Erratum: Causality-violating Higgs singlets at the LHC [Phys. Rev. D 87, 045004 (2013)]

Chiu Man Ho and Thomas J. Weiler

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In our original paper, we present a simple 5D metric that allows closed timelike curves (CTCs) as viewed by an observer confined to our 4D brane. The metric mixes time t and the fifth dimension u, as

$$d\tau^2 = dt^2 + 2g(u)dtdu - h(u)du^2 - d\vec{r}^2. \tag{1}$$

For definiteness, we assume a "constraint" equation, $|\bar{g}| < 1$, where $\bar{g}$ is the value of the metric element g when averaged over the extra dimension. In a comment on our paper, Gielen [1] has shown that our assumed relation is not consistent with the timelike condition of the energy-momentum vector in the 5D space. Using "diagonalized" coordinates $\tilde{t}$ and $\tilde{u}$, defined in our paper, and his, Gielen shows, in a simple way, that $|\bar{g}| > \bar{D}$ must hold for massless particles, where D is the square root of the metric's determinant $\mathrm{Det} = g^2 + h$ and $\bar{D}$ is its averaged value. In our work, we set Det equal to unity, its Minkowski value on the brane. Thus, Gielen's condition for us becomes $|\bar{g}| \geq 1$.

In this erratum, we rederive Gielen's constraint equation using standard t and u coordinates. Then, with this correct constraint in hand, we correct the few parts of our original paper that require change. These parts are the examples presented in Figs. 1 and 2, the parameter region that allows CTCs presented in Fig. 5, and the piece of Sec. VIIB in which we compute the energy dispersion relation for the time-traveling Higgs singlet particles. The main conclusions of our original paper are not changed.

We begin with the proof that $|\bar{g}| \geq 1$ must hold for particle geodesics enabling CTCs or prearrivals. In our published paper, we identified three constants of geodesic motion, given in Eqs. (3.1), (3.2), and (3.10) as $\dot{t} + g(u)\dot{u}$, $\dot{\vec{r}}$, and $\dot{u}$, respectively, where the overdot denotes a derivative with respect to the particle's proper time τ. The first two constants are the inevitable results of the metric depending only on the coordinate u, while the latter constant occurs when our ansatz $\mathrm{Det} = 1$ is implemented. Equivalent to any one of the geodesic constants of the motion is the "first integral" constructed by dividing the line element in Eq. (1) by $d\tau^2$:

$$\zeta = \dot{t}^2 + 2g(u)\dot{t}\,\dot{u} - h(u)\dot{u}^2 - \dot{\vec{r}}^2, \tag{2}$$

where $\zeta = 1$ for matter and $\zeta = 0$ for photons. Substituting in the first two constants of motion, and rearranging the right-hand terms a bit, one gets

$$\zeta = (\dot{t} + g(u)\dot{u})^2 - (\mathrm{Det}(u)\dot{u}^2 + \dot{\vec{r}}^2). \tag{3}$$

Then, substituting Eq. (3.6) from our original article, we arrive at

$$\zeta = \left(\dot{t} + \left(\frac{g(u)}{\sqrt{\mathrm{Det}}}\dot{u}_0\right)\right)^2 - (\dot{u}_0^2 + \dot{\vec{r}}^2). \tag{4}$$

Rearranging terms again and then taking the square root gives

$$\left|\dot{t} + \left(\frac{g(u)}{\sqrt{\mathrm{Det}}}\right)\dot{u}_0\right| = \sqrt{\zeta + \dot{u}_0^2 + \dot{\vec{r}}^2}. \tag{5}$$

Next, we take the average over the extra-dimensional transit to get

$$\left|\langle\dot{t}\rangle + \left\langle\frac{g}{\sqrt{\mathrm{Det}}}\right\rangle\dot{u}_0\right| = \sqrt{\zeta + \dot{u}_0^2 + \dot{\vec{r}}^2}. \tag{6}$$

