Neutron β^- decay as a laboratory for testing the standard model

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We analyze the sensitivity of all experimentally observable asymmetries and energy distributions for the neutron β^- decay with a polarized neutron and an unpolarized decay proton and electron, and the lifetime of the neutron to contributions of order 10^{-4} of interactions beyond the standard model (SM). We analyze the contributions of interactions beyond the SM in the linear approximation with respect to the Herczeg phenomenological coupling constants, introduced at the hadronic level. Such an approximation is good enough for the analysis of contributions of order 10^{-4} of interactions beyond the SM. We show that in such an approximation the correlation coefficients depend only on the axial coupling constant, which absorbs the contributions of the Herczeg left-left and left-right lepton-nucleon current-current interactions (vector and axial-vector interactions beyond the SM), and the Herczeg scalar and tensor coupling constants. In the lifetime of the neutron, in addition to the axial coupling constant, the contributions of the Herczeg left-left and left-right lepton-nucleon current-current interactions (vector and axial-vector interactions beyond the SM) are absorbed by the Cabibbo–Kobayashi–Maskawa matrix element.

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I. INTRODUCTION

In this paper we propose a consistent analysis of the sensitivity of all observable asymmetries and energy distributions of the neutron β^- decay $n \to p + e^- + \bar{\nu}_e$ with a polarized neutron and an unpolarized decay proton and electron, and the lifetime of the neutron [1,2] to contributions of order 10^{-4} of interactions beyond the standard model (SM), described at the phenomenological level. Such an order of corrections beyond the SM has been pointed out by Ramsey-Musolf and Su [3] within the minimal supersymmetric extension of the SM (MSSM). As has been shown in [4–7], at the phenomenological level the neutron β^- decay may be described by eight complex coupling constants determining the strength of interactions beyond the SM. As a result possible deviations from the SM, causing the contributions of order 10^{-4} to the correlation coefficients of the neutron β^- decay and the lifetime of the neutron, may be determined in terms of vector, axial-vector, scalar and tensor lepton-baryon weak coupling constants [8].

It is well known [1,2] that in the nonrelativistic approximation to leading order in the large proton mass expansion, which is equivalent to the leading order of the heavy-baryon approximation, and in the rest frame of the neutron the SM with weak $V - A$ interactions [9] describes the β^- decay of the neutron in terms of two coupling constants G_V and G_A [1,2] (see also [10]). The coupling constant G_V is defined by the product $G_V = G_F V_{ud}$ of the Fermi coupling constant $G_F = g_W^2/8M_W^2$ [9], where g_W and M_W are the electroweak coupling constant and the W-boson mass, and V_{ud} is the matrix element of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix [9]. The coupling constant G_A is equal to $G_A = \lambda G_V$, where $\lambda = G_A/G_V$ is the axial coupling constant, induced by renormalization of the axial hadronic current by strong low-energy interactions [11]. For the weak interactions, invariant under time reversal, the coupling constant λ is real.

The observables of the neutron β^- decay with unpolarized particles are the lifetime of the neutron τ_n and the correlation coefficient a_0, describing correlations between 3-momenta of the electron and antineutrino to leading order in the large proton mass expansion in the rest frame of the neutron. Experimentally, the correlation coefficient a_0 can be determined, for example, by measuring either the electron-proton energy distribution $a(E_e, T_p)$, where E_e and T_p are the total and kinetic energies of the electron and proton, respectively, or the proton-energy spectrum $a(T_p)$. As a function of the electron energy E_e the correlation coefficient of correlations between the electron and antineutrino 3-momenta we denote as $a(E_e)$.

The neutron β^- decay with a polarized neutron and an unpolarized decay proton and electron is characterized also by two additional observable correlation coefficients A_0 and B_0, describing to leading order in the large proton mass expansion correlations between the neutron spin and 3-momenta of the electron and antineutrino, respectively. As functions of the electron energy E_e the correlation coefficients of correlations between the neutron spin and 3-momenta of the electron and antineutrino we denote as $A(E_e)$ and $B(E_e)$, respectively. The lifetime of the neutron and the correlation coefficients under consideration are of

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order $\tau_n \sim 880$ s, $a_0 \sim a(E_e) \sim A_0 \sim A(E_e) \sim -0.1$ and $B_0 \sim B(E_e) \sim 1$ [9]. The coupling constants G_V and λ define the main contributions to the lifetime of the neutron and the correlation coefficients [1,2] (see also [10]).

However, for the description of the neutron β^- decay at the modern level of experimental accuracies when the experimental data on the axial coupling constant $\lambda = -1.2750(9)$ [1] and the lifetime of the neutron $\tau_n^{(\rm exp)} = 878.5(8)$ s [12] have been obtained with accuracies 0.07% and 0.09%, respectively, two coupling constants G_V and λ are not enough for the correct description of the properties of the neutron β^- decay. Indeed, it is well known that the radiative corrections to the lifetime of the neutron [13–33] calculated to leading order in the large proton mass expansion within the SM, with the $V - A$ weak interactions and quantum electrodynamics, give a contribution of about 3.9% [28] (see also [1]). This allows us to correctly describe the lifetime of the neutron [10]. The radiative corrections are very important also for the correct determination of the Fermi coupling constant G_F. It may be extracted from the experimental data on the weak coupling constant G_μ of the μ^-–decay $\mu^- \to e^- + \nu_\mu + \bar{\nu}_e$, taking into account the radiative corrections [13–16]. The contributions of the radiative corrections to the ratios of the correlation coefficients $a(E_e)/a_0$ and $A(E_e)/A_0$ are an order of magnitude smaller compared with the contribution to the lifetime of the neutron, whereas the correlation coefficient $B(E_e)$ has no radiative corrections to order α/π [20,34,35]. We would like to note that Glück [33] has used the Monte Carlo simulation method for the numerical calculation of the rates of nuclear and hadronic radiative β^- decays in order to correctly take into account the contributions of the nucleus-photon and hadron-photon correlations.

Another type of corrections, which may be calculated within the SM and should be taken into account for the description of the neutron β^- decay on the same footing as the radiative corrections, are the contributions of the "weak magnetism" and the proton recoil [36,37] (see also [35]), calculated to next-to-leading order in the large proton mass expansion.

The radiative corrections and the corrections from the weak magnetism and the proton recoil, calculated to leading order and to next-to-leading order in the large proton mass expansion, respectively, define a complete set of corrections to the observables of the neutron β^- decay, which should be taken into account within the SM as a background for the analysis of contributions of order 10^{-4} of interactions beyond the SM.

Since the observable asymmetries and energy distributions are defined in terms of the correlation coefficients [1], for the aim of analyzing the sensitivity of the observables of the neutron β^- decay to contributions of order 10^{-4} of interactions beyond the SM, we revise the calculation of the correlation coefficients of the neutron β^- decay with a polarized neutron and an unpolarized decay proton and electron, and the lifetime of the neutron within the SM with $V - A$ weak interactions. In addition to the nonrelativistic terms a_0, A_0 and B_0, calculated to leading order in the large proton mass expansion, we take into account (i) the contributions of the weak magnetism and the proton recoil to next-to-leading order in the large M expansion, which provide a complete set of corrections of order $1/M$, where M is the average mass of the neutron and proton, $M = (m_n + m_p)/2$, and (ii) the radiative corrections of order α/π, calculated to leading order in the large M expansion, where $\alpha = 1/137.036$ is the fine-structure constant [9]. The small parameter of the large M expansion is $E_0/M \sim 10^{-3}$, where $E_0 \sim 1$ MeV is the end-point energy of the electron-energy spectrum. The parameter $E_0/M \sim 10^{-3}$ is commensurable with the parameter of the radiative corrections $\alpha/\pi \sim 10^{-3}$. As we show below, due to the strong dependence on the axial coupling constant λ, the numerical values of the $1/M$ corrections vary from 10^{-5} to 10^{-1}. In turn, the contributions of the radiative corrections do not depend on the axial coupling constant λ. Following Sirlin [18] we show that the contributions of the radiative corrections, depending on the axial coupling constant λ, may be absorbed by renormalization of the Fermi G_F and axial λ coupling constants. The contributions of radiative corrections to the ratios $a(E_e)/a_0$ and $A(E_e)/A_0$ are of order 10^{-3}. The correlation coefficients, calculated within the SM to order $1/M$ and α/π, determine the theoretical background for the analysis of contributions of order 10^{-4} beyond the SM.

A phenomenological analysis of contributions of interactions beyond the SM model shows that these interactions induce only (i) the energy independent contributions and (ii) the contributions proportional to m_e/E_e, where m_e is the electron mass. We show below that the contributions of the weak magnetism and the proton recoil into the terms proportional to m_e/E_e of the correlation coefficients $a(E_e)$, $A(E_e)$ and $B(E_e)$ are of order 10^{-3}–10^{-4}. Thus, the obtained theoretical expressions for the correlation coefficients should be taken into account for a correct experimental determination of contributions of order 10^{-4} beyond the SM.

The radiative corrections to the correlation coefficients $a(E_e)/a_0$ and $A(E_e)/A_0$ determine the most important and complicated part of the corrections of order 10^{-3}. In the neutron β^- decay the radiative corrections are defined by (i) the contributions to the continuum-state β^--decay mode from one-virtual photon exchanges, electroweak-boson exchanges and QCD corrections [24,27–30] and (ii) the contribution of the radiative β^--decay mode $n \to p + e^- + \bar{\nu}_e + \gamma$ with emission of a real photon [13–30] (see also [34,35]). The sum of the electron-energy and angular distributions of these two decay modes does not suffer from infrared divergences of virtual and real photons.

We calculate the radiative corrections to the β^- decay of the neutron within the standard finite-photon mass (FPM) regularization of infrared divergences [13–30]. As has been shown by Marciano and Sirlin [22], the FPM regularization is equivalent to the dimensional regularization [38].

The radiative corrections to the lifetime of the neutron, calculated within the FPM regularization, obey the Kinoshita-Lee-Nauenberg theorem [39]. According to the Kinoshita-Lee-Nauenberg theorem, the radiative corrections to the lifetime of the neutron, integrated over the phase volume of the final state of the neutron β^- decay in the limit of the massless electron $m_e \to 0$, should not depend on the electron mass m_e. For the first time such an independence of the electron mass in the limit $m_e \to 0$ has been demonstrated by Kinoshita and Sirlin [15] (see also [40]).

We reproduce fully the radiative corrections to the lifetime of the neutron and the correlation coefficients $a(E_e)$ and $A(E_e)$, calculated in [16–30,34,35], respectively [see Eq. (7) and Appendix D of Ref. [40]]. We show also that the correlation coefficient $B(E_e)$ has no radiative corrections to order α/π, calculated to leading order in the large M expansion. This agrees well with the results obtained by Gudkov *et al.* [35]. We calculate the corrections of the weak magnetism and the proton recoil in complete agreement with the results obtained in [35] (see also [37]). Above the background of these corrections the contributions of interactions beyond the SM are calculated to leading order in the large M expansion and in the linear approximation with respect to the Herczeg phenomenological coupling constants, introduced at the hadronic level in terms of the lepton-nucleon current-current interactions (see Appendix G of Ref. [40]). Such an approximation is good enough for the analysis of the sensitivity of the observables of the neutron β^- decay to contributions of order 10^{-4} beyond the SM.

The obtained results are applied to the theoretical analysis of (i) the asymmetries $A_{\rm exp}(E_e)$, $B_{\rm exp}(E_e)$, the electron-proton energy distribution $a(E_e, T_p)$, the proton-energy spectrum $a(T_p)$, and the proton recoil asymmetry $C_{\rm exp}$, used for the experimental determination of the axial coupling constant λ and the correlation coefficients A_0, B_0, a_0 and $C_0 = -x_C(A_0 + B_0)$, where $x_C = 0.27591$ is a theoretical numerical factor (see Sec. VII), respectively; (ii) the lifetime of the neutron; and (iii) the sensitivity of the electron-proton energy distribution $a(E_e, T_p)$, the proton-energy spectrum $a(T_p)$, the asymmetries $A_{\rm exp}(E_e)$, $B_{\rm exp}(E_e)$ and $C_{\rm exp}$, and the lifetime of the neutron τ_n to contributions of order 10^{-4} of interactions beyond the SM. The experimental analysis of the asymmetries $A_{\rm exp}(E_e)$, $B_{\rm exp}(E_e)$, the proton-energy spectrum $a(T_p)$, and the proton recoil asymmetry $C_{\rm exp}$ has been carried out in [1,41–44] (see also [45]) and [46], respectively.

II. ORGANIZATION OF PAPER

The paper is organized as follows. In Sec. III we calculate the electron-energy and angular distribution of the neutron β^- decay with a polarized neutron and an unpolarized decay proton and electron. We take into account a complete set of corrections of order $1/M$, caused by the weak magnetism and the proton recoil, and the radiative corrections of order α/π. Then we analyze (i) in Sec. IV the electron asymmetry $A_{\rm exp}(E_e)$, which has been used in [1,41] for the experimental determination of the axial coupling constant $\lambda = -1.2750(9)$ and the correlation coefficient $A_0^{(\rm exp)} = -0.11933(34)$, (ii) in Sec. V the antineutrino asymmetry $B_{\rm exp}(E_e)$, which has been used for the experimental determination of the correlation coefficient $B_0^{(\rm exp)} = 0.9802(50)$ in [42,43], (iii) in Sec. VI the electron-proton energy distribution $a(E_e, T_p)$ and the proton-energy spectrum $a(T_p)$, which may be used for the experimental determination of the axial coupling constant λ and the correlation coefficient a_0, (iv) in Sec. VII the proton recoil asymmetry $C_{\rm exp}$, which has been used for the experimental determination of the correlation coefficient $C_0^{(\rm exp)} = -x_C(A_0 + B_0) = -0.2377(26)$ [46], and (v) in Sec. VIII the lifetime of the neutron. In Sec. IX we propose the theoretical analysis of the sensitivity of the electron-proton energy distribution $a(E_e, T_p)$, the proton-energy spectrum $a(T_p)$, the asymmetries $A_{\rm exp}(E_e)$, $B_{\rm exp}(E_e)$ and $C_{\rm exp}$, and the lifetime of the neutron to contributions of order 10^{-4} of interactions beyond the SM. We show that to linear approximations with respect to the Herczeg phenomenological coupling constants, the Herczeg coupling constants a^h_{LL} and a^h_{LR} of the left-left and left-right lepton-nucleon current-current interactions (vector and axial-vector interactions beyond the SM) can be fully absorbed by the axial coupling constants and cannot be determined from the experimental data on the electron-proton energy distribution $a(E_e, T_p)$, the proton-energy spectrum $a(T_p)$, and the asymmetries $A_{\rm exp}(E_e)$, $B_{\rm exp}(E_e)$ and $C_{\rm exp}(E_e)$. This agrees well with the results obtained in [47–49]. In the lifetime of the neutron the contributions of the Herczeg left-left and left-right lepton-nucleon current-current interactions (vector and axial-vector interactions beyond the SM) become unobservable after the renormalization of the CKM matrix element $V_{ud} \to (V_{ud})_{\rm eff} = V_{ud}(1 + a^h_{LL} + a^h_{LR})$ (see Sec. IX), in agreement with [47–49]. In Sec. X we summarize the obtained results. In the Appendix we given a short derivation of the functions $g_n(E_e)$ and $f_n(E_e)$, describing radiative corrections to the lifetime of the neutron and the correlation coefficients a_0 and A_0. A detailed calculation of the $1/M$ corrections and the radiative corrections of order α/π to the observables of the neutron β^-–decay is given in Appendixes A, B, C, D, E, F, H, and I of Ref. [40].

III. ELECTRON-ENERGY AND ANGULAR DISTRIBUTION OF NEUTRON β^- DECAY IN THE STANDARD MODEL

For the analysis of the electron-energy and angular distribution of the continuum-state β^- decay of the neutron, we use the Hamiltonian of $V - A$ interactions with a real axial coupling constant λ and the contribution of the weak magnetism [38,50],

$$\mathcal{H}_W(x) = \frac{G_F}{\sqrt{2}} V_{ud}\{[\bar{\psi}_p(x)\gamma_\mu(1 + \lambda\gamma^5)\psi_n(x)] + \frac{\kappa}{2M}\partial^\nu[\bar{\psi}_p(x)\sigma_{\mu\nu}\psi_n(x)]\} \times [\bar{\psi}_e(x)\gamma^\mu(1 - \gamma^5)\psi_\nu(x)], \quad (1)$$

which is invariant under time reversal. Here $\psi_p(x)$, $\psi_n(x)$, $\psi_e(x)$ and $\psi_\nu(x)$ are the field operators of the proton, neutron, electron and antineutrino, respectively; γ^μ, γ^5 and $\sigma^{\mu\nu} = \frac{i}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$ are the Dirac matrices [38]; and $\kappa = \kappa_p - \kappa_n = 3.7058$ is the isovector anomalous magnetic moment of the nucleon, defined by the anomalous magnetic moments of the proton $\kappa_p = 1.7928$ and the neutron $\kappa_n = -1.9130$ and measured in the nuclear magneton [9].

For numerical calculations we use $G_F = 1.1664 \times 10^{-11}\ \mathrm{MeV}^{-2}$ and $|V_{ud}| = 0.97428(15)$ [9]. The value of the CKM matrix element $|V_{ud}| = 0.97428(15)$ agrees well with $|V_{ud}| = 0.97425(22)$, measured from the superallowed $0^+ \to 0^+$ nuclear β^- decays [51]. It also satisfies well the unitarity condition $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.99999(41)$ for the CKM matrix elements [9]. The error $\Delta_{\mathrm{U}} = \pm 0.00041$ of the unitarity condition is determined by the errors of the CKM matrix elements $|V_{ud}| = 0,97427 \pm 0.00015$, $|V_{us}| = 0.22534 \pm 0.00065$ and $|V_{ub}| = 0.00351^{+0.00015}_{-0.00014}$ [see Eq. (11.27) on p. 162 of Ref. [9]]. As a result the error of the unitarity condition is equal to

$$\Delta_{\mathrm{U}} = \sqrt{\sum_q |2V_{uq}\Delta V_{uq}|^2} = 0.00041,$$

where $q = d, s, b$ and $\Delta V_{ud} = 0.00015$, $\Delta V_{us} = 0.00065$ and $\Delta V_{ub} = 0.00015$, respectively.

