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Tree-level contribution to $\bar{B} \to X_d\gamma$ using fragmentation functions

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We evaluate the most important tree-level contributions connected with the $b \to u\bar{u}d\gamma$ transition to the inclusive radiative decay $\bar{B} \to X_d\gamma$ using fragmentation functions. In this framework the singularities arising from collinear photon emission from the light quarks (u, $\bar{u}$, and d) can be absorbed into the (bare) quark-to-photon fragmentation function. We use as input the fragmentation function extracted by the ALEPH group from the two-jet cross section measured at the large electron positron (LEP) collider, where one of the jets is required to contain a photon. To get the quark-to-photon fragmentation function at the fragmentation scale $\mu_F \sim m_b$, we use the evolution equation, which we solve numerically. We then calculate the (integrated) photon energy spectrum for $b \to u\bar{u}d\gamma$ related to the operators $P_{1,2}^u$. For comparison, we also give the corresponding results when using nonzero (constituent) masses for the light quarks.

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I. INTRODUCTION

Rare B-meson decays are known to be a unique source of indirect information about physics at scales of several hundred GeV. To make a rigorous comparison between experiment and theory, one needs precise theoretical predictions for them.

The first estimate of the $\bar{B} \to X_s\gamma$ branching ratio within the Standard Model (SM) at the next-to-next-to-leading logarithmic (NNLL) level was published some years ago [1]. The branching ratio of the $\bar{B} \to X_s\gamma$ decay has been measured at the B factories [2]. Both theory and experiment have acquired a precision of a few percent.

Another interesting process is $\bar{B} \to X_d\gamma$. In Ref. [3] its decay rate was calculated in next-to-leading logarithmic (NLL) order (earlier in Ref. [4] a partial NLL result was found). Since then, not much theoretical work was done on $\bar{B} \to X_d\gamma$ because the corresponding measurement seemed very difficult. A few years ago, however, the *BABAR* collaboration managed to measure this branching ratio [5,6], yielding the *CP*-averaged result $\mathrm{Br}[\bar{B} \to X_d\gamma]_{E_\gamma > 1.6\,\mathrm{GeV}} = (1.41 \pm 0.57) \times 10^{-5}$ [7,8]. Extrapolation factors [9] were used to get from the experimental data to this result, which, as indicated by the notation, corresponds to a photon energy cut of 1.6 GeV.

Nonperturbative contributions related to u-quark loops are not Cabibbo-Kobayashi-Maskawa suppressed in $\bar{B} \to X_d\gamma$ (unlike in $\bar{B} \to X_s\gamma$) and therefore potentially limit the theoretical precision. It was, however, realized recently that most of these uncertainties drop out in the *CP*-averaged branching ratio [10,11]. This implies that the SM prediction for $\bar{B} \to X_d\gamma$ can in principle be calculated with similar accuracy as $\bar{B} \to X_s\gamma$. To fully exploit the information from $\bar{B} \to X_d\gamma$, we plan to derive in the near future a NNLL prediction of its *CP*-averaged branching ratio.

When going to this precision with the $b \to d\gamma$ transition, one has to take into account also the contributions from the tree-level transitions $b \to u\bar{u}d\gamma$, which lead to a nonnegligible contribution to the inclusive photon energy spectrum in $\bar{B} \to X_d\gamma$ in the considered photon energy window. Analogous tree-level contributions were considered some time ago for the full-inclusive photon energy spectrum of B decays [12] and more recently for the $\bar{B} \to X_s\gamma$ decay in Ref. [13]. In the calculation of the Feynman diagrams singularities arise which are related to collinear photon emission from one of the light quarks q ($q = u$, $\bar{u}$ or d), leading to single logarithms of the form $\ln(m_b^2/m_q^2)$. In the references just mentioned, the current quark masses m_q are replaced by a common constituent mass, providing an effective treatment of the collinear configurations.

In the present paper, we evaluate the mentioned tree-level transitions to $\bar{B} \to X_d\gamma$ associated with the operators P_1^u and P_2^u [see Eq. (1.1)], using the fragmentation function approach. In this framework the singularities arising from collinear photon emission from the light quarks (u, $\bar{u}$, and d) can be absorbed into the (bare) quark-to-photon fragmentation function.

Using this framework, one often starts with a nonperturbative initial condition for the fragmentation function at a low scale $\mu_F = \mu$ (μ of order Λ_{QCD}) and then evolves it up to the scale $\mu_F = m_b$, using the inhomogeneous evolution equation. When using a zero initial condition, the (leading logarithmic) evolution equation resums the terms $[\alpha_s \log(m_b^2/\mu^2)]^n$. This resummation was done in Ref. [14] in connection with the P_8 contribution to $\bar{B} \to X_s\gamma$. It was found that the effect of resummation suppresses the corresponding single logarithm present in lowest order perturbation theory for $E_\gamma > 1.6$ GeV, which is the relevant range in our application. In Ref. [15], dealing also with the P_8 contribution, a model based on vector meson dominance was used for the initial condition of the quark-to-photon fragmentation function [16]. As in our application, the logarithmic term (at lowest order) is numerically not much larger than the nonlogarithmic one, and because the two contributions have a different sign, we have some doubts to get reasonable predictions along these lines. This feature related to size and sign can also be seen in Table II of Ref. [13].

