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CP violation for $B^{0,\pm} \to \pi^{0,\pm}\pi^+\pi^-$ in perturbative QCD

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In the perturbative QCD (PQCD) approach, we study the direct CP violation in $B^{0,\pm} \to \rho^0(\omega)\pi^{0,\pm} \to \pi^+\pi^-\pi^{0,\pm}$ via the ρ-ω mixing mechanism. We find that the CP violation can be enhanced due to a large strong phase difference when the masses of the $\pi^+\pi^-$ pairs are in the vicinity of the ω resonance. Taking into account the ρ-ω mixing, we also compare the CP violation from the naive factorization approach, QCD factorization approach, and PQCD approach. Based on the $B^0\bar{B}^0$ mixing, we discuss the effects of $\bar{B}^0B^0$ mixing for the total CP violation of $B^0 \to \rho^0(\omega)\pi^0 \to \pi^+\pi^-\pi^0$, which is time dependent.

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I. INTRODUCTION

CP violation has been an open topic in recent years. In the Standard Model, CP violation is related to weak complex phases in the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1,2]. CP violations have been observed in the decays of B mesons [3]. In the past few years, more attention has been focused on the decays of the B-meson system, both theoretically and experimentally.

Currently, the hadronic matrix elements can be calculated from the first principles in the decays of the B meson. Exclusive two-body B-decay amplitudes have been estimated using the naive factorization approach [4], QCD factorization (QCDF) [5], perturbative QCD (PQCD) [6,7], and soft-collinear effective theory [8]. Due to the power expansion of $1/m_b$ (m_b is the b-quark mass), all of the theories of factorization are shown to deal with the hadronic matrix elements in the leading power of $1/m_b$. However, these methods are different significantly due to the collinear degree or transverse momenta. The power counting is different from the hard kernels between QCDF and PQCD. It is important to extract the strong phase difference for CP violation. A very different feature of QCDF and PQCD is the strong interaction scale, which is low in PQCD, typically of the order 1–2 GeV; in the case of QCDF, it is of the order $O(m_b)$ for the Wilson coefficients.

Direct CP violation occurs through the interference of two amplitudes with different weak phases and strong phases. The weak phase difference is directly determined by the CKM matrix elements, while the strong phase is usually difficult to control. However, the strong phase is not well determined from the theoretical approach. The B-meson decay amplitude involves the hadronic matrix elements whose computation is nontrivial. The different methods may present different strong phases. Meanwhile, we can also obtain a large strong phase difference by some phenomenological mechanism. ρ-ω mixing has been used for this purpose in the past few years [9–17] and focuses on the naive factorization and QCD factorization approaches. In this paper, we will investigate the CP violation via ρ-ω mixing in the PQCD approach.

In the PQCD approach, at the rest frame of a heavy B meson, the B-meson decays into two light mesons with large momenta that move very fast. The hard interaction dominates the decay amplitude from a short distance because there is not enough time to exchange the soft gluons with the final mesons. Since the final mesons move very fast, a hard gluon kicks the light spectator quark of the B meson to form a fast moving final meson. Hence, the hard interaction consists of six quark operators. The nonperturbative dynamics is included in the meson wave function, which can be extracted from an experiment. The hard one can be calculated by the perturbation theory.

The remainder of this paper is organized as follows. In Sec. II, we present the form of the effective Hamiltonian and the values of the Wilson coefficients. A discussion about PQCD is also presented. In Sec. III, we give the formalism and numerical results for the CP violation in $B^{0,\pm} \to \rho^0(\omega)\pi^{0,\pm} \to \pi^+\pi^-\pi^{0,\pm}$ via ρ-ω mixing. In Sec. IV, we calculate the total CP violation for $B^0 \to \rho^0(\omega)\pi^0 \to \pi^+\pi^-\pi^0$ including $B^0\bar{B}^0$ mixing. A summary and discussion are given in the Sec. V. The related functions defined in the text are given in the Appendix.

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II. THE EFFECTIVE HAMILTONIAN

With the operator product expansion, the effective weak Hamiltonian can be written as [18]

$$\mathcal{H}_{\Delta B=1} = \frac{G_F}{\sqrt{2}}\Big[V_{ub}V_{ud}^*(c_1O_1^u + c_2O_2^u) - V_{tb}V_{td}^*\sum_{i=3}^{10} c_iO_i\Big] + \text{H.c.}, \tag{1}$$

where G_F represents the Fermi constant, c_i ($i = 1, \ldots, 10$) are the Wilson coefficients, and V_{ub}, V_{ud}, V_{tb}, and V_{td} are the CKM matrix elements. The operators O_i have the following forms:

$$\begin{aligned}
O_1^u &= \bar{d}_\alpha\gamma_\mu(1-\gamma_5)u_\beta\bar{u}_\beta\gamma^\mu(1-\gamma_5)b_\alpha,\\
O_2^u &= \bar{d}\gamma_\mu(1-\gamma_5)u\bar{u}\gamma^\mu(1-\gamma_5)b,\\
O_3 &= \bar{d}\gamma_\mu(1-\gamma_5)b\sum_{q'}\bar{q}'\gamma^\mu(1-\gamma_5)q',\\
O_4 &= \bar{d}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{q'}\bar{q}'_\beta\gamma^\mu(1-\gamma_5)q'_\alpha,\\
O_5 &= \bar{d}\gamma_\mu(1-\gamma_5)b\sum_{q'}\bar{q}'\gamma^\mu(1+\gamma_5)q',\\
O_6 &= \bar{d}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{q'}\bar{q}'_\beta\gamma^\mu(1+\gamma_5)q'_\alpha,\\
O_7 &= \frac{3}{2}\bar{d}\gamma_\mu(1-\gamma_5)b\sum_{q'}e_{q'}\bar{q}'\gamma^\mu(1+\gamma_5)q',\\
O_8 &= \frac{3}{2}\bar{d}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{q'}e_{q'}\bar{q}'_\beta\gamma^\mu(1+\gamma_5)q'_\alpha,\\
O_9 &= \frac{3}{2}\bar{d}\gamma_\mu(1-\gamma_5)b\sum_{q'}e_{q'}\bar{q}'\gamma^\mu(1-\gamma_5)q',\\
O_{10} &= \frac{3}{2}\bar{d}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{q'}e_{q'}\bar{q}'_\beta\gamma^\mu(1-\gamma_5)q'_\alpha,
\end{aligned} \tag{2}$$

where α and β are color indices, and $q' = u$, d, or s quarks. In Eq. (2), O_1^u and O_2^u are tree operators, O_3–O_6 are QCD penguin operators, and O_7–O_{10} are the operators associated with electroweak penguin diagrams.

