the $O(p^2)$ level. This is, however, a subleading electroweak effect which we neglect for simplicity in this paper. One naively expects $c_i(M) \sim 1/16\pi^2$ for operators generated at the scale M with an extra loop suppression, as in the case of O_4^- for a minimally coupled UV theory. Hence, if the RG running was nonvanishing, the leading contribution to the Wilson coefficient could come from long-distance (log-enhanced) effects rather than from high-energy threshold corrections. For P_{LR}-even operators generated at tree level, like O_3^+ for example, one instead estimates $c_i(M) \sim 1/g_*^2$, so that the threshold contribution dominates over the RG evolution as long as $g_* \lesssim 4\pi/\sqrt{\log(M/\mu)}$.

If P_{LR} is an exact invariance of the strong dynamics, then $c_4^-(M)$ vanishes for unbroken $SO(5)$. In this case the only source of P_{LR} breaking stems from the couplings of the elementary gauge and fermion fields to the strong sector, and the $hZ\gamma$ contact interaction will be suppressed by a factor $(g_{\acute{G}}/g_*)^2$, as in the case of $h\gamma\gamma$ and hgg. We will thus focus on the case in which the strong dynamics explicitly breaks the P_{LR} symmetry, so that O_4^- is generated at the scale M even in the limit of unbroken $SO(5)$. A LR-violating strong dynamics generically leads to dangerously large corrections to the $Zb\bar{b}$ vertex [14], but there are special cases where the LR breaking is communicated to the $Zb\bar{b}$ coupling in a suppressed way. For example, it has been pointed out by the authors of Ref. [15] that if the fermionic resonances form (only) fundamental representations of $SO(5)$, as in the minimal composite Higgs model MCHM5 [16], then P_{LR} is an accidental invariance of the lowest derivative fermionic operators relevant for $Zb\bar{b}$ (small P_{LR}-breaking effects suppressed by m_b/m_t are present but can be neglected). In this case the spin-1 sector of resonances can be maximally LR violating, and thus generate an unsuppressed $hZ\gamma$ interaction, without leading to excessively large shifts in the $Zb\bar{b}$ coupling. Another possibility is that the sector of fermionic resonances maximally breaks P_{LR} but the shift of $Zb\bar{b}$ is suppressed by a small coupling. A minimal realization of this case can be obtained for example if the spectrum of fermionic resonances contains both fundamental and antisymmetric representations of $SO(5)$. In the following section we will provide two explicit models realizing these possibilities and compute the contribution of the resonances to the $h \to Z\gamma$ decay rate.

[3]From the viewpoint of the SILH Lagrangian of Ref. [7], this argument shows that there cannot be any contribution to the RG evolution of $O_{HW} - O_{HB}$ and $O_W - O_B$ from O_H. In fact, the authors of Ref. [12] showed that even the combination $O_{HW} + O_{HB}$ is not renormalized by O_H.

[4]For the calculation of the RG running relevant to $h \to \gamma\gamma$ and $h \to Z\gamma$ see Refs. [12,13].

III. $h \to Z\gamma$ FROM PURE COMPOSITE STATES

We calculate the contribution of pure composite states to $h \to Z\gamma$ by focusing on the lightest modes and describing their dynamics by means of a low-energy effective theory. For our description to be valid we assume that these states are lighter than the cutoff scale Λ where other resonances occur, and that the derivative expansion of the effective theory is controlled by ∂/Λ. Notice, however, that whenever it arises at the 1-loop level, the contribution of the lightest modes to $h \to Z\gamma$ is parametrically of the same order as that of the cutoff states. These latter are heavier but are also expected to be more strongly coupled than the lighter modes, so that both effects are naively of order $(v/f)^2$, as shown by Eq. (1.6). In this case our calculation should be considered as a more quantitative estimate of the contribution of the strong dynamics rather than a precise prediction of a model. For a more detailed discussion about the validity of this effective description we refer the reader to Ref. [10], whose approach we follow in this paper (see also Ref. [17]).

In the fermionic sector we assume the existence of linear elementary-composite couplings, which leads to partial compositeness [17,18]. We are, however, interested in the effects of pure composite states, hence in the following we work at lowest order in the elementary couplings and set them to zero. Before EWSB the composite states fill multiplets of the linearly realized subgroup $SO(4) \times U(1)_X \sim SU(2)_L \times SU(2)_R \times U(1)_X$. We will consider spin-1 resonances in the $(\mathbf{3},\mathbf{1})_0$ and $(\mathbf{1},\mathbf{3})_0$ representations (denoted, respectively, ρ^L and ρ^R) and fermionic resonances transforming as $(\mathbf{1},\mathbf{1})$, $(\mathbf{2},\mathbf{2})$, $(\mathbf{1},\mathbf{3})$, and $(\mathbf{3},\mathbf{1})$ representations with arbitrary $U(1)_X$ assignments.

In the following parts of this section we first derive the $hZ\gamma$ contact coupling by integrating out the composite states and matching to the low-energy theory. We will then illustrate two minimal models where maximal P_{LR} breaking can occur without generating a large modification of the $Zb\bar{b}$ coupling.

A. Tree-level exchange of spin-1 resonances

We begin by considering the contribution from the tree-level exchange of spin-1 composites.[5] We follow the vector formalism where the ρ transforms nonhomogeneously under $SO(5)$ transformations. Neglecting CP-odd operators for simplicity, the effective Lagrangian for

[5]See also Ref. [19] for a calculation of the 1-loop contribution to $h \to Z\gamma$ from vector resonances.

$\rho_\mu^L = \rho_\mu^{a_L} T^{a_L}$ and $\rho_\mu^R = \rho_\mu^{a_R} T^{a_R}$ can be written as follows (see Ref. [10] for more details):[6]

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4g_{\rho_L}^2}\mathrm{Tr}(\rho^L_{\mu\nu}\rho^{L\mu\nu}) + \frac{m_{\rho_L}^2}{2g_{\rho_L}^2}\mathrm{Tr}(\rho_\mu^L - E_\mu^L)^2 \\ &+ \alpha_{1L}Q_{1L} + \alpha_{2L}Q_{2L} \\ &-\frac{1}{4g_{\rho_R}^2}\mathrm{Tr}(\rho^R_{\mu\nu}\rho^{R\mu\nu}) + \frac{m_{\rho_R}^2}{2g_{\rho_R}^2}\mathrm{Tr}(\rho_\mu^R - E_\mu^R)^2 \\ &+ \alpha_{1R}Q_{1R} + \alpha_{2R}Q_{2R},\end{aligned} \tag{3.2}$$

where we have neglected subleading terms in the derivative expansion and have defined

$$\begin{aligned}Q_{1r} &= \mathrm{Tr}(\rho^r_{\mu\nu} i[d^\mu, d^\nu]), \\ Q_{2r} &= \mathrm{Tr}(\rho^{r\mu\nu} E^r_{\mu\nu}), \qquad r = L, R.\end{aligned} \tag{3.3}$$

It is straightforward to integrate out the spin-1 resonances at tree level by using the equations of motion: $\rho_\mu = E_\mu + O(p^3)$. One obtains the low-energy Lagrangian (2.3) with

$$\begin{aligned}c_3^\pm &= \frac{1}{2}\left[\left(\alpha_{2L} - \frac{1}{4g_{\rho_L}^2}\right) \pm \left(\alpha_{2R} - \frac{1}{4g_{\rho_R}^2}\right)\right], \\ c_4^\pm &= \frac{1}{2}(\alpha_{1L} \pm \alpha_{1R}).\end{aligned} \tag{3.4}$$

The S parameter receives a correction both from the ρ mass terms and from Q_{2L}, Q_{2R} [10].[7] The vertex $hZ\gamma$, on the other hand, follows from the operators Q_{1L}, Q_{1R} due to the ρ-photon mixing induced by the ρ mass term:

$$c_{Z\gamma} = \frac{g^2}{2}\sin^2\theta(\alpha_{1L} - \alpha_{1R}). \tag{3.6}$$

[6]At the level of leading terms in the derivative expansion there are four additional CP-odd operators:

$$\begin{aligned}\tilde{Q}_{1r} &= \epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}(\rho^r_{\mu\nu} i[d_\alpha, d_\beta]), \\ \tilde{Q}_{2r} &= \epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}(\rho^r_{\mu\nu} E^r_{\alpha\beta}), \qquad r = L, R.\end{aligned} \tag{3.1}$$

For simplicity we will concentrate on CP-even operators in the following, although the inclusion of the CP-odd ones is straightforward. Notice also that we use a slightly different basis of operators Q_i compared to Ref. [10], so as to match more easily with the low-energy Lagrangian (2.3).

[7]The authors of Ref. [20] pointed out that Q_{2L}, Q_{2R} modify the high-energy dependence of the current-current vacuum polarizations at tree level and can be made consistent with the UV behavior of the OPE only if an additional contribution to the operators $O_3^\pm$ exist, $\Delta c_3^\pm = -(\alpha_{2L}^2 g_{\rho_L}^2 \pm \alpha_{2R}^2 g_{\rho_R}^2)$. The expression of the S parameter thus reads [20]

$$S = 4\pi\sin^2\theta\left[\left(\frac{1}{g_{\rho_L}} - 2g_{\rho_L}\alpha_{2L}\right)^2 + \left(\frac{1}{g_{\rho_R}} - 2g_{\rho_R}\alpha_{2R}\right)^2\right]. \tag{3.5}$$

No similar issue arises with Q_{1L}, Q_{1R}.

After rotating to the basis of mass eigenstates, $\rho_{\mu\nu}$ gives a photon field strength $\gamma_{\mu\nu}$, while $d_\mu d_\nu$ gives $Z_\mu \partial_\nu h$.[8] In this sense the operators Q_{1r}, unlike Q_{2r}, give nonminimal couplings of the photon to neutral particles.

The size of the correction to the $h \to Z\gamma$ decay rate depends on the value of the parameters α_{1r}. By assuming partial UV completion [10], so that the strength of the interactions mediated by Q_{1r} becomes of order $g_* \equiv \Lambda/f$ at the cutoff scale Λ, one estimates $\alpha_{1r} \lesssim 1/(g_\rho g_*) < 1/g_\rho^2$. In a minimally coupled theory, on the other hand, the operators Q_{1r} carry a further loop suppression from which the more conservative estimate $\alpha_{1r} \sim 1/(16\pi^2)$, and in turn Eq. (1.6), follow.

B. Loops of fermionic resonances

Composite fermions can generate the vertex $hZ\gamma$ at 1-loop level. Let us consider for example the case of fermions transforming as $(\mathbf{1},\mathbf{1})$, $(\mathbf{2},\mathbf{2})$, $(\mathbf{1},\mathbf{3})$, and $(\mathbf{3},\mathbf{1})$ under $SO(4) \sim SU(2)_L \times SU(2)_R$ and with arbitrary common $U(1)_X$ charge. At leading order in the derivative expansion, the Lagrangian reads

$$\begin{aligned}\mathcal{L} = &\sum_r \bar{\chi}_r(i\not{\nabla} - m_r)\chi_r - [\zeta_{11}\bar{\chi}_{(\mathbf{2},\mathbf{2})}\not{d}\chi_{(\mathbf{1},\mathbf{1})} \\ &+ \zeta_{13}\bar{\chi}_{(\mathbf{2},\mathbf{2})}\not{d}\chi_{(\mathbf{1},\mathbf{3})} + \zeta_{31}\bar{\chi}_{(\mathbf{2},\mathbf{2})}\not{d}\chi_{(\mathbf{3},\mathbf{1})} + \mathrm{H.c.}],\end{aligned} \tag{3.7}$$

where r runs over all $SO(4)$ representations, $\zeta_{11}, \zeta_{13}, \zeta_{31}$ are $O(1)$ complex coefficients, and $\nabla_\mu = \partial_\mu + i(E_\mu^L + E_\mu^R)$ is the covariant derivative on $SO(5)/SO(4)$. By integrating out the fermions and matching with the low-energy Lagrangian (2.3), the contribution to $c_4^\pm$ comes from the 1-loop diagram of Fig. 1 plus its crossing, where one has to sum over all possible representations r, r'.