As we will describe in detail in Sec. II, we use as input the fragmentation function extracted by the ALEPH group from the two-jet cross section measured at the LEP, where one of the jets is required to contain a photon [17]. The theoretical framework needed for the extraction of the fragmentation function at the scale m_Z is described in Ref. [18]. Radiative corrections are considered in Refs. [19–21]. To get the quark-to-photon fragmentation function at the fragmentation scale $\mu_F \sim m_b$, we use the (inhomogeneous) evolution equation [21,22], which we solve numerically. We then calculate the (integrated) photon energy spectrum for $b \to u\bar{u}d\gamma$ related to $P_{1,2}^u$. Then, for comparison, we also give the corresponding results when using nonzero (constituent) masses for light quarks.

As just mentioned, we consider in this paper the contributions to $\bar{B} \to X_d\gamma$ originating from the four-quark operators P_1^u and P_2^u,

$$
\begin{aligned}
P_1^u &= (\bar{d}_L\gamma_\mu T^a u_L)(\bar{u}_L\gamma_\mu T^a b_L), \\
P_2^u &= (\bar{d}_L\gamma_\mu u_L)(\bar{u}_L\gamma_\mu b_L),
\end{aligned}
\tag{1.1}
$$

appearing in the effective weak Hamiltonian for the process $\bar{B} \to X_d\gamma$

$$
\mathcal{H}_{\mathrm{weak}} = -\frac{4G_F}{\sqrt{2}}\left[\xi_t\sum_{i=1}^{8} C_iP_i + \xi_u\sum_{i=1}^{2} C_i(P_i - P_i^u)\right], \tag{1.2}
$$

with $\xi_t = V_{tb}V_{td}^*$, $\xi_u = V_{ub}V_{ud}^*$. The other operators can be found in Ref. [13]. While ξ_t and ξ_u are numerically of the same order, the Wilson coefficients of the four quark operators P_i, $(i = 3, \ldots, 6)$ are smaller than those of $P_{1,2}^u$. We therefore only use the latter for our estimate.

The paper is organized as follows. In Sec. II we provide some details about our calculation and present our analytic result for the $P_{1,2}^u$ contribution to the (integrated) photon energy spectrum, using the fragmentation function approach. In Sec. III we give the corresponding results when using nonzero (constituent) masses for the light quarks. Finally, we present our conclusions in Sec. IV.

II. FRAGMENTATION FUNCTION APPROACH TO ESTIMATE THE $P_{1,2}^u$ CONTRIBUTIONS

In this section we work out the upper end of the photon energy spectrum (typically above 1.6 GeV) resulting from the tree-level transitions $b \to u\bar{u}d\gamma$ associated with the operators P_1^u and P_2^u. The corresponding Feynman diagrams are shown in Fig. 1. The kinematical configurations in which the photon is emitted (almost) collinear with d, u, or $\bar{u}$ lead to uncancelled singularities. This signals that there is another, nonperturbative contribution. Indeed, the photon energy spectrum $d\Gamma(b \to u\bar{u}d\gamma)/dE_\gamma$ (in shorthand notation $d\Gamma/dE_\gamma$ in the following) can be written as

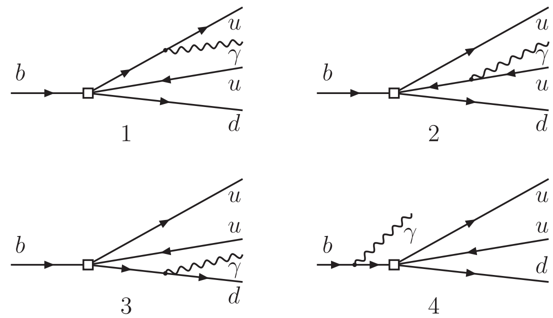

FIG. 1. Tree level Feyman diagrams corresponding to the transition $b \to u\bar{u}d\gamma$. The white square symbolizes the operators $P_{1,2}^u$.

$$
\frac{d\Gamma}{dE_\gamma} = \frac{d\hat{\Gamma}}{dE_\gamma} + \sum_{q=u,\bar{u},d}\int \frac{d\hat{\Gamma}}{dE_q} dE_q dz\,\delta(E_\gamma - zE_q)D_{q\to\gamma}(z), \tag{2.1}
$$

where the first term comes from the diagrams in Fig. 1, while the second term involves the inclusive fragmentation functions $D_{q\to\gamma}(z)$ for the partonic transitions $q \to q\gamma$, as well as the energy spectrum $\frac{d\hat{\Gamma}}{dE_q}$ of the parton q in the process $b \to u\bar{u}d$. The argument z of the fragmentation function $D_{q\to\gamma}$ denotes the energy fraction of the photon relative to the energy of the parent parton q.