The amplitude of the continuum-state β^- decay of the neutron, calculated in the rest frame of the neutron and to next-to-leading order in the large M expansion, taking into account the contributions of the weak magnetism and the proton recoil, is (see Appendix A of Ref. [40])

$$\begin{aligned} M(n \to p e^- \bar{\nu}_e) = -2m_n \frac{G_F}{\sqrt{2}} V_{ud}\{&[\varphi_p^\dagger\varphi_n][\bar{u}_e\gamma^0(1 - \gamma^5)v_{\bar{\nu}}] - \tilde{\lambda}[\varphi_p^\dagger\vec{\sigma}\varphi_n]\cdot[\bar{u}_e\vec{\gamma}(1 - \gamma^5)v_{\bar{\nu}}] \\ &- \frac{m_e}{2M}[\varphi_p^\dagger\varphi_n][\bar{u}_e(1 - \gamma^5)v_{\bar{\nu}}] + \frac{\tilde{\lambda}}{2M}[\varphi_p^\dagger(\vec{\sigma}\cdot\vec{k}_p)\varphi_n][\bar{u}_e\gamma^0(1 - \gamma^5)v_{\bar{\nu}}] \\ &- i\frac{\kappa + 1}{2M}[\varphi_p^\dagger(\vec{\sigma}\times\vec{k}_p)\varphi_n]\cdot[\bar{u}_e\vec{\gamma}(1 - \gamma^5)v_{\bar{\nu}}]\}, \end{aligned} \quad (2)$$

where φ_p and φ_n are the Pauli spinorial wave functions of the proton and neutron and u_e and $v_{\bar{\nu}}$ are the Dirac bispinor wave functions of the electron and antineutrino, respectively. Then, $\vec{k}_p$ is the 3-momentum of the proton related to the 3-momenta of the electron $\vec{k}_e$ and antineutrino $\vec{k}$ as $\vec{k}_p = -\vec{k}_e - \vec{k}$, $\tilde{\lambda} = \lambda(1 - E_0/2M)$, where $E_0 = (m_n^2 - m_p^2 + m_e^2)/2m_n = 1.2927$ MeV is the end-point energy of the electron-energy spectrum, calculated for $m_n = 939.5654$ MeV, $m_p = 938.2720$ MeV and $m_e = 0.5110$ MeV [9]. From Eq. (2) one may see that the parameter of the large M expansion or the $1/M$ corrections to the amplitude of the β^- decay of the neutron is $k_p/M \sim E_0/M \sim 10^{-3}$. The detailed calculation of the amplitude, Eq. (2), is given in Appendix A of Ref. [40].

The electron-energy and angular distribution of the neutron β^- decay takes the form [1,35,40]

$$\begin{aligned} \frac{d^5\lambda_n(E_e, \vec{k}_e, \vec{k}, \vec{\xi}_n)}{dE_e d\Omega_e d\Omega} = {}& (1 + 3\lambda^2)\frac{G_F^2|V_{ud}|^2}{32\pi^5}(E_0 - E_e)^2\sqrt{E_e^2 - m_e^2}E_e F(E_e, Z = 1)\Phi_{\beta_c^-}(\vec{k}_e, \vec{k})\tilde{\zeta}(E_e) \\ &\times\Big(1 + \tilde{a}(E_e)\frac{\vec{k}_e\cdot\vec{k}}{E_e E} + \tilde{A}(E_e)\frac{\vec{\xi}_n\cdot\vec{k}_e}{E_e} + \tilde{B}(E_e)\frac{\vec{\xi}_n\cdot\vec{k}}{E} + \tilde{K}_n(E_e)\frac{(\vec{\xi}_n\cdot\vec{k}_e)(\vec{k}_e\cdot\vec{k})}{E_e^2 E} \\ &+ \tilde{Q}_n(E_e)\frac{(\vec{\xi}_n\cdot\vec{k})(\vec{k}_e\cdot\vec{k})}{E_e E^2} + \tilde{D}(E_e)\frac{\vec{\xi}_n\cdot(\vec{k}_e\times\vec{k})}{E_e E}\Big), \end{aligned} \quad (3)$$

where $E = E_0 - E_e$ is the antineutrino energy and $d\Omega_e$ and $d\Omega$ are the infinitesimal elements of the solid angles of the electron and antineutrino 3-momenta relative to the neutron spin, respectively.

The function $\Phi_{\beta_c^-}(\vec{k}_e, \vec{k})$ is defined by the contribution of the proton recoil. To next-to-leading order in the large M expansion, it is equal to (see Appendix A of Ref. [40])

$$\Phi_{\beta_c^-}(\vec{k}_e, \vec{k}) = 1 + \frac{3}{M}\Big(E_e - \frac{\vec{k}_e\cdot\vec{k}}{E}\Big). \quad (4)$$

The function $F(E_e, Z = 1)$ is the relativistic Fermi function [52,53] (see also [37])

$$F(E_e, Z=1) = \Big(1+\frac{1}{2}\gamma\Big)\frac{4(2r_p m_e\beta)^{2\gamma}}{\Gamma^2(3+2\gamma)}\frac{e^{\pi\alpha/\beta}}{(1-\beta^2)^\gamma}\Big|\Gamma\Big(1+\gamma+i\frac{\alpha}{\beta}\Big)\Big|^2, \quad (5)$$

where $\beta = k_e/E_e = \sqrt{E_e^2 - m_e^2}/E_e$ is the electron velocity, $\gamma = \sqrt{1-\alpha^2} - 1$, r_p is the electric radius of the proton and $\alpha = 1/137.036$ is the fine-structure constant. In numerical calculations we will use $r_p = 0.841$ fm [54].

Following [35] we transcribe the right-hand side (r.h.s.) of Eq. (3) into the form

$$\begin{aligned}\frac{d^5\lambda_n(E_e,\vec{k}_e,\vec{k},\vec{\xi}_n)}{dE_e d\Omega_e d\Omega} &= (1+3\lambda^2)\frac{G_F^2|V_{ud}|^2}{32\pi^5}(E_0-E_e)^2\sqrt{E_e^2-m_e^2}E_e F(E_e,Z=1)\zeta(E_e)\\ &\times\Big\{1 + a(E_e)\frac{\vec{k}_e\cdot\vec{k}}{E_e E} + A(E_e)\frac{\vec{\xi}_n\cdot\vec{k}_e}{E_e} + B(E_e)\frac{\vec{\xi}_n\cdot\vec{k}}{E} + K_n(E_e)\frac{(\vec{\xi}_n\cdot\vec{k}_e)(\vec{k}_e\cdot\vec{k})}{E_e^2 E}\\ &+ Q_n(E_e)\frac{(\vec{\xi}_n\cdot\vec{k})(\vec{k}_e\cdot\vec{k})}{E_e E^2} + D(E_e)\frac{\vec{\xi}_n\cdot(\vec{k}_e\times\vec{k})}{E_e E} - 3\frac{E_e}{M}\frac{1-\lambda^2}{1+3\lambda^2}\Big(\frac{(\vec{k}_e\cdot\vec{k})^2}{E_e^2E^2} - \frac{1}{3}\frac{k_e^2}{E_e^2}\Big)\Big\}. \end{aligned} \quad (6)$$

The correlation coefficients are given by (see Appendixes A, B, C and D of Ref. [40])

$$\begin{aligned}\zeta(E_e) &= \Big(1+\frac{\alpha}{\pi}g_n(E_e)\Big) + \frac{1}{M}\frac{1}{1+3\lambda^2}\Big[-2\lambda(\lambda-(\kappa+1))E_0 + (10\lambda^2 - 4(\kappa+1)\lambda+2)E_e - 2\lambda(\lambda-(\kappa+1))\frac{m_e^2}{E_e}\Big],\\ \zeta(E_e)a(E_e) &= a_0\Big(1+\frac{\alpha}{\pi}g_n(E_e)+\frac{\alpha}{\pi}f_n(E_e)\Big) + \frac{1}{M}\frac{1}{1+3\lambda^2}[2\lambda(\lambda-(\kappa+1))E_0 - 4\lambda(3\lambda-(\kappa+1))E_e],\\ \zeta(E_e)A(E_e) &= A_0\Big(1+\frac{\alpha}{\pi}g_n(E_e)+\frac{\alpha}{\pi}f_n(E_e)\Big) + \frac{1}{M}\frac{1}{1+3\lambda^2}[(\lambda^2-\kappa\lambda-(\kappa+1))E_0 - (5\lambda^2-(3\kappa-4)\lambda-(\kappa+1))E_e],\\ \zeta(E_e)B(E_e) &= B_0\Big(1+\frac{\alpha}{\pi}g_n(E_e)\Big) + \frac{1}{M}\frac{1}{1+3\lambda^2}\Big[-2\lambda(\lambda-(\kappa+1))E_0 + (7\lambda^2-(3\kappa+8)\lambda+(\kappa+1))E_e\\ &\quad - (\lambda^2-(\kappa+2)\lambda+(\kappa+1))\frac{m_e^2}{E_e}\Big],\\ \zeta(E_e)K_n(E_e) &= \frac{1}{M}\frac{1}{1+3\lambda^2}(5\lambda^2-(\kappa-4)\lambda-(\kappa+1))E_e,\\ \zeta(E_e)Q_n(E_e) &= \frac{1}{M}\frac{1}{1+3\lambda^2}[(\lambda^2-(\kappa+2)\lambda+(\kappa+1))E_0 - (7\lambda^2-(\kappa+8)\lambda+(\kappa+1))E_e],\\ \zeta(E_e)D(E_e) &= 0, \end{aligned} \quad (7)$$

where the correlation coefficients a_0, A_0 and B_0 are determined by [1] (see also [10])

$$a_0 = \frac{1-\lambda^2}{1+3\lambda^2}, \qquad A_0 = -2\frac{\lambda(1+\lambda)}{1+3\lambda^2}, \qquad B_0 = -2\frac{\lambda(1-\lambda)}{1+3\lambda^2}. \quad (8)$$

The radiative corrections are determined by the functions $g_n(E_e)$ and $f_n(E_e)$, which are given in the Appendix and derived in Appendix D of Ref. [40].

The functions $(\alpha/\pi)g_n(E_e)$ and $(\alpha/\pi)f_n(E_e)$ describe the radiative corrections to the lifetime of the neutron and the correlation coefficients $a(E_e)$ and $A(E_e)$, respectively. They are equal to the radiative corrections, calculated in [18–30,34,35], respectively. For the first time the radiative corrections to the correlation coefficients $a(E_e)$ and $A(E_e)$ have been calculated by Shann [20]. The function $(\alpha/\pi)f_n(E_e)$ is in analytical agreement with the results obtained by Shann [20]. We show in Appendix D of Ref. [40] that the correlation coefficient $B(E_e)$ has no radiative corrections to order α/π. This also agrees well with the results obtained in [35]. The densities of the radiative corrections $(\alpha/\pi)g_n(E_e)\rho_{\beta_c^-}(E_e)$ and $(\alpha/\pi)f_n(E_e)\rho_{\beta_c^-}(E_e)$, where $\rho_{\beta_c^-}(E_e)$ is the electron-energy spectrum density, Eq. (A8), are plotted in Fig. 1.

The coefficients $K_n(E_e)$ and $Q_n(E_e)$ have been introduced in [34,35] and calculated within the effective quantum field theory, based on the heavy-baryon chiral perturbation theory (HBChPT). Our results of the calculation of the correlation coefficients, carried out to next-to-leading order in the large M expansion (see Appendix A of Ref. [40]), agree fully with the expressions calculated in [35].

The correlation coefficient $D(E_e)$ relates to a violation of time reversal invariance. In the SM a nonvanishing correlation coefficient $D(E_e)$ may appear due to long-range [55–58] (see also [36]) and short-range [59] mechanisms of time reversal violation. In the long-range mechanism of

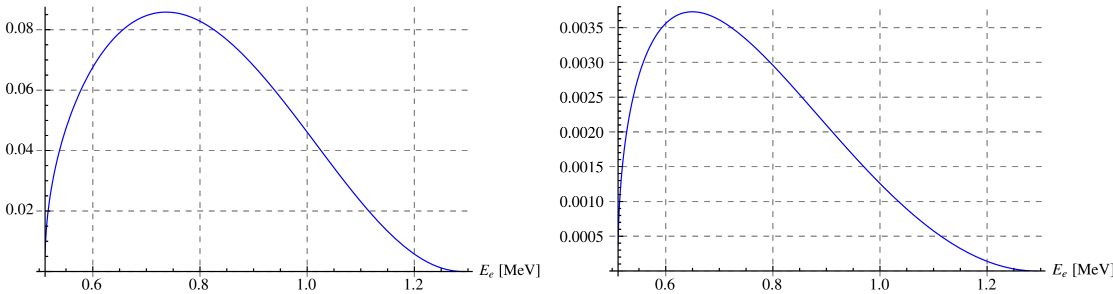

FIG. 1 (color online). The densities $(\alpha/\pi)g_n(E_e)\rho_{\beta^-_c}(E_e)$ (left panel) and $(\alpha/\pi)f_n(E_e)\rho_{\beta^-_c}(E_e)$ (right panel), measured in MeV^{-1}, of the radiative corrections to the lifetime of the neutron and the correlation coefficients $a(E_e)$ and $A(E_e)$, where $\rho_{\beta^-_c}(E_e)$ is the electron-energy spectrum density, Eq. (A8).

time reversal violation the correlation coefficient $D(E_e)$ is induced by the electron-proton interaction in the final state of the decay due to the distortion of the electron wave function in the Coulomb field of the proton [52,53], the weak magnetism and the proton recoil. In the short-range mechanism of time reversal violation the correlation coefficient $D(E_e)$ takes a contribution from the CP-violating phase δ of the CKM quark mixing matrix [9]. According to [59], the contribution of the long-range mechanism of time reversal violation dominates by many orders of magnitude in comparison with the contribution of the short-range one. As has been shown in [55–58] the correlation coefficient $D(E_e)$ is a function of the electron energy E_e. Using the results obtained in [55], for the electron kinetic energies $250\ \mathrm{keV} \le T_e \le 455\ \mathrm{keV}$ [1,41,43] and the axial coupling constant $\lambda = -1.2750$ we obtain that $D(E_e) \sim 10^{-5}$. Hence the contribution of the long-range mechanism of time reversal violation to the correlation coefficient $D(E_e)$ is smaller compared with contributions of order 10^{-4}, which may be induced by interactions beyond the SM. Recently the correlation coefficient $D(E_e)$ has been calculated within heavy-baryon effective field theory by Ando *et al.* [60]. The authors have reproduced the result obtained by Callan and Treiman [55] and have found a correction, which is smaller compared with 10^{-7} in the experimental region of electron kinetic energies $250\ \mathrm{keV} \le T_e \le 455\ \mathrm{keV}$ [1,41,43].

From Eq. (7) we define the correlation coefficients under consideration, taking into account the contributions of order $1/M$ and α/π, as follows:

$$
\begin{aligned}
a(E_e) &= a_0\Big(1 + \frac{\alpha}{\pi} f_n(E_e)\Big) + \frac{1}{M}\Big[\frac{2\lambda(\lambda - (\kappa+1))}{1+3\lambda^2} E_0 - \frac{4\lambda(3\lambda - (\kappa+1))}{1+3\lambda^2} E_e\Big] - a_0\,\delta\zeta(E_e),\\
A(E_e) &= A_0\Big(1 + \frac{\alpha}{\pi} f_n(E_e)\Big) + \frac{1}{M}\Big[\frac{\lambda^2 - \kappa\lambda - (\kappa+1)}{1+3\lambda^2} E_0 - \frac{5\lambda^2 - (3\kappa-4)\lambda - (\kappa+1)}{1+3\lambda^2} E_e\Big] - A_0\,\delta\zeta(E_e),\\
B(E_e) &= B_0 + \frac{1}{M}\Big[-\frac{2\lambda(\lambda-(\kappa+1))}{1+3\lambda^2} E_0 + \frac{7\lambda^2 - (3\kappa+8)\lambda + (\kappa+1)}{1+3\lambda^2} E_e - \frac{\lambda^2 - (\kappa+2)\lambda + (\kappa+1)}{1+3\lambda^2}\frac{m_e^2}{E_e}\Big] - B_0\,\delta\zeta(E_e),\\
K_n(E_e) &= \frac{1}{M}\frac{5\lambda^2 - (\kappa-4)\lambda - (\kappa+1)}{1+3\lambda^2} E_e,\\
Q_n(E_e) &= \frac{1}{M}\Big[\frac{\lambda^2 - (\kappa+2)\lambda + (\kappa+1)}{1+3\lambda^2} E_0 - \frac{7\lambda^2 - (\kappa+8)\lambda + (\kappa+1)}{1+3\lambda^2} E_e\Big].
\end{aligned}
\tag{9}
$$

Using the following expansion,

$$
\begin{aligned}
&\frac{1}{\zeta(E_e)} = \Big(1 - \frac{\alpha}{\pi} g_n(E_e)\Big) - \delta\zeta(E_e), \qquad \delta\zeta(E_e) = \frac{1}{M}\frac{1}{1+3\lambda^2}\Big(\zeta_1 E_0 + \zeta_2 E_e + \zeta_3\frac{m_e^2}{E_e}\Big),\\
&\zeta_1 = -2\lambda(\lambda - (\kappa+1)), \qquad \zeta_2 = 10\lambda^2 - 4(\kappa+1)\lambda + 2, \qquad \zeta_3 = -2\lambda(\lambda - (\kappa+1)),
\end{aligned}
\tag{10}
$$

we transcribe the correlation coefficients $a(E_e)$, $A(E_e)$ and $B(E_e)$ into a form that is similar to the one proposed by Wilkinson for the correlation coefficient $A^{(W)}(E_e)$ [see Eq. (20)]. We get

$$a(E_e) = a_0\Big\{1 + \frac{1}{M}\frac{1}{(1-\lambda^2)(1+3\lambda^2)} \times \Big(a_1 E_0 + a_2 E_e + a_3\frac{m_e^2}{E_e}\Big)\Big\}\Big(1 + \frac{\alpha}{\pi}f_n(E_e)\Big),$$
$$a_1 = 4\lambda(\lambda^2+1)(\lambda-(\kappa+1)),$$
$$a_2 = -26\lambda^4 + 8(\kappa+1)\lambda^3 - 20\lambda^2 + 8(\kappa+1)\lambda - 2,$$
$$a_3 = -2\lambda(\lambda^2-1)(\lambda-(\kappa+1)) \quad (11)$$

and

$$A(E_e) = A_0\Big\{1 - \frac{1}{M}\frac{1}{2\lambda(1+\lambda)(1+3\lambda^2)} \times \Big(A_1 E_0 + A_2 E_e + A_3\frac{m_e^2}{E_e}\Big)\Big\}\Big(1 + \frac{\alpha}{\pi}f_n(E_e)\Big),$$
$$A_1 = -\lambda^4 + \kappa\lambda^3 + (\kappa+2)\lambda^2 - \kappa\lambda - (\kappa+1),$$
$$A_2 = 5\lambda^4 + \kappa\lambda^3 - (5\kappa+6)\lambda^2 + 3\kappa\lambda + (\kappa+1),$$
$$A_3 = -4\lambda^2(\lambda+1)(\lambda-(\kappa+1)) \quad (12)$$

and

$$B(E_e) = B_0\Big\{1 - \frac{1}{M}\frac{1}{2\lambda(1-\lambda)(1+3\lambda^2)} \times \Big(B_1 E_0 + B_2 E_e + B_3\frac{m_e^2}{E_e}\Big)\Big\},$$
$$B_1 = -2\lambda(\lambda+1)^2(\lambda-(\kappa+1)),$$
$$B_2 = \lambda^4 - (\kappa-4)\lambda^3 - (5\kappa+2)\lambda^2 - (3\kappa+4)\lambda + (\kappa+1),$$
$$B_3 = (\lambda^2-1)(\lambda-1)(\lambda-(\kappa+1)). \quad (13)$$

For the derivation of Eqs. (11)–(13) we have neglected the terms of order $(\alpha/\pi)(E_0/M) \sim 10^{-6}$, which are smaller compared with contributions of order 10^{-4} beyond the SM.