For a given diagram with fermions in the representations r and r' of $SO(4)$, the Feynman amplitude can be expressed as

$$M^{a\hat{a}\,\hat{b}}_{\alpha\mu\nu} = N_\chi \omega^{a\hat{a}\,\hat{b}}_{[r,r']}|\zeta_{[r,r']}|^2 I_{\alpha\mu\nu}(p_1, p_2; m_r, m_{r'}), \tag{3.8}$$

where p_1 and p_2 are the momenta of $d_\mu^{\hat{a}}$ and $d_\nu^{\hat{b}}$, respectively (defined to be flowing into the corresponding

[8]It is possible to diagonalize the mixing of the ρ with the elementary gauge fields by making the field redefinition $\bar{\rho}_\mu = \rho_\mu - E_\mu$, where $\bar{\rho}$ transforms as a simple adjoint of $SO(4)$. In this mass eigenstate basis the tree-level exchange of $\bar{\rho}$ does not generate an $hZ\gamma$ vertex, due to the simple fact that no appropriate Feynman diagrams can be constructed. Instead, the $hZ\gamma$ interaction arises directly from the contribution to O_4^- that follows from Q_{1r} after replacing $\rho_{\mu\nu} = \nabla_{[\mu}\bar{\rho}_{\nu]} + E_{\mu\nu} + i[\bar{\rho}_\mu, \bar{\rho}_\nu]$. This is analogous to what happens for the S parameter and in fact for any observable at leading order in the derivative expansion. Indeed, it is easy to check that integrating out the $\bar{\rho}$ by means of the equations of motion generates only $O(p^6)$ operators, i.e. operators with more than four derivatives. This shows that the Lagrangian written in terms of $\bar{\rho}$ must be properly supplemented with additional four-derivative terms, among which is O_4^-, in order to match the original one [21].

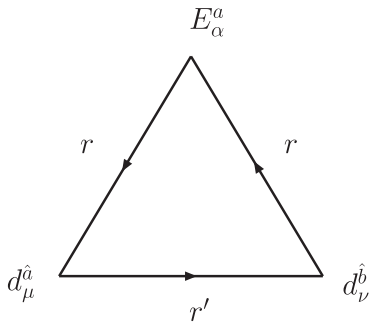

FIG. 1. 1-loop contribution to the Green function $\langle E^a_\alpha d^{\hat a}_\mu d^{\hat b}_\nu\rangle$ from composite fermions in the representations r and r' of $SO(4)$.

vertices), the index a runs over the adjoint of $SU(2)_L \times SU(2)_R$, and N_χ is the fermion multiplicity. For example, for three families of colored fermions (heavy quarks) one has $N_\chi = N_c N_F = 9$ (with $N_c = 3$, $N_F = 3$), while $N_\chi = 12$ if there are three additional families of colorless fermions (i.e. heavy leptons). Here $\zeta_{[r,r']} = \zeta^*_{[r',r]}$ denotes the coupling strength of d_μ with fermions in the representations r and r': $\zeta_{[(\mathbf{2},\mathbf{2}),(\mathbf{1},\mathbf{1})]} = \zeta_{11}$, $\zeta_{[(\mathbf{2},\mathbf{2}),(\mathbf{1},\mathbf{3})]} = \zeta_{13}$, and $\zeta_{[(\mathbf{2},\mathbf{2}),(\mathbf{3},\mathbf{1})]} = \zeta_{31}$, as in Eq. (3.7). By $SO(4)$ covariance, the second factor of Eq. (3.8) is proportional to the $SO(4)$ generator $t^a_{\hat a\hat b}$,

$$\omega^{a_L \hat a \hat b}_{[r,r']} = l^L_{[r,r']} t^{a_L}_{\hat a\hat b}, \qquad \omega^{a_R \hat a \hat b}_{[r,r']} = l^R_{[r,r']} t^{a_R}_{\hat a\hat b}, \tag{3.9}$$

where the coefficients $l^{L,R}_{[r,r']}$ are reported in Table I for the fermion representations under study. The contribution to $c_4^\pm$ can be extracted by expanding the loop function $I_{\alpha\mu\nu}$ at first order in the external momenta. It is easy to show that $I_{\alpha\mu\nu}$ is antisymmetric under the exchange $\{\mu, p_1\} \leftrightarrow \{\nu, p_2\}$, so there are three possible Lorentz structures at linear order in the external momenta:

$$\begin{aligned} I_{\alpha\mu\nu} = {} & A(m_r, m_{r'})\eta_{\mu\nu}(p_1 - p_2)_\alpha \\ & + B(m_r, m_{r'})(p_{1\mu}\eta_{\nu\alpha} - p_{2\nu}\eta_{\mu\alpha}) \\ & + C(m_r, m_{r'})[\eta_{\nu\alpha}(p_1 + p_2)_\mu \\ & - \eta_{\mu\alpha}(p_1 + p_2)_\nu] + O(p^3). \end{aligned} \tag{3.10}$$

The functions A, B, C are logarithmically divergent and their expression is given in Appendix C. The terms proportional to A and B renormalize, respectively, the operators $O_{d1} = \mathrm{Tr}[(\nabla_\mu d_\nu)^2]$ and $O_{d2} = \mathrm{Tr}[(\nabla_\mu d^\mu)^2]$, which contain terms with zero, one, and two E_μ's from the covariant derivative. In fact, the same function $I_{\alpha\mu\nu}$ accounts for the 1-loop contribution to the three-point Green function $\langle J^a_\alpha d^{\hat a}_\mu d^{\hat b}_\nu\rangle$, where J^a_μ is the $SO(4)$ conserved current [see Eq. (D3)]. It is thus subject to the Ward identity

$$i(p_1 + p_2)^\alpha I_{\alpha\mu\nu} = G_{\mu\nu}(p_1^2) - G_{\mu\nu}(p_2^2), \tag{3.11}$$

where $G_{\mu\nu}(p^2)$ is the d_μ self-energy:

$$\begin{aligned} \langle d^{\hat a}_\mu d^{\hat b}_\nu\rangle|_{\mathrm{amp}} & \equiv \delta^{\hat a\hat b} G_{\mu\nu}(p^2) \\ & = \delta^{\hat a\hat b}\eta_{\mu\nu}\Pi_0(p^2) + \delta^{\hat a\hat b} p_\mu p_\nu \Pi_1(p^2). \end{aligned} \tag{3.12}$$

From Eq. (3.11) it follows that

$$A(m_r, m_{r'}) = \Pi_0'(0), \qquad B(m_r, m_{r'}) = \Pi_1(0). \tag{3.13}$$

We have checked these identities by explicitly computing the self-energy $G_{\mu\nu}$. The coefficients of the operators O_{d1}, O_{d2} are given by

$$\begin{aligned} c_{d1} &= \frac{N_\chi}{4}\sum_{r,r'} A(m_r, m_{r'})(l^L_{[r,r']} + l^R_{[r,r']})|\zeta_{[r,r']}|^2, \\ c_{d2} &= \frac{N_\chi}{4}\sum_{r,r'} B(m_r, m_{r'})(l^L_{[r,r']} + l^R_{[r,r']})|\zeta_{[r,r']}|^2, \end{aligned} \tag{3.14}$$

where the sums are over all possible fermion representations r, r' contributing to the 1-loop diagram of Fig. 1.[9] Neither of the operators O_{d1}, O_{d2} contributes to $h \to Z\gamma$: O_{d2} can be redefined away in terms of higher-derivative operators by using the equations of motion $\nabla_\mu d^\mu = 0$, while O_{d1} can be rewritten as

$$\begin{aligned} \mathrm{Tr}[(\nabla_\mu d_\nu)^2] & \equiv O_{d1} \\ & = \frac{1}{2}\mathrm{Tr}[F^2_{\mu\nu}] - \frac{1}{2}O_3^+ + \frac{1}{4}(O_2 - O_1), \end{aligned} \tag{3.15}$$

and thus contributes to the S parameter. We will discuss this further in the next section, where we perform a detailed calculation of S.

Finally, the term proportional to C in Eq. (3.10) renormalizes the operator $\mathrm{Tr}[E_{\mu\nu} d^\mu d^\nu]$ and thus contributes to $c_4^\pm$. We find:

$$c_4^\pm = \frac{N_\chi}{4}\sum_{r,r'} C(m_r, m_{r'})(l^L_{[r,r']} \pm l^R_{[r,r']})|\zeta_{[r,r']}|^2. \tag{3.16}$$

Using the coefficients of Table I and Eqs. (C3) and (2.6), one can derive the fermionic contribution to the $hZ\gamma$

TABLE I. Value of the coefficients $l^{L,R}_{[r,r']}$ defined in Eq. (3.9) for diagrams with fermions in the $(\mathbf{1},\mathbf{1})$, $(\mathbf{2},\mathbf{2})$, $(\mathbf{1},\mathbf{3})$ and $(\mathbf{3},\mathbf{1})$ of $SO(4) \sim SU(2)_L \times SU(2)_R$. The coefficients not shown in the Table are vanishing.

$[r, r']$	$[(\mathbf{2},\mathbf{2}),(\mathbf{1},\mathbf{1})]$	$[(\mathbf{2},\mathbf{2}),(\mathbf{1},\mathbf{3})]$	$[(\mathbf{1},\mathbf{3}),(\mathbf{2},\mathbf{2})]$	$[(\mathbf{2},\mathbf{2}),(\mathbf{3},\mathbf{1})]$	$[(\mathbf{3},\mathbf{1}),(\mathbf{2},\mathbf{2})]$
$l^L_{[r,r']}$	1	$+3/4$	0	$-1/4$	1
$l^R_{[r,r']}$	1	$-1/4$	1	$+3/4$	0

[9]Notice that the expressions of c_{d1} and c_{d2} are LR symmetric, as required since the operators O_{d1}, O_{d2} are even under P_{LR}. The corresponding LR-odd combinations vanish because the functions $A(m_r, m_{r'})$, $B(m_r, m_{r'})$ are symmetric under the exchange $r \leftrightarrow r'$ and due to the sum rule (3.17).

vertex. In particular, in a theory with composite fermions only in the $(\mathbf{1},\mathbf{1})$ and $(\mathbf{2},\mathbf{2})$ representations, the contribution to c_4^- (hence to $c_{Z\gamma}$) vanishes identically, since $l^L_{[(\mathbf{2},\mathbf{2}),(\mathbf{1},\mathbf{1})]} = l^R_{[(\mathbf{2},\mathbf{2}),(\mathbf{1},\mathbf{1})]}$ and $l^{L,R}_{[(\mathbf{1},\mathbf{1}),(\mathbf{2},\mathbf{2})]} = 0$. This is expected, since the fermionic sector in this case possesses an accidental P_{LR} invariance. When fermions in the $(\mathbf{1},\mathbf{3})$ and $(\mathbf{3},\mathbf{1})$ are present, however, the contribution to c_4^- is nonvanishing provided P_{LR} is broken either by the couplings ($\zeta_{13} \neq \zeta_{31}$) or in the spectrum ($m_{(\mathbf{1},\mathbf{3})} \neq m_{(\mathbf{3},\mathbf{1})}$).

