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Chern-Simons terms in Lifshitz-like quantum electrodynamics

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In this work the generation of generalized Chern-Simons terms in three-dimensional quantum electrodynamics with high spatial derivatives is studied. We analyze the self-energy corrections to the gauge field propagator by considering an expansion of the corresponding amplitudes up to third order in the external momenta. The divergences of the corrections are determined and explicit forms for the Chern-Simons terms with high derivatives are obtained. Some unusual aspects of the calculation are stressed and the existence of a smooth isotropic limit is proved. The transversality of the anisotropic gauge propagator is also discussed.

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I. INTRODUCTION

A great deal of attention has been devoted to the analysis of possible effects of the spacetime anisotropy [1–3]. In the context of quantum field theory, this possibility appears as a tool to study nonrenormalizable theories since it improves the ultraviolet behavior of the perturbative series, in spite of violating the Lorentz symmetry which in turn has been considered in several situations [4–7].

The different behavior between space and time coordinates $x^i \to bx^i$, $t \to b^z t$ [8] ameliorates the ultraviolet behavior and it has been argued that four-dimensional gravity becomes renormalizable when $z = 3$ [9]. However, the implementation of this kind of anisotropy introduces unusual aspects and therefore it is important to carefully investigate the consequences of this new approach [10,11].

One special situation concerns the Chern-Simons (CS) term [12,13]; when it is added to the quantum electrodynamics (QED) Lagrangian, the generation of mass for the gauge field happens without gauge symmetry breaking and it naturally emerges from quantum corrections to the gauge field propagator [14]. Beyond that, applications have been devised in diverse areas [6,15–18]. Therefore, the study of the spacetime anisotropy in the theories involving the CS term is certainly relevant.

In this work we will analyze the new contributions to the self-energy of the gauge field in the $z = 2$ case up to one-loop order in the small-momenta regime. In particular, corrections to the CS term will be studied. On general grounds we expect that the leading CS corrections have the following form:

$$\mathcal{L}_{\rm CS} = a\epsilon^{\mu\rho\nu}A_\mu\partial_\rho A_\nu + b\epsilon^{\mu\rho\nu}\Delta A_\mu\partial_\rho A_\nu + c\epsilon^{\mu\rho\nu}\partial_0^2 A_\mu\partial_\rho A_\nu, \quad (1)$$

where Δ denotes the Laplacian. In an interesting work [19] where the isotropic spacetime was considered, a CS term with a structure similar to Eq. (1) was employed. There, the Laplacian was replaced by a D'Alambertian and $c = 0$, so that the gauge propagator shows two massive excitations, one of them being ghost like.

By starting from Eq. (1) with $c = 0$, so that there is at most one time derivative, and adding the Maxwell term,

$$\mathcal{L}_A = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\lambda}{2}(\partial_\mu A^\mu)^2, \quad (2)$$

we obtain the propagator

$$D^{\mu\nu}(k) = \frac{-i}{[k^2-(a-b\vec{k}^2)^2]}\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} - \frac{i(a-b\vec{k}^2)\epsilon^{\mu\rho\nu}k_\rho}{k^2}\right) + -\frac{ik^\mu k^\nu}{\lambda k^2[k^2-(a-b\vec{k}^2)^2]} + \frac{i(a-b\vec{k}^2)^2 k^\mu k^\nu}{\lambda k^2[k^2-(a-b\vec{k}^2)^2]}, \quad (3)$$

which does not contain particle like poles and, for small momenta, indicates disturbances propagating with squared velocity (1–2ab). In this work we will also check the consistency with respect to the gauge symmetry of these corrections through the verification of the transversality of the self-energy contributions to the gauge field propagator. Our calculations show that, analogously to the relativistic situation, although finite the coefficient a is regularization dependent. Actually, if dimensional reduction (see Appendix A) is employed, the constant a turns out to be equal to zero whereas in the relativistic situation it is nonvanishing.

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This work is organized as follows. In Sec. II we introduce high-derivative terms in the Dirac and Maxwell Lagrangians and analyze the CS generation and the characteristics of the self-energy of the gauge field. For simplicity, our calculations will be restricted to the small-momenta regime. In Sec. III we discuss the transversality of the corrections to the gauge field propagator. Section IV presents some concluding remarks. Two appendices are dedicated to detail some aspects of the calculations.