To make the paper self-contained, we briefly show in Sec. II A that the mentioned collinear singularities in the first term of Eq. (2.1), which we regulate dimensionally ($d = 4 - 2\varepsilon$), can be factorized into the fragmentation function. Doing so, we closely follow the formalism in Ref. [18]. We then give the result for $d\Gamma(b \to u\bar{u}d\gamma)/dE_\gamma$ in a way in which the singularities are manifestly cancelled. Sec. II B then deals with the details of the fragmentation function $D(z, \mu_F)$. Finally, in Sec. II C the numerical results for the (integrated) photon energy spectrum are given.

A. Analytic results for $d\Gamma(b \to u\bar{u}d\gamma)/dE_\gamma$

For the following it is useful to decompose the first term in Eq. (2.1) according to

$$
\frac{d\hat{\Gamma}}{dE_\gamma} = \frac{d\hat{\Gamma}^{uu}}{dE_\gamma} + \frac{d\hat{\Gamma}^{\bar{u}\bar{u}}}{dE_\gamma} + \frac{d\hat{\Gamma}^{dd}}{dE_\gamma} + \frac{d\hat{\Gamma}^{bb}}{dE_\gamma} + \frac{d\hat{\Gamma}^{\mathrm{int}}}{dE_\gamma}, \tag{2.2}
$$

where $\frac{d\hat{\Gamma}^{uu}}{dE_\gamma}$ corresponds to the self-interference of diagram 1 in Fig. 1, i.e., with photon emission from the u quark; $\frac{d\hat{\Gamma}^{\bar{u}\bar{u}}}{dE_\gamma}$ corresponds to the self-interference of diagram 2; etc. When summing over the transverse polarizations of the photon, we used the general expression

$$
\sum_{r=1}^{2}\epsilon_{r,\mu}\epsilon_{r,\nu} = -g_{\mu\nu} + \frac{t_\mu k_\nu + k_\mu t_\nu}{k\cdot t} - \frac{t^2 k_\mu k_\nu}{(k\cdot t)^2}, \tag{2.3}
$$

where the polarization vectors ϵ_r, the momentum k of the photon, and the arbitrary vector t are chosen such that

$$\epsilon_r \cdot k = 0, \quad \epsilon_r \cdot \epsilon_{r'} = -\delta_{rr'}, \quad \epsilon_r \cdot t = 0 \quad (r, r' = 1, 2). \quad (2.4)$$

The vector t is then identified with the momentum of the b quark, i.e., $t = p_b = m_b(1, 0, 0, 0)$. It turns out that in this setup, the collinear singularities are contained in the contributions $\frac{d\hat{\Gamma}^{uu}}{dE_\gamma}$, $\frac{d\hat{\Gamma}^{\bar{u}\bar{u}}}{dE_\gamma}$, and $\frac{d\hat{\Gamma}^{dd}}{dE_\gamma}$, whereas the sum of the other two terms define the nonsingular (ns) contribution $\frac{d\Gamma^{\rm ns}}{dE_\gamma}$,

$$\frac{d\Gamma^{\rm ns}}{dE_\gamma} = \frac{d\hat{\Gamma}^{bb}}{dE_\gamma} + \frac{d\hat{\Gamma}^{\rm int}}{dE_\gamma}. \quad (2.5)$$

As a consequence of this specific singularity structure, it is useful to consider the combinations $\frac{d\Gamma^{qq}}{dE_\gamma}$ (with $q = u, \bar{u}, d$) defined as

$$\frac{d\Gamma^{qq}}{dE_\gamma} = \frac{d\hat{\Gamma}^{qq}}{dE_\gamma} + \int \frac{d\hat{\Gamma}}{dE_q} dE_q dz \delta(E_\gamma - zE_q) D_{q\to\gamma}(z). \quad (2.6)$$

To be concrete, we work out in the following $d\Gamma^{uu}/dE_\gamma$ (the other two quantities can then be obtained by obvious replacements). Defining $s = (p_u + p_\gamma)^2$, we can split the perturbative part $d\hat{\Gamma}^{uu}/dE_\gamma$ according to

$$\frac{d\hat{\Gamma}^{uu}}{dE_\gamma} = \frac{d\hat{\Gamma}^{uu,\rm res}}{dE_\gamma} + \frac{d\hat{\Gamma}^{uu,\rm coll}}{dE_\gamma}, \quad (2.7)$$