We can obtain the numerical values of c_i. When $c_i(m_b)$ [7],

$$\begin{aligned}
&c_1 = -0.2703, \quad c_2 = 1.1188, \quad c_3 = 0.0126,\\
&c_4 = -0.0270, \quad c_5 = 0.0085, \quad c_6 = -0.0326,\\
&c_7 = 0.0011, \quad c_8 = 0.0004, \quad c_9 = -0.0090,\\
&c_{10} = 0.0022.
\end{aligned} \tag{3}$$

The CKM matrix, which should be determined from the experiments, can be expressed in terms of the Wolfenstein parameters A, λ, ρ, and η [19]:

$$\begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}, \tag{4}$$

where the $O(\lambda^4)$ corrections are neglected. The latest values for the parameters in the CKM matrix are [3]

$$\begin{aligned}
&\lambda = 0.2272 \pm 0.0010, \qquad A = 0.818^{+0.007}_{-0.017},\\
&\bar{\rho} = 0.221^{+0.064}_{-0.028}, \qquad \bar{\eta} = 0.340^{+0.017}_{-0.045},
\end{aligned} \tag{5}$$

where

$$\bar{\rho} = \rho\Big(1-\frac{\lambda^2}{2}\Big), \qquad \bar{\eta} = \eta\Big(1-\frac{\lambda^2}{2}\Big). \tag{6}$$

From Eqs. (5) and (6) we have

$$0.198 < \rho < 0.293, \qquad 0.302 < \eta < 0.366. \tag{7}$$

In the PQCD approach, there are three scales involved: the W-boson mass m_W associated with weak interaction, the hard scale t, and the factorization scale $1/b$ (b is the conjugate variable of the parton transverse momenta k_T). The leading logarithm order contribution of the short distance associated with the Wilson coefficients $C(t)$ can be resumed from the order m_W down to hadronic scale t by a renormalization group equation. The interaction associated with the scale below the factorization scale k_T is not calculable in perturbation theory, which can be involved in hadronic wave function $\Phi(x)$. Taking into account the high-order power correction in the meson wave function, the overlap of the soft and collinear divergence will produce double logarithms $\ln^2(Pb)$, where P denotes the light-cone component of the meson momentum. The resummation of the double logarithms leads to a Sudakov factor $\exp[-s(P,b)]$, which suppresses the long-distance contribution in the large b region and vanishes as $b > 1/\Lambda_{\rm QCD}$. The remaining finite contributions are absorbed into a hard subamplitude $H(x,t)$.

The three scale factorization formula is written as

$$C(t)\otimes H(x,t)\otimes\Phi(x)\otimes\exp\Big[-s(P,b) - 2\int_{1/b}^{t}\frac{d\mu}{\mu}\gamma_q(\alpha_s(\mu))\Big], \tag{8}$$

where $\gamma_q = -\alpha_s/\pi$ is the quark anomalous dimension in the axial gauge. The hard amplitude $H(x,t)$ can be calculated in perturbative theory. However, the double logarithms $\alpha_s\ln^2(x)$ will appear and cannot expand as the momentum fraction x is at the end-point region. Then, the threshold resummation is presented due to the resummation of all orders of double logarithms that are associated with a jet function $S_t(x)$.

III. CP VIOLATION IN $B^{0,+} \to \rho^0(\omega)\pi^{0,+} \to \pi^+\pi^-\pi^{0,+}$

A. Formalism

The amplitude A ($\bar{A}$) for the decay $B^0 \to \pi^+\pi^-\pi^0$ ($\bar{B}^0 \to \pi^+\pi^-\pi^0$) can be written as (for simplicity, we neglect the calculating formalism of CP violation for the process of $B^+ \to \pi^+\pi^-\pi^+$, which is similar to the case of $B^0 \to \pi^+\pi^-\pi^0$)

$$A = \langle \pi^+\pi^-\pi^0|H^T|B^0\rangle + \langle \pi^+\pi^-\pi^0|H^P|B^0\rangle, \quad (9)$$

$$\bar{A} = \langle \pi^+\pi^-\pi^0|H^T|\bar{B}^0\rangle + \langle \pi^+\pi^-\pi^0|H^P|\bar{B}^0\rangle, \quad (10)$$

with H^T and H^P being the Hamiltonians for the tree and penguin operators, respectively.

We can define the relative magnitudes and phases between the tree and penguin operator contributions as follows:

$$A = \langle \pi^+\pi^-\pi^0|H^T|B^0\rangle[1 + re^{i(\delta+\phi)}], \quad (11)$$

$$\bar{A} = \langle \pi^+\pi^-\pi^0|H^T|\bar{B}^0\rangle[1 + re^{i(\delta-\phi)}], \quad (12)$$

where δ and ϕ are the strong and weak phases, respectively. ϕ arises from the CP-violating phase in the CKM matrix, which is $\arg[V_{tb}V_{td}^*/(V_{ub}V_{ud}^*)]$. The parameter r is the absolute value of the ratio of the penguin and tree amplitudes:

$$r \equiv \left| \frac{\langle \pi^+\pi^-\pi^0|H^P|B^0\rangle}{\langle \pi^+\pi^-\pi^0|H^T|B^0\rangle} \right|. \quad (13)$$

The CP-violating asymmetry a can be written as

$$a \equiv \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} = \frac{-2r\sin\delta\sin\phi}{1 + 2r\cos\delta\cos\phi + r^2}. \quad (14)$$

From Eq. (14), one can find that the CP violation depends on the weak phase difference and the strong phase difference. The weak phase is determined for a specific decay process. Hence, in order to obtain a large CP violation, we need some mechanism to make $\sin\delta$ large. It has been found that ρ-ω mixing (which was proposed based on vector meson dominance [20]) leads to a large strong phase difference [10–17]. Based on ρ-ω mixing and working to the first order of isospin violation, we have the following results:

$$\langle \pi^+\pi^-\pi^0|H^T|B^0\rangle = \frac{g_\rho}{s_\rho s_\omega}\tilde{\Pi}_{\rho\omega}t_\omega + \frac{g_\rho}{s_\rho}t_\rho, \quad (15)$$

$$\langle \pi^+\pi^-\pi^0|H^P|B^0\rangle = \frac{g_\rho}{s_\rho s_\omega}\tilde{\Pi}_{\rho\omega}p_\omega + \frac{g_\rho}{s_\rho}p_\rho, \quad (16)$$

where $t_\rho(p_\rho)$ and $t_\omega(p_\omega)$ are the tree (penguin) amplitudes for $B^0 \to \rho^0\pi^0$ and $B^0 \to \omega\pi^0$, respectively; g_ρ is the coupling for $\rho^0 \to \pi^+\pi^-$; $\tilde{\Pi}_{\rho\omega}$ is the effective ρ-ω mixing amplitude, which also effectively includes the direct coupling $\omega \to \pi^+\pi^-$. s_V, m_V, and Γ_V ($V = \rho$ or ω) are the inverse propagator, mass, and decay rate of the vector meson V, respectively, and

$$s_V = s - m_V^2 + \mathrm{i}m_V\Gamma_V, \quad (17)$$

with $\sqrt{s}$ being the invariant masses of the $\pi^+\pi^-$ pairs.