II. GENERATION OF CHERN-SIMONS TERMS IN THE ANISOTROPIC QED

The modified Dirac Lagrangian containing a high spatial derivative of second order is

$$\mathcal{L} = \bar{\psi}(i\gamma^0 D_0)\psi + b_1\bar{\psi}(i\gamma^i D_i)\psi + b_2\bar{\psi}(i\gamma^i D_i)^2\psi - m\bar{\psi}\psi, \tag{4}$$

where $i = 1, 2$ and $D^\mu = \partial^\mu - ieA^\mu$ ($\mu = 0, 1, 2$) is the covariant derivative and γ^μ indicates a 2×2 representation of the Dirac gamma matrices. To obtain the QED Lagrangian we add to Eq. (4) the Maxwell term with high derivatives,

$$\mathcal{L}_{\rm M} = \frac{1}{4}(F_{ij}\Delta F_{ij} + 2F_{0i}F_{0i}) = \frac{1}{4}F_{ij}\Delta F_{ij} + \frac{1}{2}\partial_i A_0\partial_i A_0 - \partial_0 A_i\partial_i A_0 + \frac{1}{2}\partial_0 A_i\partial_0 A_i, \tag{5}$$

in which $F_{ij} = \partial_i A_j - \partial_j A_i$. The above contribution exhibits a mixture among space and time components, which may cause complications in the calculations involving the gauge field propagator. It is possible to avoid the mixed propagators by conveniently choosing the gauge fixing [11]. As the gauge field propagator does not appear in our calculations we will keep an nonspecific gauge fixing, $\mathcal{L}_{\rm F}$. Despite the mixing terms or gauge choice, notice that the determination of the transversal and longitudinal parts of the gauge field propagator is highly nontrivial in the anisotropic theory, and thus we will dedicate a section to an analysis of the transversality of the corrections to the gauge field propagator.

We can rewrite Eq. (4) and define the Dirac anisotropic Lagrangian,

$$\mathcal{L}_{\rm D} = \bar{\psi}(i\gamma^0\partial_0)\psi + b_1\bar{\psi}(i\gamma^i\partial_i)\psi + b_2\bar{\psi}(i\gamma^i\partial_i)^2\psi - m\bar{\psi}\psi, \tag{6}$$

and, up to irrelevant surface terms, the interaction Lagrangian

$$\mathcal{L}_{\rm I} = e\bar{\psi}(\gamma^0 A_0 + b_1\gamma^i A_i)\psi + e^2 b_2\bar{\psi}(\gamma^i A_i)^2\psi + ieb_2(\bar{\psi}\gamma^j\gamma^i\partial_i\psi - \partial_i\bar{\psi}\gamma^i\gamma^j\psi)A_j = V_1 + V_2 + V_5 + V_4 + V_3, \tag{7}$$

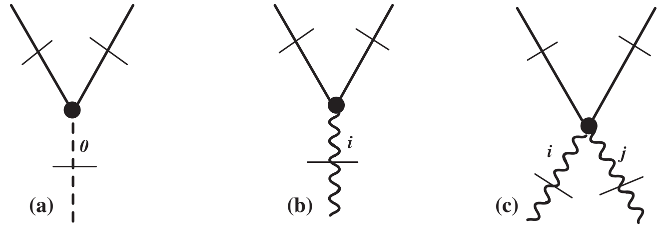

FIG. 1. Graphical representation of the interaction vertices. The continuos line stands for the fermion field, the dashed and wavy lines are for the A_0 and A_i components, respectively. The diagram (a) is the trilinear vertex with two fermion fields and the A_0 component. Figure (b) is the vertex with two fermion fields and the A_i component; besides that, it may contain a spatial derivative. Graph (c) is the quadrilinear vertex with two fermion fields and two A_i components.

such that the total Lagrangian is $\mathcal{L}_{\rm T} = \mathcal{L}_{\rm M} + \mathcal{L}_{\rm F} + \mathcal{L}_{\rm D} + \mathcal{L}_{\rm I}$. In Eq. (7) we introduced a notation for the vertices where $V_1 = e\bar{\psi}(\gamma^0 A_0)\psi$, $V_2 = eb_1\bar{\psi}(\gamma^i A_i)\psi$, etc. These vertices, fixed by $\mathcal{L}_{\rm I}$, are graphically represented in Fig. 1. For the free fermion propagator we obtain

$$S(k) = \frac{i(\hat{k\!\!\!/} + b_1\bar{k\!\!\!/} + b_2\mathbf{k}^2 + m)}{k_0^2 - (b_1^2 + 2Mb_2)\mathbf{k}^2 - b_2^2\mathbf{k}^4 - m^2},$$

where the hat and the bar denote the time and space components, respectively, i.e., $\hat{k\!\!\!/} = \gamma^0 k_0$, $\bar{k\!\!\!/} = \gamma^i k_i$ and $\mathbf{k}^2 = k^i k^i$ (we will also use the notations $\hat{k}_0 = k_0$ and $\bar{k}_i = k_i$, and thus $\bar{k}^2 = \bar{k}^i\bar{k}_i$).