In order to estimate the values of the obtained $1/M$ corrections we calculate them at $\lambda = -1.2750$. This gives

$$\delta\zeta(E_e) = \frac{1}{M}\frac{1}{1+3\lambda^2}\Big(\zeta_1 E_0 + \zeta_2 E_e + \zeta_3\frac{m_e^2}{E_e}\Big) = -3.57\times 10^{-3} + 9.90\times 10^{-3}\frac{E_e}{E_0} - 1.41\times 10^{-3}\frac{m_e}{E_e},$$
$$\frac{\delta a(E_e)}{a_0} = \frac{1}{M}\frac{1}{(1-\lambda^2)(1+3\lambda^2)}\Big(a_1 E_0 + a_2 E_e + a_3\frac{m_e^2}{E_e}\Big) = -3.00\times 10^{-2} + 8.59\times 10^{-2}\frac{E_e}{E_0} + 1.41\times 10^{-3}\frac{m_e}{E_e},$$
$$\frac{\delta A(E_e)}{A_0} = -\frac{1}{M}\frac{1}{2\lambda(1+\lambda)(1+3\lambda^2)}\Big(A_1 E_0 + A_2 E_e + A_3\frac{m_e^2}{E_e}\Big) = 3.44\times 10^{-4} + 1.46\times 10^{-2}\frac{E_e}{E_0} + 1.41\times 10^{-3}\frac{m_e}{E_e},$$
$$\frac{\delta B(E_e)}{B_0} = -\frac{1}{M}\frac{1}{2\lambda(1-\lambda)(1+3\lambda^2)}\Big(B_1 E_0 + B_2 E_e + B_3\frac{m_e^2}{E_e}\Big) = -4.66\times 10^{-5} - 2.97\times 10^{-4}\frac{E_e}{E_0} + 1.36\times 10^{-4}\frac{m_e}{E_e},$$
$$K_n(E_e) = 7.14\times 10^{-4}\frac{E_e}{E_0},$$
$$Q_n(E_e) = 3.19\times 10^{-3} - 7.27\times 10^{-3}\frac{E_e}{E_0}. \quad (14)$$

Because of a strong dependence on the axial coupling constant λ, the numerical values of the contributions of the "weak magnetism" and the proton recoil, calculated at $\lambda = -1.2750$, vary from 10^{-5} to 10^{-1} for energy independent and energy dependent terms.

In summary, the correlation coefficients of the electron-energy and angular distribution of the neutron β^- decay with a polarized neutron and an unpolarized decay proton and electron are calculated by taking into account a complete set of the $1/M$ corrections, caused by the weak magnetism and the proton recoil, and the radiative corrections of order α/π, calculated to leading order in the large M expansion. The obtained expressions for the correlation coefficients should be used as a theoretical background for contributions of order 10^{-4} of interactions beyond the SM, calculated in Appendix G of Ref. [40]. Contributions of order 10^{-4} of interactions beyond the SM may be determined by measuring the asymmetries $A_{\rm exp}(E_e)$, $B_{\rm exp}(E_e)$ and $C_{\rm exp}$ between the neutron spin and the 3-momenta of the decay particles, the electron-proton energy distribution $a(E_e, T_p)$ and the proton-energy spectrum $a(T_p)$, related to correlations between the 3-momenta of the electron and proton, and the lifetime of the neutron τ_n (see Sec. IX).

IV. STANDARD MODEL ANALYSIS OF EXPERIMENTAL DETERMINATION OF CORRELATION COEFFICIENT A_0. ELECTRON ASYMMETRY $A_{\rm exp}(E_e)$

For the experimental determination of the correlation coefficient A_0, defining correlations between the neutron spin and the electron 3-momentum in the SM to leading order in the large M expansion [1], the directions of the emission of the antineutrino are not fixed and one has to integrate over the antineutrino 3-momentum $\vec{k}$. As a result we arrive at the following electron-energy and angular distribution [1,41],

$$\frac{d^2\lambda_n(E_e,\vec{k}_e,\vec{\xi}_n)}{dE_e d\Omega_e} = (1+3\lambda^2)\frac{G_F^2|V_{ud}|^2}{8\pi^4}(E_0-E_e)^2 \times \sqrt{E_e^2-m_e^2}E_eF(E_e,Z=1)\zeta(E_e) \times \Big(1 + A^{(W)}(E_e)\Big(1+\frac{\alpha}{\pi}f_n(E_e)\Big)\vec{\xi}_n\cdot\vec{\beta}\Big), \tag{15}$$

where we have denoted

$$A(E_e) + \frac{1}{3}Q_n(E_e) = A^{(W)}(E_e)\Big(1+\frac{\alpha}{\pi}f_n(E_e)\Big), \tag{16}$$

$d\Omega_e = 2\pi\sin\theta_e d\theta_e$ is an infinitesimal solid angle of the electron 3-momentum with respect to the neutron spin and $\vec{\xi}_n\cdot\vec{\beta} = P\beta\cos\theta_e$ with the neutron polarization $P = |\vec{\xi}_n| \le 1$. The correlation coefficients $A(E_e)$ and $Q_n(E_e)$ are given in Eq. (12) and (9), respectively. We obtain the contribution, proportional to $Q_n(E_e)$, with structure $\vec{\xi}_n\cdot\vec{k}_e/E_e$, having integrated the term with the structure $(\vec{\xi}_n\cdot\vec{k})(\vec{k}_e\cdot\vec{k})/E_eE^2$ in Eq. (6) over directions of the antineutrino 3-momentum $\vec{k}$.

The asymmetry, which may be used for the experimental determination of the axial coupling constant λ and the correlation coefficient A_0, takes the form

$$A_{\rm exp}(E_e) = \frac{N^+(E_e)-N^-(E_e)}{N^+(E_e)+N^-(E_e)} = \frac{1}{2}A^{(W)}(E_e)\Big(1+\frac{\alpha}{\pi}f_n(E_e)\Big)P\beta(\cos\theta_1+\cos\theta_2), \tag{17}$$

where $N^{\pm}(E_e)$ are the numbers of events of the emission of the electron forward $(+)$ and backward $(-)$ with respect to the neutron spin into the solid angle $\Delta\Omega_{12} = 2\pi(\cos\theta_1 - \cos\theta_2)$ with $0\le\varphi\le 2\pi$ and $\theta_1\le\theta_e\le\theta_2$. They are determined by [61]

$$\begin{aligned} N^+(E_e) &= 2\pi N(E_e)\int_{\theta_1}^{\theta_2}\Big(1 + A^{(W)}(E_e)\Big(1+\frac{\alpha}{\pi}f_n(E_e)\Big)P\beta\cos\theta_e\Big)\sin\theta_e d\theta_e \\ &= 2\pi N(E_e)\Big(1 + \frac{1}{2}A^{(W)}(E_e)\Big(1+\frac{\alpha}{\pi}f_n(E_e)\Big)P\beta(\cos\theta_1+\cos\theta_2)\Big)(\cos\theta_1-\cos\theta_2), \\ N^-(E_e) &= 2\pi N(E_e)\int_{\pi-\theta_1}^{\pi-\theta_2}\Big(1 + A^{(W)}(E_e)\Big(1+\frac{\alpha}{\pi}f_n(E_e)\Big)P\beta\cos\theta_e\Big)\sin\theta_e d\theta_e \\ &= 2\pi N(E_e)\Big(1 - \frac{1}{2}A^{(W)}(E_e)\Big(1+\frac{\alpha}{\pi}f_n(E_e)\Big)P\beta(\cos\theta_1+\cos\theta_2)\Big)(\cos\theta_1-\cos\theta_2), \end{aligned} \tag{18}$$

where $N(E_e)$ is the normalization factor equal to

$$N(E_e) = (1+3\lambda^2)\frac{G_F^2|V_{ud}|^2}{8\pi^4}(E_0-E_e)^2\sqrt{E_e^2-m_e^2}E_eF(E_e,Z=1)\zeta(E_e). \tag{19}$$

The correlation coefficient $A^{(W)}(E_e)$ is

$$\begin{aligned} A^{(W)}(E_e) &= A_0\Big\{1 - \frac{1}{M}\frac{1}{2\lambda(1+\lambda)(1+3\lambda^2)}\Big(A_1^{(W)}E_0 + A_2^{(W)}E_e + A_3^{(W)}\frac{m_e^2}{E_e}\Big)\Big\}, \\ A_1^{(W)} &= \frac{2}{3}(-3\lambda^3 + (3\kappa+5)\lambda^2 - (2\kappa+1)\lambda - (\kappa+1)) = -2(\lambda-(\kappa+1))\Big(\lambda^2 - \frac{2}{3}\lambda - \frac{1}{3}\Big), \\ A_2^{(W)} &= \frac{2}{3}(-3\lambda^4 + (3\kappa+12)\lambda^3 - (9\kappa+14)\lambda^2 + (5\kappa+4)\lambda + (\kappa+1)) = -2(\lambda-(\kappa+1))\Big(\lambda^3 - 3\lambda^2 + \frac{5}{3}\lambda + \frac{1}{3}\Big), \\ A_3^{(W)} &= -4\lambda^2(\lambda+1)(\lambda-(\kappa+1)). \end{aligned} \tag{20}$$

It agrees well with the result obtained by Wilkinson [37,62]. We note that the correlation coefficient $A(E_e) + \frac{1}{3}Q_n(E_e)$ differs from the Wilkinson correlation coefficient $A^{(W)}(E_e)$ by the contribution of the radiative corrections, described by the function $(\alpha/\pi)f_n(E_e)$. In the replacement $A(E_e) + \frac{1}{3}Q_n(E_e) \to A^{(W)}(E_e)(1 + (\alpha/\pi)f_n(E_e))$ we have neglected the contributions of order $(\alpha/\pi)(E_0/M) \sim 10^{-6}$, which are smaller compared with contributions of order 10^{-4} in which we are interested. The contribution to the correlation coefficient $A^{(W)}(E_e)$ of the weak magnetism and the proton recoil, calculated at $\lambda = -1.2750$, is equal to

$$\frac{\delta A^{(W)}(E_e)}{A_0} = -\frac{1}{M}\frac{1}{2\lambda(1+\lambda)(1+3\lambda^2)} \times \Big(A^{(W)}_1 E_0 + A^{(W)}_2 E_e + A^{(W)}_3 \frac{m_e^2}{E_e}\Big) = -8.56\times 10^{-3} + 3.49\times 10^{-2}\frac{E_e}{E_0} + 1.41\times 10^{-3}\frac{m_e}{E_e}. \tag{21}$$

In the experimentally used region of electron kinetic energies 250 keV $\le T_e \le$ 455 keV [1,41,43], the radiative corrections $(\alpha/\pi)f_n(E_e)$ vary over the region $1.53\times 10^{-3} \ge (\alpha/\pi)f_n(E_e) \ge 1.04\times 10^{-3}$ and increase the absolute value of the correlation coefficient $A^{(W)}(E_e)$.

In summary, the Wilkinson expression for the correlation coefficient $A^{(\mathrm{W})}(E_e)$ [see Eq. (20)], taking into account a complete set of the $1/M$ corrections from the weak magnetism and the proton recoil, is improved by accounting for the radiative corrections, described by the function $2.81\times 10^{-3} \ge (\alpha/\pi)f_n(E_e) \ge 0.62\times 10^{-3}$ for the electron energies $m_e \le E_e \le E_0$ [see Eq. (D.57) of Ref. [40]]. The contribution of the radiative corrections increases the absolute value of the correlation coefficient $A^{(\mathrm{W})}(E_e)$. The results obtained in this section should be used as a theoretical background for the experimental determination of contributions of order 10^{-4} of interactions beyond the SM from the experimental data on the electron asymmetry $A_{\exp}(E_e)$ (see Sec. IX).

V. STANDARD MODEL ANALYSIS OF EXPERIMENTAL DETERMINATION OF CORRELATION COEFFICIENT B_0. ANTINEUTRINO ASYMMETRY $B_{\exp}(E_e)$

In the SM to leading order in the large M expansion the correlations between the neutron spin and the antineutrino 3-momentum are defined by the correlation coefficient B_0 [1], which may be determined from the experimental data on the asymmetry $B_{\exp}(E_e)$, defined by [42]

$$B_{\exp}(E_e) = \frac{N^{--}(E_e) - N^{++}(E_e)}{N^{--}(E_e) + N^{++}(E_e)}. \tag{22}$$

It defines the asymmetry of the emission of the antineutrinos into the forward and backward hemispheres with respect to the neutron spin, where $N^{\mp\mp}(E_e)$ is the number of events of the emission of the electron-proton pairs as functions of the electron energy E_e. The signs $(++)$ and $(--)$ show that the electron-proton pairs were emitted parallel $(++)$ and antiparallel $(--)$ to a direction of the neutron spin. This means that antineutrinos were emitted antiparallel $(++)$ and parallel $(--)$ to a direction of the neutron spin. The numbers of events $N^{--}(E_e)$ and $N^{++}(E_e)$ are defined by the electron-energy and angular distribution of the neutron β^- decay, integrated over the forward and backward hemispheres relative to the neutron spin, respectively.

The integration region for the electron-proton pairs, emitted parallel to a direction $(++)$ of the neutron spin, is defined by the following constraints [63]: $\vec{\xi}_n\cdot\vec{k}_e = Pk_e\cos\theta_e > 0$ and $\vec{\xi}_n\cdot\vec{k}_p = \vec{\xi}_n\cdot(-\vec{k}_e-\vec{k}) = PE(-r\cos\theta_e - \cos\theta) > 0$ or $-r\cos\theta_e > \cos\theta$, where $r = k_e/E = \sqrt{E_e^2-m_e^2}/(E_0-E_e)$ and $P = |\vec{\xi}_n| \le 1$ is the neutron polarization. For $N^{++}(E_e)$ and $r<1$ we obtain the following expression:

$$N^{++}(E_e) = 2\pi N(E_e)\Big\{\Big(1 - \frac{1}{4}a\beta + \frac{1}{2}P\beta\Big(A - \frac{1}{3}K_n\beta\Big) - \frac{1}{2}P\Big(B - \frac{1}{3}Q_n\beta\Big)\Big) - \frac{1}{2}r\Big(1 + \frac{2}{3}PA\beta\Big) + \frac{1}{8}r^2\Big(a\beta + \frac{4}{3}PB + \frac{4}{5}PK_n\beta^2\Big) - \frac{1}{15}r^3PQ_n\beta - \frac{1}{8}a_0\beta^2 r(1-r^2)\frac{E_e}{M}\Big\}, \tag{23}$$

where $N(E_e)$ is the normalization factor Eq. (19). For $r>1$ the upper limit of the integration over $\cos\theta_e$ is restricted by $\cos\theta_e \le 1/r$. The result of the interaction is

$$N^{++}(E_e) = 2\pi N(E_e)\Big\{\frac{1}{2}\frac{1}{r}\Big(1 - \frac{2}{3}PB\Big) - \frac{1}{8}\frac{1}{r^2}\Big(a\beta - \frac{4}{3}PA\beta - \frac{4}{5}PQ_n\beta\Big) - \frac{1}{15}\frac{1}{r^3}PK_n\beta^2 + \frac{1}{8}a_0\beta^2\frac{1}{r}\Big(1 - \frac{1}{r^2}\Big)\frac{E_e}{M}\Big\}. \tag{24}$$

For the calculation of $N^{--}(E_e)$ we have to integrate over the region following [63]: $\vec{\xi}_n\cdot\vec{k}_e = Pk_e\cos\theta_e < 0$ and $\vec{\xi}_n\cdot\vec{k}_p = PE(-r\cos\theta_e - \cos\theta) < 0$ or $\cos\theta > -r\cos\theta_e$. The number of events $N^{--}(E_e)$ for $r<1$ is given by

$$N^{--}(E_e) = 2\pi N(E_e)\Big\{\Big(1 - \frac{1}{4}a\beta - \frac{1}{2}P\beta\Big(A - \frac{1}{3}K_n\beta\Big) + \frac{1}{2}P\Big(B - \frac{1}{3}Q_n\beta\Big)\Big) - \frac{1}{2}r\Big(1 - \frac{2}{3}PA\beta\Big) + \frac{1}{8}r^2\Big(a\beta - \frac{4}{3}PB - \frac{4}{5}PK_n\beta^2\Big) + \frac{1}{15}r^3PQ_n\beta - \frac{1}{8}a_0\beta^2 r(1-r^2)\frac{E_e}{M}\Big\}. \tag{25}$$