It is interesting to notice that although the function C is logarithmically divergent, the contribution to c_4^- from Eq. (3.16) is finite, since the coefficients $l^{L,R}_{[r,r']}$ satisfy the sum rule

$$(l^L_{[r,r']} - l^R_{[r,r']}) + (l^L_{[r',r]} - l^R_{[r',r]}) = 0 \quad \text{for any } r, r'. \tag{3.17}$$

This identity can be directly checked on the coefficients of Table I, and a simple argument shows that it holds in general for any pair (r, r'). The proof goes as follows. When computing the 1-loop diagram of Fig. 1, it is useful to treat E_μ and d_μ as external backgrounds coupled to the fermions. Let us then turn on E_μ along the diagonal $U(1)_{L+R}$ subgroup of $SO(4) \sim SU(2)_L \times SU(2)_R$, under which $d^1_\mu \pm i d^2_\mu$ has charge ± 1 and d^3_μ and d^4_μ have charge 0. By charge conservation there are only two possible diagrams (plus their crossings) as in Fig. 1: one with d^1_μ and d^2_ν at the two lower vertices, the other with d^3_μ and d^4_ν. Since $d^{1,2,3}_\mu$ are odd while d^4_μ is even under P_{LR}, the second diagram contributes to O_4^-, while the first renormalizes O_4^+.[10] We thus concentrate on the diagram with $d^3_\mu d^4_\nu$ and notice that the fermions circulating in the loop must all have the same $U(1)_{L+R}$ charge. Let λ^3_{ij} and λ^4_{ij} be the coupling strengths of two same-charge fermions i and j, respectively, to d^3_μ and d^4_μ (in the fermions' mass eigenbasis). For a given diagram with fermions i and j in the loop, the log-divergent part is thus proportional to $(\lambda^3_{ij}\lambda^4_{ji})$. Due to the antisymmetry of the loop function, $I_{\alpha\mu\nu}(p_1, p_2, m_i, m_j) = -I_{\alpha\nu\mu}(p_2, p_1, m_i, m_j)$, the log-divergent part of the crossed diagram is instead proportional to $-(\lambda^4_{ij}\lambda^3_{ji})$. The sum then vanishes after summing over all fermions i, j with the same charge. This proves that there is no log-divergent contribution to c_4^- from 1-loop fermion diagrams, hence the sum rule (3.17) must hold for any pair of $SO(4)$ representations (r, r'). In general, a logarithmic divergence $\log(\Lambda/m)$ is associated with the running of a Wilson coefficient from the cutoff scale Λ down to the fermion mass scale m. The above argument thus shows that there is no RG running of c_4^- induced by 1-loop diagrams of fermions above their mass scale, while such running is present in general for c_4^+.

[10] A more direct way to see this is the following: the $SO(4)$ generators satisfy $t^{3_L}_{34} = -t^{3_R}_{34}$, $t^{3_L}_{12} = t^{3_R}_{12}$, which implies $O_4^- = -i(t^{a_L}_{\hat{a}\hat{b}} E^{a_L}_{\mu\nu} - t^{a_R}_{\hat{a}\hat{b}} E^{a_R}_{\mu\nu}) d^{\hat{a}}_\mu d^{\hat{b}}_\nu \supset v_{\mu\nu} d^3_\mu d^4_\nu$, $O_4^+ = -i(t^{a_L}_{\hat{a}\hat{b}} E^{a_L}_{\mu\nu} + t^{a_R}_{\hat{a}\hat{b}} E^{a_R}_{\mu\nu}) d^{\hat{a}}_\mu d^{\hat{b}}_\nu \supset v_{\mu\nu} d^1_\mu d^2_\nu$ in the background $v_\mu = E^{3L}_\mu = E^{3R}_\mu$. In terms of physical fields, O_4^- contains a term $\gamma^{\mu\nu}(\partial_\mu h) Z_\nu$, while O_4^+ contains $\gamma^{\mu\nu} W^+_\mu W^-_\nu$.

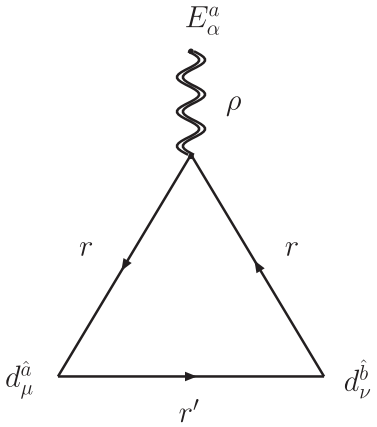

FIG. 2. Mixed rho-fermion contribution to the Green function $\langle E^a_\alpha d^{\hat{a}}_\mu d^{\hat{b}}_\nu \rangle$ which arises at the 1-loop level.

So far we have considered 1-loop diagrams with only composite fermions. There are also diagrams where both fermions and spin-1 resonances can circulate. At the 1-loop level, however, $\rho^{L,R}$ can only appear external to the loop due to the mixing with E_μ from its mass term and a coupling to the fermions of the form

$$\bar{\chi}_r(\rho_\mu - E_\mu)\gamma^\mu \chi_r. \tag{3.18}$$

The corresponding diagram is shown in Fig. 2. It is easy to see that its contribution to c_4^- vanishes at leading order: integrating out the ρ through the equations of motion generates only four-fermion operators, which in turn do not contribute at the 1-loop level. In general, the tree-level exchange of the ρ in the diagram of Fig. 2 leads to a form-factor correction to the vertex of E_μ with the fermionic current. For $q^2 = (p_1 + p_2)^2 \ll m_\rho^2$, such a form factor correction is of order q^2/m_ρ^2 and is thus suppressed compared to the direct interaction from the fermions' kinetic terms.

C. Two models

We have already alluded to the fact that a generic P_{LR}-violating strong dynamics can lead to unacceptably large corrections to the $Z\bar{b}b$ vertex [14]. Here we sketch two simple models where the breaking of P_{LR} is communicated to the $Z\bar{b}b$ vertex in a suppressed way, such that a sizable correction to $h \to Z\gamma$ is phenomenologically allowed.

1. Model 1

In the first model, which is a low-energy simplified version of the MCHM5 [16], the composite fermions fill

two fundamental representations of $SO(5)$, with $U(1)_X$ charge $+2/3$ and $-1/3$, respectively:

$$\begin{aligned}\chi_5 &= (\mathbf{1},\mathbf{1})_{2/3} + (\mathbf{2},\mathbf{2})_{2/3} \\ \chi_5' &= (\mathbf{1},\mathbf{1})_{-1/3} + (\mathbf{2},\mathbf{2})_{-1/3}.\end{aligned} \tag{3.19}$$

The spectrum of composite states also includes a ρ^L and ρ^R, while we omit for simplicity spin-1 states transforming as bifundamentals of $SU(2)_L \times SU(2)_R$. The Lagrangian can be written as $\mathcal{L} = \mathcal{L}_{\rm elem} + \mathcal{L}_{\rm comp} + \mathcal{L}_{\rm mix}$, where $\mathcal{L}_{\rm elem}$ describes the elementary fields in isolation and the expression of the composite Lagrangian $\mathcal{L}_{\rm comp}$ is as in Eqs. (3.2) and (3.7). The term $\mathcal{L}_{\rm mix}$ accounts for the mixing of the elementary to composite fermions:

$$\begin{aligned}\mathcal{L}_{\rm mix} = {}& \lambda_q \bar{q}_L P_q U(\pi)\chi_5 + \lambda_q' \bar{q}_L P_q U(\pi)\chi_5' \\ & + \lambda_t \bar{t}_R P_t U(\pi)\chi_5 + \lambda_b \bar{b}_R P_b U(\pi)\chi_5' + \text{H.c.},\end{aligned} \tag{3.20}$$

where $P_{q,t,b}$ project out the components of the composite fields with the electroweak quantum numbers of the corresponding elementary fields. The P_{LR} invariance is taken to be maximally violated in the spin-1 sector, but is accidentally preserved in the fermion sector. If $\lambda_q' \ll \lambda_q$, then the $Z\bar{b}b$ coupling is protected from large corrections since for $\lambda_q' = 0$ there is no operator at leading order in the derivative expansion which can modify it [15]. A small λ_q'/λ_q can in fact naturally arise from the RG running of the full theory and explain the hierarchy between the top and bottom masses if $\lambda_t \simeq \lambda_b$ [16]. We note in passing that at tree level the correction to $Z\bar{b}b$ from a $\rho^{L,R}$ is always vanishing at leading order in the derivative expansion, as can be easily checked by using the equations of motion $\rho_\mu = E_\mu + O(p^3)$ in Eq. (3.18). This is because the shift induced by the exchange of the ρ is exactly compensated by the additional interaction $\bar{\chi}\gamma^\mu E_\mu \chi = \bar{\chi}\gamma^\mu (H^\dagger i\overleftrightarrow{D}_\mu H + \cdots)\chi$ required by $SO(5)$ invariance and included in the term of Eq. (3.18). A nonvanishing $Z\bar{b}b$ will, however, arise in general at the 1-loop level in the absence of a symmetry protection. In the model under consideration such a protection comes from the accidental P_{LR} symmetry of the fermionic sector, which also implies that the vertex $hZ\gamma$ in this case is generated only by the ρ exchange; the value of $c_{Z\gamma}$ is thus given by Eq. (3.6).

2. Model 2

In the second model the composite fermions fill one fundamental plus one antisymmetric representation of $SO(5)$, with $U(1)_X$ charge $+2/3$ and $-1/3$, respectively:

$$\begin{aligned}\chi_5 &= (\mathbf{1},\mathbf{1})_{2/3} + (\mathbf{2},\mathbf{2})_{2/3} \\ \chi_{10} &= (\mathbf{2},\mathbf{2})_{-1/3} + (\mathbf{1},\mathbf{3})_{-1/3} + (\mathbf{3},\mathbf{1})_{-1/3}.\end{aligned} \tag{3.21}$$

As before, the Lagrangian can be divided into an elementary and a composite part plus a mixing term

$$\begin{aligned}\mathcal{L}_{\rm mix} = {}& \lambda_q^{10} \bar{q}_L P_q U(\pi)\chi_{10} + \lambda_q^5 \bar{q}_L P_q U(\pi)\chi_5 \\ & + \lambda_t^{10} \bar{t}_R P_t U(\pi)\chi_{10} + \lambda_t^5 \bar{t}_R P_t U(\pi)\chi_5 \\ & + \lambda_b \bar{b}_R P_b U(\pi)\chi_{10} + \text{H.c.}\end{aligned} \tag{3.22}$$

In this case the fermionic sector is not in general P_{LR} invariant, so both loops of composite fermions and the tree-level exchange of the ρ can contribute to generate the $hZ\gamma$ vertex. Using Eqs. (3.6), (3.16), and (2.6) we find

$$\begin{aligned}c_{Z\gamma} = {}& \frac{g^2}{2}\sin^2\theta(\alpha_{1L} - \alpha_{1R}) \\ & + \frac{g^2}{4} N_\chi \sin^2\theta [|\zeta_{13}|^2 (C(m_{(\mathbf{2},\mathbf{2})}, m_{(\mathbf{1},\mathbf{3})}) \\ & - C(m_{(\mathbf{1},\mathbf{3})}, m_{(\mathbf{2},\mathbf{2})})) - |\zeta_{31}|^2 (C(m_{(\mathbf{2},\mathbf{2})}, m_{(\mathbf{3},\mathbf{1})}) \\ & - C(m_{(\mathbf{3},\mathbf{1})}, m_{(\mathbf{2},\mathbf{2})}))].\end{aligned} \tag{3.23}$$

The shift to $Z\bar{b}b$ is suppressed for λ_q^{10} small, since no effect can arise from the P_{LR}-preserving coupling λ_q^5 [14]. As before, a small $\lambda_q^{10}/\lambda_q^5$ can arise naturally from the RG flow of the full theory, and can explain the hierarchy between the top and bottom masses if $\lambda_t^{10} \ll \lambda_t^5 \simeq \lambda_b$.