The corrections to the gauge field propagator at one loop are represented in Fig. 2, and their analytical expressions are

$$\Pi^{\mu\nu}_{ab} = C_{ab}\int d^z\hat{k}d^d\bar{k}\,{\rm Tr}[V^\mu_a S(k)V^\nu_b S(k+p)], \tag{8}$$

$$\Pi^{ij}_5 = C_5\int d^z\hat{k}d^d\bar{k}\,{\rm Tr}[\gamma^i\gamma^j S(k)], \tag{9}$$

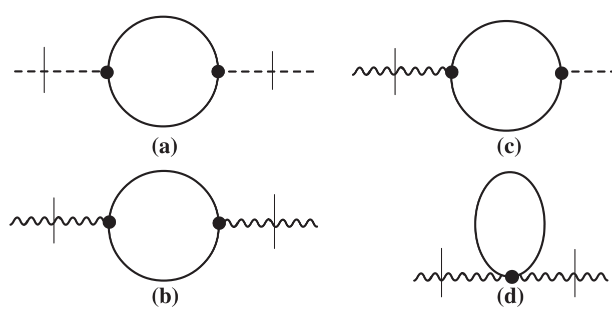

FIG. 2. One-loop contributions to the gauge field self-energy. The figures (a), (b) and (c) involve two triple vertices with external fields (A_0, A_0), (A_0, A_i) and (A_i, A_j), respectively. The diagram (d) contains the quadrilinear vertex with (A_i, A_j) external fields.

where V_a^μ is the time or spatial component ($\mu = 0$, i) associated to the vertices as defined in Eq. (7), and thus a, $b = 1, \ldots, 4$ and C_{ab} is the momentum-independent coefficient corresponding to each amplitude. The development of these expressions are discussed in Appendices A and B.

By power counting we find that the Fig. 2(c) is linearly or logarithmically divergent if the vertex attached to the wavy line has or does not have a derivative; the divergences of Fig. 2(b) are quadratic if both vertices have one derivative, are linear when one vertex has a derivative and the other does not, and finally are logarithmic when no derivative appears in the vertices, just like in the Fig. 2(a), which is composed of two external $A^0(p)$ components.

In the sequel we will consider the small-momenta regime and Taylor expand the self-energy corrections. The usual CS term [i.e. the first term in Eq. (1)] is obtained from the first-order terms of the Taylor derivative expansion, while the extended CS term [which has a form like the second and third terms in Eq. (1)] is given by the third-order terms. Considering that the highest divergence is quadratic, the extended CS contributions, which are of third order in the Taylor expansion, are of course finite. Differently, for the usual CS terms, the algebra reduces the degree of divergence of some diagrams only to logarithmic so that they require the introduction of a regularization scheme. Similarly to the relativistic theory, in the sense that there is a regularization dependence, there is an ambiguity in the induced CS term.

For the even terms in the momentum expansion, which do not contribute to the CS terms, the integration of the momenta furnishes hypergeometric functions, as indicated in Appendix A. In this case, the divergences are characterized by the Schwinger parameter, x, which receives contributions from the hypergeometric functions and from the coefficients associated to them [see Eq. (A2)]. To integrate the x parameter, we will power expand the hypergeometric function for small x and $d = 2$, until its exponent becomes non-negative. Observe that here the isotropic limit, $b_2 \to 0$, cannot be taken alone because it is inconsistent with the small-x expansion. However, although the divergent terms individually exhibit poles for $d = 2$, they are canceled when summed to produce the total self-energy correction, allowing for a smooth isotropic limit.

III. TRANSVERSALITY OF THE GAUGE SELF-ENERGY

The conservation of Noether's current,

$$J^0 = \bar\psi \gamma^0 \psi,$$

$$J^k = b_1 \bar\psi \gamma^k \psi - i b_2 [(\partial_i \bar\psi)\gamma^i \gamma^k \psi - \bar\psi \gamma^k \gamma^i (\partial_i \psi)] + e b_2 [\bar\psi \gamma^i \gamma^k \psi + \bar\psi \gamma^k \gamma^i \psi] A_i, \quad (10)$$

allows us to prove the transversality of the self-energy corrections. Indeed, we may write $\mathcal{L}_\mathrm{I}$ in terms of the above current as

$$\mathcal{L}_\mathrm{I} = \frac{e}{2}[A_\mu J^\mu_{(A^\rho=0)} + A_\mu J^\mu], \quad (11)$$

where in the first term the argument of the current (A^ρ) must be taken equal to zero.