In our published paper we showed that CTCs and prearrivals require that $\bar{g} \equiv \langle g(u)\rangle$ and $\dot{u}_0$ have the same sign, which we conventionally choose to be positive (so "corotating" particles may experience CTCs and prearrivals, whereas "counterrotating" particles may not). We note that $\langle\dot{t}\rangle = 0$ for a CTC and $\langle\dot{t}\rangle < 0$ for a prearriving particle, so writing $\dot{u}_0$ as $(du/dt)_0(dt/d\tau)_0 \equiv \beta_0\gamma_0$, and $\dot{\vec{r}}_0$ as $(d\vec{r}/dt)_0(dt/d\tau)_0 \equiv \vec{v}_0\gamma_0$, we arrive at the final expression for the necessary and sufficient condition for a CTC or a prearrival to occur:

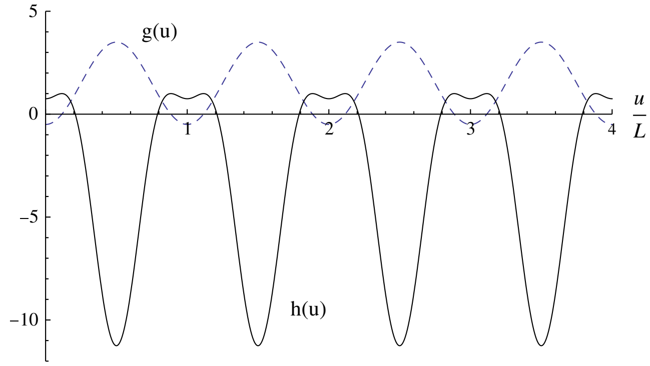

FIG. 1 (color online). $g(u)$ (dashed line) and $h(u) = 1 - g^2(u)$ (solid line) versus u/L, for parameter choices $g_0 = -0.5$, $a_1 = A = 2$, and $a_{n\neq 1} = b_n = 0$. (Refer to our original paper for the definition of the Fourier mode notation.)

$$\left\langle \frac{g}{\sqrt{\mathrm{Det}}} \right\rangle \geq \sqrt{1 + \frac{(\zeta/\gamma_0^2) + \vec{v}_0{}^2}{\beta_0^2}}. \tag{7}$$

(We do not consider here the alternate mathematical solution with very large, negative $\langle \dot{t} \rangle$; this solution does not connect continuously to the $\langle \dot{t} \rangle = 0$ CTC condition.)

The particle's boost factor γ_0 may greatly exceed unity, and the particle's velocity $\vec{v}_0$ along the brane may be zero. Thus, we may write the necessary but insufficient condition for a CTC or prearrival as simply

$$\left\langle \frac{g}{\sqrt{\mathrm{Det}}} \right\rangle > 1. \tag{8}$$

For a photon (with $\zeta = 0$) but not for massive particles such as Higgs singlet Kaluza-Klein modes (with $\zeta = 1$) of interest to us, the inequality becomes "$\geq$." Thus, with Det taken to be unity, we arrive at the constraint $\bar{g} \geq 1$ for photons, in agreement with Gielen's result.

The corrected Figs. 1 and 2 that accommodate the constraint $|\bar{g}| = |g_0 + A| > 1$ are given here [$g_0 \equiv g(0)$ and $A \equiv \bar{g} - g_0$]. One can easily see from the example in Fig. 2 that Gielen's constraint $|\bar{g}| > 1$ allows negative travel time.

Next, we correct the energy dispersion relation for the viable time-traveling particles. In Sec. VIIB of [1], the dispersion relation was (correctly) obtained from the equation of motion for the free ϕ field:

$$G^{AB}\partial_A\partial_B\phi + m^2\phi = 0 \quad \text{[5D Klein-Gordon (KG) equation]}. \tag{9}$$

The general solution to this equation for the nth energy eigenfunction takes the form

$$\phi_n^{(\mathrm{KG})} = e^{-iE_n[t + \int_0^u g(u)du]} e^{i\vec{p}\cdot\vec{x}} e^{i\xi u}, \tag{10}$$

where E_n is the energy of the nth mode (at fixed $\vec{p}$) and $\vec{p}$ is the standard three-momentum along the brane direction. Since the extra dimension is compactified, we require $\phi_n(u + L) = \phi_n(u)$, which in turn requires that