The direct coupling $\omega \to \pi^+\pi^-$ has been effectively absorbed into $\tilde{\Pi}_{\rho\omega}$, which leads to the explicit s dependence of $\tilde{\Pi}_{\rho\omega}$ [21]. However, the s dependence of $\tilde{\Pi}_{\rho\omega}$ is negligible in practice. We can make the expansion $\tilde{\Pi}_{\rho\omega}(s) = \tilde{\Pi}_{\rho\omega}(m_\omega^2) + (s - m_\omega)\tilde{\Pi}'_{\rho\omega}(m_\omega^2)$. The $\rho - \omega$ mixing parameters were determined per the fit of Gardner and O'Connell [22]:

$$\begin{aligned} \Re\mathfrak{e}\tilde{\Pi}_{\rho\omega}(m_\omega^2) &= -3500 \pm 300\ \mathrm{MeV}^2, \\ \Im\mathfrak{m}\tilde{\Pi}_{\rho\omega}(m_\omega^2) &= -300 \pm 300\ \mathrm{MeV}^2, \\ \tilde{\Pi}'_{\rho\omega}(m_\omega^2) &= 0.03 \pm 0.04. \end{aligned} \quad (18)$$

From Eqs. (9), (11), (15), and (16), one has

$$re^{i\delta}e^{i\phi} = \frac{\tilde{\Pi}_{\rho\omega}p_\omega + s_\omega p_\rho}{\tilde{\Pi}_{\rho\omega}t_\omega + s_\omega t_\rho}. \quad (19)$$

Defining

$$\frac{p_\omega}{t_\rho} \equiv r'e^{i(\delta_q+\phi)}, \quad \frac{t_\omega}{t_\rho} \equiv \alpha e^{i\delta_\alpha}, \quad \frac{p_\rho}{p_\omega} \equiv \beta e^{i\delta_\beta}, \quad (20)$$

where δ_α, δ_β, and δ_q are strong phases. One finds the following expression from Eqs. (19) and (20):

$$re^{i\delta} = r'e^{i\delta_q}\frac{\tilde{\Pi}_{\rho\omega} + \beta e^{i\delta_\beta}s_\omega}{\tilde{\Pi}_{\rho\omega}\alpha e^{i\delta_\alpha} + s_\omega}. \quad (21)$$

In order to get the CP-violating asymmetry in Eq. (14), $\sin\phi$ and $\cos\phi$ are needed. The weak phase ϕ is fixed by the CKM matrix elements. In the Wolfenstein parametrization [19], one has

$$\begin{aligned} \sin\phi &= \frac{\eta}{\sqrt{[\rho(1-\rho) - \eta^2]^2 + \eta^2}}, \\ \cos\phi &= \frac{\rho(1-\rho) - \eta^2}{\sqrt{[\rho(1-\rho) - \eta^2]^2 + \eta^2}}. \end{aligned} \quad (22)$$

B. Calculational details

From Eqs. (14), (19), and (20), in order to obtain the formulas of the CP violation, we calculate the amplitudes t_ρ, t_ω, p_ρ, and p_ω in the PQCD approach, which can be decomposed in terms of tree-level and penguin-level amplitudes due to the CKM matrix elements of $V_{ub}V_{ud}^*$ and $V_{tb}V_{td}^*$. In the following, we calculate the decay amplitudes for $B^0 \to \rho^0(\omega)\pi^0$ and $B^+ \to \rho^0(\omega)\pi^+$, which we will use in the next paragraph. The PQCD functions of F and M can be found in the Appendix.

1. The decay amplitudes of $B^0 \to \rho^0(\omega)\pi^0$

With the Hamiltonian (1) depending on the CKM matrix elements of $V_{ub}V_{ud}^*$ and $V_{tb}V_{td}^*$, the decay amplitudes for $B^0 \to \rho^0\pi^0$ in PQCD can be written as

$$-2M(B^0 \to \rho^0\pi^0) = V_{ub}V_{ud}^* t_\rho - V_{tb}V_{td}^* p_\rho, \tag{23}$$

where

$$t_\rho = (F_e + F_{e\rho})\left(C_1 + \frac{1}{3}C_2\right) + (M_e + M_{e\rho} - M_a - M_{a\rho})C_2, \tag{24}$$

and

$$\begin{aligned} p_\rho = {}& F_e\left(-\frac{1}{3}C_3 - C_4 + \frac{3}{2}C_7 + \frac{1}{2}C_8 + \frac{5}{3}C_9 + C_{10}\right) + F_{e\rho}\left(-\frac{1}{3}C_3 - C_4 - \frac{3}{2}C_7 - \frac{1}{2}C_8 + \frac{5}{3}C_9 + C_{10}\right) \\ & - F_{e\rho}^P\left(\frac{1}{3}C_5 + C_6 - \frac{1}{6}C_7 - \frac{1}{2}C_8\right) + M_e\left(-C_3 - \frac{3}{2}C_8 + \frac{1}{2}C_9 + \frac{3}{2}C_{10}\right) + M_{e\rho}\left(-C_3 + \frac{3}{2}C_8 + \frac{1}{2}C_9 + \frac{3}{2}C_{10}\right) \\ & - (M_a + M_{a\rho})\left(C_3 + 2C_4 - 2C_6 - \frac{1}{2}C_8 - \frac{1}{2}C_9 + \frac{1}{2}C_{10}\right) - (M_e^P + 2M_a^P)\left(C_5 - \frac{1}{2}C_7\right). \end{aligned} \tag{25}$$