For $r>1$ the lower limit of the integration over $\cos\theta_e$ is restricted by $\cos\theta_e > -1/r$. The number of events $N^{--}(E_e)$, calculated for $r>1$, is equal to

$$N^{--}(E_e) = 2\pi N(E_e)\Big\{\frac{1}{2}\frac{1}{r}\Big(1 + \frac{2}{3}PB\Big) - \frac{1}{8}\frac{1}{r^2}\Big(a\beta + \frac{4}{3}PA\beta + \frac{4}{5}PQ_n\beta\Big) + \frac{1}{15}\frac{1}{r^3}PK_n\beta^2 + \frac{1}{8}a_0\beta^2\frac{1}{r}\Big(1 - \frac{1}{r^2}\Big)\frac{E_e}{M}\Big\}. \tag{26}$$

Using our formulas for the numbers of events we calculate the asymmetry $B_{\rm exp}(E_e)$. For $r \le 1$ or $0 \le T_e \le (E_0 - m_e)^2/2E_0 = 236\,\mathrm{keV}$ and for $r \ge 1$ or $(E_0 - m_e)^2/2E_0 = 236$ keV $\le T_e \le E_0 - m_e$ the asymmetry $B_{\rm exp}(E_e)$ is equal to

$$B^{(r<1)}_{\rm exp}(E_e) = \frac{2P}{3}\,\frac{(3 - r^2)B - (3 - 2r)A\beta + (1 - \frac{3}{5}r^2)K_n\beta^2 - (1 - \frac{2}{5}r^3)Q_n\beta}{(4 - 2r) - (1 - \frac{1}{2}r^2)a\beta - \frac{1}{2}a_0\beta^2 r(1 - r^2)\frac{E_e}{M}} \tag{27}$$

and

$$B^{(r>1)}_{\rm exp}(E_e) = \frac{2P}{3}\,\frac{B - \frac{1}{2}(A + \frac{3}{5}Q_n)\frac{\beta}{r} + \frac{1}{5}K_n\frac{\beta^2}{r^2}}{1 - a\frac{\beta}{4r} + \frac{1}{4}a_0\beta^2(1 - \frac{1}{r^2})\frac{E_e}{M}}, \tag{28}$$

respectively. At $r = 1$ or $T_e = (E_0 - m_e)^2/2E_0 = 236$ keV the asymmetry $B_{\rm exp}(E_e)$ is continuous. To leading order in the large M expansion the asymmetry $B_{\rm exp}(E_e)$ reduces to the form

$$B_{\rm exp}(E_e)\big|_{M\to\infty} = \frac{2P}{3}\begin{cases} \dfrac{B(\frac{3}{2} - \frac{1}{2}r^2) - (\frac{3}{2} - r)A\beta}{(2 - r) - \frac{1}{2}(1 - \frac{1}{2}r^2)a\beta} & r \le 1 \\ \dfrac{B - \frac{1}{2}A\frac{\beta}{r}}{1 - \frac{1}{4}a\frac{\beta}{r}} & r \ge 1, \end{cases} \tag{29}$$

where the correlation coefficients B, A and a are equal to

$$B = B_0, \quad A = A_0\Big(1 + \frac{\alpha}{\pi}f_n(E_e)\Big), \quad a = a_0\Big(1 + \frac{\alpha}{\pi}f_n(E_e)\Big). \tag{30}$$

In Fig. 2 we plot the asymmetry $B_{\rm exp}(E_e)$, given by Eqs. (27) and (28) and obtained in the SM, accounting for the contributions of the $1/M$ corrections, caused by the weak magnetism and the proton recoil and the radiative corrections of order α/π, calculated to leading order in the large M expansion and described by the function $(\alpha/\pi)f_n(E_e)$. In [42,43], for the experimental determination of the correlation coefficient B_0, the experimental data on the asymmetry $B_{\rm exp}(E_e)|_{M\to\infty}$ were fitted by Eq. (29) with the replacements $B \to B_0$, $A \to A_0$ and $a \to a_0$, respectively. Such an asymmetry was calculated in [63]. Following [64] we may rewrite the asymmetry $B_{\rm exp}(E_e)$ in Eqs. (27) and (28) as follows:

$$B_{\rm exp}(E_e) = B_{\rm exp}(E_e)\Big|_{M\to\infty}(1 + R(E_e)), \tag{31}$$

where $B_{\rm exp}(E_e)|_{M\to\infty}$ is given by Eq. (29) with $B = B_0$, $A = A_0$ and $a = a_0$. The function $R(E_e)$ is defined for $r < 1$ and $r > 1$ by the functions $R_{r<1}(E_e)$ and $R_{r>1}(E_e)$ equal to

$$\begin{aligned} R_{r<1}(E_e) &= \frac{(3 - r^2)(B - B_0) - (3 - 2r)(A - A_0)\beta + (1 - \frac{3}{5}r^2)K_n\beta^2 - (1 - \frac{2}{5}r^3)Q_n\beta}{(3 - r^2)B_0 - (3 - 2r)A_0\beta} \\ &\quad + \frac{(2 - r^2)(a - a_0)\beta + a_0\beta^2 r(1 - r^2)\frac{E_e}{M}}{(8 - 4r) - (2 - r^2)a_0\beta}, \\ R_{r>1}(E_e) &= \frac{(B - B_0) - \frac{1}{2}(A - A_0 + \frac{3}{5}Q_n)\frac{\beta}{r} + \frac{1}{5}K_n\frac{\beta^2}{r^2}}{B_0 - \frac{1}{2}A_0\frac{\beta}{r}} + \frac{(a - a_0)\frac{\beta}{4r} - \frac{1}{4}a_0\beta^2(1 - \frac{1}{r^2})\frac{E_e}{M}}{1 - a_0\frac{\beta}{4r}}. \end{aligned} \tag{32}$$

In Fig. 3 we plot the function $R(E_e)$ in the electron kinetic energy region 100 keV $\le T_e \le$ 700 keV, corresponding to the energy interval used in Table II in [64]. One may see that in our approach the function $R(E_e)$, caused by the $1/M$ contributions of the weak magnetism and the proton recoil and the radiative corrections of order α/π, is positive, $0.17\% < R(E_e) < 0.28\%$. Our result disagrees with the numerical corrections calculated in [64]. Our corrections are of order of magnitude larger compared with the absolute values of the corrections in [63], which become negative for $T_e > 470$ keV.

The theoretical value $B_0 = 0.9871(1)$ of the correlation coefficient B_0, calculated for $\lambda = -1.2750(9)$, agrees within 1.5 standard deviations with the experimental values $B_0^{(\rm exp)} = 0.9802(50)$, $B_0^{(\rm exp)} = 0.9821(40)$ and $B_0^{(\rm exp)} = 0.9894(83)$, obtained in [42,43] (see also [1]), [65,66], respectively, and within 2 standard deviations with the experimental one $B_0^{(\rm exp)} = 0.967(12)$ obtained in [67].

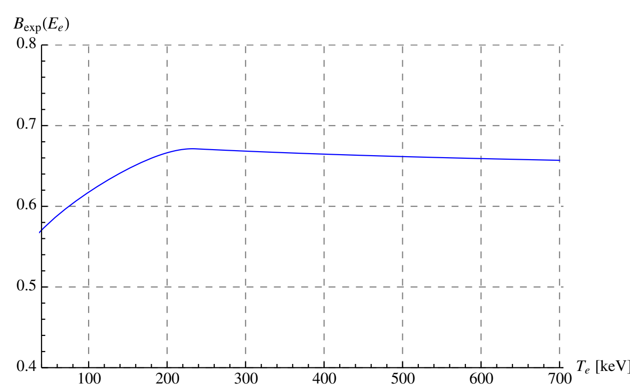

FIG. 2 (color online). The antineutrino asymmetry $B_{\rm exp}(E_e)$, including a complete set of the $1/M$ corrections, caused by the weak magnetism and the proton recoil, and radiative corrections of order α/π, calculated to leading order in the large M expansion; $T_e = E_e - m_e$ is the electron kinetic energy.

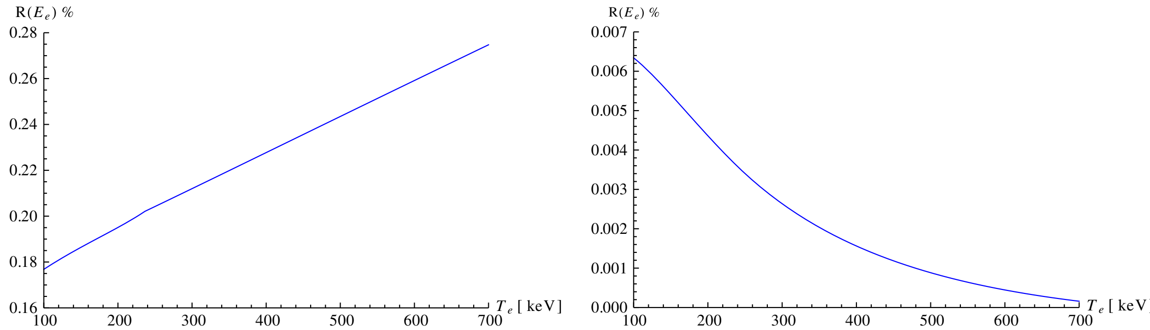

FIG. 3 (color online). The function $R(E_e)$ defining corrections to the asymmetry $B_{\rm exp}(E_e)|_{M\to\infty}$ (see Eq. (29)) with $B = B_0$, $A = A_0$ and $a = a_0$, including (i) the $1/M$ corrections of the "weak magnetism" and the proton recoil and the radiative corrections of order α/π (left) and (ii) the radiative corrections of order α/π only (right).

In summary, the antineutrino asymmetry $B_{\rm exp}(E_e)$, calculated in [63] and used in [42,43] for the experimental determination of the correlation coefficient $B_0^{(\rm exp)} = 0.9802(50)$, is improved by the contributions of a complete set of the $1/M$ corrections, caused by the weak magnetism and the proton recoil, and the radiative corrections $(\alpha/\pi)f_n(E_e)$, calculated to leading order in the large M expansion. The obtained expression of the antineutrino asymmetry $B_{\rm exp}(E_e)$ should be used as a theoretical background for the experimental determination of contributions of order 10^{-4} of interactions beyond the SM (see Sec. IX).

VI. STANDARD MODEL ANALYSIS OF EXPERIMENTAL DETERMINATION OF CORRELATION COEFFICIENT a_0. ELECTRON-PROTON ENERGY DISTRIBUTION $a(E_e, T_p)$ AND PROTON-ENERGY SPECTRUM $a(T_p)$

For the experimental determination of the correlation coefficient a_0, the correlation coefficient $a(E_e)$, given by Eq. (11), can hardly be used since the antineutrino is difficult to detect. Thus, for the determination of a_0 and the contributions of interactions beyond the SM, one should measure correlations of the 3-momenta of the decay charged particles. Using the results obtained in Appendix I of Ref. [40], the electron-proton energy spectrum for the neutron β^- decay with unpolarized particles can be given in the following form:

$$\frac{d^2{\rm BR}_{\beta_c^-}(E_e, T_p)}{dE_e dT_p} = (\tau_n)_{\rm SM} M(1 + 3\lambda^2)\frac{G_F^2|V_{ud}|^2}{4\pi^3} a(E_e, T_p) \times \Big(1 + \frac{\alpha}{\pi} g_n(E_e)\Big) F(E_e, Z = 1) E_e, \tag{33}$$

where $(\tau_n)_{\rm SM}$ is the theoretical lifetime of the neutron, calculated in the SM (see Sec. VIII) and T_p is the kinetic energy of the proton varying from zero to its maximal value $(T_p)_{\rm max} = (m_n - m_p)^2 - m_e^2/2m_n = (E_0^2 - m_e^2)/2M = 0.751\,{\rm keV}$, i.e., $0 \le T_p \le 0.751$ keV. The limits of the integration over the electron energy E_e, i.e., $(E_e)_{\rm min} \le E_e \le (E_e)_{\rm max}$, are the functions of the proton kinetic energy T_p. They are adduced in Appendix I of Ref. [40]. The electron-proton energy distribution $a(E_e, T_p)$ is defined by (see Appendix I of Ref. [40])

$$a(E_e, T_p) = \zeta_1(E_e, T_p) + a_0\Big(1 + \frac{1}{1 - \lambda^2}\frac{E_0}{M}\Big) \times \Big(1 + \frac{\alpha}{\pi} f_n(E_e)\Big)\zeta_2(E_e, T_p). \tag{34}$$

The functions $\zeta_1(E_e, T_p)$ and $\zeta_2(E_e, T_p)$ are given in Appendix I of Ref. [40]. They are calculated to next-to-leading order in the large M expansion and do not contain the radiative corrections. In the electron energy region $(E_e)_{\rm min} \le E_e \le (E_e)_{\rm max}$ the contribution of the radiative corrections $(\alpha/\pi)f_n(E_e)$ to the electron-proton energy distribution $a(E_e, T_p)$ relative to the correlation coefficient a_0 is of order 10^{-3}. Accounting for the radiative corrections is very important for a correct experimental determination of contributions of order 10^{-4} of interactions beyond the SM.

Integrating the electron-proton energy spectrum Eq. (33) over the electron energy $(E_e)_{\rm min} \le E_e \le (E_e)_{\rm max}$ we obtain the proton-energy spectrum

$$\frac{d{\rm BR}_{\beta_c^-}(T_p)}{dT_p} = (\tau_n)_{\rm SM} M(1 + 3\lambda^2)\frac{G_F^2|V_{ud}|^2}{4\pi^3} a(T_p), \tag{35}$$

where $a(T_p)$ is defined by

$$a(T_p) = g_1(T_p) + a_0\Big(1 + \frac{1}{1 - \lambda^2}\frac{E_0}{M}\Big) g_2(T_p). \tag{36}$$

The functions $g_1(T_p)$ and $g_2(T_p)$ are given in Appendix I of Ref. [40]. Recently the correlation coefficient a_0 and the axial coupling constant λ have been determined from the proton-energy spectrum by Byrne *et al.* [44]. The obtained value $a_0^{(\rm exp)} = -0.1054(55)$ can be fitted by the axial coupling constant $\lambda = -1.271(18)$. In turn, the experimental value $a_0^{(\rm exp)} = -0.1017(51)$, obtained in [45], defines the axial coupling constant equal to $\lambda = -1.259(17)$. These experimental values agree with the theoretical value of the correlation coefficient $a_0 = -0.1065(3)$, calculated at $\lambda = -1.2750(9)$, and with the axial coupling constant $\lambda = -1.2750(9)$ within 1 standard deviation.

From the point of view of the experimental determination of contributions of order 10^{-4} of interactions beyond the SM (see a discussion in Sec. VII) the experimental accuracy of the determination of the axial coupling constant λ by measuring the proton-energy spectrum as well as the electron-proton energy distribution $a(E_e, T_p)$ should be improved by more than 2 orders of magnitude in comparison with the accuracy of Byrne's experiment [44]. There are three major, funded experiments that are currently attempting to do this. They are (i) the aSPECT experiment at Institute Laue-Langevin (ILL) in Grenoble, invented to perform precise measurements of the correlation coefficient a_0 by measuring the proton-energy spectrum in the decay of unpolarized neutrons, (ii) aCORN at the National Institute of Standards and Technology (NIST), and (iii) Nab at the new Spallation Neutron Source (SNS) in Oak Ridge, Tennessee [68]. The expected experimental accuracy of the determination of the correlation coefficient a_0 in the aSPECT, aCORN, and Nab experiments is better than 1% (see also [69]).

We have discussed the experimental determination of the correlation coefficient a_0 by measuring the electron-proton energy distribution $a(E_e, T_p)$ and the proton-energy spectrum $a(T_p)$. However, there are other possibilities to experimentally determine the correlation coefficient a_0. It can be extracted from the experimental data on the $(T_e, \cos\theta_{e\bar{\nu}})$ and $(T_e, \cos\theta_{ep})$ distributions [70], where $\theta_{e\bar{\nu}}$ and θ_{ep} are angles of the electron-antineutrino and electron-proton correlations, respectively.

In summary, the electron-proton energy distribution $a(E_e, T_p)$ is calculated by taking into account a complete set of the $1/M$ corrections, caused by the weak magnetism and the proton recoil, and the radiative corrections of order α/π, calculated to leading order in the large M expansion. The proton-energy spectrum is improved in comparison with that used in [44] by taking into account the $1/M$ and radiative corrections. The obtained expression for the electron-proton energy distribution $a(E_e, T_p)$ as well as the proton-energy spectrum $a(T_p)$ should provide a theoretical background for the experimental determination of contributions of order 10^{-4} of interactions beyond the SM (see Sec. IX). For the analysis of contributions of interactions beyond the SM we propose to multiply the electron-proton energy spectrum of the rate of the neutron β^- decay by the lifetime of the neutron τ_n, calculated by taking into account the contributions of interactions beyond the SM (see Appendix G of Ref. [40]). As a result the analysis of the contributions of vector and axial-vector interactions beyond the SM by means of the electron-proton energy distribution $\tau_n a(E_e, T_p)$ and the proton-energy spectrum $\tau_n a(T_p)$ reduces to the analysis of these contributions to the axial coupling constant λ only (see Sec. IX).