Now, in computing the self-energy contributions notice that it is equal to

$$\left\langle T \frac{\delta \mathcal{S}_I}{\delta A_\mu} \frac{\delta \mathcal{S}_I}{\delta A_\nu} \right\rangle \propto \langle J^\mu J^\nu \rangle, \quad (12)$$

where $\mathcal{S}_\mathrm{I} = \int \mathcal{L}_\mathrm{I}$ is the interaction action; Eq. (12) must be transversal due to the conservation of the current.

IV. CONCLUDING REMARKS

In this work we studied the effects of the anisotropy of the spacetime on the generation of the CS term and on the self-energy correction to the gauge field propagator. We proved that this correction is transversal by constructing the conserved Noether's current which interacts with the gauge field.

Some interesting aspects of the calculation are related to the structure of the divergences. For the even part of the gauge field self-energy, there are seven amplitudes involving the new vertices which are divergent and their pole terms break gauge invariance. Nevertheless, a contribution coming from the usual vertices cancel these divergences.

There are two types of CS terms that may be induced by the radiative corrections: the usual CS term which contains just one derivative and the extended CS term with three derivatives. Whereas the second type of CS term is always finite, the usual one presents results which are divergent, but these divergences notably cancel among themselves when the total contribution is considered. Furthermore, the coefficient of the usual CS term is regularization dependent and vanishes if dimensional reduction is adopted. In this situation, one is forced to consider the extended CS term which then constitutes the dominant contribution.

When referring to the usual CS terms, we notice that the contributions coming from the usual vertices are finite and therefore are free from ambiguities caused by the regularization. On the other hand, the contributions coming from the new vertices are regularization dependent. Besides that, the explicit b_2 factor from these vertices is canceled because it is multiplied by amplitudes which diverge as $b_2 \to 0$; this furnishes nonzero contributions when the isotropic limit is considered. Consequently, the regularization dependence is restored and in the isotropic limit unexpected results—such as the just-mentioned vanishing of the usual CS term if dimensional reduction is employed—are obtained.

Considering the extended CS, all amplitudes associated to it are finite even if $b_2 \to 0$. The isotropic limit for these terms is smooth with only the contributions coming from the usual vertices remaining.

By considering Eqs. (B1) and (B2), the induced CS terms can be written as

$$\Pi^{\mu\nu}_{\rm CS} = -\frac{[\alpha_{\mu\nu} + \hat{p}^2 + \bar{p}^2(b_1^2 + 4mb_2)]e^2 b_1^2 \epsilon^{\mu\nu\rho} p_\rho}{48\pi m^2(b_1^2 + 4mb_2)}, \quad (13)$$

in accord with our proposal (1). Notice that the breaking of the Lorentz invariance is a very simple function of b_1 and b_2 and that in the isotropic limit with $b_1 = 1$ the Lorenz symmetry is restored.

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APPENDIX A: THE CALCULATION PROCEDURE

In this appendix we will describe the procedure employed to calculate the Feynman diagrams. As an example, we will consider the amplitude given by the vertices $(eb_1 \bar{\psi}\gamma^i \psi A_i)$ and $(eb_1 \bar{\psi}\gamma^j \psi A_j)$, which leads us to

$$\Pi^{ij}(p) = \frac{(eb_1)^2}{(2\pi)^3} \int d\hat{k} d^d \bar{k}\, {\rm Tr}[\gamma^i S(k) \gamma^j S(k-p)]. \quad (A1)$$

To solve Eq. (A1) we adopt the dimensional reduction scheme, in which all the algebra of the gamma matrices is done in $d = 2$ and afterwards the integral is promoted to d dimensions [20]. Therefore, the leading term in the Taylor expansion, i.e. the term with $(\hat{p} = 0,\ \bar{p} = 0)$ is

$$-\frac{(eb_1)^2}{(2\pi)^3} \int d\hat{k} d^d \bar{k} \left[\frac{2g^{ij}[m^2 - \hat{k}^2 - b_2 \mathbf{k}^2(-\mathbf{k}^2 b_2 - 2M)]}{(\hat{k}^2 - \mathbf{k}^2(b_1^2 + 2Mb_2) - \mathbf{k}^4 b_2^2 - m^2)^2}\right].$$