The decay amplitudes for $B^0 \to \omega\pi^0$ can be written as

$$2M(B^0 \to \omega\pi^0) = V_{ub}V_{ud}^* t_\omega - V_{tb}V_{td}^* p_\omega, \tag{26}$$

where

$$t_\omega = (F_{e\rho} - F_e)\left(C_1 + \frac{1}{3}C_2\right) + (M_{e\rho} - M_e + M_a + M_{a\rho})C_2, \tag{27}$$

and

$$\begin{aligned} p_\omega = {}& F_e\left(-\frac{7}{3}C_3 - \frac{5}{3}C_4 - 2C_5 - \frac{2}{3}C_6 - \frac{1}{2}C_7 - \frac{1}{6}C_8 - \frac{1}{3}C_9 + \frac{1}{3}C_{10}\right) + F_{e\rho}\left(-\frac{1}{3}C_3 - C_4 - \frac{3}{2}C_7 - \frac{1}{2}C_8 + \frac{5}{3}C_9 + C_{10}\right) \\ & - F_{e\rho}^P\left(C_6 + \frac{1}{3}C_5 - \frac{1}{6}C_7 - \frac{1}{2}C_8\right) + M_e\left(-C_3 - 2C_4 + 2_6 + \frac{1}{2}C_8 + \frac{1}{2}C_9 - \frac{1}{2}C_{10}\right) \\ & + M_{e\rho}\left(-C_3 + \frac{3}{2}C_8 + \frac{1}{2}C_9 + \frac{3}{2}C_{10}\right) + (M_a + M_{a\rho})\left(-C_3 - \frac{3}{2}C_8 + \frac{1}{2}C_9 + \frac{3}{2}C_{10}\right) - (M_e^P + 2M_a^P)\left(C_5 - \frac{1}{2}C_7\right). \end{aligned} \tag{28}$$

Based on the definition of Eq. (20), we can get

$$\alpha e^{i\delta_\alpha} = \frac{t_\omega}{t_\rho}, \tag{29}$$

$$\beta e^{i\delta_\beta} = \frac{p_\rho}{p_\omega}, \tag{30}$$

$$r' e^{i\delta_q} = \frac{p_\omega}{t_\rho} \times \left|\frac{V_{tb}V_{td}^*}{V_{ub}V_{ud}^*}\right|, \tag{31}$$

where

$$\left|\frac{V_{tb}V_{td}^*}{V_{ub}V_{ud}^*}\right| = \frac{\sqrt{(1-\rho)^2 + \eta^2}}{(1-\lambda^2/2)\sqrt{\rho^2 + \eta^2}}. \tag{32}$$

2. The decay amplitudes of $B^+ \to \rho^0(\omega)\pi^+$

The decay amplitudes for $B^+ \to \rho^0\pi^+$ can be written as

$$\sqrt{2}M(B^+ \to \rho^0\pi^+) = V_{ub}V_{ud}^* t_\rho - V_{tb}V_{td}^* p_\rho, \tag{33}$$

where

$$\begin{aligned} t_\rho = {}& F_e\left(C_1 + \frac{1}{3}C_2\right) + (F_{e\rho} - 2F_a)\left(\frac{1}{3}C_1 + C_2\right) \\ & + M_e C_2 + (M_{e\rho} - M_a + M_{a\rho})C_1, \end{aligned} \tag{34}$$

and

$$p_\rho = F_e\Big(-\frac{1}{3}C_3 - C_4 + \frac{3}{2}C_7 + \frac{1}{2}C_8 + \frac{5}{3}C_9 + C_{10}\Big) + (F_{e\rho} - 2F_a)\Big(\frac{1}{3}C_3 + C_4 + \frac{1}{3}C_9 + C_{10}\Big) + (F^P_{e\rho} - 2F^P_a)\Big(\frac{1}{3}C_5 + C_6 + \frac{1}{3}C_7 + C_8\Big) + M_e\Big(-C_3 - \frac{3}{2}C_8 + \frac{1}{2}C_9 + \frac{3}{2}C_{10}\Big) + (M_{e\rho} - M_a + M_{a\rho})(C_3 + C_9) - M^P_e\Big(C_5 - \frac{1}{2}C_7\Big). \tag{35}$$

The decay amplitudes for $B^+ \to \omega\pi^+$ can be written as

$$\sqrt{2}M(B^+ \to \omega\pi^+) = V_{ub}V^*_{ud}t_\omega - V_{tb}V^*_{td}p_\omega, \tag{36}$$

where

$$t_\omega = F_e\Big(C_1 + \frac{1}{3}C_2\Big) + F_{e\rho}\Big(\frac{1}{3}C_1 + C_2\Big) + M_eC_2 + (M_{e\rho} + M_a + M_{a\rho})C_1, \tag{37}$$

and

$$p_\omega = F_e\Big(\frac{7}{3}C_3 + \frac{5}{3}C_4 + 2C_5 + \frac{2}{3}C_6 + \frac{1}{2}C_7 + \frac{1}{6}C_8 + \frac{1}{3}C_9 - \frac{1}{3}C_{10}\Big) + F_{e\rho}\Big(\frac{1}{3}C_3 + C_4 + \frac{1}{3}C_9 + C_{10}\Big) + F^P_{e\rho}\Big(\frac{1}{3}C_5 + C_6 + \frac{1}{3}C_7 + C_8\Big) + M_e\Big(C_3 + 2C_4 - 2C_6 - \frac{1}{2}C_8 - \frac{1}{2}C_9 + \frac{1}{2}C_{10}\Big) + (M_{e\rho} + M_a + M_{a\rho})(C_3 + C_9) + (M^P_a + M^P_{a\rho})(C_5 + C_7) + M^P_e\Big(C_5 - \frac{1}{2}C_7\Big). \tag{38}$$

Similarly, we can also obtain the strong phase from Eqs. (29)–(32).

C. Numerical results

In our numerical calculation, we use the following input parameters [3,23]:

$$m_B = 5.28\ \text{GeV}, \quad \tau_{B^+} = 1.641\ \text{ps},$$
$$\tau_{B^0} = 1.519\ \text{ps}, \quad f_\pi = 0.13\ \text{GeV},$$
$$f_\rho = 0.216\ \text{GeV}, \quad f_{\omega^T} = 0.145\ \text{GeV},$$
$$f_\omega = 0.195\ \text{GeV}, \quad f_{\rho^T} = 0.165\ \text{GeV},$$
$$\Lambda^{(4)}_{\text{QCD}} = 0.148\ \text{GeV}.$$

CP violation depends on the CKM matrix elements and the strong phases. The CKM matrix elements, which relate to ρ, η, λ, and A are given by Eq. (5). For our results, we find the CP violation is not sensitive to the CKM matrix elements for the different ρ, η, λ, and A. Hence, we present the CP violation as a function of the central CKM matrix elements in Fig. 1. From the numerical results, we find that there is a maximum CP-violating parameter value, $a_{\max}$, when the invariant masses of the $\pi^+\pi^-$ pairs are in the vicinity of the ω resonance. For example, the maximum CP-violating parameter can reach 73% for the decay channel of $B^+ \to \pi^+\pi^-\pi^+$ when $\sqrt{s} = 0.785$ GeV. In the case of $B^0 \to \pi^+\pi^-\pi^0$, the maximum CP-violating parameter can also reach 74% when $\sqrt{s} = 0.782$ GeV. This is shown in Fig. 1. One can see that the CP violations are enhanced for above two decay channels when the invariant masses of the $\pi^+\pi^-$ pairs are in the vicinity of the ω resonance. The qualities of ρ and ω are very close to each other. On the other hand, they have very different decay widths. The width of ρ is much larger than that of ω. Consequently, the Breit-Wigner of ρ will look very smooth even in the vicinity of ρ. As a result, the phase in the $\pi\pi$ scattering caused by the ρ resonance will change very slowly even in the vicinity of the ρ meson. This is also the reason why we do not see a rapid change of CP asymmetry in the vicinity of the ρ meson in Fig. 1.