VII. STANDARD MODEL ANALYSIS OF EXPERIMENTAL DETERMINATION OF CORRELATION COEFFICIENT C_0. PROTON ASYMMETRY $C_{\rm exp}$

The correlations between the neutron spin and the proton 3-momentum are described by the correlation coefficient C [71–75]. It defines the proton recoil asymmetry $C_0 = -x_C(A_0 + B_0)$, where x_C is the theoretical numerical factor, calculated within the SM to leading order in the large M expansion. The first measurement of the correlation coefficient $C_0^{(\rm exp)} = -0.2377(26)$ has been performed by Schumann *et al.* [46]. The angular distribution of the probability of the neutron β^- decay, related to the proton recoil asymmetry, is given by [71]

$$4\pi\frac{dW(\theta_p)}{d\Omega_p} = 1 + 2PC\cos\theta_p, \tag{37}$$

where $d\Omega_p = 2\pi\sin\theta_p d\theta_p$ is the infinitesimal solid angle of the proton 3-momentum with respect to the neutron polarization $\vec{\xi}_n$, i.e., $\vec{\xi}_n\cdot\vec{k}_p = Pk_p\cos\theta_p$, and $P = |\vec{\xi}_n|$ is the neutron polarization. The correlation coefficient C, calculated accounting for the $1/M$ and α/π-radiative corrections, is equal to (see Appendix I of Ref. [40])

$$\begin{aligned}C = {}& -\frac{1}{2}\frac{X_8}{X_1}(A_0 + B_0) + \frac{1}{2}\frac{X_9}{X_1}A_0 - \frac{\alpha}{\pi}\frac{1}{2}\frac{X_{\rm eff}}{X_1}(A_0 + B_0) + \frac{1}{M}\frac{1}{1+3\lambda^2}\Big(\lambda\frac{1}{2}\frac{X_{12}}{X_1} - (\kappa+1)\lambda\frac{1}{2}\frac{X_{13}}{X_1} - (2\kappa+1)\lambda\frac{1}{2}\frac{X_{14}}{X_1}\\ & - \lambda(1+\lambda)\frac{1}{2}\frac{X_{15}+Y_3}{X_1} + \lambda(1-\lambda)\frac{1}{2}\frac{X_{16}+Y_4}{X_1}\Big) + (A_0 + B_0)\frac{X_8}{X_1}\Big\{\frac{\alpha}{\pi}\frac{1}{2}\frac{X_2}{X_1} + \frac{1}{M}\frac{1}{1+3\lambda^2}\Big(\frac{1}{2}\frac{X_3}{X_1} + (1+3\lambda^2)\frac{1}{2}\frac{X_4+Y_1}{X_1}\\ & - (1-\lambda^2)\frac{1}{2}\frac{X_5+Y_2}{X_1} + (\lambda^2 + 2(\kappa+1)\lambda)\frac{1}{2}\frac{X_6}{X_1} - (\lambda^2 - 2(\kappa+1)\lambda)\frac{1}{2}\frac{X_7}{X_1}\Big)\Big\},\end{aligned} \tag{38}$$

where the numerical factors X_j ($j = 1, \ldots, 14$) and Y_j ($j = 1, 2, 3, 4$) are calculated in Appendix I of Ref. [40]. The factor $x_C = X_8/2X_1 = 0.27591$ agrees well with the factor $x_C = 0.27594$, calculated by Glück [72]. As a result the correlation coefficient C_0 is equal to

$$C_0 = -0.27591(A_0 + B_0). \tag{39}$$

The numerical value $C_0 = -0.2386$, calculated at $\lambda = -1.2750$, agrees well with the experimental one $C_0^{\rm exp} = -0.2377(26)$ [46]. The term proportional to

$X_{\rm eff}/2X_1 = 4.712120$ contains the contributions of the proton-photon correlations in the radiative β^- decay of the neutron, calculated in [76]. The contributions of the proton-photon correlations make the contributions of the radiative corrections to the correlation coefficient C symmetric with respect to a change $A_0 \leftrightarrow B_0$ as well as the main term $C_0 = -x_C(A_0 + B_0)$, calculated for the first time by Treiman [71].

Defining the proton recoil asymmetry $C_{\rm exp}$ as we have defined the asymmetry $A_{\rm exp}(E_e)$ [see Eq. (17)] we obtain

$$C_{\rm exp} = PC(\cos\vartheta_1 + \cos\vartheta_2), \tag{40}$$

where the polar angles θ_1 and θ_2 define the solid angle $\Delta\Omega_{12} = 2\pi(\cos\theta_1 - \cos\theta_2)$ of the proton emission to the forward and backward hemispheres with respect to the neutron spin.

In summary, the correlation coefficient C, describing correlations between the neutron spin and the proton 3-momentum, is calculated by taking into account a complete set of the $1/M$ corrections, caused by the weak magnetism and the proton recoil, and the radiative corrections of order α/π, calculated to leading order in the large M expansion. The obtained result should be used as a theoretical background for the experimental determination of contributions of order 10^{-4} of interactions beyond the SM (see Sec. IX).

VIII. STANDARD MODEL ANALYSIS OF THE LIFETIME OF A NEUTRON

Having integrated the electron-energy and angular distribution, Eq. (6), over the directions of the electron 3-momentum $\vec{k}_e$, the antineutrino 3-momentum $\vec{k}$ and the electron energy E_e within the limits $m_e \le E_e \le E_0$, we obtain the rate of the neutron β^- decay. It is equal to [10]

$$(\lambda_n)_{\rm SM} = (1 + 3\lambda^2)\frac{G_F^2|V_{ud}|^2}{2\pi^3} f_n(E_0, Z = 1), \tag{41}$$

where the Fermi integral $f_n(E_0, Z = 1)$, given by

$$\begin{aligned} f_n(E_0, Z=1) = &\int_{m_e}^{E_0}(E_0 - E_e)^2\sqrt{E_e^2 - m_e^2}E_eF(E_e, Z=1)\\ &\times\Big(1 + \frac{\alpha}{\pi}g_n(E_e)\Big)\Big\{1 + \frac{1}{M}\frac{1}{1+3\lambda^2}\\ &\times\Big[(10\lambda^2 - 4(\kappa+1)\lambda + 2)E_e - 2\lambda\\ &\times(\lambda - (\kappa+1))\Big(E_0 + \frac{m_e^2}{E_e}\Big)\Big]\Big\}dE_e, \end{aligned} \tag{42}$$

contains the contributions of the weak magnetism, the proton recoil and the radiative corrections, described by the function $g_n(E_e)$. The calculation of the lifetime of the neutron with the Fermi function, determined by Eq. (5), the axial coupling constant $\lambda = -1.2750(9)$ and the CKM matrix element $V_{ud} = 0.97428(15)$, gives $(\tau_n)_{\rm SM} = 879.6(1.1)$ s. The error bars ± 1.1 s are defined by the error bars of the experimental value of the coupling constant and the CKM matrix element. The theoretical value of the lifetime of the neutron, $(\tau_n)_{\rm SM} = 879.6(1.1)$ s, agrees well with the experimental values $\tau_n^{(\rm exp)} = 878.5(8)$ s, $\tau_n^{(\rm exp)} = 880.7(1.8)$ s and $\tau_n^{(\rm exp)} = 881.6(2.1)$ s, measured by Serebrov *et al.* [12], Pichlmaier *et al.* [77] and Arzumanov *et al.* [78], respectively. It also agrees well with the new world average values (w.a.v.) of the neutron lifetime, $\tau_n^{(\rm w.a.v.)} = 880.1(1.1)$ s [9], $\tau_n^{(\rm w.a.v.)} = 880.0(9)$ s [79] and $\tau_n^{(\rm w.a.v.)} = 881.9(1.3)$ s [80], respectively.

In summary, the lifetime of the neutron is calculated by taking into account the radiative corrections by Sirlin *et al.*, calculated to leading order in the large M expansion, and a complete set of the $1/M$ corrections, caused by the weak magnetism and the proton recoil. The obtained result should be used as a theoretical background for the experimental determination of contributions of order 10^{-4} of interactions beyond the SM (see Sec. IX). The numerical value of the lifetime of the neutron, $(\tau_n)_{\rm SM} = 879.6(1.1)$ s, calculated in this section is left unchanged after the absorption of the Herczeg phenomenological coupling constants a^h_{LL} and a^h_{LR} of the left-left and left-right lepton-nucleon current-current interactions (vector and axial-vector interactions beyond the SM) by the axial coupling constant $\lambda \to \lambda_{\rm eff} = (\lambda - a^h_{LL} + a^h_{LR})/(1 + a^h_{LL} + a^h_{LR}) = \lambda - a^h_{LL} + a^h_{LR} - \lambda(a^h_{LL} + a^h_{LR})$ and the CKM matrix element $V_{ud} \to (V_{ud})_{\rm eff} = V_{ud}(1 + a^h_{LL} + a^h_{LR})$ (see Sec. IX).

IX. SENSITIVITY OF THE ELECTRON-PROTON ENERGY DISTRIBUTION, ASYMMETRIES $A_{\rm exp}(E_e)$, $B_{\rm exp}(E_e)$ AND $C_{\rm exp}$, AND LIFETIME OF NEUTRON τ_n TO CONTRIBUTIONS OF ORDER 10^{-4} OF INTERACTIONS BEYOND THE STANDARD MODEL

In this section we propose a theoretical analysis of the sensitivity of the electron-proton energy distribution $a(E_e, T_p)$, the proton-energy spectrum $a(T_p)$, the asymmetries $A_{\rm exp}(E_e)$, $B_{\rm exp}(E_e)$ and $C_{\rm exp}$, and the lifetime of the neutron τ_n to contributions of order 10^{-4} of interactions beyond the SM.

The electron-proton energy distribution $a(E_e, T_p)$ and the correlation coefficients $a(E_e)$, $A(E_e)$ and $B(E_e)$, including the contributions of interactions beyond the SM, can be written in the form

$$\begin{aligned} a(E_e, T_p) &= \zeta_1(E_e, T_p)_{\rm eff}\\ &\quad + \bar{a}_{\rm eff}(E_e)\Big(1 + \frac{\alpha}{\pi}f_n(E_e)\Big)\zeta_2(E_e, T_p),\\ a(E_e) &= a_{\rm eff}(E_e)\Big(1 + \frac{\alpha}{\pi}f_n(E_e)\Big),\\ A(E_e) &= A_{\rm eff}(E_e)\Big(1 + \frac{\alpha}{\pi}f_n(E_e)\Big),\\ B(E_e) &= B_{\rm eff}(E_e), \end{aligned} \tag{43}$$

where $\zeta_1(E_e, T_p)_{\rm eff}$, $\bar{a}_{\rm eff}(E_e)$, $a_{\rm eff}(E_e)$, $A_{\rm eff}(E_e)$ and $B_{\rm eff}(E_e)$ are the electron-proton energy distribution and the correlation coefficients, defined by the contributions of the weak magnetism, the proton recoil and interactions beyond the SM only. They are given by

$$\begin{aligned}\zeta_1(E_e, T_p)_{\rm eff} &= \Big(1 - \frac{a_4}{a_0}\Big(\frac{m_e}{E_e} - \Big\langle\frac{m_e}{E_e}\Big\rangle_{\rm SM}\Big)\Big)\zeta_1(E_e, T_p),\\ \bar{a}_{\rm eff}(E_e) &= (a_0)_{\rm eff} + \frac{1}{M}\frac{1}{1+3\lambda^2}E_0 + a_4\Big\langle\frac{m_e}{E_e}\Big\rangle_{\rm SM},\\ a_{\rm eff}(E_e) &= (a_0)_{\rm eff} + \frac{1}{M}\frac{1}{(1+3\lambda^2)^2}\\ &\quad\times\Big(a_1E_0 + a_2E_e + a_3\frac{m_e^2}{E_e}\Big) + a_4\frac{m_e}{E_e},\\ A_{\rm eff}(E_e) &= (A_0)_{\rm eff} + \frac{1}{M}\frac{1}{(1+3\lambda^2)^2}\\ &\quad\times\Big(A_1E_0 + A_2E_e + A_3\frac{m_e^2}{E_e}\Big) + A_4\frac{m_e}{E_e},\\ B_{\rm eff}(E_e) &= (B_0)_{\rm eff} + \frac{1}{M}\frac{1}{(1+3\lambda^2)^2}\\ &\quad\times\Big(B_1E_0 + B_2E_e + B_3\frac{m_e^2}{E_e}\Big) + B_4\frac{m_e}{E_e},\end{aligned} \tag{44}$$

where $\langle m_e/E_e\rangle_{\rm SM}$ is the average value calculated in the SM with the electron-energy spectrum density, Eq. (A8). The terms proportional to $\langle m_e/E_e\rangle_{\rm SM}$ come from the lifetime of the neutron, calculated accounting for the contributions of interactions beyond the SM (see Appendix G of Ref. [40]). The coefficients a_j, A_j and B_j for $j = 1, 2, 3$ are given in Eqs. (11)–(13), respectively. They are calculated to next-to-leading order in the large M expansion and include the contributions of the weak magnetism and the proton recoil only. The numerical values of these corrections are estimated in Eq. (14) at $\lambda = -1.2750$. The terms $(a_0)_{\rm eff}$, $(A_0)_{\rm eff}$ and $(B_0)_{\rm eff}$ are the sums of the correlation coefficients a_0, A_0 and B_0 and energy independent contributions of interactions beyond the SM (see Appendix G of Ref. [40]). They read

$$\begin{aligned}(a_0)_{\rm eff} &= a_0 + \frac{1}{(1+3\lambda^2)^2}(4\lambda^2\,{\rm Re}(\delta C_V - \delta\bar{C}_V)\\ &\quad + 4\lambda\,{\rm Re}(\delta C_A - \delta\bar{C}_A)),\\ (A_0)_{\rm eff} &= A_0 + \frac{1}{(1+3\lambda^2)^2}(-\lambda(3\lambda^2 - 2\lambda - 1)\\ &\quad\times{\rm Re}(\delta C_V - \delta\bar{C}_V) - (3\lambda^2 - 2\lambda - 1)\\ &\quad\times{\rm Re}(\delta C_A - \delta\bar{C}_A)),\\ (B_0)_{\rm eff} &= B_0 + \frac{1}{(1+3\lambda^2)^2}(-\lambda(3\lambda^2 + 2\lambda - 1)\\ &\quad\times{\rm Re}(\delta C_V - \delta\bar{C}_V) - (3\lambda^2 + 2\lambda - 1)\\ &\quad\times{\rm Re}(\delta C_A - \delta\bar{C}_A)).\end{aligned} \tag{45}$$

The coefficients a_4, A_4 and B_4 are induced by interactions beyond the SM only. They are equal to

$$\begin{aligned}a_4 &= -\frac{1-\lambda^2}{(1+3\lambda^2)^2}({\rm Re}(C_S - \bar{C}_S) + 3\lambda\,{\rm Re}(C_T - \bar{C}_T)),\\ A_4 &= 2\frac{\lambda(1+\lambda)}{(1+3\lambda^2)^2}({\rm Re}(C_S - \bar{C}_S) + 3\lambda\,{\rm Re}\,(C_T - \bar{C}_T)),\\ B_4 &= \frac{(1+\lambda)(1-3\lambda)}{(1+3\lambda^2)^2}(\lambda\,{\rm Re}(C_S - \bar{C}_S) - {\rm Re}(C_T - \bar{C}_T)).\end{aligned} \tag{46}$$

For the derivation of Eqs. (45) and (46) we have used the results obtained in Appendix G of Ref. [40], with the coupling constants $C_V = 1 + \delta C_V$, $\bar{C}_V = -1 + \delta\bar{C}_V$, $C_A = -\lambda + \delta C_A$ and $\bar{C}_A = \lambda + \delta\bar{C}_A$, and we have kept only the linear contributions of the deviations from the coupling constants of the SM. By assumption, the axial coupling constant λ in Eqs. (45) and (46) is determined by all interactions within the SM only.

The differences of the phenomenological coupling constants $\delta C_V - \delta\bar{C}_V$, $\delta C_A - \delta\bar{C}_A$, $C_S - \bar{C}_S$ and $C_T - \bar{C}_T$, calculated in terms of the Herczeg phenomenological lepton-nucleon coupling constants [6] (see Appendix G of Ref. [40]), take the form

$$\begin{aligned}\delta C_V - \delta\bar{C}_V &= 2(a^h_{LL} + a^h_{LR}),\\ \delta C_A - \delta\bar{C}_A &= 2(a^h_{LL} - a^h_{LR}),\\ C_S - \bar{C}_S &= 2(A^h_{LL} + A^h_{LR}) = 4a_S,\\ C_T - \bar{C}_T &= 4\alpha^h_{LL} = 4a_T.\end{aligned} \tag{47}$$

This means that the phenomenological coupling constants $\delta C_V - \delta\bar{C}_V$, $\delta C_A - \delta\bar{C}_A$, $C_S - \bar{C}_S$ and $C_T - \bar{C}_T$ describe interactions beyond the SM of left-handed leptonic currents and left (right)-handed hadronic currents, i.e., $L\otimes L$ and $L\otimes R$, respectively.