IV. *S* PARAMETER FROM LOOPS OF FERMIONIC RESONANCES

In the previous section we have seen that loops of composite fermions generate the operator O_{d1} through the triangle diagram of Fig. 1; the value of the corresponding coefficient c_{d1} is given by Eq. (3.14). Since O_{d1} can be rewritten in terms of O_3^+ as in Eq. (3.15), it contributes to the S parameter. As implied by the Ward identity (3.11), the same contribution to c_{d1}, hence to S, can be derived by considering the $\langle d_\mu d_\nu \rangle$ self-energy diagram shown on the left of Fig. 3, where two different $SO(4)$ representations of fermions circulate in the loop. There is, however, an additional direct contribution to $O_3^\pm$ which comes from the $\langle E_\mu E_\nu \rangle$ self-energy diagram shown on the right of Fig. 3, where a single fermion representation r appears in the loop. Summing over r, we find

$$c_3^\pm = \frac{N_\chi}{8}\sum_r (C_L[r] \pm C_R[r]) A(m_r, m_r), \tag{4.1}$$

where for the fundamental representation $C_L[(\mathbf{2},\mathbf{2})] = C_R[(\mathbf{2},\mathbf{2})] = 1$, for the adjoint $C_R[(\mathbf{1},\mathbf{3})] = C_L[(\mathbf{3},\mathbf{1})] = 2$

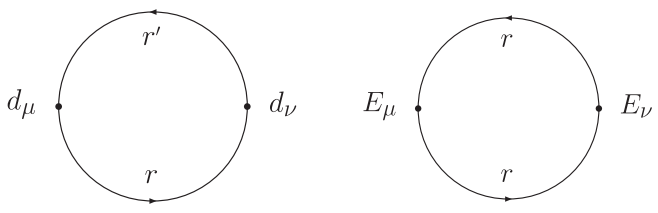

FIG. 3. 1-loop diagrams contributing to the $\langle d_\mu d_\nu \rangle$ (left) and $\langle E_\mu E_\nu \rangle$ (right) self-energies.

and $C_L[(\mathbf{1},\mathbf{3})] = C_R[(\mathbf{3},\mathbf{1})] = 0$, while $C_{L,R}[(\mathbf{1},\mathbf{1})] = 0$. The total contribution to the S parameter from loops of composite fermions is thus

$$\Delta S = -4\pi N_\chi \sin^2\theta \Big[\sum_r (C_L[r] + C_R[r]) A(m_r, m_r) - \sum_{r,r'} |\zeta_{[r,r']}|^2 C[r,r'] A(m_r, m_{r'}) \Big], \tag{4.2}$$

where we have conveniently defined $C[r,r'] = (1/2)\times(l^L_{[r,r']} + l^R_{[r,r']} + l^L_{[r',r]} + l^R_{[r',r]})$ and used the fact that the function $A(m_r, m_{r'})$ is symmetric in its arguments. For example, in the first model discussed in Sec. III C with fermions in the $(\mathbf{1},\mathbf{1})$ and $(\mathbf{2},\mathbf{2})$ of $SO(4)$ one has

$$\begin{aligned}\Delta S &= -8\pi N_\chi \sin^2\theta (A(m_{(\mathbf{2},\mathbf{2})}, m_{(\mathbf{2},\mathbf{2})}) - |\zeta_{11}|^2 A(m_{(\mathbf{2},\mathbf{2})}, m_{(\mathbf{1},\mathbf{1})})) \\ &= \frac{2}{3}\frac{N_\chi}{\pi}\frac{v^2}{f^2}(1-|\zeta_{11}|^2)\log\left(\frac{\Lambda^2}{\bar m^2}\right) + \text{finite terms},\end{aligned} \tag{4.3}$$

where in the second expression $\bar m$ denotes an average mass and the finite terms include the proper ratios of fermion masses. In the second model with fermions in the $(\mathbf{2},\mathbf{2})$, $(\mathbf{1},\mathbf{3})$, and $(\mathbf{3},\mathbf{1})$ we find

$$\begin{aligned}\Delta S = &-8\pi N_\chi \sin^2\theta \Big[A(m_{(\mathbf{2},\mathbf{2})}, m_{(\mathbf{2},\mathbf{2})}) + A(m_{(\mathbf{3},\mathbf{1})}, m_{(\mathbf{3},\mathbf{1})}) \\ &+ A(m_{(\mathbf{1},\mathbf{3})}, m_{(\mathbf{1},\mathbf{3})}) - \frac{3}{2}|\zeta_{13}|^2 A(m_{(\mathbf{2},\mathbf{2})}, m_{(\mathbf{1},\mathbf{3})}) \\ &- \frac{3}{2}|\zeta_{31}|^2 A(m_{(\mathbf{2},\mathbf{2})}, m_{(\mathbf{3},\mathbf{1})}) \Big] \\ = &\frac{1}{2}\frac{N_\chi}{\pi}\frac{v^2}{f^2}(2-|\zeta_{13}|^2-|\zeta_{31}|^2)\log\left(\frac{\Lambda^2}{\bar m^2}\right) \\ &+ \text{finite terms}.\end{aligned} \tag{4.4}$$

From Eqs. (4.2), (4.3), and (4.4) one can see that the S parameter is in general logarithmically divergent, as expected on dimensional grounds. The coefficient of the log can be either positive or negative depending on the value of the parameters ζ. Analogous results were first obtained in the context of technicolor theories in Ref. [22] and later rederived for $SO(4)/SO(3)$ Higgsless models by Ref. [23]. More recently, the case of $SO(5)/SO(4)$ composite Higgs theories has been discussed in Ref. [24].[11] A simple way to understand why the log divergence vanishes if the parameters ζ are equal to 1 is by noticing that in this limit the Lagrangian (3.7) can be rewritten, through a field redefinition, as the Lagrangian of a two-site model where the Higgs couplings to the composite fermions are nonderivative.[12] This implies, by simple inspection of the relevant 1-loop diagrams, that the S parameter is finite in this case. For completeness we report in Appendix E a short discussion on the connection between the CCWZ Lagrangian (3.7) and that of the two-site model.

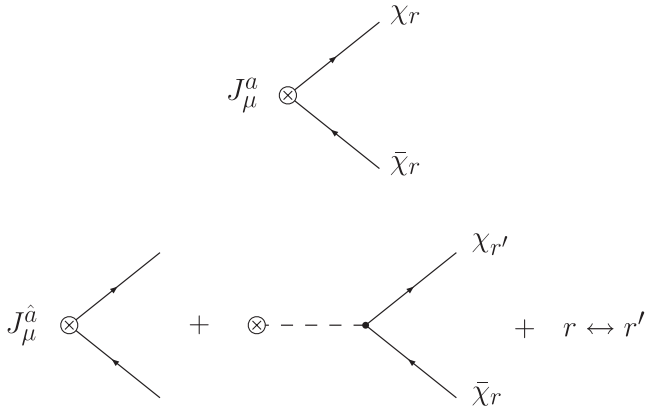

FIG. 4. Two-particle contribution to the spectral function of unbroken (upper row) and broken (lower row) currents from the composite fermions. The dashed line denotes the propagator of a NG boson.

The fact that the overall sign of S is controlled by the coefficients ζ and can be negative is more clearly understood by considering the dispersion relation obeyed by S [20]:

$$S = 4\pi \sin^2\theta \int \frac{ds}{s}[\rho_{LL}(s) + \rho_{RR}(s) - 2\rho_{BB}(s)], \tag{4.5}$$

where ρ_{LL}, ρ_{RR} and ρ_{BB} are the spectral functions, respectively, of two unbroken $[SU(2)_L$ and $SU(2)_R]$ and broken $[SO(5)/SO(4)]$ conserved currents of the strong sector. The definition of the spectral function $\rho(s)$ and the expression of the currents is reported in Appendix D for completeness. From Eq. (4.5) and from the positivity of each individual spectral function, it is clear that a negative S can occur if ρ_{BB} is sufficiently large. The leading contribution of the fermions to the spectral functions can be easily computed from the diagrams shown in Fig. 4. We find

$$\begin{aligned}\rho_{LL,RR}(q^2) &= \frac{1}{12\pi^2}\sum_r C_{L,R}[r]\lambda(q^2, m_r, m_r), \\ \rho_{BB}(q^2) &= \frac{1}{24\pi^2}\sum_{r,r'}|\zeta_{[r,r']}|^2 C[r,r']\lambda(q^2, m_r, m_{r'}),\end{aligned} \tag{4.6}$$

[11]The same results hold in five-dimensional holographic Higgs models. This can be most easily shown by solving the bulk dynamics and deriving the holographic action on the boundary where the elementary fields live; see for example Ref. [25]. In the absence of boundary terms, the 4D holographic action for the fermions has the CCWZ form with $\zeta = 1$. This is indeed the reason why previous 1-loop calculations in the context of 5D models found a finite S parameter; see for example Ref. [26]. Values $\zeta \neq 1$ can be obtained by introducing the boundary term $F^{\hat a}_{\mu 5}\bar\psi T^{\hat a}\gamma^\mu\psi$, since by using the equations of motion in the bulk it follows $F^{\hat a}_{\mu 5} \propto d^{\hat a}_\mu(\pi)$.

[12]The same observation was recently made by Ref. [24].

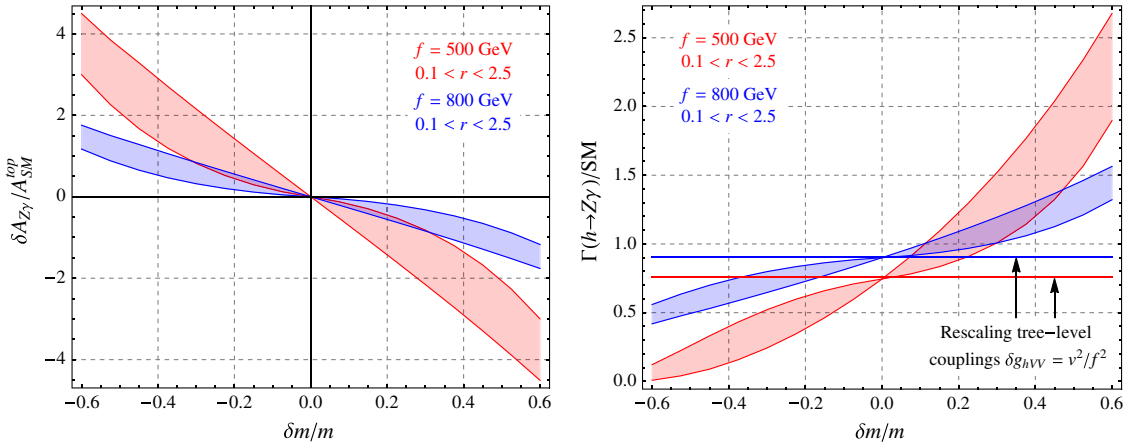

FIG. 5 (color online). Left: shift of the $h \to Z\gamma$ decay amplitude in units of the SM top contribution, $\delta A/A_{\rm SM}^{\rm top}$, in the second model of Sec. III C as a function of the LR mass splitting. Right: total decay rate of $h \to Z\gamma$ normalized to its SM value, $\Gamma/\Gamma_{\rm SM}$, in the same model. The left plot assumes one family of colored fermions ($N_\chi = 3$), while the right plot assumes three degenerate families of composites ($N_\chi = 9$). The horizontal lines indicate the value obtained by including only the effect of the modified tree-level Higgs couplings.

where we have defined

$$\lambda(q^2, m_r, m_{r'}) = \left(1 - \frac{(m_r - m_{r'})^2}{q^2}\right)\left(1 + \frac{(m_r + m_{r'})^2}{2q^2}\right) \times \sqrt{\left(1 + \frac{m_r^2 - m_{r'}^2}{q^2}\right)^2 - 4\frac{m_r^2}{q^2}}. \quad (4.7)$$

By inserting these expressions into the dispersion relation (4.5), one reobtains the result of Eq. (4.2). Since ρ_{BB} is proportional to $|\zeta_{[r,r']}|^2$, it is clear that for sufficiently large $|\zeta_{[r,r']}|$ the S parameter will become negative.