where the resolved (collinear) contribution corresponds to $s > s_{\rm min}$ ($s < s_{\rm min}$). Needless to say, the double differential decay width $\frac{d\hat{\Gamma}^{uu}}{dsdE_\gamma}$ needs to be worked out to obtain this splitting. The resolved part is finite for each $s_{\rm min} > 0$. If $s_{\rm min}$ is sufficiently small, the collinear piece can be worked out using the collinear approximation of the matrix element and the phase space, leading to

$$\frac{d\hat{\Gamma}^{uu,\rm coll}}{dE_\gamma} = \int \frac{d\hat{\Gamma}}{dE_u} dE_u dz \delta(E_\gamma - zE_u)\left[-\frac{1}{\varepsilon}\left(\frac{4\pi\mu^2}{s_{\rm min}}\right)^\varepsilon \times \frac{[z(1-z)]^{-\varepsilon}}{\Gamma(1-\varepsilon)} \frac{\alpha e_u^2}{2\pi} P(z)\right], \quad (2.8)$$

where the d-dimensional splitting function $P(z)$ reads

$$P(z) = \frac{1 + (1-z)^2 - \varepsilon z^2}{z}. \quad (2.9)$$

From the explicit structure of $\frac{d\hat{\Gamma}^{uu,\rm coll}}{dE_\gamma}$, we see that the $1/\varepsilon$ pole can be factorized into the bare fragmentation function $D_{u\to\gamma}(z)$ at the fragmentation scale μ_F such that in the $\overline{\rm MS}$ scheme

$$D_{u\to\gamma}(z) = D_{u\to\gamma}(z, \mu_F) + \frac{1}{\varepsilon}\left(\frac{4\pi\mu^2}{\mu_F^2}\right)^\varepsilon \frac{1}{\Gamma(1-\varepsilon)} \frac{\alpha e_u^2}{2\pi} P^{(0)}(z), \quad (2.10)$$

where the four-dimensional splitting function $P^{(0)}(z)$ reads

$$P^{(0)}(z) = \frac{1 + (1-z)^2}{z}. \quad (2.11)$$

The result for $\frac{d\Gamma^{uu}}{dE_\gamma}$ is then

$$\frac{d\Gamma^{uu}}{dE_\gamma} = \frac{d\hat{\Gamma}^{uu,\rm res}}{dE_\gamma} + \int \frac{d\hat{\Gamma}}{dE_u} dE_u dz \delta(E_\gamma - zE_u) \times \frac{\alpha e_u^2}{2\pi}\left[P^{(0)}(z) \ln\left(\frac{s_{\rm min} z(1-z)}{\mu_F^2}\right) + z\right] + \int \frac{d\hat{\Gamma}}{dE_u} dE_u dz \delta(E_\gamma - zE_u) D_{u\to\gamma}(z, \mu_F). \quad (2.12)$$

The treatment of the collinear phase space becomes exact when we take the limit $s_{\rm min} \to 0$. After working out the integral in the second term, we get in this limit (using $e_\gamma = E_\gamma/m_b$)

$$\frac{d\Gamma^{uu}}{dE_\gamma} = \frac{\alpha e_u^2}{2\pi} \frac{G_F^2 m_b^4 |\xi_u|^2}{288\pi^3} \frac{(9C_2^2 + 2C_1^2)}{3} \frac{1}{e_\gamma}[2(24e_\gamma^2 - 10e_\gamma + 3)\ln(e_\gamma) + (1-2e_\gamma)(16e_\gamma^3 - 16e_\gamma^2 + 4e_\gamma - 3)\ln(\mu_F^2/m_b^2) - (1-2e_\gamma)(16e_\gamma^3 - 16e_\gamma^2 + 4e_\gamma - 3)\ln(1-2e_\gamma) - 16e_\gamma^3(3+e_\gamma) + e_\gamma^2(38 + 48\ln(2)) + e_\gamma(1 - 20\ln(2)) - 3 + 6\ln(2)] + \int \frac{d\hat{\Gamma}}{dE_u} dE_u dz \delta(E_\gamma - zE_u) D_{u\to\gamma}(z, \mu_F), \quad (2.13)$$

where

$$\frac{d\hat{\Gamma}}{dE_u} = G_F^2 m_b |\xi_u|^2 \frac{(9C_2^2 + 2C_1^2)}{3} \frac{E_u^2(3m_b - 4E_u)}{12\pi^3}. \quad (2.14)$$

We notice that from this procedure, we could easily read off the renormalization equation (2.10). Assuming that this universal equation is known already (which is of course the case), we could have obtained the final result (2.13) without separating the phase space into a resolved and a collinear region, which has the advantage that the double differential decay width (in E_γ and s) is not needed. Indeed, as a check, we followed this procedure and got the same result.