The experimental determination of the correlation coefficients a_0, A_0 and B_0 is as follows. Using the theoretical expressions for the electron-proton energy distribution $a(E_e, T_p)$ and the asymmetries $A_{\rm exp}(E_e)$ and $B_{\rm exp}(E_e)$, their experimental data can be fit by a tuning of the axial coupling constant [1]. Such a procedure gives the axial-coupling constants λ_a, λ_A and λ_B, obtained from the fit of the experimental data on $a(E_e, T_p)$, $A_{\rm exp}(E_e)$ and $B_{\rm exp}(E_e)$, respectively. In terms of these axial coupling constants one may define the correlation coefficients $(a_0)^{(\rm exp)}_{\rm eff}$, $(A_0)^{(\rm exp)}_{\rm eff}$ and $(B_0)^{(\rm exp)}_{\rm eff}$, respectively, as follows:

$$\begin{aligned}(a_0)^{(\rm exp)}_{\rm eff} &= \frac{1-\lambda_a^2}{1+3\lambda_a^2}, \qquad (A_0)^{(\rm exp)}_{\rm eff} = -2\frac{\lambda_A(1+\lambda_A)}{1+3\lambda_A^2},\\ (B_0)^{(\rm exp)}_{\rm eff} &= -2\frac{\lambda_B(1-\lambda_B)}{1+3\lambda_B^2}.\end{aligned} \tag{48}$$

The theoretical expressions for the axial coupling constants λ_a, λ_A and λ_B in terms of the axial coupling constant λ, defined by interactions within the SM only, and the contributions of interactions beyond the SM are

$$\lambda_a = \lambda - \frac{1}{2}(\lambda\,\mathrm{Re}(\delta C_V - \delta\bar{C}_V) + \mathrm{Re}(\delta C_A - \delta\bar{C}_A)),$$
$$\lambda_A = \lambda - \frac{1}{2}(\lambda\,\mathrm{Re}(\delta C_V - \delta\bar{C}_V) + \mathrm{Re}(\delta C_A - \delta\bar{C}_A)),$$
$$\lambda_B = \lambda - \frac{1}{2}(\lambda\,\mathrm{Re}(\delta C_V - \delta\bar{C}_V) + \mathrm{Re}(\delta C_A - \delta\bar{C}_A)). \tag{49}$$

Thus, in the linear approximation with respect to the deviations of the phenomenological coupling constants of interactions beyond the SM from the coupling constants of the SM, we obtain that $\lambda_a = \lambda_A = \lambda_B$. This agrees well with the results obtained in [47–49]. Replacing λ_a, λ_A and λ_B by $\lambda_{\rm eff}$, which includes the contributions of vector and axial-vector interactions beyond the SM in addition to the contributions of interactions within the SM, and denoting

$$b_F = \frac{1}{1+3\lambda^2_{\rm eff}}(\mathrm{Re}(C_S - \bar{C}_S) + 3\lambda_{\rm eff}\,\mathrm{Re}(C_T - \bar{C}_T)) = \frac{4}{1+3\lambda^2_{\rm eff}}(\mathrm{Re}(a_S) + 3\lambda_{\rm eff}\,\mathrm{Re}(a_T)) \tag{50}$$

and

$$c_{ST} = \frac{1}{1+3\lambda^2_{\rm eff}}(\lambda_{\rm eff}\,\mathrm{Re}(C_S - \bar{C}_S) - \mathrm{Re}(C_T - \bar{C}_T)) = \frac{4}{1+3\lambda^2_{\rm eff}}(\lambda_{\rm eff}\,\mathrm{Re}(a_S) - \mathrm{Re}(a_T)), \tag{51}$$

where b_F is the Fierz term (see Appendix G of Ref. [40]), we obtain the electron-proton energy distribution $a(E_e, T_p)$ and the correlation coefficients $a(E_e)$, $A(E_e)$ and $B(E_e)$ in the following form:

$$\begin{aligned}
a(E_e,T_p) &= \Big(1 - b_F\Big\langle\frac{m_e}{E_e}\Big\rangle_{\rm SM}\Big)\Big\{\Big(1 + b_F\frac{m_e}{E_e}\Big)\zeta_1(E_e,T_p) + a_0\Big(1 + \frac{1}{1-\lambda^2_{\rm eff}}\frac{E_0}{M}\Big)\Big(1 + \frac{\alpha}{\pi}f_n(E_e)\Big)\zeta_2(E_e,T_p)\Big\},\\
a(E_e) &= a_0\Big(1 - b_F\frac{m_e}{E_e}\Big)\Big(1 + \frac{\alpha}{\pi}f_n(E_e)\Big)\Big\{1 + \frac{1}{M}\frac{1}{(1-\lambda^2_{\rm eff})(1+3\lambda^2_{\rm eff})}\Big(a_1E_0 + a_2E_e + a_3\frac{m_e^2}{E_e}\Big)\Big\},\\
A(E_e) &= A_0\Big(1 - b_F\frac{m_e}{E_e}\Big)\Big(1 + \frac{\alpha}{\pi}f_n(E_e)\Big)\Big\{1 - \frac{1}{M}\frac{1}{2\lambda_{\rm eff}(1+\lambda_{\rm eff})(1+3\lambda^2_{\rm eff})}\Big(A_1E_0 + A_2E_e + A_3\frac{m_e^2}{E_e}\Big)\Big\},\\
B(E_e) &= B_0\Big(1 - \frac{(1+\lambda_{\rm eff})(1-3\lambda_{\rm eff})}{2\lambda_{\rm eff}(1-\lambda_{\rm eff})}c_{ST}\frac{m_e}{E_e}\Big)\Big\{1 - \frac{1}{M}\frac{1}{2\lambda_{\rm eff}(1-\lambda_{\rm eff})(1+3\lambda^2_{\rm eff})}\Big(B_1E_0 + B_2E_e + B_3\frac{m_e^2}{E_e}\Big)\Big\},
\end{aligned} \tag{52}$$

where the coefficients a_j, A_j and B_j for $j = 1, 2, 3$ are defined in Eqs. (11)–(13), respectively, with the replacement $\lambda \to \lambda_{\rm eff}$. The same replacement defines the correlation coefficients a_0, A_0 and B_0 in terms of $\lambda_{\rm eff}$ [see Eq. (8)].

The correlation coefficients $a(E_e)$, $A(E_e)$ and $B(E_e)$, extended by the contributions of interactions beyond the SM, together with the correlation coefficients $K_n(E_e)$ and $Q_n(E_e)$, caused by the contributions of the $1/M$ corrections from the weak magnetism and the proton recoil only, determine the asymmetries $A_{\rm exp}(E_e)$ and $B_{\rm exp}(E_e)$. For the calculation of the electron asymmetry $A_{\rm exp}(E_e)$, taking into account the contributions of interactions beyond the SM, we have to replace the correlation coefficient $A^{(\rm W)}(E_e)$ in Eq. (17) by the expression

$$A^{(\rm W)}(E_e) = A_0\Big(1 - b_F\frac{m_e}{E_e}\Big)\Big\{1 - \frac{1}{M}\frac{1}{2\lambda_{\rm eff}(1+\lambda_{\rm eff})(1+3\lambda^2_{\rm eff})} \times\Big(A_1^{(\rm W)}E_0 + A_2^{(\rm W)}E_e + A_3^{(\rm W)}\frac{m_e^2}{E_e}\Big)\Big\}, \tag{53}$$

where the coefficients $A_j^{(\rm W)}$ are given in Eq. (20) with the replacement $\lambda \to \lambda_{\rm eff}$.

The antineutrino asymmetry $B_{\rm exp}(E_e)$ is defined by Eqs. (27) and (28) for $r \le 1$ and $r \ge 1$, respectively. For the account of the contributions of interactions beyond the SM, the correlation coefficients $a(E_e)$, $A(E_e)$ and $B(E_e)$ should be taken in the form given by Eq. (52).

The proton recoil asymmetry $C_{\rm exp}$ completes the set of asymmetries that can be measured in the neutron β^- decay with a polarized neutron and an unpolarized proton and electron. The correlation coefficient $C_{\rm eff}$ (see Appendix I of Ref. [40]), defining the asymmetry $C_{\rm exp}$ and extended by the contributions of interactions beyond the SM, takes the form

$$C_{\rm eff} = -x_C(A_0 + B_0) + \frac{\lambda(1+\lambda_{\rm eff})}{1+3\lambda^2_{\rm eff}}b_F\frac{X_{17}}{X_1} - \frac{1}{2}\frac{(1+\lambda_{\rm eff})(1-3\lambda_{\rm eff})}{1+3\lambda^2_{\rm eff}}c_{ST}\frac{X_{18}}{X_1} + C_{\rm SM}, \tag{54}$$

where $C_{\rm SM} = C + x_C(A_0 + B_0)$ and $x_C = 0.27591$ [see Eqs. (38) and (39)]. The numerical factors $X_{17}/X_1 = -0.90187$ and $X_{18}/X_1 = 0.39806$ are calculated in Appendix I of Ref. [40].

Using the results obtained in Appendix G of Ref. [40], the theoretical expression for the rate of the neutron β^- decay, including the contributions of interactions beyond the SM, is

$$\lambda_n = (\lambda_n)_{\rm SM}\Big(1 + \frac{1}{1+3\lambda^2}({\rm Re}(\delta C_V - \delta\bar{C}_V) - 3\lambda\,{\rm Re}(\delta C_A - \delta\bar{C}_A)) + b_F\Big\langle\frac{m_e}{E_e}\Big\rangle_{\rm SM}\Big)$$
$$= \frac{G_F^2|V_{ud}|^2}{2\pi^3} f_n(E_0, Z=1)(1+3\lambda^2)\Big(1 + \frac{1}{1+3\lambda^2}({\rm Re}(\delta C_V - \delta\bar{C}_V) - 3\lambda\,{\rm Re}(\delta C_A - \delta\bar{C}_A)) + b_F\Big\langle\frac{m_e}{E_e}\Big\rangle_{\rm SM}\Big), \quad (55)$$

where $(\lambda_n)_{\rm SM}$ is the lifetime of the neutron, calculated within the SM [see Eqs. (41) and (42)], and $\langle m_e/E_e\rangle_{\rm SM}$ is the average value, calculated with the electron-energy spectrum density, Eq. (A8). Now we have to define the rate of the neutron β^- decay, Eq. (55), in terms of the axial coupling constant $\lambda_{\rm eff}$, which is related to the axial coupling constant λ as

$$\lambda = \lambda_{\rm eff} + \frac{1}{2}(\lambda_{\rm eff}\,{\rm Re}(\delta C_V - \delta\bar{C}_V) + {\rm Re}(\delta C_A - \delta\bar{C}_A)). \quad (56)$$

Substituting Eq. (56) into Eq. (55) and keeping only the linear terms in powers of ${\rm Re}(\delta C_V - \delta\bar{C}_V)$ and ${\rm Re}(\delta C_A - \delta\bar{C}_A)$, one may show that

$$(1+3\lambda^2)\Big(1 + \frac{1}{1+3\lambda^2}({\rm Re}(\delta C_V - \delta\bar{C}_V) - 3\lambda\,{\rm Re}(\delta C_A - \delta\bar{C}_A)) + b_F\Big\langle\frac{m_e}{E_e}\Big\rangle_{\rm SM}\Big)$$
$$= (1 + {\rm Re}(\delta C_V - \delta\bar{C}_V))(1+3\lambda^2_{\rm eff})\Big(1 + b_F\Big\langle\frac{m_e}{E_e}\Big\rangle_{\rm SM}\Big). \quad (57)$$

This gives the rate of the neutron β^- decay equal to

$$\lambda_n = \frac{G_F^2|V_{ud}|^2}{2\pi^3} f_n(E_0, Z=1)(1 + {\rm Re}(\delta C_V - \delta\bar{C}_V)) \times (1+3\lambda^2_{\rm eff})\Big(1 + b_F\Big\langle\frac{m_e}{E_e}\Big\rangle_{\rm SM}\Big), \quad (58)$$

where the Fermi integral is given by Eq. (42) with the replacement $\lambda \to \lambda_{\rm eff}$.

If we introduce again $(\lambda_n)_{\rm SM}$, defined by Eqs. (41) and (42) with the replacement $\lambda \to \lambda_{\rm eff}$, we get the following expression for the rate of the neutron β^- decay, corrected by the contributions of interactions beyond the SM taken to linear approximation with respect to the Herczeg phenomenological coupling constants,

$$\lambda_n = (\lambda_n)_{\rm SM}(1 + 2\,{\rm Re}(a^h_{LL} + a^h_{LR}))\Big(1 + b_F\Big\langle\frac{m_e}{E_e}\Big\rangle_{\rm SM}\Big), \quad (59)$$

where we have set ${\rm Re}(\delta C_V - \delta\bar{C}_V) = 2\,{\rm Re}(a^h_{LL} + a^h_{LR})$. Thus, at first glance a deviation of the experimental values $\tau_n = 1/\lambda_n$ of the lifetime of the neutron considered relative to the theoretical value of the lifetime of the neutron $(\tau_n)_{\rm SM} = 1/(\lambda_n)_{\rm SM}$, calculated in the SM at zero Herczeg coupling constants, may give information about the contribution of the Herczeg left-left and left-right lepton-nucleon current-current interactions (vector and axial-vector interactions beyond the SM) by using the experimental value of the Fierz term b_F, determined from the experimental data on the electron-proton energy distribution $a(E_e, T_p)$, the proton-energy spectrum $a(T_p)$, and the asymmetries $A_{\rm exp}(E_e)$, $B_{\rm exp}(E_e)$ and $C_{\rm exp}$. However, the problem is, in the following, that the Herczeg phenomenological interactions beyond the SM, when they exist, should always exist together with the interactions of the SM, and a separation of these interactions is rather artificial.

Hence, using the result obtained in Appendix G of Ref. [40], we may redefine the axial coupling constant and the CKM matrix element as follows: $\lambda_{\rm eff} = (\lambda - a^h_{LL} + a^h_{LR})/(1 + a^h_{LL} + a^h_{LR})$ and $(V_{ud})_{\rm eff} = V_{ud}(1 + a^h_{LL} + a^h_{LR})$, as proposed in [47–49]. After such a change one may show that the rate of the neutron β^- decay may contain only the contribution of the Fierz term,

$$\lambda_n = (\lambda_n)_{\rm SM}\Big(1 + b_F\Big\langle\frac{m_e}{E_e}\Big\rangle_{\rm SM}\Big), \quad (60)$$

where $(\lambda_n)_{\rm SM}$ is given by Eqs. (41) and (42) with the replacements $\lambda \to \lambda_{\rm eff}$ and $V_{ud} \to (V_{ud})_{\rm eff}$.

The definition of the effective coupling constant $\lambda_{\rm eff} = (\lambda - a^h_{LL} + a^h_{LR})/(1 + a^h_{LL} + a^h_{LR})$ and the CKM matrix element $(V_{ud})_{\rm eff} = V_{ud}(1 + a^h_{LL} + a^h_{LR})$ at the Hamiltonian level introduces the imaginary parts to the axial coupling constant and the CKM matrix element,

$$\lambda_{\rm eff} = {\rm Re}\lambda_{\rm eff} + i\,{\rm Im}\lambda_{\rm eff},$$
$$(V_{ud})_{\rm eff} = V_{ud}(1 + {\rm Re}(a^h_{LL} + a^h_{LR}))(1 + i\,{\rm Im}(a^h_{LL} + a^h_{LR})), \quad (61)$$

where ${\rm Im}\lambda_{\rm eff}$ is equal to

$${\rm Im}\lambda_{\rm eff} = -(1 + {\rm Re}\lambda_{\rm eff}){\rm Im}(a^h_{LL}) + (1 - {\rm Re}\lambda_{\rm eff}){\rm Im}(a^h_{LR}). \quad (62)$$

One may obtain information about the imaginary part of the axial coupling constant $\lambda_{\rm eff}$ by measuring the correlation coefficient $D(E_e)$, describing a violation of time reversal invariance. From Eq. (62) and the results obtained in Appendix G of Ref. [40], we get

$$\begin{aligned} D(E_e) &= D_{\rm SM}(E_e) - \frac{2}{1+3\lambda^2_{\rm eff}}(\lambda_{\rm eff}\,{\rm Im}(a^h_{LL}+a^h_{LR}) \\ &\quad + {\rm Im}(a^h_{LL}-a^h_{LR})) \\ &= D_{\rm SM}(E_e) - \frac{2}{1+3\lambda^2_{\rm eff}}((1+\lambda_{\rm eff}){\rm Im}(a^h_{LL}) \\ &\quad - (1-\lambda_{\rm eff}){\rm Im}(a^h_{LR})) \\ &= D_{\rm SM}(E_e) + \frac{2\,{\rm Im}\lambda_{\rm eff}}{1+3\lambda^2_{\rm eff}}, \end{aligned} \tag{63}$$

where we have replaced ${\rm Re}\lambda_{\rm eff}$ by $\lambda_{\rm eff}$, having neglected the contribution of the imaginary part, and $D_{\rm SM}(E_e)$ is the contribution to the correlation coefficient $D(E_e)$ calculated within the SM [55–60]. As we have estimated in Sec. III, such a contribution, caused by the electron-proton interaction in the final state [55–58,60], is of order 10^{-5} for the electron kinetic energies $250\ {\rm keV} \le T_e \le 455\ {\rm keV}$. Thus, the correlation coefficient $D(E_e)$, defined in the same energy region, should be sensitive to the contributions beyond the SM of 10^{-4}. Recently, the experimental value $D_{\rm exp}(E_e) = (-4\pm 6)\times 10^{-4}$ of the correlation coefficient $D(E_e)$ [1,9] has been substantially improved, with the result $D_{\rm exp}(E_e) = (-0.96 \pm 1.89_{\rm stat} \pm 1.01_{\rm syst})\times 10^{-4}$ [81]. However, the new experimental value, as well as the old one, still implies that to order 10^{-4} the correlation coefficient $D(E_e)$ is commensurable with zero. Hence, to order 10^{-4} the imaginary part of the axial coupling constant $\lambda_{\rm eff}$ is also commensurable with zero, i.e., ${\rm Im}\lambda_{\rm eff} = 0$. Nevertheless, setting $\lambda_{\rm eff} = {\rm Re}\lambda_{\rm eff} = -1.2750$ and using the relation ${\rm Im}\lambda_{\rm eff} = 0$, we may obtain ${\rm Im}(a^h_{LR}) = -0.12\,{\rm Im}(a^h_{LL})$. This adds an additional phase shift $e^{i0.88\,{\rm Im}(a^h_{LL})}$ to the CKM matrix element $(V_{ud})_{\rm eff}$.

The axial coupling constant λ_a and the correlation coefficient a_0 may also be determined by measuring the proton-energy spectrum, Eq. (35) (see [44]). The proton-energy spectrum, taking into account the contributions of interactions beyond the SM, is

$$\begin{aligned} a_{\rm eff}(T_p) &= \Big(1 - b_F\Big\langle\frac{m_e}{E_e}\Big\rangle_{\rm SM}\Big)\Big\{g_1(T_p)_{\rm eff} \\ &\quad + a_0\Big(1+\frac{1}{1-\lambda^2}\frac{E_0}{M}\Big)g_2(T_p)\Big\}, \end{aligned} \tag{64}$$

where the functions $g_1(T_p)_{\rm eff}$ and $g_2(T_p)$ are defined by the integrals

$$\begin{aligned} g_1(T_p)_{\rm eff} &= \int^{(E_e)_{\rm max}}_{(E_e)_{\rm min}}\Big(1+b_F\frac{m_e}{E_e}\Big)\Big(1+\frac{\alpha}{\pi}g_n(E_e)\Big) \\ &\quad \times \zeta_1(E_e,T_p)F(E_e,Z=1)E_e dE_e, \\ g_2(T_p) &= \int^{(E_e)_{\rm max}}_{(E_e)_{\rm min}}\Big(1+\frac{\alpha}{\pi}g_n(E_e)+\frac{\alpha}{\pi}f_n(E_e)\Big) \\ &\quad \times \zeta_2(E_e,T_p)F(E_e,Z=1)E_e dE_e, \end{aligned} \tag{65}$$

where the limits of integration $(E_e)_{\rm max/min}$ are given in Appendix I of Ref. [40].