To perform the above integral, we use the Schwinger representation and integrate over the momenta. The result is very extensive, and therefore we will consider just one of the terms which leads to a divergent result,

$$e^{-iM^2 x} x^{-d/4}\, {}_1F_1\left(\frac{d+2}{4}; \frac{1}{2}; \frac{ix(b_1^2 + 2Mb_2)^2}{4b_2^2}\right), \quad (A2)$$

where x is the Schwinger parameter and ${}_1F_1$ denotes the hypergeometric function. Note that this function is divergent in the limit $x \to 0$, so we will Taylor expand it, up to a non-negative power of x for $d = 2$. Thus we obtain

$$x^{-d/4}\, {}_1F_1\left(\frac{d+2}{4}; \frac{1}{2}; \frac{ix(b_1^2 + 2Mb_2)^2}{4b_2^2}\right) \to x^{-d/4}\left(1 + 2\frac{d+2}{4}\frac{ix(b_1^2 + 2Mb_2)^2}{4b_2^2}\right). \quad (A3)$$

The final result is obtained by integrating over x and expanding the result around $d = 2$, such that the general result for this amplitude is given by

$$\frac{ie^2 b_1^2 g^{ij}}{2\pi b_2(d-2)} + \text{finite}. \quad (A4)$$

Notice that this divergence is absent in the relativistic theory because it clearly comes from modifications introduced by the anisotropy. Observe also that from the argument of the hypergeometric function in Eq. (A2) the isotropic limit, $(b_2 \to 0)$, is incompatible with the adopted expansion for $x \to 0$.

On the other hand, if we consider the subleading term in the Taylor expansion, responsible for the usual CS term, i.e. $\frac{\partial \Pi^{ij}}{\partial \hat{p}}|_{\hat{p}=\bar{p}=0}$, the result is

$$\frac{(eb_1)^2}{(2\pi)^3} \int d\hat{k} d^d \bar{k} \left[\frac{2i\epsilon^{ij0}(m - b_2 \bar{k}^2)}{(\hat{k}^2 - \mathbf{k}^2(b_1^2 + 2Mb_2) - \mathbf{k}^4 b_2^2 - m^2)^2}\right]. \quad (A5)$$

To compute the above integral, which is finite by power counting, we simply take $d = 2$ and integrate over the momenta.

APPENDIX B: INTERACTION VERTICES

In this appendix we will present the one-loop contributions, for small momenta, to the self-energy of the gauge field. From the interaction Lagrangian we observe that we have a total of 12 different amplitudes coming from the Wick contractions of the vertices. The sum of all contributions gives (further details will be presented elsewhere)

$$\Pi^{ij}_{\rm CS} = -\frac{[\alpha_{ij} + \bar{p}^2(b_1^2 + 4mb_2) + \hat{p}^2]e^2 b_1^2 \epsilon^{ij0} \hat{p}_0}{48\pi m^2(b_1^2 + 4mb_2)} \quad (B1)$$

and

$$\Pi^{0i}_{\rm CS} = -\frac{[\alpha_{0i} + \bar{p}^2(b_1^2 + 4mb_2) + \hat{p}^2]e^2 b_1^2 \epsilon^{0ia} \bar{p}_a}{48\pi m^2(b_1^2 + 4mb_2)}, \quad (B2)$$

where α_{ij} and α_{0i} are constant parameters introduced in Eqs. (B1) and (B2), respectively, to denote the ambiguity coming from the regularization scheme. We may note that there is a cancellation of the usual CS term, leaving only regularization-dependent terms. This occurs due to the contributions of the new vertices introduced by the anisotropy and by the fact that their b_2 vertex factor is eliminated after performing the momenta integrals.

In the isotropic limit $b_2 \to 0$ of Eqs. (B1) and (B2) all the extended CS terms coming from the new vertices' contributions are cancelled and we get

$$\Pi^{ij}_{\rm CS} = -\frac{e^2(\alpha_{ij} + b_1^2\bar{p}^2 + \hat{p}^2)\epsilon^{ij0}\hat{p}_0}{48\pi m^2}$$

and

$$\Pi^{0i}_{\rm CS} = -\frac{e^2(\alpha_{0i} + b_1^2\bar{p}^2 + \hat{p}^2)\epsilon^{0ia}\bar{p}_a}{48\pi m^2}.$$

By taking $b_1 = 1$ we obtain an expression similar to that introduced in Ref. [19].

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