V. NUMERICAL RESULTS AND DISCUSSION

In this paper we have focused on the virtual effects due to purely composite states. The Higgs decay rate to $Z\gamma$ and the S parameter are two low-energy observables extremely sensitive to such effects.[13] It is well know that the tree-level contribution to S from spin-1 resonances is large and poses tight constraints on the scale of compositeness. We have seen that the exchange of ρ^L and ρ^R generates the effective interaction $hZ\gamma$ also at tree level, provided their masses and couplings are not P_{LR} symmetric. This leads to a correction to the $h \to Z\gamma$ decay rate that is potentially larger than that due to the $O(v^2/f^2)$ shifts in the tree-level Higgs couplings from the nonlinear σ-model Lagrangian. This is the case unless the coefficients of the operators Q_{1L} and Q_{1R} are loop suppressed, as happens for example in Holographic Higgs theories. The contribution from fermionic resonances arises at the 1-loop level, and can be numerically large. The main reason for this is that loops of pure composites are sensitive to the multiplicity of states arising from the strong dynamics. In particular all the composite fermion species, including the partners of SM light quarks and leptons, will circulate in the loop regardless of how strongly mixed with the elementary fermions they are. The multiplicity factor N_χ can then partly compensate for the 1-loop suppression, giving large shifts to both the S parameter and the $h \to Z\gamma$ rate.[14]

To illustrate the size of the effects we have been discussing, the left plot of Fig. 5 shows the shift to the $h \to Z\gamma$ decay amplitude in units of the SM top contribution, $\delta A/A_{\rm SM}^{\rm top}$, due to one family of colored fermions (composite quarks) transforming as a **10** + **5** of $SO(5)$ (second model of Sec. III C with $N_\chi = 3$). As discussed in Sec. III B, the correction comes entirely from the **10**, hence the relevant parameters are the following: the scale of compositeness f, the coefficients ζ_{13}, ζ_{31}, and two ratios of masses which we conveniently define to be $\delta m/m \equiv (m_{(\mathbf{3},\mathbf{1})} - m_{(\mathbf{1},\mathbf{3})})/(m_{(\mathbf{3},\mathbf{1})} + m_{(\mathbf{1},\mathbf{3})})$ and $r \equiv m_{(\mathbf{2},\mathbf{2})}/(m_{(\mathbf{3},\mathbf{1})} + m_{(\mathbf{1},\mathbf{3})})$. For simplicity we fix $\zeta_{13} = \zeta_{31} = 1$, so that the amount of P_{LR} breaking is fully controlled by $\delta m/m$. The plot shows the relative shift $\delta A/A_{\rm SM}^{\rm top}$ as a function of $\delta m/m$ for two representative values $f = 500$ GeV and $f = 800$ GeV. The red and blue bands are obtained by varying r in the interval $0.1 < r < 2.5$. By rescaling ζ_{13} and ζ_{31} by

[13]We are particularly grateful to John Terning for drawing our attention to the possibility of correlation between these two effects.

[14]One might worry that a large multiplicity factor N_χ could invalidate the perturbative expansion. However, the light Higgs mass already indicates that composite fermions must be somewhat more weakly coupled than other resonances; see for example Refs. [27,28]. With ~1 TeV fermion masses and $f = 500$–800 GeV, for example, the coupling strength $g_* = M/f$ is sufficiently small to allow a perturbative expansion controlled by the loop parameter $N_\chi(g_*^2/16\pi^2)$.

a common factor ζ, δA goes like ζ^2, though even without such an enhancement we see that shifts of several times the SM top amplitude are possible for large mass splittings. The right plot of Fig. 5 shows the total decay rate normalized to its SM value, this time for three degenerate families of colored fermions (second model of Sec. III C with $N_\chi = 9$). The horizontal lines indicate the value obtained by including only the effect of the modified tree-level Higgs couplings discussed above. Since in the SM the W loop contribution largely dominates that of the top quark, the effect from the modified tree-level couplings is a suppression of the decay rate by a factor $(g_{WWh}/g^{\rm SM}_{WWh})^2 = (1 - v^2/f^2)$. The correction from the 1-loop exchange of composite fermions is included in addition to this effect, and can further suppress or enhance the decay rate depending on the sign of the mass splitting $\delta m/m$.

It is interesting to derive the contribution to the S parameter in this model and analyze the impact of a sizable correction to the $h \to Z\gamma$ decay rate on the EWPT. This is illustrated by Fig. 6 in the (S, T) plane.[15] The plot shows the region spanned by varying f and $\zeta \equiv \zeta_{13} = \zeta_{31} = \zeta_{11}$ due to the IR correction to S and T from modified Higgs couplings and to the 1-loop correction to S from three degenerate families of composite fermions [Eqs. (4.3) and (4.4) with $N_\chi = 9$]. We have fixed the cutoff scale to $\Lambda = 5$ TeV and have chosen the following spectrum of composite masses: $m_{(1,1)} = 1.5$ TeV, $m_{(2,2)} = 2.0$ TeV, $m_{(3,1)} = 3.4$ TeV, $m_{(1,3)} = 1.0$ TeV, so that $r = 0.45$ and $\delta m/m = 0.55$. Even in the absence of additional contributions to T, the correction to S from loops of composite fermions can compensate the shift due to the modified couplings of the Higgs to the SM vector bosons and bring the theory point back into the 95% probability contour. For example, for $f = 800$ GeV [i.e. $(v/f)^2 \simeq 0.09$)] one has $\Delta S \simeq 0.83(1 - \zeta^2)$ from composite fermions, so that $\zeta \sim 1.1$ gives $\Delta S \sim -0.2$ as required to offset the IR shift. Correspondingly, the correction to the $h \to Z\gamma$ rate is sizable and of order 70% of the SM value. In general, the 1-loop contribution to S is large and only values $\zeta \simeq 1$ are viable. The fact that EWPT select a narrow range of ζ is directly relevant for the experimental searches of the fermionic resonances, since ζ controls their single production [28].[16] The exact allowed range depends, however, on possible additional contributions to S and T. For example, for $g_{\rho_L} = 3$ and $f = 800$ GeV the contribution to S from a ρ^L is $\Delta S \simeq 0.13$ [see Eq. (3.5)], which increases the preferred value of ζ by only 5%–10%, which in turn corresponds to an increase of the $h \to Z\gamma$ rate by 10%–20%.

[15]The probability contours have been derived by using the fit on (S, T) performed by the GFitter Collaboration [29]. Similar results are obtained by using the more recent analysis of Ref. [30].

[16]We thank Minho Son for drawing our attention to this point.

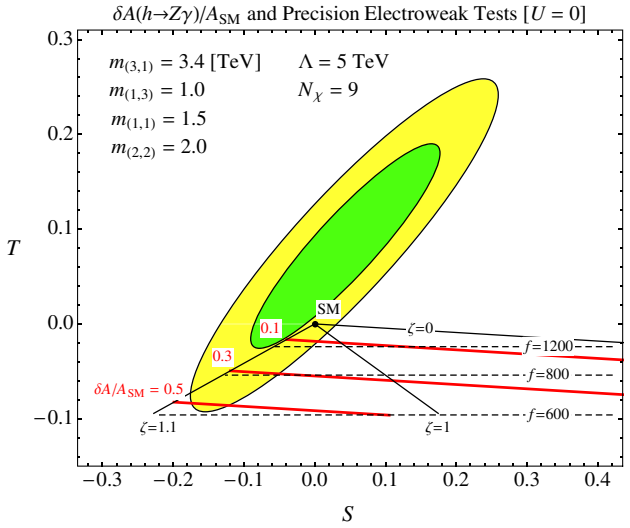

FIG. 6 (color online). Region spanned in the plane (S, T) when varying f and $\zeta = \zeta_{13} = \zeta_{31} = \zeta_{11}$ in the second model of Sec. III C, as due to ΔS from loops of composite fermions and to the IR correction to S and T from modified Higgs couplings. The green (inner) and yellow (outer) areas indicate the regions with 68% and 95% probability [29]. Dashed (solid black) lines indicate the trajectories of fixed f (ζ). The thicker solid red lines indicate the isocurves of constant $h \to Z\gamma$ decay amplitude.

The tuning required to comply with the EW precision tests can be alleviated if an additional positive contribution to T is present. This can arise from loops of fermionic resonances, as recently discussed in Ref. [24]; see also Ref. [31]. Unlike the S parameter, however, T is generated only if the custodial invariance of the strong dynamics is broken, and therefore no correction can come from purely composite states. In theories with partial compositeness and flavor anarchy of the strong sector, the leading contribution arises from loops of elementary top quarks. For example, if t_R mixes with a composite singlet of $SU(2)_L \times SU(2)_R$, as in model 1 of Sec. III C, the only breaking of custodial symmetry in the fermionic sector comes from λ_q. As a spurion analysis shows [7], one needs four powers of λ_q to generate T, which implies a finite result (i.e. independent of the cutoff scale Λ). A subleading contribution comes from loops of spin-1 resonances and elementary hypercharge vector bosons. In this case the breaking of custodial symmetry comes from the hypercharge coupling, and two powers of g' are sufficient to generate T. Table II summarizes the naive estimates of the corrections to $\hat{S} \equiv (\alpha_{\rm em}/4\sin^2\theta_W)S$ and $\hat{T} \equiv \alpha_{\rm em} T$. As before, $g_* \sim M/f \sim g_\rho$ denotes the coupling strength of the composite states and M their mass scale. The first two lines show the corrections discussed above that arise from the exchange of composite fermions and spin-1 resonances. These are short-distance effects at the scale M, which in the language of the Higgs effective Lagrangian correspond to threshold corrections to the Wilson coefficients $\bar{c}_W + \bar{c}_B$

TABLE II. Naive estimates of the UV corrections (from composite fermions χ and spin-1 resonances ρ) and IR corrections (from NG bosons and the top quark) to $\hat{S} \equiv (\alpha_{\rm em}/4\sin^2\theta_W)S$ and $\hat{T} \equiv \alpha_{\rm em} T$.

		$\Delta\hat{S}$	$\Delta\hat{T}$
UV	χ	$N_\chi \frac{g^2}{16\pi^2}\left(\frac{v^2}{f^2}\right)\log\left(\frac{\Lambda}{M}\right)$	$N_c \frac{\lambda_q^2}{16\pi^2}\left(\frac{v^2}{f^2}\right)\frac{\lambda_q^2}{g_*^2}$
	ρ	$\left(\frac{v^2}{f^2}\right)\left(\frac{g^2}{g_*^2}\right)$	$\frac{g'^2}{16\pi^2}\left(\frac{v^2}{f^2}\right)\log\left(\frac{\Lambda}{M}\right)$
IR	NGB	$\frac{g^2}{16\pi^2}\left(\frac{v^2}{f^2}\right)\log\left(\frac{M}{\mu}\right)$	$\frac{g'^2}{16\pi^2}\left(\frac{v^2}{f^2}\right)\log\left(\frac{M}{\mu}\right)$
	top	$N_c \frac{g^2}{16\pi^2}\left(\frac{v^2}{f^2}\right)\frac{\lambda_q^2}{g_*^2}\log\left(\frac{M}{\mu}\right)$	$N_c \frac{y_t^2}{16\pi^2}\left(\frac{v^2}{f^2}\right)\frac{\lambda_q^2}{g_*^2}\log\left(\frac{M}{\mu}\right)$

and $\bar{c}_T$; see Ref. [6]. There are, however, additional contributions which are generated by the exchange of light SM fields below M and are thus associated with the RG evolution of the Wilson coefficients down to IR scales $\mu \approx m_Z$. The largest corrections arise from loops of NG bosons (i.e. longitudinally polarized W and Z and the Higgs boson) and of top quarks, and correspond to the RG evolution of $\bar{c}_W + \bar{c}_B$ and $\bar{c}_T$ due to $\bar{c}_H$ and $\bar{c}_{H\psi}$, respectively [6]. Their naive estimates are reported in the last two lines of Table II. Loops of transverse gauge bosons also lead to IR corrections which are subleading. For example, as recently pointed out by the authors of Ref. [32], 1-loop diagrams featuring one insertion of the effective $hZ\gamma$ vertex (induced by the operator $O_{HW} - O_{HB}$) give a correction to the $\hat{S}$ parameter of order

$$\Delta\hat{S} \sim \left(\frac{g^2}{16\pi^2}\right)^2\left(\frac{v^2}{f^2}\right)\log\left(\frac{M}{\mu}\right)^2. \tag{5.1}$$

Although directly linked to $h \to Z\gamma$, this is a 2-loop EW effect which is parametrically subleading compared to other IR effects and numerically smaller than the UV corrections from pure composite fermions (see Table II).