From Eq. (14), one can see that the CP-violating parameter is dependent on $\sin\delta$ and r. In our calculation, it is found that the ρ-ω mixing mechanism produces a large $\sin\delta$ which is independent of the CKM matrix elements. The plot of $\sin\delta$ as a function of $\sqrt{s}$ is shown in Fig. 2. One can see that the ρ-ω mixing mechanism leads to the strong phase δ reaching -90° ($\sin\delta = -1$) at the ω resonance for the processes of $B^+ \to \pi^+\pi^-\pi^+$ and $B^0 \to \pi^+\pi^-\pi^0$. From Fig. 3, it can be seen that r increases rapidly for the channel of $B^+ \to \pi^+\pi^-\pi^+$ when the invariant masses of the $\pi^+\pi^-$ pairs are in the vicinity of the ω resonance. However, in the case of $B^0 \to \pi^+\pi^-\pi^0$, r changes slightly.

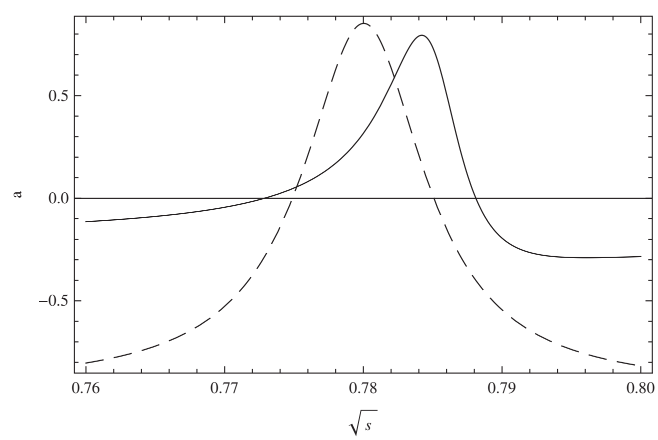

FIG. 1. The CP-violating asymmetry, a, as a function of $\sqrt{s}$ for central CKM matrix elements. The solid (dashed) line corresponds to the decay channel of $B^+ \to \pi^+\pi^-\pi^+$ ($B^0 \to \pi^+\pi^-\pi^0$).

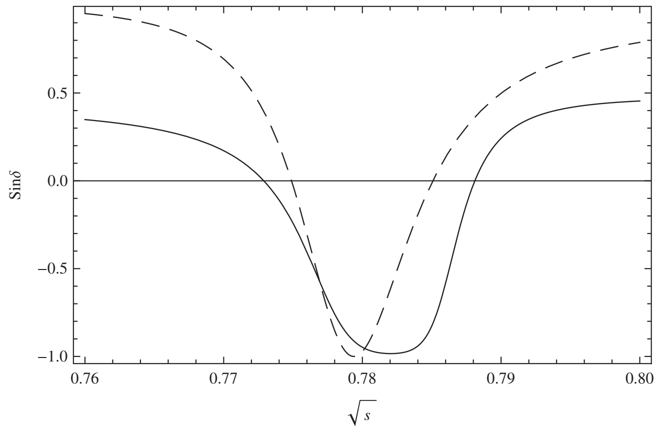

FIG. 2. $\sin\delta$ as a function of $\sqrt{s}$ for central CKM matrix elements. The solid (dashed) line corresponds to the decay channel of $B^+ \to \pi^+\pi^-\pi^+$ ($B^0 \to \pi^+\pi^-\pi^0$).

For the process of the above decay channels, we find the ρ-ω mixing presents a strong phase so as to make $\sin\delta$ big. The involvement of the ρ-ω mixing can also change the value of r. However, as found from our detailed analysis, the effect of the change of r on a is small compared with the case of $\sin\delta$.

In Table I, we compare the maximum CP violation obtained via the ρ-ω mixing by naive factorization [12], QCD factorization [24], and the perturbative QCD approaches. One can find that the different approaches give different results. However, the CP violations are all enhanced via the ρ-ω mixing by the three methods, which, however, are significantly different due to the collinear degrees, transverse momenta, and strong interaction scale. Since the b quark is heavy and energetic, the naive factorization approximation is expected to be a good method. The uncertainties come mainly from the color-octet and nonfactorizable contribution that are effectively absorbed into N_c [12]. The QCD factorization scheme adds α_s

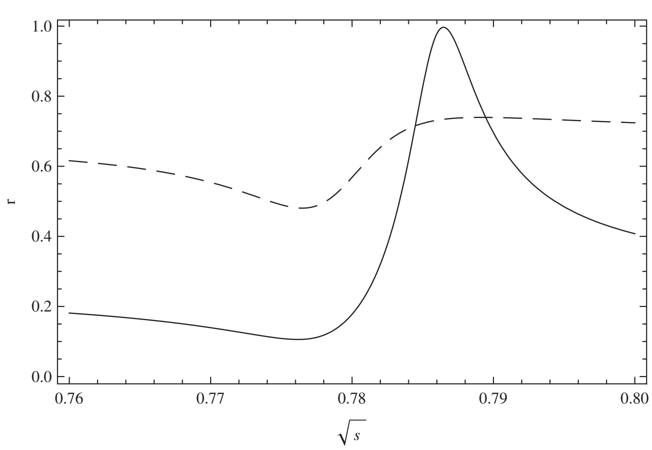

FIG. 3. r as a function of $\sqrt{s}$ for central CKM matrix elements. The solid (dashed) line corresponds to the decay channel of $B^+ \to \pi^+\pi^-\pi^+$ ($B^0 \to \pi^+\pi^-\pi^0$).

TABLE I. The maximum CP violation for different approaches via ρ-ω mixing.

	$B^+ \to \pi^+\pi^-\pi^+$	$B^0 \to \pi^+\pi^-\pi^0$
Naive factorization	-0.23– -0.57	-0.42– -0.81
QCD factorization	-0.02–0.05	-0.3–0.05
Perturbative QCD	0.69–0.73	0.72–0.74

corrections. In this framework, there is a cancellation of the scale and renormalization scheme dependence between the Wilson coefficients and the hadronic matrix elements. The QCD factorization suffers from end-point singularities, which are not well controlled. The CP violation depends on the unknown parameters associated with such end-point singularities. The uncertainties of PQCD come from the variation of the hard scattering scale together with the uncertainty on $\Lambda_{\rm QCD}$ and the CKM matrix elements. The different methods present different strong phases which lead to different CP violation. However, the case is the same in that the ρ-ω mixing provides new strong phases. Hence, large CP violation can be obtained using different factorization approaches.