In an analogous way, we obtain $\frac{d\Gamma^{\bar{u}\bar{u}}}{dE_\gamma}$:

$$\frac{d\Gamma^{\bar{u}\bar{u}}}{dE_\gamma} = \frac{\alpha e_u^2}{2\pi}\frac{G_F^2 m_b^4|\xi_u|^2}{96\pi^3}\frac{(9C_2^2+2C_1^2)}{3}\frac{1}{e_\gamma}[2(12e_\gamma^2-4e_\gamma+1)\ln(e_\gamma)-(1-2e_\gamma)^2(8e_\gamma^2+1)\ln(\mu_F^2/m_b^2) + (1-2e_\gamma)^2(8e_\gamma^2+1)\ln(1-2e_\gamma)-4e_\gamma^3(5+6e_\gamma)+6e_\gamma^2(3+4\ln(2))+e_\gamma(1-8\ln(2))-1+2\ln(2)] + \int\frac{d\hat{\Gamma}}{dE_{\bar{u}}}dE_{\bar{u}}dz\delta(E_\gamma-zE_{\bar{u}})D_{\bar{u}\to\gamma}(z,\mu_F), \tag{2.15}$$

where

$$\frac{d\hat{\Gamma}}{dE_{\bar{u}}} = G_F^2 m_b|\xi_u|^2\frac{(9C_2^2+2C_1^2)}{3}\frac{E_{\bar{u}}^2(m_b-2E_{\bar{u}})}{2\pi^3}. \tag{2.16}$$

We note that in our approximation the fragmentation functions $D_{q\to\gamma}(z,\mu_F)$ are the same for $q=u,\ \bar{u},\ d$ up to obvious charge factors.

$\frac{d\Gamma^{dd}}{dE_\gamma}$ is easily obtained as it is related to $\frac{d\Gamma^{uu}}{dE_\gamma}$ through a Fierz identity. One obtains

$$\frac{d\Gamma^{dd}}{dE_\gamma} = \frac{e_d^2}{e_u^2}\frac{d\Gamma^{uu}}{dE_\gamma}. \tag{2.17}$$

Finally, the explicit expression for the nonsingular part $d\Gamma^{\rm ns}/dE_\gamma$, defined in Eq. (2.5), reads

$$\frac{d\Gamma^{\rm ns}}{dE_\gamma} = \frac{\alpha}{2\pi}\frac{G_F^2 m_b^4|\xi_u|^2}{2592\pi^3}\frac{(9C_2^2+2C_1^2)}{3}\frac{(1-2e_\gamma)}{e_\gamma}\times(14e_\gamma^3+39e_\gamma^2-27e_\gamma+14). \tag{2.18}$$

To summarize this subsection, the photon energy spectrum $d\Gamma(b\to u\bar{u}d\gamma)/dE_\gamma$ can be written as

$$\frac{d\Gamma(b\to u\bar{u}d\gamma)}{dE_\gamma} = \frac{d\Gamma^{uu}}{dE_\gamma}+\frac{d\Gamma^{\bar{u}\bar{u}}}{dE_\gamma}+\frac{d\Gamma^{dd}}{dE_\gamma}+\frac{d\Gamma^{\rm ns}}{dE_\gamma}, \tag{2.19}$$

where the individual terms are given in Eqs. (2.13), (2.15), (2.17), and (2.18).

B. Parametrization of the fragmentation function

To obtain a numerical prediction for $d\Gamma(b\to u\bar{u}d\gamma)/dE_\gamma$, we need the nonperturbative fragmentation function $D_{q\to\gamma}(z,\mu_F)$ at a fragmentation scale μ_F, which is typically of order m_b. As we are not aware that this function has been directly extracted at the factorization scale $\mu_F\sim m_b$, i.e., from B decays, we use as an input the ALEPH measurement of the fragmentation function obtained for $\mu_F=m_Z$ [17].

We should stress here that this measurement was induced by a theoretical paper of Glover and Morgan [18], who suggested to extract this function from the normalized two-jet cross section

$$\frac{1}{\sigma_{\rm had}}\frac{d\sigma(2\text{-jets})}{dz_\gamma} \tag{2.20}$$

in e^+e^- collisions at the Z pole. To be more precise, the photon is understood to be within one of the two jets ($E_\gamma>5$ GeV), carrying at least 70% of the total energy of the jet. The fractional energy, z_γ, of such a photon within a jet is defined as

$$z_\gamma = \frac{E_\gamma}{E_\gamma+E_{\rm had}}, \tag{2.21}$$

where $E_{\rm had}$ is the energy of all accompanying hadrons in the "photon jet."

For the extraction of the fragmentation function $D_{q\to\gamma}(z,\mu_F)$ at $\mu_F=m_Z$, the experimental paper [17] follows exactly the procedure described in full detail in the theoretical work [18]. As a result, the ALEPH experiment found that the simple ansatz

$$D_{q\to\gamma}(z,m_Z) = \frac{\alpha e_q^2}{2\pi}\left[P^{(0)}(z)\ln\left(\frac{m_Z^2}{\mu_0^2(1-z)^2}\right)-1-\ln\left(\frac{m_Z^2}{2\mu_0^2}\right)\right], \tag{2.22}$$

which contains a single parameter (μ_0), leads to a reasonable description of the two-jet cross section (2.20) for

$$\mu_0 = (0.14^{+0.21+0.22}_{-0.08-0.04})\ \text{GeV}. \tag{2.23}$$

This analysis was done for values of $z_\gamma>0.7$, which means that the measurement of the fragmentation function is restricted to values $z>0.7$.