We briefly summarize our findings with the following conclusions.

(i) The decay mode $h \to Z\gamma$, unlike other loop-mediated processes of a Nambu-Goldstone composite Higgs boson, is subject to NP corrections that are not suppressed by the Goldstone symmetry itself. While new contributions to the hgg and $h\gamma\gamma$ contact interactions of the effective Lagrangian are typically (and observably) small, a highly nonstandard $hZ\gamma$ interaction is possible and consistent with the symmetry that is assumed to be responsible for stabilizing the weak scale.

(ii) Generating a large $hZ\gamma$ interaction in the absence of significant breaking of the Goldstone symmetry relies on the intervention of states arising from a strong sector that breaks a left-right symmetry, P_{LR}. Provided this breaking is mediated in a suppressed way to the $Zb\bar{b}$ coupling, as in the case of the models presented above, enhancements of $h \to Z\gamma$ remain phenomenologically viable.

(iii) There are two operators contributing to the S parameter that are closely related to those governing $h \to Z\gamma$, and a naive prediction would be for a tight correlation between these two observables. However, the composite Higgs boson can couple to fermions through interactions that contribute only to the two-point function of two broken currents, allowing an offsetting (negative) contribution to S such that again the viability of large corrections in $h \to Z\gamma$ is retained.

In this paper we have highlighted the anatomy of the $h \to Z\gamma$ channel, one in which a composite Higgs boson might naturally interact in a novel way that could help shed light on its origins in the absence of other, more obvious, clues. As such, this channel deserves our full attention in the continuation of Higgs study at the LHC.

ACKNOWLEDGMENTS

We would like to thank David Marzocca, Riccardo Rattazzi, Slava Rychkov, Marco Serone, Minho Son, John Terning, and Enrico Trincherini for useful discussions. We also thank Leandro Da Rold and Eduardo Pontón for pointing out a few typos in the first version of the paper. The work of A. A., R. C., and J. Ġ. was partly supported by the ERC Advanced Grant No. 267985 *Electroweak Symmetry Breaking, Flavour and Dark Matter: One Solution for Three Mysteries (DaMeSyFla).*

APPENDIX A: FORMULAS FOR THE $h \to Z\gamma$ DECAY RATE

We collect here the formulas useful for the calculation of the decay rate $h \to Z\gamma$. The partial width is given by

$$\Gamma(h \to Z\gamma) = \frac{1}{32\pi}\frac{m_h^3}{v^2}\left(1 - \frac{m_Z^2}{m_h^2}\right)^3 |A|^2, \tag{A1}$$

where A is the total decay amplitude. The SM contribution arises from loops of W vector bosons and fermions:

$$\begin{aligned} A_{\rm SM} &= A_F + A_W, \\ A_F &= -\frac{\alpha_{\rm em}}{\pi}\sum_f N_{cf} Q_f \frac{(T_f^{3L} - 2Q_f \sin^2\theta_W)}{\sin\theta_W \cos\theta_W} \\ &\quad \times [I_1(\tau_f, \lambda_f) - I_2(\tau_f, \lambda_f)], \\ A_W &= -\frac{\alpha_{\rm em}}{2\pi}\cot\theta_W \Big\{ 4(3 - \tan^2\theta_W) I_2(\tau_W, \lambda_W) \\ &\quad + \left[\left(1 + \frac{2}{\tau_W}\right)\tan^2\theta_W - \left(5 + \frac{2}{\tau_W}\right)\right] I_1(\tau_W, \lambda_W)\Big\}, \end{aligned} \tag{A2}$$

where N_{cf} and Q_f are, respectively, the number of color and the electromagnetic charge of the fermion f, and we have defined

$$\tau_f \equiv \frac{4m_f^2}{m_h^2}, \qquad \lambda_f = \frac{4m_f^2}{m_Z^2},$$
$$\tau_W = \frac{4m_W^2}{m_h^2}, \qquad \lambda_W = \frac{4m_W^2}{m_Z^2}. \tag{A3}$$

The loop functions are equal to

$$I_1(a,b) = \frac{ab}{2(a-b)} + \frac{a^2b^2}{2(a-b)^2}(f(a)-f(b)) + \frac{a^2b}{(a-b)^2}(g(a)-g(b)),$$
$$I_2(a,b) = -\frac{ab}{2(a-b)}(f(a)-f(b)), \tag{A4}$$
$$g(\tau) = \begin{cases} \sqrt{\tau-1}\arcsin(1/\sqrt{\tau}), & \tau \geq 1, \\ \frac{1}{2}\sqrt{1-\tau}[\log(\eta_+/\eta_-) - i\pi], & \tau < 1, \end{cases}$$
$$f(\tau) = \begin{cases} [\arcsin(1/\sqrt{\tau})]^2, & \tau \geq 1, \\ -\frac{1}{4}[\log(\eta_+/\eta_-) - i\pi]^2, & \tau < 1, \end{cases}$$

where $\eta_\pm \equiv (1 \pm \sqrt{1-\tau})$. In the limit in which the new physics effect can be parametrized by the effective Lagrangian of Eq. (1.3), the contribution to the decay amplitude is given by

$$A_{\rm NP} = -2c_{Z\gamma}. \tag{A5}$$

Numerically evaluating the SM contribution one finally obtains [6]

$$\frac{\Gamma(h\to Z\gamma)}{\Gamma(h\to Z\gamma)_{\rm SM}} \simeq \left| 1 + 0.01\frac{4\pi}{\alpha_{\rm em}\cos\theta_w}c_{Z\gamma}\right|^2 \simeq 1 + 0.02\frac{4\pi}{\alpha_{\rm em}\cos\theta_w}c_{Z\gamma}. \tag{A6}$$

APPENDIX B: RELATION BETWEEN DIFFERENT BASES OF OPERATORS

In this appendix we discuss the relations between our basis of operators (2.3) and those adopted in Ref. [10] (the CMPR basis for short) and Ref. [7] (the SILH Lagrangian).

The CMPR list of CP-even operators is given by

$$\mathcal{O}_3 = {\rm Tr}[(E^L_{\mu\nu})^2 - (E^R_{\mu\nu})^2],$$
$$\mathcal{O}_4^\pm = {\rm Tr}[(f^L_{\mu\nu} \pm f^R_{\mu\nu})i[d^\mu, d^\nu]],$$
$$\mathcal{O}_5^+ = {\rm Tr}((f^-_{\mu\nu})^2), \qquad \mathcal{O}_5^- = {\rm Tr}[(f^L_{\mu\nu})^2 - (f^R_{\mu\nu})^2], \tag{B1}$$

plus other two operators, $\mathcal{O}_1 = O_1$ and $\mathcal{O}_2 = O_2$, whose expansion in terms of NG bosons starts at dimension 8. Here $f^{L,R}_{\mu\nu}$ and $f^-_{\mu\nu}$ are the dressed field strengths along the $SU(2)_L \times SU(2)_R$ and $SO(5)/SO(4)$ directions [10]. We can relate the CMPR set to our basis by using the identity

$$f^L_{\mu\nu} + f^R_{\mu\nu} = E^L_{\mu\nu} + E^R_{\mu\nu} + i[d_\mu, d_\nu], \tag{B2}$$

which holds for $SO(5)/SO(4)$. We find

$$\mathcal{O}_3 = O_3^-,$$
$$\mathcal{O}_4^+ = O_4^+ - \frac{1}{2}O_2 + \frac{1}{2}O_1, \qquad \mathcal{O}_4^- = O_4^- - O_5,$$
$$\mathcal{O}_5^+ = O_3^+ + 2O_4^+ + \frac{1}{2}O_1 - \frac{1}{2}O_2, \quad \mathcal{O}_5^- = O_3^- + 2O_4^- - O_5. \tag{B3}$$

The advantage of our basis over the CMPR one is that the connection to the SILH Lagrangian is more straightforward, since only four operators start at dimension 6 when expanded in powers of the NG bosons. Also, only one operator gives an $hZ\gamma$ contact interaction.

At the dimension-6 level, the connection between our operators and those of the SILH Lagrangian is given by

$$-\frac{4f^2}{m_W^2}O_4^\pm = O_{HW} \pm O_{HB} + \cdots$$
$$-\frac{f^2}{m_W^2}O_3^\pm = O_W \pm O_B + \cdots, \tag{B4}$$

where the dots stand for dimension-8 terms. The SILH operators are defined in Eqs. (1.1) and (1.2).

APPENDIX C: LOOP FUNCTIONS

We collect here the expression of the loop functions $A(m_r, m_{r'})$, $B(m_r, m_{r'})$, $C(m_r, m_{r'})$ defined in Eq. (3.10):

$$A(m_r,m_{r'}) = \frac{1}{24\pi^2}\left[-\log\left(\frac{\Lambda^4}{m_r^2 m_{r'}^2}\right) - \frac{m_r m_{r'}(-4m_r m_{r'} + 3m_{r'}^2 + 3m_r^2)}{(m_r^2 - m_{r'}^2)^2} + \frac{(6m_r^3 m_{r'}^3 - 3m_r^2 m_{r'}^2(m_{r'}^2 + m_r^2) + m_{r'}^6 + m_r^6)\log(m_r^2/m_{r'}^2)}{(m_r^2 - m_{r'}^2)^3}\right], \tag{C1}$$

$$B(m_r, m_{r'}) = \frac{1}{24\pi^2}\frac{1}{(m_r^2 - m_{r'}^2)^3}\left[2m_{r'}^4(m_{r'}^2 - 3m_r^2)\log\left(\frac{m_{r'}^2}{\Lambda^2}\right) - 2m_r^4(m_r^2 - 3m_{r'}^2)\log\left(\frac{m_r^2}{\Lambda^2}\right) - 7m_r^4 m_{r'}^2 + 7m_r^2 m_{r'}^4 - m_{r'}^6 + m_r^6\right], \tag{C2}$$

$$C(m_r, m_{r'}) = \frac{1}{24\pi^2}\frac{1}{(m_r^2 - m_{r'}^2)^3}\left[2(3m_r^2 m_{r'} + 3m_r m_{r'}^2 - 2m_{r'}^3 - 3m_r^3)m_{r'}^3\log\left(\frac{m_{r'}^2}{\Lambda^2}\right) + 2m_r(-6m_r^3 m_{r'}^2 + 3m_r^2 m_{r'}^3 + 3m_r m_{r'}^4 - 3m_{r'}^5 + 2m_r^5)\log\left(\frac{m_r^2}{\Lambda^2}\right) + (m_r^2 - m_{r'}^2)(-6m_r^3 m_{r'} - 3m_r^2 m_{r'}^2 + 6m_r m_{r'}^3 + m_{r'}^4 + 4m_r^4)\right]. \tag{C3}$$

Unlike $C(m_r, m_{r'})$, the functions $A(m_r, m_{r'})$ and $B(m_r, m_{r'})$ are symmetric in their arguments, as can be easily verified by inspection. The effective vertex $hZ\gamma$ is proportional to the antisymmetric combination [see for example Eq. (3.23)]

$$C(m_r, m_{r'}) - C(m_{r'}, m_r) = \frac{1}{8\pi^2}\frac{1}{(m_r^2 - m_{r'}^2)^2}\left[(m_r^2 - m_{r'}^2)(-4m_r m_{r'} + m_{r'}^2 + m_r^2) + 2m_r m_{r'}(-m_r m_{r'} + m_{r'}^2 + m_r^2)\log\left(\frac{m_r^2}{m_{r'}^2}\right)\right], \tag{C4}$$

which is finite (i.e. cutoff independent) as expected by the argument of Sec. III B.