IV. CP VIOLATION FOR $B^0 \to \pi^0\pi^+\pi^-$

Generally, the contribution of neutral meson mixing may be small for CP violation, which can be neglected. In our calculating formalism, the case may be different due to the involvement of ρ-ω mixing. The decay process can be produced as $B^0 \to f$ or $B^0 \to \bar{B}^0 \to f$. Hence, we also consider the effect of $B^0\bar{B}^0$ mixing for the process $B^0 \to \pi^0\pi^+\pi^-$ via ρ-ω mixing. Based on the $B^0\bar{B}^0$ mixing, the time-dependent CP violation of neutral B decays to a final state f given by [25]

$$\begin{aligned} a(t) &= \frac{\Gamma(B^0(t) \to f) - \Gamma(\bar{B}^0(t) \to f)}{\Gamma(B^0(t) \to f) + \Gamma(\bar{B}^0(t) \to f)} \\ &= a\cos(\Delta m t) + S_f \sin(\Delta m t), \end{aligned} \tag{39}$$

where Δm is the mass difference of the two mass eigenstates of the neutral B meson. The direct CP-violating parameter a is defined in Eq. (14). The mixing CP-violating parameter is defined as

$$S_f = -\frac{2\,\mathrm{Im}(\lambda_{CP})}{1 + |\lambda_{CP}|^2}, \tag{40}$$

where

$$\lambda_{CP} = \frac{V_{tb}^* V_{td} \langle f|H|\bar{B}^0\rangle}{V_{tb} V_{td}^* \langle f|H|B^0\rangle}, \tag{41}$$

and we can get

$$\lambda_{CP} = e^{2i\phi}\frac{1 + re^{i(\delta-\phi)}}{1 + re^{i(\delta+\phi)}}. \tag{42}$$

Including the $B\bar{B}$ mixing effect and by integrating over time t, the total CP violation can be written as [7]

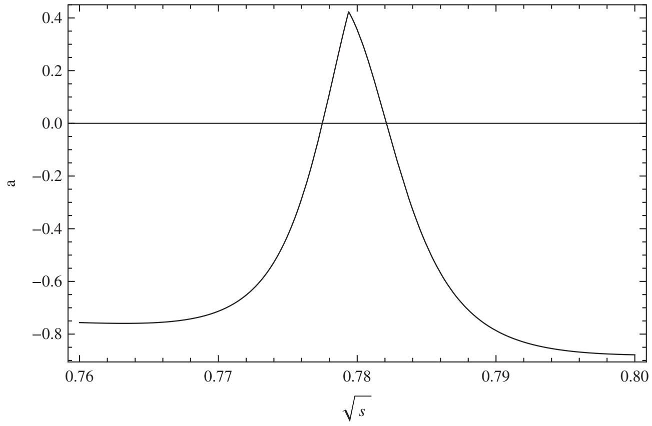

FIG. 4. The total *CP*-violating asymmetry, a, as a function of $\sqrt{s}$ for central CKM matrix elements for $B^0 \to \rho^0\pi^0 \to \pi^0\pi^+\pi^-$.

$$a_{\text{total}} = \frac{1}{1+x^2}a + \frac{x}{1+x^2}S_f, \tag{43}$$

with $x = \Delta m/\Gamma \simeq 0.723$.

The predictions for the total *CP* violation of $B^0 \to \rho^0\pi^0 \to \pi^0\pi^+\pi^-$ are presented in Fig. 4. One can find that the total *CP* violation is also enhanced when the invariant masses of the $\pi^+\pi^-$ pairs are in the vicinity of the ω resonance, which is suppressed when compared with direct *CP* violation. However, the maximum total *CP* violation can also reach 41% including the ρ-ω mixing.

V. SUMMARY AND DISCUSSION

We have studied the *CP* violation in the decay of $B^{0,\pm} \to \pi^{0,\pm}\pi^+\pi^-$ due to the contribution of ρ-ω mixing in the PQCD approach. It was found that ρ-ω mixing can cause a large strong phase difference so that large *CP* violation can be obtained at the ω resonance. As a result, it was found that the *CP* violation could reach 74%.

The LHC is a proton-proton collider with the designed center-of-mass energy of 14 TeV and luminosity of $L = 10^{34}\ \text{cm}^{-2}\,\text{s}^{-1}$. The production rates for heavy quark flavors will be large at the LHC, and the $b\bar{b}$ production cross section will be of the order of 0.5 mb, providing as many as 0.5×10^{12} bottom events per year [26]. In particular, the LHCb detector is designed to exploit a large number of b hadrons produced at the LHC in order to make precise studies on *CP* asymmetries and on rare decays in b-hadron systems. The other two experiments, ATLAS and CMS, are optimized for discovering new physics and will complete most of their B physics program within the first few years [26,27]. Recently, the LHCb Collaboration found clear evidence for direct *CP* violation in some three-body decay channels in charmless decays of a B meson. Meanwhile, large *CP* violation was obtained in $B^+ \to K^+K^-\pi^+$, $B^\pm \to \pi^\pm\pi^+\pi^-$ in the region of $m^2_{\pi^+\pi^-\text{low}} < 0.4\ \text{GeV}^2$ and $m^2_{\pi^+\pi^-\text{low}} > 15\ \text{GeV}^2$ [28]. The LHCb experiment may perhaps collect data in the region of the invariant masses of $\pi^+\pi^-$ associated ω resonance for detecting our prediction of *CP* violation. Meanwhile, the experiment results can also detect the different factorization methods and the ρ-ω mixing mechanism.

For the decay processes of bottom mesons, the traditional factorization approach was the leading order of the QCD factorization scheme with the $1/m_b$ corrections being neglected. The QCD factorization scheme suffers from end-point singularities, which are not well controlled. Next-to-leading order (NLO) corrections for the decay of $B \to \pi\rho(\omega)$ have recently been implemented using the PQCD approach [29]. In Ref. [30], the author discusses some problems with the NLO accurate calculation in the PQCD approach. A complete NLO calculation in the PQCD approach requires a calculation of all one-loop spectator-scattering diagrams. However, Ref. [29] only considers the one-loop BBNS kernel (similar to the kernel in the QCD factorization) for the decay of $B \to \pi\rho(\omega)$. The vertex diagram can be far off shell as a subdiagram in a large diagram with hard-collinear line exchanges. Moreover, the Wilson coefficients are evaluated at very low scales that perturbation theory breaks down. An unphysical enhancement of the Wilson coefficients at small scales is also the origin of the large penguin and annihilation. Based on the above discussion, we investigate the *CP* violation of $B^{0,\pm} \to \pi^{0,\pm}\pi^+\pi^-$ via ρ-ω mixing from the leading-order contribution in PQCD.