In our problem we need the fragmentation functions $D_{q\to\gamma}(z,\mu_F)$ at the low scale μ_b, where μ_b is of order m_b. To this end we solve the corresponding evolution equation. In leading logarithmic precision (with respect to QCD), $D_{q\to\gamma}(z,\mu_F)$ satisfied the inhomogeneous integrodifferential equation (see, e.g., Refs. [21,22]),

$$\mu_F \frac{\partial D_{q\to\gamma}(z,\mu_F)}{\partial \mu_F} = \frac{\alpha e_q^2}{\pi} P^{(0)}(z) + \frac{\alpha_s(\mu_F)}{\pi} \int_z^1 \frac{dy}{y} D_{q\to\gamma}\left(\frac{z}{y},\mu_F\right) P^{(0)}_{q\to q}(y), \tag{2.24}$$

where the Altarelli–Parisi splitting function $P^{(0)}_{q\to q}(y)$ reads

$$P^{(0)}_{q\to q}(y) = C_F\left[\frac{1+y^2}{1-y}\right]_+ \tag{2.25}$$

and the function $P^{(0)}(z)$ is given in Eq. (2.11).

From the structure of this equation, it is clear that the fragmentation functions $D_{q\to\gamma}(z_0,\mu_b)$ at a given value of z_0 only depend on the initial condition $D_{q\to\gamma}(z,m_Z)$ for values of z satisfying $z \geq z_0$. This is important because the initial condition extracted from experiment is only known above $z > 0.7$. This then means that we can determine $D_{q\to\gamma}(z,\mu_b)$ for values of $z \geq 0.7$, which is sufficient for our application.

We solved this equation numerically, using Eq. (2.22) as an initial condition. By doing so, we performed the integration with respect to μ_F using 4000 steps (at step 0 $\mu_F = m_Z$, and at step 4000 $\mu_F = \mu_b$). After each step, we fitted the z dependence to a set of 15 "basis functions." At the end of this procedure, we got the fragmentation function at the low scale μ_b in a version where the z dependence is given in a parametrized form.

As our application is rather sensitive to the fragmentation function near $z = 1$, we also solved (as a check) the evolution equation in moment space which we could basically do in an analytic way. Through this check, we are sure that the purely numerical uncertainties in our prediction of $d\Gamma(b\to u\bar{u}d\gamma)/dE_\gamma$ are negligible.

With the fragmentation function at the low scale $\mu_F = \mu_b$ at hand, we can numerically evaluate, using Eq. (2.19), the tree-level contributions of P_1^u and P_2^u to the (integrated) photon energy spectrum. We are mostly interested in an upper limit for these contributions, which amounts to using a small value for μ_0 in Eq. (2.22). Therefore, we choose $\mu_0 = 0.02$ GeV, which is still compatible with the range in Eq. (2.23) obtained through the two-jet cross section at the LEP. This compatibility is illustrated in Fig. 2.

The z dependence of the resulting fragmentation function $D_{q\to\gamma}(z,\mu_F)$ is shown in Fig. 3 for various values of μ_F.

C. Numerical results

With Eq. (2.19) and the fragmentation function $D_{q\to\gamma}(z,\mu_F)$ at the low scale $\mu_F \sim m_b$, we have all the ingredients to do the numerics for $d\Gamma(b\to u\bar{u}d\gamma)/dE_\gamma$ associated with the operators P_1^u and P_2^u. Unless stated otherwise, we use the value $\mu_0 = 0.02$ GeV in Eq. (2.22) because our aim is to give an estimate for the upper limit of this contribution. For the other input parameters, we use $m_b = 4.68$ GeV, $|\xi_u|^2 = 1.114\times 10^{-5}$, $|\xi_t|^2 = 7.530\times 10^{-5}$, and for the Wilson coefficients in leading logarithmic approximation (which are always taken at the scale 2.5 GeV in this paper), we use, as in Ref. [13], $C_1 = -0.8144$, $C_2 = 1.0611$, and $C_7 = -0.3688$.