In summary, we have analyzed the sensitivity of the electron-proton energy distribution $a(E_e, T_p)$, the proton-energy spectrum $a(T_p)$, the asymmetries $A_{\rm exp}(E_e)$, $B_{\rm exp}(E_e)$, $C_{\rm exp}$, and the lifetime of the neutron τ_n to contributions of order 10^{-4} of interactions beyond the SM, taken to a linear approximation with respect to the Herczeg phenomenological coupling constants of weak lepton-nucleon current-current interactions. We have shown that, in such an approximation, the axial coupling constant $\lambda_{\rm eff}$ and the CKM matrix element $(V_{ud})_{\rm eff}$ absorb the contributions of the Herczeg left-left and left-right lepton-nucleon current-current interactions with the coupling constants a^h_{LL} and a^h_{LR}. In this approximation the axial coupling constant does not acquire an imaginary part to order 10^{-4}, but the CKM matrix element becomes an additional phase $e^{i\,{\rm Im}(a^h_{LL}+a^h_{LR})}$. Thus, after the measurements of the electron-proton energy spectrum $a(E_e, T_p)$, the proton-energy spectrum $a(T_p)$, the asymmetries $A_{\rm exp}(E_e)$, $B_{\rm exp}(E_e)$ and $C_{\rm exp}$, and the lifetime of the neutron τ_n, one may determine the axial coupling constant $\lambda_{\rm eff}$ and the real parts of the scalar and tensor coupling constants a_S and a_T, defined in Eq. (47),

$$\begin{aligned} {\rm Re}(a_S) + 3\lambda_{\rm eff}\,{\rm Re}(a_T) &= \frac{1+3\lambda^2_{\rm eff}}{4}b_F, \\ \lambda_{\rm eff}\,{\rm Re}(a_S) - {\rm Re}(a_T) &= \frac{1+3\lambda^2_{\rm eff}}{4}c_{ST}, \end{aligned} \tag{66}$$

where in the r.h.s. of the algebraical equations, $\lambda_{\rm eff}$, b_F and c_{ST} are the experimental values with their experimental errors.

X. CONCLUSION

We have analyzed the sensitivity of the electron-proton energy distribution $a(E_e, T_p)$, the proton-energy spectrum $a(T_p)$, and the asymmetries $A_{\rm exp}(E_e)$, $B_{\rm exp}(E_e)$ and $C_{\rm exp}$ of the correlations between the neutron spin and 3-momenta of the decay electron, antineutrino and proton, respectively, for the neutron β^- decay with a polarized neutron and an unpolarized proton and electron to contributions of order 10^{-4} of interactions beyond the SM. For the analysis of contributions of order 10^{-4} we have used the linear approximation for the correlation coefficient with respect to the Herczeg phenomenological coupling constants of weak lepton-nucleon current-current interactions. We have shown that, in such an approximation, the Herczeg right-left and right-right lepton-nucleon current-current interactions with the coupling constants a^h_{RL} and a^h_{RR} give no contributions to the correlation coefficients of the neutron β^- decay and the lifetime of the neutron. Then, the contributions of the Herczeg left-left and left-right lepton-nucleon current-current interactions with the coupling constants a^h_{LL} and a^h_{LR} may be absorbed by the axial coupling constant, which we denote as $\lambda_{\rm eff} = \lambda - {\rm Re}(a^h_{LL} - a^h_{LR}) + \lambda\,{\rm Re}(a^h_{LL} + a^h_{LR})$, and the CKM matrix element $(V_{ud})_{\rm eff} = V_{ud}(1 + {\rm Re}(a^h_{LL} + a^h_{LR}))$. We have shown that the Herczeg coupling constants a^h_{LL} and a^h_{LR} in the effective Hamiltonian of weak interactions may be absorbed by the axial coupling constant $\lambda_{\rm eff} = (\lambda - a^h_{LL} + a^h_{LR})/(1 + a^h_{LL} + a^h_{LR})$ and the CKM matrix element $(V_{ud})_{\rm eff} = V_{ud}(1 + a^h_{LL} + a^h_{LR})$. Using the experimental data on the correlation coefficient $D(E_e)$, we have shown that at the level of 10^{-4} the imaginary part of the axial coupling constant $\lambda_{\rm eff}$ is equal to zero, whereas the CKM matrix element acquires an additional phase $e^{i\,{\rm Im}(a^h_{LL}+a^h_{LR})}$. This shows that, in addition to the background calculated in the SM, the correlation coefficients of the neutron β^- decay and the lifetime of the neutron, calculated to a linear approximation with respect to the Herczeg coupling constants of lepton-nucleon current-current interactions, depend on the contributions of the scalar and tensor interactions only. This agrees with recent results obtained in [47–49].

We have shown that the contributions of the scalar and tensor interactions beyond the SM are described by the Fierz term b_F and the coupling constant c_{ST}. The Fierz term may be determined from the experimental data on the asymmetry $A_{\rm exp}(E_e)$ and the electron-proton energy distribution $a(E_e, T_p)$ [or the proton-energy spectrum $a(T_p)$]. The coupling constant c_{ST} may be determined from the experimental data on the asymmetry $B_{\rm exp}(E_e)$ and the proton recoil asymmetry $C_{\rm exp}$. This allows us to determine the scalar ${\rm Re}(a_S)$ and tensor ${\rm Re}(a_T)$ coupling constant by solving the system of algebraical equations [see Eq. (66)]

$$\begin{aligned} {\rm Re}(a_S) + 3\lambda_{\rm eff}\,{\rm Re}(a_T) &= \frac{1 + 3\lambda^2_{\rm eff}}{4}\,b_F, \\ \lambda_{\rm eff}\,{\rm Re}(a_S) - {\rm Re}(a_T) &= \frac{1 + 3\lambda^2_{\rm eff}}{4}\,c_{ST}. \end{aligned}$$

The lifetime of the neutron is defined by the background calculated in the SM, and the Fierz term [see Eq. (60)]

$$\tau_n = (\tau_n)_{\rm SM}\Big(1 - b_F\Big\langle\frac{m_e}{E_e}\Big\rangle_{\rm SM}\Big),$$

where $(\tau_n)_{\rm SM}$ and $\langle m_e/E_e\rangle_{\rm SM}$ are calculated in the SM.

We would like to note that the experimental analysis of the neutron β^- decay with a polarized neutron and an unpolarized proton and electron may be carried out in terms the electron-proton energy distribution $a(E_e, T_p)$, the proton-energy spectrum $a(T_p)$, the asymmetries $A_{\rm exp}(E_e)$, $B_{\rm exp}(E_e)$, $C_{\rm exp}$, and the lifetime of the neutron τ_n. From the fit of the experimental data on these energy distributions, asymmetries and the lifetime, we determine three parameters, i.e., the axial coupling constant $\lambda_{\rm eff}$, the Fierz term b_F and the coupling constant c_{ST}. In order to determine these parameters without correlations between them, it suffices to use experimental data obtained only in three out of five independent experiments. This means that experimental data obtained from two other independent experiments should be described well by the parameters $(\lambda_{\rm eff}, b_F, c_{ST})$ determined from the first three experiments. The deviations from the predicted values should be much smaller compared with 10^{-4}, since they may be explained only by the contributions of higher powers of the Herczeg coupling constants.

We note that the theoretical analysis of the sensitivity of the electron-proton energy distribution $a(E_e, T_p)$, the proton-energy spectrum $a(T_p)$, the asymmetries $A_{\rm exp}(E_e)$, $B_{\rm exp}(E_e)$ and $C_{\rm exp}$ of the neutron β^- decay, and the lifetime of the neutron to contributions of order 10^{-4} of interactions beyond the SM has been carried out above the background, calculated within the SM. We have taken into account a complete set of the $1/M$ corrections, caused by the weak magnetism and the proton recoil, calculated to next-to-leading order in the large M or the large proton mass expansion, and the radiative corrections of order α/π, calculated to leading order in the large M or the large proton mass expansion.

The corrections caused by the weak magnetism and the proton recoil are calculated in analytical agreement with the results obtained by Wilkinson [37] and Gudkov *et al.* [35]. We have given the radiative corrections to the lifetime of the neutron and the correlation coefficients of the neutron β^- decay with a polarized neutron and an unpolarized decay proton and electron, calculated in this paper, in terms of two functions, $(\alpha/\pi)g_n(E_e)$ and $(\alpha/\pi)f_n(E_e)$, given in Appendix D of Ref. [40]. They are in analytical agreement with the radiative corrections calculated in [18–30] and in [34,35], respectively. We would like to note that the radiative corrections to order α/π to the correlation coefficients $a(E_e)$ and $A(E_e)$ were calculated for the first time by Shann [20]. The function $(\alpha/\pi)f_n(E_e)$ is in analytical agreement with the results obtained by Shann [20]. We have confirmed Sirlin's assertion that an unambiguous definition of the observable radiative corrections to the lifetime of the neutron is fully caused by the requirement of gauge invariance of the amplitude of one-virtual photon exchanges of the continuum-state β^- decay of the neutron [18].

We have improved the theoretical expressions for the asymmetries $A_{\rm exp}(E_e)$, $B_{\rm exp}(E_e)$ and $C_{\rm exp}$ with respect to the expressions used in [1,41–43,46] for the experimental determination of the axial coupling constant $\lambda = -1.2750(9)$ and the correlation coefficients $A^{(\rm exp)}_0 = -0.11933(34)$, $B^{(\rm exp)}_0 = 0.9802(50)$ and $C^{(\rm exp)}_0 = -0.2377(26)$, respectively. We have added the radiative corrections to the electron asymmetry $A_{\rm exp}(E_e)$ and the $1/M$ and radiative corrections to the antineutrino $B_{\rm exp}(E_e)$ and proton $C_{\rm exp}$ asymmetries. In connection with the experimental analysis of contributions of order 10^{-4} of interactions beyond the SM to the neutron β^- decay, we have calculated the electron-proton energy distribution $a(E_e, T_p)$ and the proton-energy spectrum $a(T_p)$ by taking into account the complete set of $1/M$ corrections, caused by the weak magnetism and the proton recoil, and the radiative corrections of order α/π.

As has been pointed out by Glück [33], the contributions of the radiative β^- decay of the neutron to the proton-energy spectrum and angular distribution demand a detailed analysis of the proton-photon correlations, which appear in the proton recoil energy and angular distribution of the radiative β^- decay of the neutron. For the aim of a consistent calculation of the contributions of the nucleus-photon and hadron-photon correlations in the radiative nuclear and hadronic β decays, Glück has used the Monte Carlo simulation method. Recently the calculation of the proton-photon correlations has been performed in [76]. There it has been shown that the contributions of the proton-photon correlations to the lifetime of the neutron τ_n, the proton-energy spectrum $a(T_p)$ and the electron-proton energy distribution $a(E_e, T_p)$ are smaller compared with the contributions of the radiative corrections described by the functions $g_n(E_e)$ and $f_n(E_e)$. At the level of 10^{-5} accuracy the contributions of the proton-photon correlations to part of the proton recoil angular distribution, independent of $\cos\theta_p$, can be neglected. In turn, the contributions of the proton-photon correlations to the proton recoil asymmetry C, i.e., in part of the proton recoil angular distribution proportional to $\cos\theta_p$, are of order 10^{-4}. The account of these contributions makes the radiative corrections to the proton recoil angular distribution and the proton recoil asymmetry C symmetric with respect to a change $A_0 \leftrightarrow B_0$, as well as the main term $C_0 = -x_C(A_0 + B_0)$. A detailed analysis of the proton-energy spectrum $a(T_p)$ has recently been carried out in [76]. In addition to the proton-energy spectrum $a(T_p)$ calculated in this paper, the authors of Ref. [76] have included the contributions of the proton-photon correlations and analyzed the proton energy regions, which are convenient for measurements of the Fierz term b_F, caused by scalar and tensor interactions beyond the SM.

We have also shown that, at the present level of experimental accuracy, the lifetime of the neutron is described well by the SM, taking into account a complete set of $1/M$ corrections, caused by the weak magnetism and the proton recoil, calculated to next-to-leading order in the large proton mass expansion, and the radiative corrections of order α/π, calculated to leading order in the large proton mass expansion. The theoretical value of the lifetime of the neutron, $(\tau_n)_{\rm SM} = 879.6(1.1)$ s, where the error bars are defined by the error bars of the axial coupling constant $\lambda = -1.2750(9)$ and the CKM matrix element $V_{ud} = 0.97428(15)$, agrees well with the experimental values $\tau_n^{(\rm exp)} = 878.5(8)$ s, $\tau_n^{(\rm exp)} = 880.7(1.8)$ s and $\tau_n^{(\rm exp)} = 881.6(2.1)$ s, measured by Serebrov *et al.* [12], Pichlmaier *et al.* [77] and Arzumanov *et al.* [78], respectively, and the world average values of the neutron lifetime, $\tau_n^{(\rm w.a.v.)} = 880.1(1.1)$ s, $\tau_n^{(\rm w.a.v.)} = 880.0(9)$ s and $\tau_n^{(\rm w.a.v.)} = 881.9(1.3)$ s, obtained in [9,79,80], respectively.

In order to demonstrate the sensitivity of the lifetime of the neutron calculated in the SM to the contributions of the radiative and $1/M$ corrections, we propose to rewrite the Fermi integral $f_n(E_0, Z = 1)$, given by Eq. (42), in the following form:

$$f_n(E_0, Z=1) = \int_{m_e}^{E_0}(E_0 - E_e)^2\sqrt{E_e^2 - m_e^2}E_eF(E_e, Z=1)\Big\{\Big(1 + k_1\frac{\alpha}{\pi}g_n(E_e)\Big) + k_2\frac{1}{M}\frac{1}{1+3\lambda^2} \times\Big[(10\lambda^2 - 4(\kappa+1)\lambda + 2)E_e - 2\lambda(\lambda - (\kappa+1))\Big(E_0 + \frac{m_e^2}{E_e}\Big)\Big]\Big\}dE_e, \qquad (67)$$

where the coefficients k_j for $j = 1, 2$ are equal to $k_j = 0$ or $k_j = 1$, which means without $k_j = 0$ and with $k_j = 1$ corresponding corrections. The numerical values of the lifetime of the neutron for different k_j are adduced in Table I. We show that the most important contributions come from the radiative corrections.

For the comparison of the theoretical lifetime of the neutron, defined by Eq. (41), with the expression that is usually used for the measurement of the CKM matrix element V_{ud} [28] (see also [82]), we transcribe Eq. (41) into the form [28,82]

$$\frac{1}{\tau_n} = C_n|V_{ud}|^2(1 + 3\lambda^2)f(1 + {\rm RC}), \qquad (68)$$

where we have denoted $C_n = G_F^2m_e^5/2\pi^3 = 1.1614 \times 10^{-4}\ {\rm s}^{-1}$ and ${\rm RC} = \langle(\alpha/\pi)g_n(E_e)\rangle = 0.03886$, defining the radiative corrections [1,28] integrated over the phase-space volume, accounting for the proton-electron final-state Coulomb interaction. Then, the phase-space factor f, including the $1/M$ corrections from the weak magnetism and the proton recoil, is determined by

$$f = \frac{1}{m_e^5}\int_{m_e}^{E_0}(E_0 - E_e)^2\sqrt{E_e^2 - m_e^2}E_e F(E_e, Z = 1)\bigg\{1 + \frac{1}{M}\frac{1}{1 + 3\lambda^2}\bigg[(10\lambda^2 - 4(\kappa + 1)\lambda + 2)E_e - 2\lambda(\lambda - (\kappa + 1))\bigg(E_0 + \frac{m_e^2}{E_e}\bigg)\bigg]\bigg\}dE_e = 1.6894. \tag{69}$$

The numerical value agrees well with the value $f = 1.6887$ calculated in [28] (see also [82]). We can represent the factor $1 + \mathrm{RC}$ in the following form: $1 + \mathrm{RC} = (1 + \delta_R) \times (1 + \Delta_R)$, where $\delta_R = \langle(\alpha/\pi)(g_n(E_e) - C_{WZ})\rangle = 0.01505$ is defined by one-photon exchange and emission only [28,29] and $\Delta_R = (\alpha/\pi)C_{WZ} = 0.02381$ is part of the radiative corrections induced by electroweak-boson exchanges and QCD corrections [28,29] (see also [35]). The phase-space factor f_R, including the contributions of the radiative corrections, caused by one-photon exchanges and emission only, is equal to $f_R = f(1 + \delta_R) = 1.71483$. It does not contradict the value $f_R = 1.71385(34)$ used in [82] (see also [83]).

Currently the lifetime of the neutron is proposed to be measured in TU München within the project PENeLOPE, using a superconducting magnetogravitational trap of ultracold neutrons (UCN) for a precise neutron lifetime measurement [84]. In this experiment the UCN are trapped in a multipole field of a flux density up to 2 T and bound by a gravitational force at the top. This makes the extraction and detection of the protons possible and allows a direct measurement of neutron decay. A planing accuracy of 0.1 s and better demands high storage times and good knowledge of systematic errors, which could result from neutron spin flip and high energetic UCN that leave the storage volume only slowly. Therefore, the neutron spectrum is cleaned by an absorber. The big storage volume of 800 dm^3 and the expected high neutron flux of FRMII give more than 10^7 neutrons per filling of the storage volume and meet statistical demands. Of course, the experimental data on the lifetime of the neutron, which should be obtained within this project with a planning accuracy better than 0.1 s, should place new constraints on contributions of interactions beyond the SM.

For the completeness of our analysis we have calculated (see Appendix H of Ref. [40]) the contributions of the proton recoil corrections of order α/M, caused by the electron-proton Coulomb interaction in the final state of the neutron β^- decay. We have shown that these corrections to the lifetime of the neutron and the correlation coefficients are of order 10^{-6}–10^{-5}. This allows us to neglect them for the analysis of contributions of order 10^{-4} of interactions beyond the SM.