ACKNOWLEDGMENTS

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APPENDIX: RELATED FUNCTIONS DEFINED IN THE TEXT

$$\begin{aligned} M_e^P = {}& \frac{64}{3}\sqrt{3}\pi C_F G_F m_\rho^2 m_B(\epsilon\cdot p_\pi)\int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1,b_1) \times \{r_\pi(x_3-x_2)[\phi_\pi^P(x_2,b_1)\phi_\rho^t(x_3,b_2) \\ & + \phi_\pi^\sigma(x_2,b_1)\phi_\rho^s(x_3,b_2)] - r_\pi(x_2+x_3)[\phi_\pi^P(x_2,b_1)\phi_\rho^s(x_3,b_2) + \phi_\pi^\sigma(x_2,b_1)\phi_\rho^t(x_3,b_2)] + x_3\phi_\pi^A(x_2,b_1)[\phi_\rho^t(x_3,b_2) \\ & - \phi_\rho^s(x_3,b_2)]\}h_d(x_1,x_2,x_3,b_1,b_2)\exp[-S_{cd}(t_d)], \end{aligned} \tag{A1}$$

$$F_e = 8\sqrt{2}\pi C_F G_F f_\rho m_\rho m_B^2(\epsilon\cdot p_\pi)\int_0^1 dx_1dx_2\int_0^\infty b_1db_1b_2db_2\phi_B(x_1,b_1)\{[(1+x_2)\phi_\pi^A(x_2,b_2) + r_\pi(1-2x_2)(\phi_\pi^P(x_2,b_2)+\phi_\pi^\sigma(x_2,b_2))]\alpha_s(t_e^1)h_e(x_1,x_2,b_1,b_2)\exp[-S_{ab}(t_e^1)] + 2r_\pi\phi_\pi^P(x_2,b_2)\alpha_s(t_e^2)h_e(x_2,x_1,b_2,b_1)\exp[-S_{ab}(t_e^2)]\}, \tag{A2}$$

$$M_e = -\frac{32}{3}\sqrt{3}\pi C_F G_F m_\rho m_B^2(\epsilon\cdot p_\pi)\int_0^1 dx_1dx_2dx_3\int_0^\infty b_1db_1b_2db_2\phi_B(x_1,b_1)x_2[\phi_\pi^A(x_2,b_1) - 2r_\pi\phi_\pi^\sigma(x_2,b_1)]\phi_\rho(x_3,b_2)h_d(x_1,x_2,x_3,b_1,b_2)\exp[-S_{cd}(t_d)], \tag{A3}$$

$$M_a = \frac{32}{3}\sqrt{3}\pi C_F G_F m_\rho m_B^2(\epsilon\cdot p_\pi)\int_0^1 dx_1dx_2dx_3\int_0^\infty b_1db_1b_2db_2\phi_B(x_1,b_1)\{[x_2\phi_\pi^A(x_2,b_2)\phi_\rho(x_3,b_2) + r_\pi r_\rho(x_2-x_3)(\phi_\pi^P(x_2,b_2)\phi_\rho^t(x_3,b_2)+\phi_\pi^\sigma(x_2,b_2)\phi_\rho^s(x_3,b_2)) + r_\pi r_\rho(x_2+x_3)(\phi_\pi^\sigma(x_2,b_2)\phi_\rho^t(x_3,b_2) + \phi_\pi^P(x_2,b_2)\phi_\rho^s(x_3,b_2))]h_f^1(x_1,x_2,x_3,b_1,b_2)\exp[-S_{ef}(t_f^1)] - [x_3\phi_\pi^A(x_2,b_2)\phi_\rho(x_3,b_2) + r_\pi r_\rho(x_3-x_2)(\phi_\pi^P(x_2,b_2)\phi_\rho^t(x_3,b_2)+\phi_\pi^\sigma(x_2,b_2)\phi_\rho^s(x_3,b_2)) + r_\pi r_\rho(2+x_2+x_3)\phi_\pi^P(x_2,b_2)\phi_\rho^s(x_3,b_2) - r_\pi r_\rho(2-x_2-x_3)\phi_\pi^\sigma(x_2,b_2)\phi_\rho^t(x_3,b_2)]h_f^2(x_1,x_2,x_3,b_1,b_2)\exp[-S_{ef}(t_f^2)]\}. \tag{A4}$$

We use a wave function for ϕ_π^A and the twist-three wave functions ϕ_π^P and ϕ_π^t from [31]

$$\phi_\pi^A(x) = \frac{3}{\sqrt{6}}f_\pi x(1-x)[1+0.44C_2^{3/2}(2x-1) + 0.25C_4^{3/2}(2x-1)], \tag{A5}$$

$$\phi_\pi^P(x) = \frac{f_\pi}{2\sqrt{6}}[1+0.43C_2^{1/2}(2x-1)+0.09C_4^{1/2}(2x-1)], \tag{A6}$$

$$\phi_\pi^t(x) = \frac{f_\pi}{2\sqrt{6}}(1-2x)[1+0.55(10x^2-10x+1)]. \tag{A7}$$

The Gegenbauer polynomials are defined by

$$C_2^{1/2}(t) = \frac{1}{2}(3t^2-1), \qquad C_4^{1/2}(t) = \frac{1}{8}(35t^4-30t^2+3),$$
$$C_2^{3/2}(t) = \frac{3}{2}(5t^2-1), \qquad C_4^{3/2}(t) = \frac{15}{8}(21t^4-14t^2+1). \tag{A8}$$

We choose the wave function of the ρ and ω meson [32]:

$$\phi_\rho(x) = \phi_\omega(x) = \frac{3}{\sqrt{6}}f_\rho x(1-x)[1+0.18C_2^{3/2}(2x-1)], \tag{A9}$$

$$\phi_\rho^t(x) = \phi_\omega^t(x) = \frac{f_\rho^T}{2\sqrt{6}}\{3(2x-1)^2+0.3(2x-1)^2[5(2x-1)^2-3] + 0.21[3-30(2x-1)^2+35(2x-1)^4]\}, \tag{A10}$$

$$\phi_\rho^s(x) = \phi_\omega^s(x) = \frac{3}{2\sqrt{6}}f_\rho^T(1-2x)[1+0.76(10x^2-10x+1)]. \tag{A11}$$

For the B meson, the wave function is chosen as

$$\phi_B(x,b) = \frac{N_B}{2\sqrt{6}}f_B x^2(1-x)^2\exp\left[-\frac{M_B^2x^2}{2\omega_b^2}-\frac{1}{2}(\omega_b b)^2\right], \tag{A12}$$

with $\omega_b = 0.4$ GeV. $N_B = 2365.57$ is a normalization factor.