In Fig. 4 we plot the normalized photon energy spectrum dR_d/de_γ

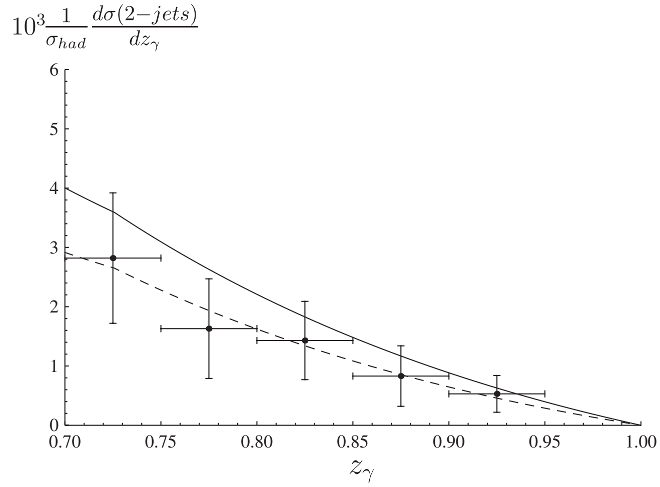

FIG. 2. Two-jet cross section $\frac{1}{\sigma_{\rm had}}\frac{d\sigma(\text{2-jets})}{dz_\gamma}$ for $y_{\rm cut} = 0.06$. The dots correspond to the ALEPH measurements [17] (see there Fig. 5 and Table 2), while the dashed line shows the theory prediction when using the central value $\mu_0 = 0.14$ GeV in the parametrization (2.22) of the fragmentation function $D_{q\to\gamma}(z,m_Z)$. The solid line corresponds to $\mu_0 = 0.02$ GeV (see the text).

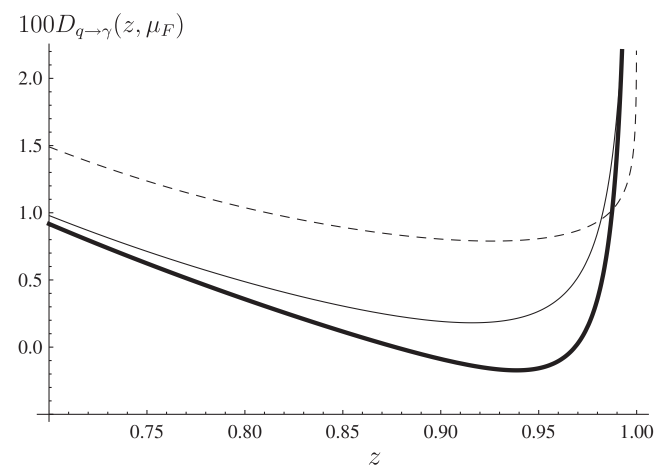

FIG. 3. Dependence of the fragmentation function $D_{q\to\gamma}(z,\mu_F)$ (for $e_q = 1$) on z and μ_F. The dashed line shows the fragmentation function for $\mu_F = m_Z$ as extracted from the ALEPH data, using $\mu_0 = 0.02$ GeV. The thick (thin) solid lines shows the corresponding fragmentation function for $\mu_F = m_b/2$ ($\mu_F = m_b$), obtained after solving the QCD evolution equation (2.24) in leading logarithmic precision (see the text).

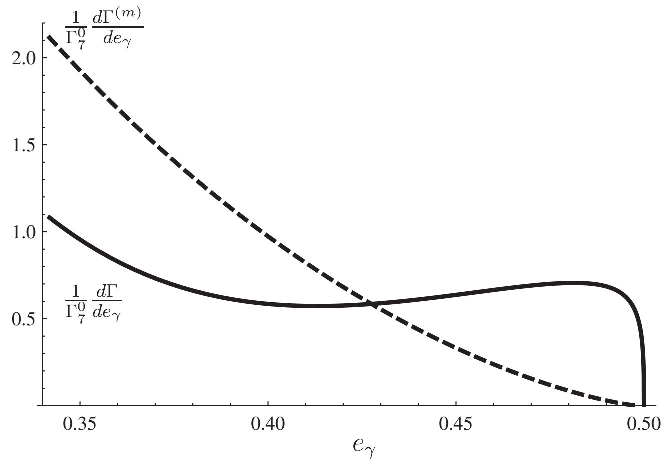

FIG. 4. Normalized photon energy distribution (2.26) due to $b \to u\bar{u}d\gamma$ associated with the operators $P_{1,2}^u$ for the two different approaches. Solid line: fragmentation function approach with fragmentation scale $\mu_F = m_b/2$; dashed line: introducing a common constituent quark mass m for the light quarks, taking $m/m_b = 1/50$.

$$\frac{dR_d}{de_\gamma} = \frac{1}{\Gamma_7^0}\frac{d\Gamma(b \to u\bar{u}d\gamma)}{de_\gamma} \tag{2.26}$$

as a function of the rescaled photon energy e_γ ($e_\gamma = E_\gamma/m_b$). Γ_7^0 corresponds to the total $b \to d\gamma$ decay width when only taking into account the tree-level contribution of the magnetic dipole operator P_7, i.e,

$$\Gamma_7^0 = \frac{G_F^2 m_b^5 |\xi_t|^2 \alpha C_7^2}{32\pi^4}. \tag{2.27}$$

The result for dR_d/de_γ is shown by the solid line in Fig. 4; the dashed line corresponds to the result when using constituent masses for the light quarks, as will be discussed in Sec. III.