APPENDIX D: SPECTRAL FUNCTIONS AND $SO(5)$ CURRENTS

For completeness we report here the definition of the spectral function of two currents. One has

$$\rho_{\mu\nu}(q) \equiv \sum_n \delta^{(4)}(q - p_n)\langle 0|J_\mu(0)|n\rangle\langle n|J_\nu(0)|0\rangle, \tag{D1}$$

where the sum is over a complete set of states. By Lorentz covariance,

$$\rho_{\mu\nu}(q) = \frac{1}{(2\pi)^3}\theta(q^0)(q_\mu q_\nu - \eta_{\mu\nu}q^2)\rho(q^2), \tag{D2}$$

where $\rho(q^2)$ is the spectral function.

At leading order in the number of fields and derivatives, the expression of the $SO(5)$ conserved currents is (we show for simplicity only terms involving the NG bosons and the fermions)

$$J_\mu^a = \sum_r \bar{\chi}_r\gamma^\mu T^a\chi_r + \cdots, \quad a = a_L, a_R,$$
$$J_\mu^{\hat{a}} = \frac{f}{\sqrt{2}}\partial_\mu\pi^{\hat{a}} - \sum_{r,r'}(\zeta_{[r,r']}\bar{\chi}_r\gamma^\mu T^{\hat{a}}\chi_{r'} + \text{H.c.}) + \cdots. \tag{D3}$$

APPENDIX E: TWO-SITE VS CCWZ FERMIONIC LAGRANGIAN

In this appendix we briefly discuss the relation between the description of the fermion interactions in the CCWZ approach and the so-called "two-site" model Lagrangian (see for example Ref. [17]) where fermions couple to the Higgs boson only through (nonderivative) Yukawa terms.

In the general case, the CCWZ Lagrangian of composite fermions is written at leading order in the derivative expansion as

$$\mathcal{L} = \sum_r \bar{\chi}_r(i\not{\nabla} - m_r)\chi_r - \sum_{r,r'}\zeta_{[r,r']}\bar{\chi}_r\not{d}\chi'_r, \tag{E1}$$

where the sums run over all possible representations r, r' of the unbroken subgroup $\mathcal{H}$. If the composite fermions can be arranged into complete multiplets of the global group $\mathcal{G}$ (which occurs if $\mathcal{G}$ is linearly realized at high energy) and all the parameters $\zeta_{[r,r']}$ are equal to 1, it is easy to show that the Lagrangian (E1) can be rewritten in terms of a "two-site" Lagrangian by means of a field redefinition (see also Ref. [24]). In this limit, Eq. (E1) becomes

$$\mathcal{L} = \bar{\psi}\gamma^\mu(iD_\mu + U(\pi)^\dagger iD_\mu U(\pi))\psi - \sum_r m_r\overline{(P_r\cdot\psi)}(P_r\cdot\psi), \tag{E2}$$

where $\psi \equiv (\chi_1, \chi_2 \ldots)$ denotes the (possibly reducible) representation of $\mathcal{G}$, and P_r is a projector on the representation r of $\mathcal{H}$, that is, $P_r\cdot\psi \equiv \chi_r$. Also, we have used the fact that $d_\mu + E_\mu = -iU(\pi)^\dagger D_\mu U(\pi)$. We then perform the field redefinition $\Psi \equiv U\psi$, so that Ψ transforms linearly under $\mathcal{G}$: $\Psi \to g\Psi$. The Lagrangian can be reexpressed as

$$\mathcal{L} = \bar{\Psi}\gamma^\mu iD_\mu\Psi - \sum_r m_r\overline{(P_r\cdot U(\pi)^\dagger\Psi)}(P_r\cdot U(\pi)^\dagger\Psi), \tag{E3}$$

so that the Higgs interactions with fermions now come entirely from the second term and are of nonderivative type.

As an illustrative example, it is instructive to consider the $SO(5)/SO(4)$ case in which the composite fermions fill a **10** of $SO(5)$, where $\mathbf{10} = (\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{1}) + (\mathbf{2},\mathbf{2})$ under $SU(2)_L \times SU(2)_R$. The field redefinition in this case reads $\Psi_{10} = U(\pi)\psi_{10}U(\pi)^\dagger$, where $\psi_{10} = (\chi_{(2,2)}, \chi_{(1,3)}, \chi_{(3,1)})$ and both Ψ_{10} and ψ_{10} are conveniently described in 5×5 matrix notation. After the field redefinition, the Lagrangian reads

$$\mathcal{L} = \mathrm{Tr}[\bar{\Psi}_{10} i\not{\partial} \Psi_{10}] - m_+ \mathrm{Tr}[\bar{\Psi}_{10}\Psi_{10}] - (m_{(\mathbf{2},\mathbf{2})} - m_+)\mathrm{Tr}[\overline{(P_{(2,2)}\cdot U^\dagger \Psi_{10} U)}(P_{(2,2)}\cdot U^\dagger \Psi_{10} U)] - m_- \mathrm{Tr}[\overline{(P_{(3,1)}\cdot U^\dagger \Psi_{10} U)}(P_{(3,1)}\cdot U^\dagger \Psi_{10} U) - \overline{(P_{(1,3)}\cdot U^\dagger \Psi_{10} U)}(P_{(1,3)}\cdot U^\dagger \Psi_{10} U)], \quad \text{(E4)}$$

where we have defined $m_\pm = (m_{(\mathbf{3},\mathbf{1})} \pm m_{(\mathbf{1},\mathbf{3})})/2$. The action of the projectors $P_{(2,2)}$, $P_{(1,3)}$, and $P_{(3,1)}$ on an element of the algebra M is defined as

$$P_{(2,2)}\cdot M \equiv \sum_{\hat{a}} T^{\hat{a}}\,\mathrm{Tr}[T^{\hat{a}} M],$$
$$P_{(3,1)}\cdot M \equiv \sum_{a_L} T^{a_L}\,\mathrm{Tr}[T^{a_L} M], \quad \text{(E5)}$$
$$P_{(1,3)}\cdot M \equiv \sum_{a_R} T^{a_R}\,\mathrm{Tr}[T^{a_R} M].$$

The term in the second line of Eq. (E4) can be more conveniently rewritten in terms of the field $\Phi = U(\pi)\Phi_0$, where $\Phi_0 = (0,0,0,0,1)^T$, by using identities between $SO(5)$ generators. One has

$$\mathrm{Tr}[\overline{(P_{(2,2)}\cdot U^\dagger \Psi_{10} U)}(P_{(2,2)}\cdot U^\dagger \Psi_{10} U)] = \sum_{\hat{a}} \mathrm{Tr}[T^{\hat{a}} U^\dagger \bar{\Psi}_{10} U]\mathrm{Tr}[T^{\hat{a}} U^\dagger \Psi_{10} U] = 2(U^\dagger \bar{\Psi}_{10}\Psi_{10} U)_{55} = 2\Phi^\dagger \bar{\Psi}_{10}\Psi_{10}\Phi, \quad \text{(E6)}$$

where in the first equality we have made use of Eq. (E5). The term proportional to $m_{(3,1)} - m_{(1,3)}$ can be rearranged by using the identities

$$\sum_{a_L} (T^{a_L})_{ij}(T^{a_L})_{kl} - \sum_{a_R} (T^{a_R})_{ij}(T^{a_R})_{kl} = -\frac{1}{2}\epsilon^{ijkl5}$$
$$\epsilon^{ijkl5} U_{i'i} U_{j'j} U_{k'k} U_{l'l} = \epsilon^{i'j'k'l'n'} U_{n'5}\,\det(U) = \epsilon^{i'j'k'l'n'} U_{n'5}. \quad \text{(E7)}$$

One can show that

$$\mathrm{Tr}[\overline{(P_{(3,1)}\cdot U^\dagger \Psi_{10} U)}(P_{(3,1)}\cdot U^\dagger \Psi_{10} U) - \overline{(P_{(1,3)}\cdot U^\dagger \Psi_{10} U)}(P_{(1,3)}\cdot U^\dagger \Psi_{10} U)] = -\frac{1}{2}\epsilon^{mnrpk}\bar{\Psi}_{10}^{mn}\Psi_{10}^{rp}\Phi^k. \quad \text{(E8)}$$

Note that this P_{LR}-violating term is invariant under $SO(5)$ but not under $O(5)$, which is expected since P_{LR} is an element of $O(5)$ but not of $SO(5)$.[17] The Lagrangian can thus be written as

$$\mathcal{L} = \mathrm{Tr}[\bar{\Psi}_{10} i\not{\partial} \Psi_{10}] - m_+ \mathrm{Tr}[\bar{\Psi}_{10}\Psi_{10}] - 2(m_{(2,2)} - m_+)\Phi^\dagger \bar{\Psi}_{10}\Psi_{10}\Phi + \frac{m_-}{2}\epsilon^{mnrpk}\bar{\Psi}_{10}^{mn}\Psi_{10}^{rp}\Phi^k. \quad \text{(E9)}$$

APPENDIX F: CALCULATION OF THE FERMIONIC CONTRIBUTION TO THE DECAY RATE $h \to Z\gamma$ IN THE MASS EIGENSTATE BASIS

In the main text we have described the calculation of the contribution of composite fermions to the $h \to Z\gamma$ decay rate by using the effective field theory approach. We have thus expanded the loop integrals keeping only the leading terms suppressed by two powers of the NP scale and neglecting more suppressed contributions. Also, we performed our calculation by neglecting the elementary-composite mixing terms in the fermionic sector, which explicitly violate the Goldstone symmetry. It is, however, possible, and somehow straightforward, to perform a complete calculation of the 1-loop contribution of heavy fermions to the decay amplitude of $h \to Z\gamma$ without making approximations. In this appendix we describe such a calculation and show that it reduces to the results presented in the text in the proper limit.

In the SM the fermionic contribution to the decay rate comes from 1-loop diagrams with only one particle species circulating in the loop. In a generic NP model on the other hand, such as the composite Higgs theories under examination in this paper, there will be several fermions with the same electromagnetic charge and off-diagonal couplings to the Z and the Higgs boson. It is thus possible to have two different species of fermions circulating in the same loop for $h \to Z\gamma$, as shown in Fig. 7. In the basis of mass eigenstates and focusing on fermions with the same electric charge, the terms of interest in the Lagrangian can be written in full generality as follows:

$$\mathcal{L} = \bar{\psi}^i (i\not{\partial} - m_i)\psi^i + \frac{1}{2}\bar{\psi}^i h(\lambda^h_{ij} + i\gamma^5 \bar{\lambda}^h_{ij})\psi^j + \frac{1}{2}\bar{\psi}^i Z_\mu \gamma^\mu (\lambda^Z_{ij} + \gamma_5 \bar{\lambda}^Z_{ij})\psi^j, \quad \text{(F1)}$$

where a sum over all mass eigenstates i, j is left understood and the matrices $\lambda^{h,Z}$, $\bar{\lambda}^{h,Z}$ are all Hermitian. Possible derivative interactions of the Higgs boson with the

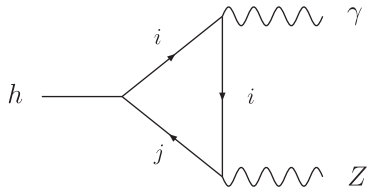

FIG. 7. 1-loop contribution to $h \to Z\gamma$ from fermion species i and j.