We show here the function of h_i's coming from the Fourier transform of $H^{(0)}$,

$$h_e(x_1,x_2,b_1,b_2) = K_0(\sqrt{x_1x_2}m_Bb_1)[\theta(b_1-b_2)\times K_0(\sqrt{x_2}m_Bb_1)I_0(\sqrt{x_2}m_Bb_2) + \theta(b_2-b_1)K_0(\sqrt{x_2}m_Bb_2)\times I_0(\sqrt{x_2}m_Bb_1)]S_t(x_2), \tag{A13}$$

$$h_d(x_1,x_2,x_3,b_1,b_2) = \alpha_s(t_d)K_0(-i\sqrt{x_2x_3}m_Bb_2)\times[\theta(b_1-b_2)K_0(\sqrt{x_1x_2}m_Bb_1)\times I_0(\sqrt{x_1x_2}m_Bb_2)+\theta(b_2-b_1)\times K_0(\sqrt{x_1x_2}m_Bb_2)I_0(\sqrt{x_1x_2}m_Bb_1)], \tag{A14}$$

$$h_f^1 \quad (x_1, x_2, x_3, b_1, b_2) = K_0(-i\sqrt{x_2 x_3} m_B b_1)\alpha_s(t_f^1) \times [\theta(b_1 - b_2) K_0(-i\sqrt{x_2 x_3} m_B b_1) \times J_0(\sqrt{x_2 x_3} m_B b_2) + \theta(b_2 - b_1) \times K_0(-i\sqrt{x_2 x_3} m_B b_2) \times J_0(\sqrt{x_2 x_3} m_B b_1)], \quad \text{(A15)}$$

$$h_f^2 \quad (x_1, x_2, x_3, b_1, b_2) = K_0(\sqrt{x_2 + x_3 - x_2 x_3} m_B b_1)\alpha_s(t_f^2) \times [\theta(b_1 - b_2) K_0(-i\sqrt{x_2 x_3} m_B b_1) \times J_0(\sqrt{x_2 x_3} m_B b_2) + \theta(b_2 - b_1) \times K_0(-i\sqrt{x_2 x_3} m_B b_2) \times J_0(\sqrt{x_2 x_3} m_B b_1)], \quad \text{(A16)}$$

$$h_a \quad (x_1, x_2, b_1, b_2) = K_0(-i\sqrt{x_1 x_2} m_B b_2) S_t(x_1)[\theta(b_1 - b_2) \times K_0(-i\sqrt{x_1} m_B b_1) J_0(\sqrt{x_1} m_B b_2) + \theta(b_2 - b_1) K_0(-i\sqrt{x_1} m_B b_2) \times J_0(\sqrt{x_1} m_B b_1)], \quad \text{(A17)}$$

where J_0 is the Bessel function, and K_0, I_0 are modified Bessel functions $K_0(-ix) = -(\pi/2)Y_0(x) + i(\pi/2)J_0(x)$. The threshold resummation form factor $S_t(x_i)$ is adopted from Ref. [33],

$$S_t(x) = \frac{2^{1+2c}\Gamma(3/2 + c)}{\sqrt{\pi}\Gamma(1 + c)}[x(1 - x)]^c, \quad \text{(A18)}$$

where the parameter $c = 0.3$. This function is normalized to unity.

The Sudakov factors used in the text are defined as

$$S_{ab}(t) = s(x_1 m_B/\sqrt{2}, b_1) + s(x_2 m_B/\sqrt{2}, b_2) \times + s((1 - x_2) m_B/\sqrt{2}, b_2) - \frac{1}{\beta_1}\left[\ln\frac{\ln(t/\Lambda)}{-\ln(b_1\Lambda)} + \ln\frac{\ln(t/\Lambda)}{-\ln(b_2\Lambda)}\right], \quad \text{(A19)}$$

$$S_{cd}(t) = s(x_1 m_B/\sqrt{2}, b_1) + s(x_2 m_B/\sqrt{2}, b_2) + s((1 - x_2) m_B/\sqrt{2}, b_2) + s(x_3 m_B/\sqrt{2}, b_1) + s((1 - x_3) m_B/\sqrt{2}, b_1) - \frac{1}{\beta_1}\left[2\ln\frac{\ln(t/\Lambda)}{-\ln(b_1\Lambda)} + \ln\frac{\ln(t/\Lambda)}{-\ln(b_2\Lambda)}\right], \quad \text{(A20)}$$

$$S_{ef}(t) = s(x_1 m_B/\sqrt{2}, b_1) + s(x_2 m_B/\sqrt{2}, b_2) + s((1 - x_2) m_B/\sqrt{2}, b_2) + s(x_3 m_B/\sqrt{2}, b_2) + s((1 - x_3) m_B/\sqrt{2}, b_2) - \frac{1}{\beta_1}\left[\ln\frac{\ln(t/\Lambda)}{-\ln(b_1\Lambda)} + 2\ln\frac{\ln(t/\Lambda)}{-\ln(b_2\Lambda)}\right], \quad \text{(A21)}$$

$$S_{gh}(t) = s(x_2 m_B/\sqrt{2}, b_1) + s(x_3 m_B/\sqrt{2}, b_2) + s((1 - x_2) m_B/\sqrt{2}, b_1) + s((1 - x_3) m_B/\sqrt{2}, b_2) - \frac{1}{\beta_1}\left[\ln\frac{\ln(t/\Lambda)}{-\ln(b_1\Lambda)} + \ln\frac{\ln(t/\Lambda)}{-\ln(b_2\Lambda)}\right], \quad \text{(A22)}$$

where the functions $s(q, b)$ are defined in Appendix A of Ref. [7]. The scale t_i's in the above equations are chosen as

$$t_e^1 = \max(\sqrt{x_2} m_B, 1/b_1, 1/b_2), \quad \text{(A23)}$$

$$t_e^2 = \max(\sqrt{x_1} m_B, 1/b_1, 1/b_2), \quad \text{(A24)}$$

$$t_d = \max(\sqrt{x_1 x_2} m_B, \sqrt{x_2 x_3} m_B, 1/b_1, 1/b_2), \quad \text{(A25)}$$

$$t_f^1 = \max(\sqrt{x_2 x_3} m_B, 1/b_1, 1/b_2), \quad \text{(A26)}$$

$$t_f^2 = \max(\sqrt{x_2 x_3} m_B, \sqrt{x_2 + x_3 - x_2 x_3} m_B, 1/b_1, 1/b_2). \quad \text{(A27)}$$

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