We would like to note that we have used the experimental value of the axial coupling constant $\lambda = -1.2750(9)$, determined from the experimental data on the electron asymmetry $A_{\exp}(E_e)$ [1,41]. Such an experimental value of the axial coupling constant has been obtained with an unprecedented accuracy of about 0.07%. The axial coupling constant $\lambda = -1.2750(9)$ agrees well with the axial coupling constants $\lambda = -1.2761^{(+14)}_{(-17)}$, $\lambda = -1.2759^{+40.9}_{-44.5}$ and $\lambda = -1.2756(30)$, obtained recently by the PERKEO (PERKEO II) Collaboration [82] and the UCNA (Ultracold Neutron Asymmetry) Collaboration [85,86], respectively, the accuracies of which are large compared with the accuracy of the axial coupling constant $\lambda = -1.2750(9)$. The lifetimes of the neutron, $\tau_n = 879(2)$ s, $\tau_n = 879(6)$ s and $\tau_n = 879(4)$ s, calculated for the axial coupling constants $\lambda = -1.2761^{(+14)}_{(-17)}$, $\lambda = -1.2759^{+40.9}_{-44.5}$ and $\lambda = -1.2756(30)$, respectively, agree with the experimental data $\tau_n^{(\exp)} = 878.5(8)$ s, $\tau_n^{(\exp)} = 880.7(1.8)$ s and $\tau_n^{(\exp)} = 881.6(2.1)$ s, measured by Serebrov *et al.* [12], Pichlmaier *et al.* [77] and Arzumanov *et al.* [78], respectively, and the world average values of the neutron lifetime, $\tau_n^{(\mathrm{w.a.v.})} = 880.1(1.1)$ s, $\tau_n^{(\mathrm{w.a.v.})} = 880.0(9)$ s and $\tau_n^{(\mathrm{w.a.v.})} = 881.9(1.3)$ s, obtained in [9,79,80], respectively.

The other experimental values of the axial coupling constant, $\lambda = -1.266(4)$, $\lambda = -1.2594(38)$ and $\lambda = -1.262(5)$, obtained in [87–89], respectively, and cited by [9], lead to the lifetimes of the neutron, $\tau_n = 890(5)$ s, $\tau_n = 898(5)$ s and $\tau_n = 895(7)$ s, which do not agree with the world average values of the neutron lifetime, $\tau_n^{(\mathrm{w.a.v.})} = 880.1(1.1)$ s, $\tau_n^{(\mathrm{w.a.v.})} = 880.0(9)$ s and $\tau_n^{(\mathrm{w.a.v.})} = 881.9(1.3)$ s, obtained in [9,79,80], respectively. Moreover, the experimental methods, used in [87–89] for the measurements of the electron asymmetry $A_{\exp}(E_e)$, have been recently criticized in [82]. As has been pointed out by Mund *et al.* [82], in the experiments [87–89] large corrections of about 15%–30% should be applied to (1) neutron polarization, (2) magnetic mirror effects, (3) solid angle and (4) background.

For a long time [13–30] (see also [34,35]), due to infrared divergences, the calculation of the radiative β^- decay of the neutron has been associated with the calculation of the radiative corrections to the neutron β^- decay. As has been shown already in [13], the sum of the rates, as well as the electron-energy and angular distributions of the continuum-state and radiative β^--decay modes of the neutron, does not suffer from infrared divergences caused by one-virtual photon exchanges in the continuum-state β^--decay mode and by the emission of real photons in the radiative β^--decay mode of the neutron.

Nevertheless, the radiative β^- decay of the neutron $n \to p + e^- + \bar{\nu}_e + \gamma$ may be treated as a physical process, which may be observed separately from the neutron β^- decay $n \to p + e^- + \bar{\nu}_e$. For the first time, the theoretical analysis of the radiative β^- decay of the neutron

$n \to p + e^- + \bar{\nu}_e + \gamma$ as a physical observable process has been carried out in [90,91]. The first reliable experimental data on the branching ratio of the radiative β^- decay of the neutron $\mathrm{BR}^{(\mathrm{exp})}_{\beta^-_c\gamma} = 3.13(35) \times 10^{-3}$, measured for the photon energy region $\omega_{\min} = 15\ \mathrm{keV} \le \omega \le \omega_{\max} = 340\ \mathrm{keV}$, have been reported by Nico *et al.* [92]. Then, this result has been updated by Cooper *et al.* [93,94], who have obtained $\mathrm{BR}^{(\mathrm{exp})}_{\beta^-_c\gamma} = 3.09(32) \times 10^{-3}$. These experimental values agree well with the theoretical value $\mathrm{BR}_{\beta^-_c\gamma} = 2.85 \times 10^{-3}$, calculated by Gardner within HBχPT for the same photon energy region [91–93]. In Appendix B of Ref. [40] we have carried out the calculation of the rate, the electron-photon energy and photon-energy spectra, and angular distributions of the radiative β^- decay of the neutron with a polarized neutron and unpolarized decay particles. Our results for the branching ratios, $\mathrm{BR}_{\beta^-_c\gamma} = 2.87 \times 10^{-3}$ and $\mathrm{BR}_{\beta^-_c\gamma} = 4.45 \times 10^{-3}$, calculated for the photon energy regions $\omega_{\min} = 15\ \mathrm{keV} \le \omega \le 350\ \mathrm{keV}$ and $\omega_{\min} = 5\ \mathrm{keV} \le \omega \le E_0 - m_e$, respectively, agree well with the results $\mathrm{BR}_{\beta^-_c\gamma} = 2.85 \times 10^{-3}$ and $\mathrm{BR}_{\beta^-_c\gamma} = 4.41 \times 10^{-3}$ obtained by Gardner [92,93] and Bernard *et al.* [91], respectively. Within 1 standard deviation the branching ratio $\mathrm{BR}_{\beta^-_c\gamma} = 2.87 \times 10^{-3}$ agrees also with the experimental data [92,93]. The Monte Carlo simulation method for the calculation of the rates of nuclear and hadronic radiative β decays has been used by Glück in [33].

The rate of the radiative β^-–decay of the neutron, depending on a photon polarization, has been calculated in [91]. We argue that the more precise theoretical and experimental analyses of the energy spectra and angular distributions of the radiative β^- decay of the neutron, depending on the polarizations of the neutron and photon, should be of great importance for a test of the SM. We are planning to perform such a theoretical analysis in our forthcoming publication.

A. Universality of radiative corrections to order α/π

The radiative corrections of order $\alpha/\pi \sim 10^{-3}$ to the electron-energy spectrum of the neutron β^- decay, described by the function $g_n(E_e)$, are universal for the electron (positron) energy spectra of nuclear and neutron β decays [95,96]. A universality of the radiative corrections to order $\alpha/\pi \sim 10^{-3}$ to neutrino (antineutrino) reactions, induced by weak charged currents, has been pointed out by Kurylov, Ramsey–Musolf and Vogel [97] using the example of the neutrino (antineutrino) disintegration of the deuteron with the electron (positron) in the final state. Such a universality has been confirmed in [98] for the cross section for the inverse β decay. As has been shown in [98] the radiative corrections calculated in [97] can be described by the function $f_A(E_{\bar{\nu}})$ of the antineutrino energy $E_{\bar{\nu}}$, calculated by Vogel [99], Fayans [100], Fukugita and Kubota [101], and Raha, Myhrer and Kudobera [102] (see also [98]) and caused by one-virtual photon exchanges and the radiative inverse β decay, and the constant part, caused by the electroweak boson exchanges. In turn, as has been shown by Sirlin [103], the radiative corrections, caused by one-virtual photon exchanges and the bremsstrahlung to neutrino (antineutrino) energy spectra of the β decays, are also described by the function $f_A(E_{\bar{\nu}})$ (see also [98]). A nice review of the radiative corrections in precision electroweak physics has recently been written by Sirlin and Ferroglia [104].

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APPENDIX: RADIATIVE CORRECTIONS TO NEUTRON β^- DECAY

The amplitude of the continuum-state β^- decay of the neutron, including a complete set of $1/M$ and radiative corrections, takes the form [40]

$$
\begin{aligned}
&M(n\to pe^-\bar{\nu}_e)\\
&= -2m_n\frac{G_F}{\sqrt{2}}V_{ud}\Big\{\Big(1+\frac{\alpha}{2\pi}f_{\beta^-_c}(E_e,\mu)\Big)[\varphi^\dagger_p\varphi_n][\bar{u}_e\gamma^0(1-\gamma^5)v_{\bar{\nu}}] - \tilde{\lambda}\Big(1+\frac{\alpha}{2\pi}f_{\beta^-_c}(E_e,\mu)\Big)[\varphi^\dagger_p\vec{\sigma}\varphi_n]\cdot[\bar{u}_e\vec{\gamma}(1-\gamma^5)v_{\bar{\nu}}]\\
&\quad -\frac{\alpha}{2\pi}g_F(E_e)[\varphi^\dagger_p\varphi_n][\bar{u}_e(1-\gamma^5)v_{\bar{\nu}}] + \frac{\alpha}{2\pi}\tilde{\lambda}g_F(E_e)[\varphi^\dagger_p\vec{\sigma}\varphi_n]\cdot[\bar{u}_e\gamma^0\vec{\gamma}(1-\gamma^5)v_{\bar{\nu}}]\\
&\quad -\frac{m_e}{2M}[\varphi^\dagger_p\varphi_n][\bar{u}_e(1-\gamma^5)v_{\bar{\nu}}] + \frac{\tilde{\lambda}}{2M}[\varphi^\dagger_p(\vec{\sigma}\cdot\vec{k}_p)\varphi_n][\bar{u}_e\gamma^0(1-\gamma^5)v_{\bar{\nu}}] - i\frac{\kappa+1}{2M}[\varphi^\dagger_p(\vec{\sigma}\times\vec{k}_p)\varphi_n]\cdot[\bar{u}_e\vec{\gamma}(1-\gamma^5)v_{\bar{\nu}}]\Big\},
\end{aligned}
\tag{A1}
$$

where $\tilde{\lambda} = \lambda(1 - E_0/2M)$ and $\vec{k}_p = -\vec{k}_e - \vec{k}$ is the proton 3-momentum in the rest frame of the neutron. The function $(\alpha/\pi)f_{\beta^-_c}(E_e, \mu)$ describes the radiative corrections induced by one-virtual photon exchanges. It is equal to [40]

$$
f_{\beta^-_c}(E_e,\mu) = \frac{3}{2}\ell n\Big(\frac{m_p}{m_e}\Big) - \frac{11}{8} + 2\ell n\Big(\frac{\mu}{m_e}\Big)\Big[\frac{1}{2\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big) - 1\Big] + \frac{1}{\beta}L\Big(\frac{2\beta}{1+\beta}\Big) - \frac{1}{4\beta}\ell n^2\Big(\frac{1+\beta}{1-\beta}\Big) + \frac{1}{2\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big),
\tag{A2}
$$

where μ is an infinitesimal photon mass and $L(x)$ is the Spence function. The function $g_F(E_e)$ is given by [40]

$$
g_F(E_e) = \frac{\sqrt{1-\beta^2}}{2\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big).
\tag{A3}
$$

From Eq. (A1) it is seen that the function $g_F(E_e)$ defines the coupling constants of effective scalar and tensor lepton-nucleon weak interactions induced by one-virtual photon exchanges [40].

The radiative corrections to the rate of the continuum-state β^- decay of the neutron, which we denote as $g_{\beta^-_c}(E_e, \mu)$, acquire an additional contribution of the electromagnetic Fierz term [40],

$$
\begin{aligned}
g_{\beta^-_c}(E_e,\mu) &= f_{\beta^-_c}(E_e,\mu) + \frac{\pi}{\alpha}b^{(\mathrm{em})}_F(E_e)\frac{m_e}{E_e} = f_{\beta^-_c}(E_e,\mu) - g_F(E_e)\frac{m_e}{E_e} = \frac{3}{2}\ell n\Big(\frac{m_p}{m_e}\Big)\\
&\quad - \frac{11}{8} + 2\ell n\Big(\frac{\mu}{m_e}\Big)\Big[\frac{1}{2\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big) - 1\Big] + \frac{1}{\beta}L\Big(\frac{2\beta}{1+\beta}\Big) - \frac{1}{4\beta}\ell n^2\Big(\frac{1+\beta}{1-\beta}\Big) + \frac{\beta}{2}\ell n\Big(\frac{1+\beta}{1-\beta}\Big),
\end{aligned}
\tag{A4}
$$

where the term proportional to $\beta/2$ is defined by

$$
\frac{\beta}{2}\ell n\Big(\frac{1+\beta}{1-\beta}\Big) = \frac{1}{2\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big) - g_F(E_e)\frac{m_e}{E_e}.
\tag{A5}
$$

The correlation coefficients of the neutron β^- decay with a polarized neutron and an unpolarized decay proton and electron, including the radiative corrections of order α/π and the $1/M$ corrections, take the form

$$
\begin{aligned}
\zeta(E_e) &= \Big(1+\frac{\alpha}{\pi}g_n(E_e)\Big) + O(1/M), \qquad a(E_e) = a_0\Big(1+\frac{\alpha}{\pi}f_n(E_e)\Big) + O(1/M),\\
A(E_e) &= A_0\Big(1+\frac{\alpha}{\pi}f_n(E_e)\Big) + O(1/M), \qquad B(E_e) = B_0 + O(1/M).
\end{aligned}
\tag{A6}
$$

The correlation coefficient $B(E_e)$ does not acquire the radiative corrections to order α/π. The exact expressions of the $1/M$ corrections are given in Eqs. (7) and (11)–(13). The functions $g_n(E_e)$ and $f_n(E_e)$, describing the radiative corrections to the rate of the neutron β^- decay and the correlation coefficients $a(E_e)$ and $A(E_e)$, are defined by

$$\begin{aligned}
g_n(E_e) &= \lim_{\mu\to 0}[g_{\beta^-_c}(E_e,\mu) + g^{(1)}_{\beta^-_c\gamma}(E_e,\mu)] \\
&= \frac{3}{2}\ell n\Big(\frac{m_p}{m_e}\Big) - \frac{3}{8} + 2\Big[\frac{1}{2\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big) - 1\Big]\Big[\ell n\Big(\frac{2(E_0-E_e)}{m_e}\Big) - \frac{3}{2} + \frac{1}{3}\frac{E_0-E_e}{E_e}\Big] + \frac{2}{\beta}L\Big(\frac{2\beta}{1+\beta}\Big) \\
&\quad + \frac{1}{2\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big)\Big[(1+\beta^2) + \frac{1}{12}\frac{(E_0-E_e)^2}{E_e^2} - \ell n\Big(\frac{1+\beta}{1-\beta}\Big)\Big] + C_{WZ}, \\
f_n(E_e) &= \lim_{\mu\to 0}[g^{(2)}_{\beta^-_c\gamma}(E_e,\mu) - g^{(1)}_{\beta^-_c\gamma}(E_e,\mu)] + g_F(E_e)\frac{m_e}{E_e} \\
&= \frac{2}{3}\frac{E_0-E_e}{E_e}\Big(1 + \frac{1}{8}\frac{E_0-E_e}{E_e}\Big)\frac{1-\beta^2}{\beta^2}\Big[\frac{1}{2\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big) - 1\Big] - \frac{1}{12}\frac{(E_0-E_e)^2}{E_e^2} + \frac{1-\beta^2}{2\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big) + C_{WZ},
\end{aligned} \tag{A7}$$

where the functions $g^{(1)}_{\beta^-_c\gamma}(E_e,\mu)$ and $g^{(2)}_{\beta^-_c\gamma}(E_e,\mu)$ appear from the contribution of the radiative β^- decay of the neutron (see Appendix B of Ref. [40]). The constant C_{WZ}, defined by the contributions of electroweak-boson exchanges and QCD corrections [28], is equal to $C_{WZ} = 10.249$. This numerical value is obtained from the fit of the radiative corrections to the lifetime of the neutron, $(\alpha/\pi)\langle g_n(E_e)\rangle = 0.03886(39)$ [1] and $(\alpha/\pi)\langle g_n(E_e)\rangle = 0.0390(8)$ [28], averaged over the phase volume of the neutron decay. The $g_n(E_e)$ and $f_n(E_e)$, multiplied by α/π, are in analytical agreement with results obtained in [18–30,34,35], respectively. The terms of order $O(1/M)$ are adduced in Eqs. (9)–(13).

The radiative corrections $(\alpha/\pi)g_n(E_e)$ and $(\alpha/\pi)f_n(E_e)$, weighted with the electron energy spectrum density [40]

$$\rho_{\beta^-_c}(E_e) = (E_0-E_e)^2\sqrt{E_e^2-m_e^2}\,E_e\zeta(E_e)\frac{F(E_e,Z=1)}{f_n(E_0,Z=1)}, \tag{A8}$$

where the functions $\zeta(E_e)$ and $F(E_e,Z=1)$ are given in Eqs. (7) and (5), respectively, and $f_n(E_0,Z=1)$ is the Fermi integral Eq. (42), are plotted in Fig. 1.

Finally we would like to note that, having attributed the terms proportional to $(\lambda+1)$ to the renormalization constants of the Fermi and axial coupling constants only, we arrive at the radiative corrections described by the function [40]

$$\begin{aligned}
g(E_e) &= g_n(E_e) + 3\ell n\Big(\frac{\Lambda}{m_p}\Big) + \frac{9}{8} - C_{WZ} \\
&= 3\ell n\Big(\frac{\Lambda}{m_p}\Big) + \frac{3}{2}\ell n\Big(\frac{m_p}{m_e}\Big) + \frac{3}{4} + 2\Big[\frac{1}{2\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big) - 1\Big]\Big[\ell n\Big(\frac{2(E_0-E_e)}{m_e}\Big) - \frac{3}{2} + \frac{1}{3}\frac{E_0-E_e}{E_e}\Big] \\
&\quad + \frac{2}{\beta}L\Big(\frac{2\beta}{1+\beta}\Big) + \frac{1}{2\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big)\Big[(1+\beta^2) + \frac{1}{12}\frac{(E_0-E_e)^2}{E_e^2} - \ell n\Big(\frac{1+\beta}{1-\beta}\Big)\Big],
\end{aligned} \tag{A9}$$

where Λ is an ultraviolet cutoff. The function $g(E_e)$, multiplied by α/π, agrees analytically with the result calculated by Kinoshita and Sirlin [15].

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