In Table I we consider the corresponding integrated quantity $R_d^{\rm cut}$

$$R_d^{\rm cut} = \int_{e_\gamma^{\rm cut}}^{1/2} \frac{dR_d}{de_\gamma}, \tag{2.28}$$

for $e_\gamma^{\rm cut} = 0.342$ (which corresponds to a photon energy cut of 1.6 GeV), using two different values for the fragmentation scale μ_F. As in Fig. 4, we also show in Table I the corresponding results when using a common constituent mass m for the light quarks, as discussed in Sec. III.

TABLE I. The ratio $R_d^{\rm cut}$ [see Eq. (2.28)] for $E_\gamma^{\rm cut} = 1.6$ GeV for different values of fragmentation scale μ_F and different values of the common constituent mass m of the light quarks.

	$\mu_F = m_b/2$	$\mu_F = m_b$	$\frac{m}{m_b} = \frac{1}{50}$	$\frac{m}{m_b} = \frac{1}{10}$
$R_d^{\rm cut}$	0.107	0.0683	0.126	0.0188

As mentioned above, the results in Table I for the fragmentation function approach are based on using the value $\mu_0 = 0.02$ GeV in Eq. (2.22) and should therefore be considered as an upper limit for $R_d^{\rm cut}$. For the central value $\mu_0 = 0.14$ GeV [see Eq. (2.23)], one gets $R_d^{\rm cut} = 0.0549$ for $\mu_F = m_b/2$ and $R_d^{\rm cut} = 0.0291$ for $\mu_F = m_b$.

One sees from Table I that these upper limits are close to the results when using a common constituent quark mass m (with $m/m_b = 1/50$) for the light quarks. These contributions are not very small; therefore, it will be necessary to take them into account when deriving a NNLL prediction of the CP-averaged branching ratio for $\bar{B} \to X_d\gamma$.

III. RESULT WHEN USING CONSTITUENT QUARK MASSES

Another possibility to effectively treat the collinear regions connected with photon emission from light quarks is to provide the latter with constituent masses [13].

Making use of Ref. [23], where useful ingredients for computing the phase space integrals with massive particles in the final state are given, we easily get the spectrum $d\Gamma^{(m)}(b \to u\bar{u}d\gamma)/dE_\gamma$ associated with the operators P_1^u and P_2^u. Providing all light quarks with the same constituent mass m and keeping the m dependence only in logarithmic terms, we obtain

$$\begin{aligned} \frac{d\Gamma^{(m)}(b \to u\bar{u}d\gamma)}{dE_\gamma} &= \frac{G_F^2 m_b^4 |\xi_u|^2 \alpha}{32\pi^4} \frac{(9C_2^2 + 2C_1^2)}{3} \frac{(1-2e_\gamma)}{972 e_\gamma} \\ &\times \Big[6(272e_\gamma^3 - 176e_\gamma^2 + 44e_\gamma - 27) \\ &\times \Big(2\ln\frac{m}{m_b} - \ln(1-2e_\gamma)\Big) + 4316e_\gamma^3 \\ &- 2138e_\gamma^2 + 422e_\gamma - 399 \Big]. \end{aligned} \tag{3.1}$$

In this formula the charge factors $e_u = 2/3$ and $e_d = -1/3$ are inserted, and e_γ stands again for the rescaled photon energy ($e_\gamma = E_\gamma/m_b$). The numerical results of this approach can be seen in Fig. 4 and in Table I.

IV. SUMMARY AND CONCLUSIONS

Using data from the two-jet cross section (where one of the jets is required to contain a photon) measured by the ALEPH experiment at the LEP [17], the quark-to-photon fragmentation function was extracted (with the help of the theoretical work [18]) at the fragmentation scale $\mu_F = m_Z$. Using this input, we determine the fragmentation function at the scale $\mu_F \sim m_b$ by numerically solving the corresponding evolution equation. Using the so-obtained fragmentation function, we worked out the upper limit of that contribution to the (integrated) photon energy spectrum for $\bar{B} \to X_d\gamma$, which stems from the tree-level transitions $b \to u\bar{u}d\gamma$ associated with the operators and $P^u_{1,2}$. This upper limit is close to the result when using a common constituent quark mass m (with $m/m_b = 1/50$) for the light quarks. We conclude that these contributions are not very small, and therefore it will be necessary to take them into account when deriving a NNLL prediction of the CP-averaged branching ratio for $\bar{B} \to X_d\gamma$. Needless to say, it would be useful to have a determination of the quark-to-photon fragmentation function which directly uses data from B-meson decays. This would obviously lead to a more precise prediction of the $b \to u\bar{u}d\gamma$ transition.

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