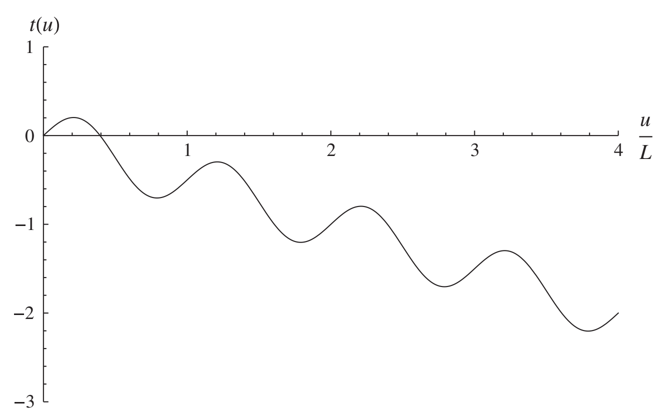

FIG. 2. $t(u)$ versus u/L, for the same parameter choices as in Fig. 1, and with $\beta_0 \equiv (\frac{du}{dt})_0 = 2/3$.

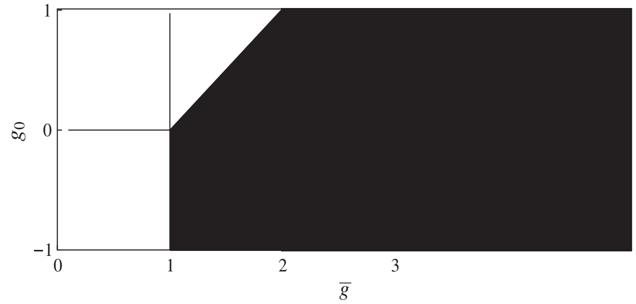

FIG. 3. The shaded region in the g_0-$\bar{g}$ plane for which CTCs are possible.

$$\xi = \bar{g}E_n - \frac{2\pi n}{L} \quad \text{with} \quad n = 0, \pm 1, \pm 2, \ldots \tag{11}$$

Thus, the solution to the KG equation is given by

$$\phi_n^{(\mathrm{KG})} = e^{-iE_n t} e^{i\vec{p}\cdot\vec{x}} e^{-iE_n \int_0^u (g-\bar{g})du} e^{-inu/R}, \tag{12}$$

where we have defined an extra-dimensional "radius" $R \equiv L/2\pi$ to streamline some notation.

To determine the energy dispersion relation, we simply need to plug Eq. (12) into the 5D KG equation above and solve for E_n. A bit of algebra yields the quadratic dispersion relation

$$(\bar{g}^2 - 1)E_n^2 - 2\bar{g}E_n \frac{n}{R} + \vec{p}^2 + \frac{n^2}{R^2} + m^2 = 0. \tag{13}$$

Solving for E_n then gives

$$E_n = \frac{\bar{g}\frac{n}{R} \pm \sqrt{\frac{n^2}{R^2} - (\bar{g}^2 - 1)(\vec{p}^2 + m^2)}}{\bar{g}^2 - 1}. \tag{14}$$

Equation (14) makes it clear that the mode energy E_n depends on $\vec{p}$ as well as on n; nevertheless, for brevity of notation, we will continue to use the "fixed $\vec{p}$" notation for both E_n and ϕ_n. In order to ensure that E_n is real, the condition $\frac{n^2}{R^2} > (\bar{g}^2 - 1)(\vec{p}^2 + m^2)$ needs to be satisfied. Also, we discard the case with $\bar{g} < 0$, which leads to negative E_n. Both the reality and positive definiteness of E_n are required to provide the stable modes for the CTC geodesics.

From Eq. (13) we may also derive a lower bound on the time-traveling particle's energy. The result is

$$E_n > \lim_{\bar{g}\to 1} E_n = \frac{1}{2}\left(\frac{n}{R} + \frac{\vec{p}^2 + m^2}{(n/R)}\right). \tag{15}$$

The ultimate conditions on the metric which guarantee CTC solutions are simple: (i) $\bar{g} > 1$ for massive particles (with $\dot{u}_0$ taken to be positive by convention), (ii) $|g_0| \le 1$, and (iii) $|A| = |\bar{g} - g_0| > 1$. The first condition is the result corrected in this erratum, and the second and third conditions are unchanged from our original paper (and derived therein). In Fig. 3 (which replaces Fig. 5 in our original paper), we display the shaded region in the g_0-$\bar{g}$ plane that allows CTCs. The allowed positive values of $\bar{g}$ extend to $+\infty$.

[1] S. Gielen, Phys. Rev. D **88**, 068701 (2013).