[17] We define $P_{LR} = \mathrm{diag}(-1,-1,-1,+1,+1)$ so that it is unbroken in the $SO(4)$ vacuum.

fermions can always be rewritten as in Eq. (F1) by integration by parts and use of the equations of motion. We will show this in detail in the following. By calculating the diagram of Fig. 7 and summing over i, j one obtains the following decay amplitude:

$$A_{\rm NP} = -\frac{eQ_\psi}{4\pi^2} v \sum_{i,j} [\lambda^h_{ij}\lambda^Z_{ji} F(m_i, m_j, m_h, m_Z) + i\bar{\lambda}^h_{ij}\bar{\lambda}^Z_{ji} F(m_i, -m_j, m_h, m_Z)]. \tag{F2}$$

The final result is thus obtained by further summing the contributions from fermions with different electric charge. The loop function is equal to [33]

$$F(m_1, m_2, m_h, m_Z) = \frac{1}{2(m_h^2 - m_Z^2)}\left\{\frac{m_Z^2(m_1+m_2)}{(m_h^2-m_Z^2)}[B_0(m_Z^2, m_1, m_2) - B_0(m_h^2, m_1, m_2)] + \left[-\frac{m_1}{2}(2m_1(m_1+m_2) - m_h^2 + m_Z^2)C_0(m_1, m_1, m_2) + (m_1 \leftrightarrow m_2)\right] - (m_1 + m_2)\right\}, \tag{F3}$$

where $B_0(p^2, m_1, m_2)$, $C_0(p_1^2, p_2^2, (p_1+p_2)^2, m_1, m_2, m_3)$ are two- and three-points Passarino-Veltman functions (for a review see Ref. [34]), and we define for convenience $C_0(m_1, m_2, m_3) \equiv C_0(0, m_Z^2, m_h^2, m_1, m_2, m_3)$. In the equal mass limit $m_1 = m_2$ the loop function reduces to the SM one [see Eqs. (A2) and (A4)]:

$$F(m, m, m_h, m_Z) = \frac{1}{2m}(I_1(\tau_f, \lambda_f) - I_2(\tau_f, \lambda_f)). \tag{F4}$$

In the limit of heavy fermions, $m_1^2, m_2^2 \gg m_h^2, m_Z^2$, the loop function reduces to

$$F(m_1, m_2, 0, 0) = -\frac{1}{8(m_1-m_2)^3(m_1+m_2)^2}\left[(m_1^2-m_2^2)(m_1^2-4m_2m_1+m_2^2) + 4m_1m_2(m_1^2-m_2m_1+m_2^2)\log\left(\frac{m_1}{m_2}\right)\right]. \tag{F5}$$

If the Higgs boson is a NG boson, its interactions to the fermions can be only of derivative type, as shown for example in Eq. (E1). In the mass-eigenstate basis the Lagrangian can thus be written as

$$\mathcal{L} = \bar{\psi}^i(i\not{\partial} - m_i)\psi^i + \frac{1}{2}\bar{\psi}^i(\partial_\mu h)\gamma^\mu(T^h_{ij} + \gamma^5\bar{T}^h_{ij})\psi^j + \frac{1}{2}\bar{\psi}^i Z_\mu \gamma^\mu(\lambda^Z_{ij} + \gamma_5\bar{\lambda}^Z_{ij})\psi^j, \tag{F6}$$

where T^h, $\bar{T}^h$ are Hermitian. By integrating by parts and using the fermions' equations of motion, the above Lagrangian can be rewritten as

$$\mathcal{L} = \bar{\psi}^i(i\not{\partial} - m_i)\psi^i + \frac{1}{2}\bar{\psi}^i h[i(m_j - m_i)T^h_{ij} + i\gamma^5(m_i + m_j)\bar{T}^h_{ij}]\psi^j + \frac{1}{2}\bar{\psi}^i Z_\mu\gamma^\mu(\lambda^Z_{ij} + \gamma_5\bar{\lambda}^Z_{ij})\psi^j + O[(h^2\psi^2), (hZ\psi^2)]. \tag{F7}$$

This is of the form (F1) upon identifying

$$\lambda^h_{ij} = i(m_j - m_i)T^h_{ij}, \qquad \bar{\lambda}^h_{ij} = (m_i + m_j)\bar{T}^h_{ij}. \tag{F8}$$

At the 1-loop level the $O(h^2\psi^2)$ terms are irrelevant for $h \to Z\gamma$ and can be safely ignored. The $O(hZ\psi^2)$ terms also do not contribute to $h \to Z\gamma$: they lead to (two-pointlike) diagrams whose loop function has a transverse Lorentz structure, $(q_\mu q_\nu - g_{\mu\nu}q^2)$, where q is the photon momentum, hence the corresponding Feynman amplitude vanishes identically for an on-shell photon. The final expression of the amplitude is thus given by Eq. (F2), with λ^h and $\bar{\lambda}^h$ given by Eq. (F8).

In Sec. III B we have computed the contribution to $h \to Z\gamma$ from pure composite fermions using an effective Lagrangian approach. For vanishing elementary-composite mixings, the composite multiplets of $SO(4)$ are mass eigenstates, and Eq. (E1) is of the form (F6) with $T^h \propto \zeta T^{\hat{4}}$ and $\bar{T}^h = 0$. The vanishing of $\bar{T}^h$ follows from our tacit assumption to have the same coupling to the Higgs boson for both left- and right-handed chiralities of composite fermions in Eq. (E1). By using the above results, in particular Eqs. (F2) and (F8), and taking the limit of heavy fermion masses, the decay amplitude reads

$$A_{\rm NP} = -\frac{eQ_\psi}{4\pi^2}\sum_{i,j} i(m_j - m_i)T^h_{ij}\lambda^Z_{ji}F(m_i, m_j, 0, 0). \tag{F9}$$

By summing the contributions from mass eigenstates with different electric charge and using the identity

$$\frac{(m_j - m_i)}{\pi^2}F(m_i, m_j, 0, 0) = C(m_i, m_j) - C(m_j, m_i), \tag{F10}$$

one finally reobtains the result of Eq. (3.16).

[1] ATLAS Collaboration, Report No. ATLAS-CONF-2013-034.

[2] CMS Collaboration, Report No. CMS-PAS-HIG-13-005.

[3] C. J. C. Burges and H. J. Schnitzer, Nucl. Phys. **B228**, 464 (1983); C. N. Leung, S. T. Love, and S. Rao, Z. Phys. C **31**, 433 (1986); W. Buchmuller and D. Wyler, Nucl. Phys. **B268**, 621 (1986).

[4] R. Rattazzi, Z. Phys. C **40**, 605 (1988); B. Grzadkowski, Z. Hioki, K. Ohkuma, and J. Wudka, Nucl. Phys. **B689**, 108 (2004); P. J. Fox, Z. Ligeti, M. Papucci, G. Perez, and M. D. Schwartz, Phys. Rev. D **78**, 054008 (2008); J. A. Aguilar-Saavedra, Nucl. Phys. **B812**, 181 (2009); **B821**, 215 (2009); C. Grojean, W. Skiba, and J. Terning, Phys. Rev. D **73**, 075008 (2006).

[5] B. Grzadkowski, M. Iskrzynski, M. Misiak, and J. Rosiek, J. High Energy Phys. 10 (2010) 085.

[6] R. Contino, M. Ghezzi, C. Grojean, M. Muhlleitner, and M. Spira, J. High Energy Phys. 07 (2013) 035.

[7] G. F. Giudice, C. Grojean, A. Pomarol, and R. Rattazzi, J. High Energy Phys. 06 (2007) 045.

[8] M. E. Peskin and T. Takeuchi, Phys. Rev. D **46**, 381 (1992); R. Barbieri, A. Pomarol, R. Rattazzi, and A. Strumia, Nucl. Phys. **B703**, 127 (2004).

[9] E. E. Jenkins, A. V. Manohar, and M. Trott, arXiv:1305.0017.

[10] R. Contino, D. Marzocca, D. Pappadopulo, and R. Rattazzi, J. High Energy Phys. 10 (2011) 081.

[11] S. R. Coleman, J. Wess, and B. Zumino, Phys. Rev. **177**, 2239 (1969); C. G. Callan, Jr., S. Coleman, J. Wess, and B. Zumino, Phys. Rev. **177**, 2247 (1969).

[12] J. Elias-Miro, J. R. Espinosa, E. Masso, and A. Pomarol, arXiv:1302.5661.

[13] C. Grojean, E. E. Jenkins, A. V. Manohar, and M. Trott, J. High Energy Phys. 04 (2013) 016.

[14] K. Agashe, R. Contino, L. Da Rold, and A. Pomarol, Phys. Lett. B **641**, 62 (2006).

[15] J. Mrazek, A. Pomarol, R. Rattazzi, M. Redi, J. Serra, and A. Wulzer, Nucl. Phys. **B853**, 1 (2011).

[16] R. Contino, L. Da Rold, and A. Pomarol, Phys. Rev. D **75**, 055014 (2007).

[17] R. Contino, T. Kramer, M. Son, and R. Sundrum, J. High Energy Phys. 05 (2007) 074.

[18] D. B. Kaplan, Nucl. Phys. **B365**, 259 (1991).

[19] H. Cai, arXiv:1306.3922.

[20] A. Orgogozo and S. Rychkov, J. High Energy Phys. 06 (2013) 014.

[21] G. Ecker, J. Gasser, H. Leutwyler, A. Pich, and E. de Rafael, Phys. Lett. B **223**, 425 (1989).

[22] M. Golden and L. Randall, Nucl. Phys. **B361**, 3 (1991).

[23] R. Barbieri, G. Isidori, and D. Pappadopulo, J. High Energy Phys. 02 (2009) 029.

[24] C. Grojean, O. Matsedonskyi, and G. Panico, arXiv:1306.4655.

[25] G. Panico and A. Wulzer, J. High Energy Phys. 05 (2007) 060.

[26] M. S. Carena, E. Ponton, J. Santiago, and C. E. M. Wagner, Nucl. Phys. **B759**, 202 (2006); Phys. Rev. D **76**, 035006 (2007); G. Panico, M. Safari, and M. Serone, J. High Energy Phys. 02 (2011) 103.

[27] G. Panico, M. Redi, A. Tesi, and A. Wulzer, J. High Energy Phys. 03 (2013) 051.

[28] A. De Simone, O. Matsedonskyi, R. Rattazzi, and A. Wulzer, J. High Energy Phys. 04 (2013) 004.

[29] M. Baak, M. Goebel, J. Haller, A. Hoecker, D. Kennedy, R. Kogler, K. Mönig, M. Schott, and J. Stelzer, Eur. Phys. J. C **72**, 2205 (2012).

[30] M. Ciuchini, E. Franco, S. Mishima, and L. Silvestrini, arXiv:1306.4644.

[31] A. Pomarol and J. Serra, Phys. Rev. D **78**, 074026 (2008).

[32] A. Falkowski, F. Riva, and A. Urbano, arXiv:1303.1812.

[33] A. Djouadi, V. Driesen, W. Hollik, and A. Kraft, Eur. Phys. J. C **1**, 163 (1998).

[34] D. Y. Bardin and G. Passarino, *The Standard Model in the Making: Precision Study of the Electroweak Interactions*, International Series of Monographs on Physics Vol. 104 (Oxford University, New York, 1999).